Status of Sterile Neutrinos

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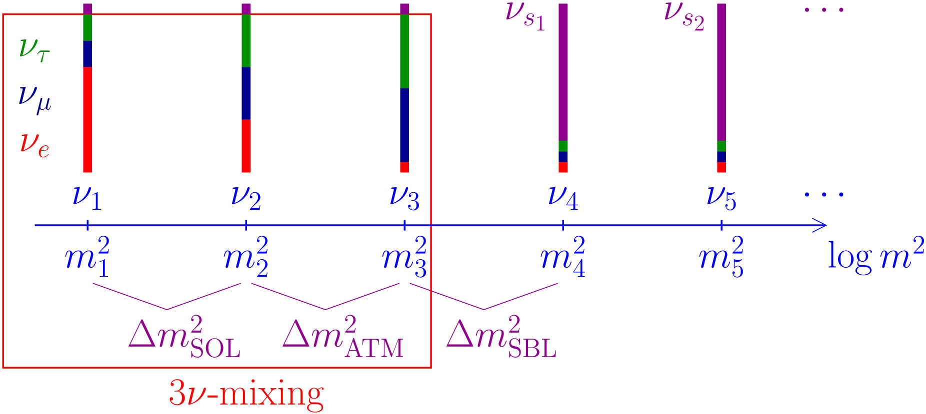
NOW 2012, Neutrino Oscillation Workshop

9-16 September 2012, Conca Specchiulla, Otranto, Italy

Beyond Three-Neutrino Mixing



Sterile Neutrinos from Physics Beyond the SM

- Neutrinos are special in the Standard Model: the only neutral fermions
- In extensions of SM neutrinos can mix with non-SM fermions
- SM: $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$ $\qquad \widetilde{\Phi} = i\sigma_2\,\Phi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$
- SM singlet $\overline{L_L}\widetilde{\Phi}$ can couple to new singlet chiral fermion field f_R related to physics beyond the SM
- Known examples: light ν_R from see-saw [Lindner @ NOW2012], SUSY (axino, modulino, ...), extra dimensions (KK modes), mirror world [Berezhiani @ NOW2012], ...
- Dirac mass term $\sim \overline{L_L}\widetilde{\Phi} f_R \quad + \quad$ Majorana mass term $\sim \overline{f_R^C} f_R$
- f_R is often called Right-Handed Neutrino: $f_R \to \nu_R$

Sterile Neutrinos

- Light anti-ν_R are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Short-baseline anomalies + 3ν-mixing:

$$\begin{array}{ccccc} & \Delta m^2_{21} \ll & |\Delta m^2_{31}| \ll & |\Delta m^2_{41}| \leq & \ldots \\ \nu_1 & \nu_2 & \nu_3 & \nu_4 & \ldots \\ \nu_e & \nu_\mu & \nu_\tau & \nu_{s_1} & \ldots \end{array}$$

- In this talk I consider sterile neutrinos with mass scale $\sim 1\,\mathrm{eV}$ in light of short-baseline LSND, MiniBooNE, Reactor Anomaly, Gallium Anomaly.

- Other possibilities (not incompatible):

  - Very light sterile neutrinos with mass scale $\ll 1\,\mathrm{eV}$: important for solar neutrino phenomenology

    [Das, Pulido, Picariello, PRD 79 (2009) 073010]
    [de Holanda, Smirnov, PRD 83 (2011) 113011]

  - Heavy sterile neutrinos with mass scale $\gg 1\,\mathrm{eV}$: could be Warm Dark Matter

    [Kusenko, Phys. Rept. 481 (2009) 1]
    [Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191]
    [Lindner @ NOW2012]

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

$$|U_{e4}|^2 \ll 1 \, , \quad |U_{\mu 4}|^2 \ll 1 \, , \quad |U_{\tau 4}|^2 \ll 1 \, , \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{array}{ccc|c|} \cline{4-4} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \\ \cline{4-4} \end{array}\right)$$

$\uparrow$

SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m^2_{kj} L / 4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 \phi_{41} + 4|U_{e5}|^2 |U_{\mu 5}|^2 \sin^2 \phi_{51}$$
$$+ 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54} \overset{(+)}{-} \eta)$$

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2 \phi_{41} + |U_{\alpha 5}|^2 \sin^2 \phi_{51})$$
$$- 4|U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \sin^2 \phi_{54}$$

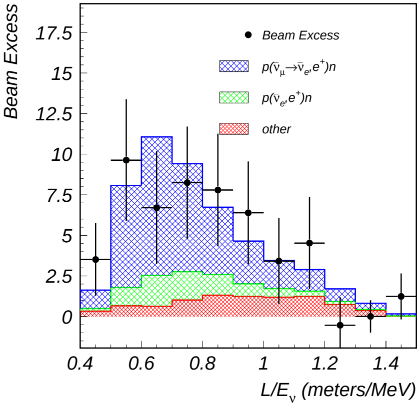
[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, arXiv:1205.5230; Conrad, Ignarra, Karagiorgi, Shaevitz, Spitz, arXiv:1207.4765] [Ignarra @ NOW2012]

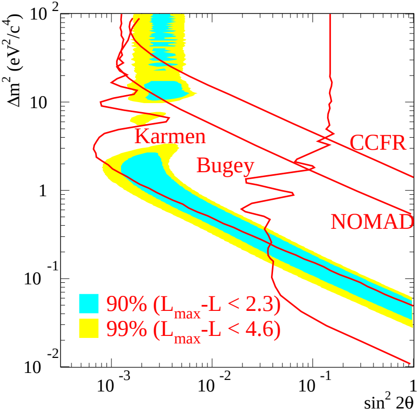
- More parameters: 7 (vs 3 in 3+1)
- CP violation
- 3+3? [Xing @ NOW2012]

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 30\,\text{m}$ $\quad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$



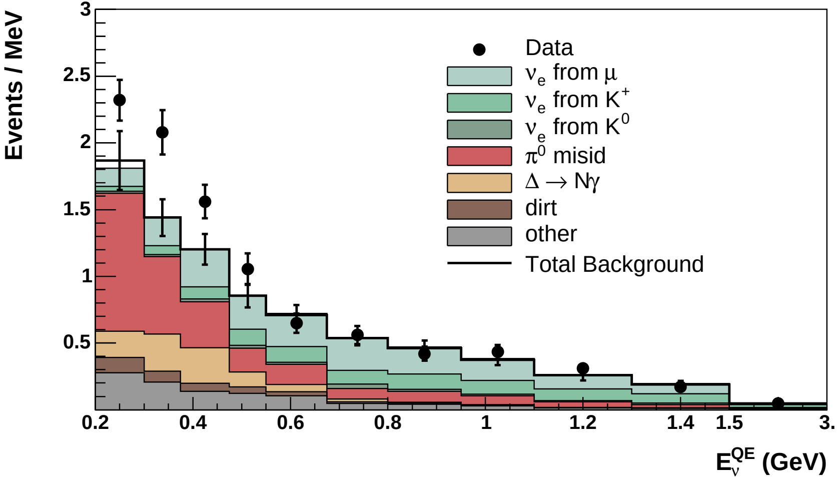


3.8σ excess $\quad \Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2 \quad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

MiniBooNE Neutrinos

[PRL 98 (2007) 231801; PRL 102 (2009) 101802]

$\nu_\mu \to \nu_e$ $L \simeq 541\,\mathrm{m}$ $200\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$

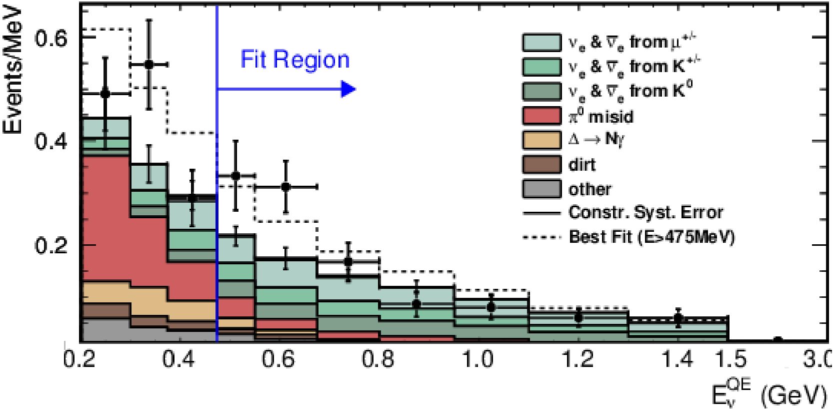


- no $\nu_\mu \to \nu_e$ signal corresponding to LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal ($E > 475\,\mathrm{MeV}$)
- low-energy anomaly

MiniBooNE Antineutrinos - 2009-2010

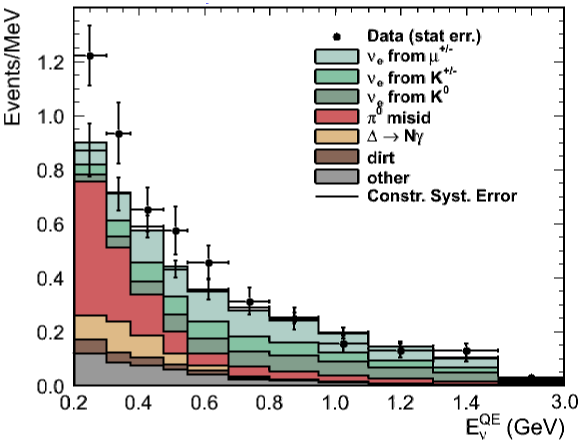
[PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ $\quad L \simeq 541\,\text{m}$ $\quad 200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$



- agreement with LSND $\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ signal ($E > 475\,\text{MeV}$)
- similar L/E but different L and E $\Longrightarrow$ oscillations

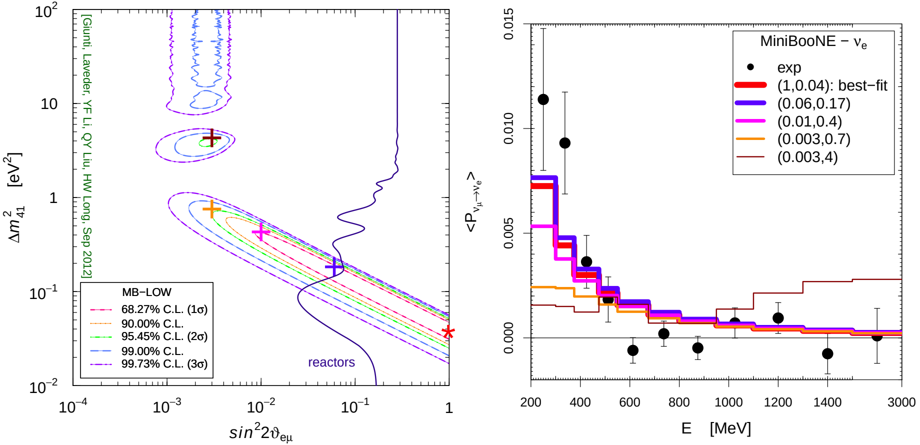
MiniBooNE $\bar{\nu}$ - Neutrino 2012 - 6 June



		1st half			2nd half	
	data	mc	excess	data	mc	excess
200-475	119	100.5±14.3	18.5 (1.3s)	138	100.0±14.1	38 (2.7s)
475-1250	120	99.1±14.0	20.9 (1.5s)	101	103.1±14.4	-2.2 (-0.2s)

agreement with LSND signal is sadly vanishing

MiniBooNE ν and $\bar{\nu}$ - arXiv:1207.4809



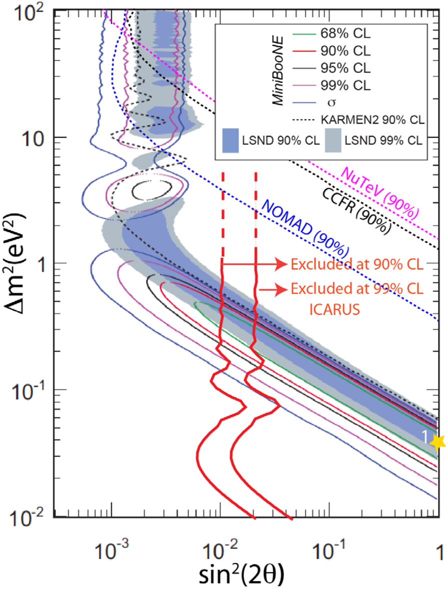
- Fit of low-energy excess is marginal
- It requires $\Delta m^2_{41} \lesssim 0.4 \text{eV}^2$
- Neutrino energy reconstruction problem?

[Martini, Ericson, Chanfray, arXiv:1202.4745]

ICARUS

[arXiv:1209.0122] [C.Rubbia @ NOW2012]

- $\nu_\mu \to \nu_e$
- $L = 730\,\text{km}$ (CNGS)
- $10 < E < 30\,\text{GeV}$
- $3 \times 10^{-3} < \frac{E}{L} < 9 \times 10^{-3}\,\text{eV}^2$
- 2 observed ν_e events
- 3.7 background ν_e events



Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

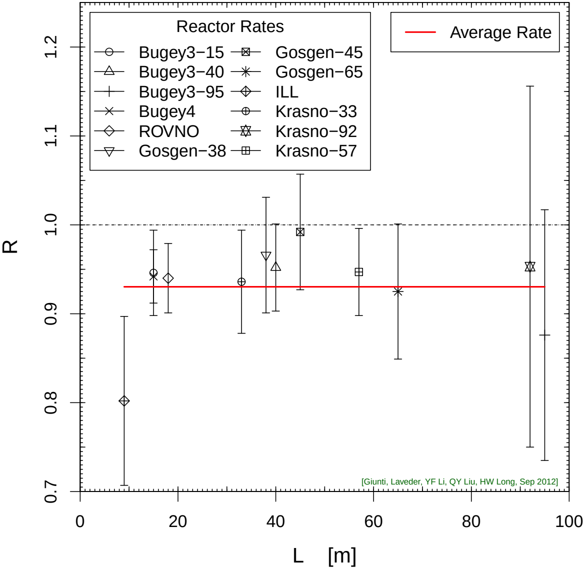
[update in White Paper, arXiv:1204.5379

new reactor $\bar{\nu}_e$ fluxes

[Mueller et al, PRC 83 (2011) 054615]

[Huber, PRC 84 (2011) 024617]

$$\boxed{\overline{R} = 0.930 \pm 0.024}$$



[Giunti, Laveder, YF Li, QY Liu, HW Long, Sep 2012]

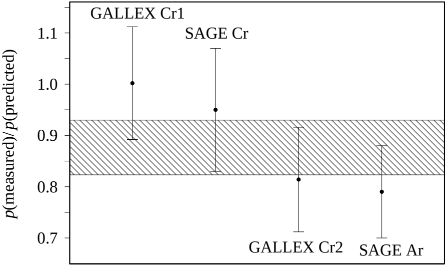
Gallium Anomaly

Gallium Radioactive Source Experiments

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

$E \sim 0.7\,\text{MeV}$

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$

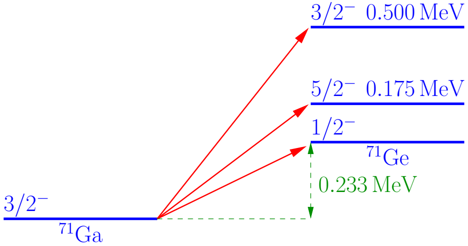
$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

$\boxed{\overline{R}_{\text{B}} = 0.86 \pm 0.05}$

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\text{G.S.}}$ from $T_{1/2}({}^{71}\text{Ge}) = 11.43 \pm 0.03\,\text{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\text{G.S.}}({}^{51}\text{Cr}) = 55.3 \times 10^{-46}\,\text{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\text{Cr}) = \sigma_{\text{G.S.}}({}^{51}\text{Cr}) \left(1 + 0.669\,\frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}} + 0.220\,\frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}}\right)$

- Contribution of Excited States only 5%!

		$\frac{\mathrm{BGT}_{175}}{\mathrm{BGT}_{\mathrm{G.S.}}}$	$\frac{\mathrm{BGT}_{500}}{\mathrm{BGT}_{\mathrm{G.S.}}}$
Krofcheck et al. PRL 55 (1985) 1051	$^{71}\mathrm{Ga}(p,n)^{71}\mathrm{Ge}$	< 0.056	0.126 ± 0.023
Haxton PLB 431 (1998) 110	Shell Model	0.19 ± 0.18	
Frekers et al. PLB 706 (2011) 134	$^{71}\mathrm{Ga}(^{3}\mathrm{He},{}^{3}\mathrm{H})^{71}\mathrm{Ge}$	0.039 ± 0.030	0.202 ± 0.016

- Haxton: [Haxton, PLB 431 (1998) 110]

  "a sophisticated shell model calculation is performed ... for the transition to the first excited state in $^{71}\mathrm{Ge}$. The calculation predicts destructive interference between the (p,n) spin and spin-tensor matrix elements"
- Does Haxton argument apply also to $(^{3}\mathrm{He},{}^{3}\mathrm{H})$ measurements?
- 2.7σ discrepancy of $\mathrm{BGT}_{500}/\mathrm{BGT}_{\mathrm{G.S.}}$ measurements!
- Anyhow, new $^{71}\mathrm{Ga}(^{3}\mathrm{He},{}^{3}\mathrm{H})^{71}\mathrm{Ge}$ data support Gallium Anomaly!

SUN&KamLAND + ϑ_{13} bound on $|U_{e4}|^2$

[Giunti, YF Li, PRD 80 (2009) 113007; Palazzo, PRD 83 (2011) 113013, Palazzo, PRD 85 (2012) 077301] [Palazzo @ NOW2012]

3+1 with simplifying assumptions: $U_{\mu 4} = U_{\tau 4} = 0$, no CP violation

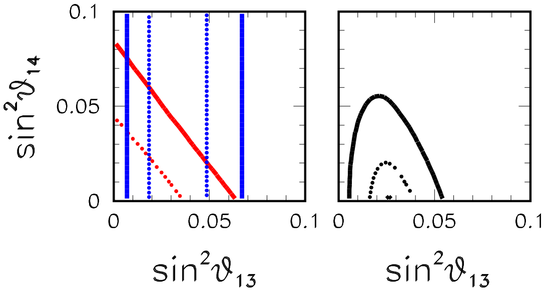
$$U_{e1} = c_{12} c_{13} c_{14} \quad U_{e2} = s_{12} c_{13} c_{14} \quad U_{e3} = s_{13} c_{14} \quad U_{e4} = s_{14}$$

$$U_{s1} = -c_{12} c_{13} s_{14} \quad U_{s2} = -s_{12} c_{13} s_{14} \quad U_{s3} = -s_{13} s_{14} \quad U_{s4} = c_{14}$$

$$P_{\nu_e \to \nu_e} \simeq \left(1 - 2 s_{13}^2\right) \left(1 - 2 s_{14}^2\right) P_{\nu_e \to \nu_e}^{2\nu}$$

$$P_{\nu_e \to \nu_s} \simeq s_{14}^2 \left(1 + P_{\nu_e \to \nu_s}^{2\nu}\right)$$

$$\begin{aligned} V &= c_{13}^2 c_{14}^2 V_{\text{CC}} - c_{13}^2 s_{14}^2 V_{\text{NC}} \\ &= (|U_{e1}|^2 + |U_{e2}|^2) V_{\text{CC}} - (|U_{s1}|^2 + |U_{s2}|^2) V_{\text{NC}} \end{aligned}$$

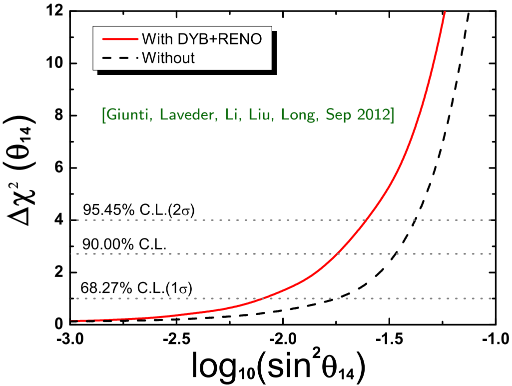


red band: SUN and KL

blue band: T2K and MINOS

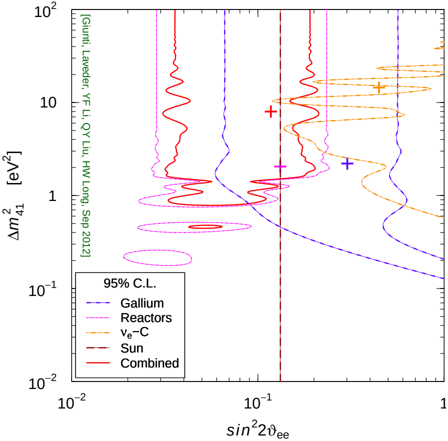
before Daya Bay and RENO

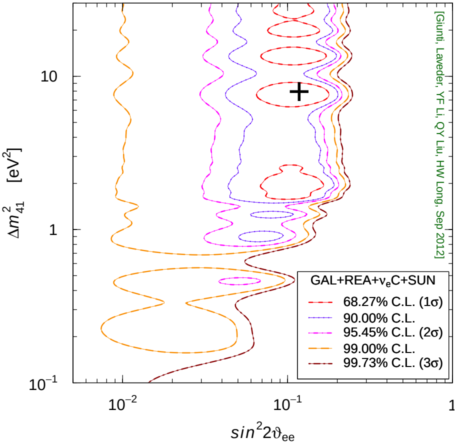
[Palazzo, PRD 85 (2012) 077301]



[Giunti, Laveder, Li, Liu, Long, Sep 2012]

Global ν_e and $\bar{\nu}_e$ Disappearance





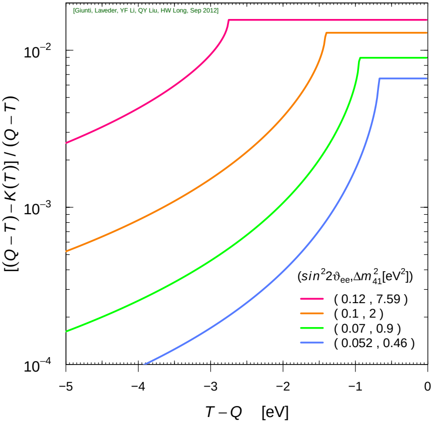
$$\nu_e + {}^{12}\mathrm{C} \to {}^{12}\mathrm{N}_{\mathrm{g.s.}} + e^-$$

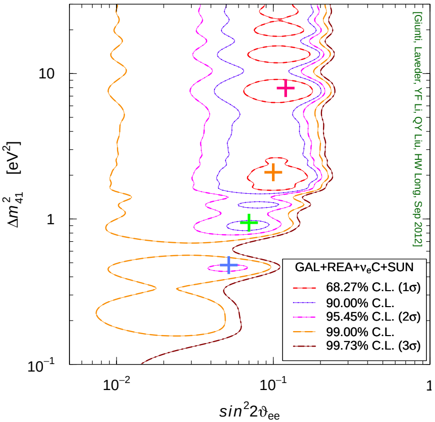
KARMEN + LSND

[Conrad, Shaevitz, PRD 85 (2012) 013017]

[Giunti, Laveder, PLB 706 (2011) 200]

Testable Implications: β Decay



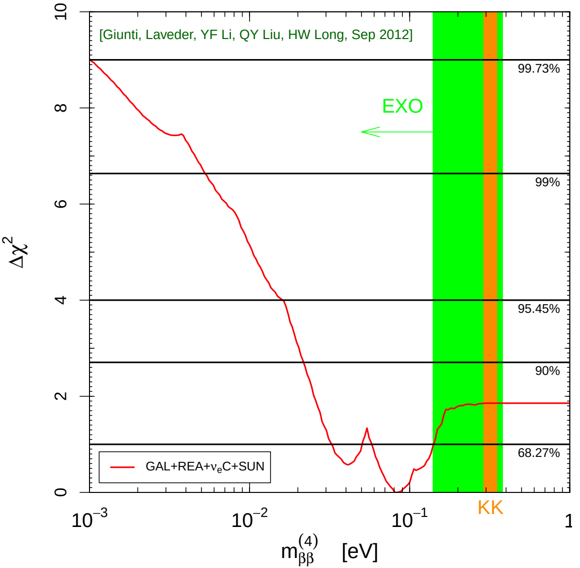


relative deviation of Kurie plot

$$\frac{(Q-T)-K(T)}{Q-T}$$

KATRIN: [Drexlin @ NOW2012]

Testable Implications: $(\beta\beta)_{0\nu}$ Decay



$$m_{\beta\beta} = \left|\sum_k U_{ek}^2 \, m_k\right|$$

$$m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

caveat:
possible cancellation
with $m_{\beta\beta}^{(3\nu-IH)}$
[Rodejohann @ NOW2012]

Global 3+1 Fit: Disappearance Constraints

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

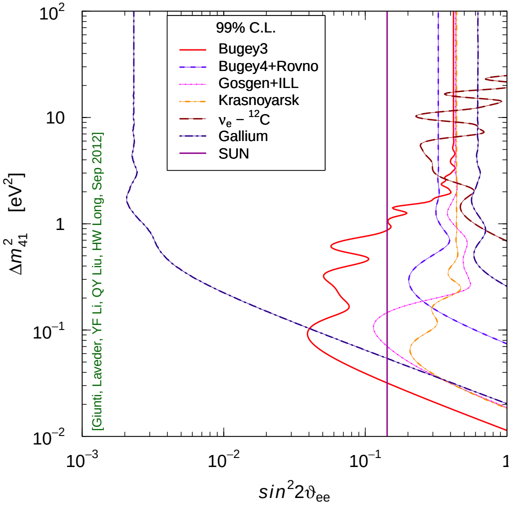
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694]

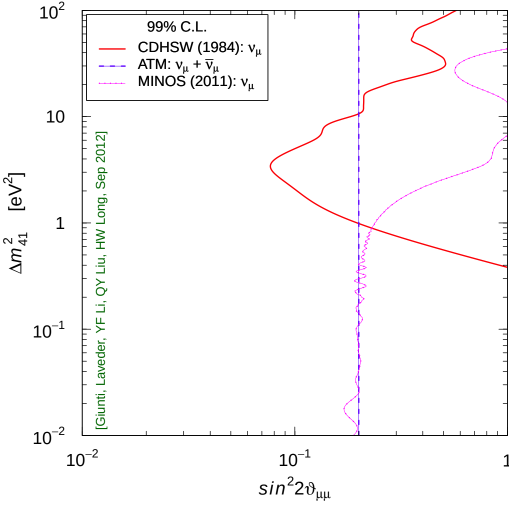
[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247]

ν_e and $\bar{\nu}_e$ Disappearance



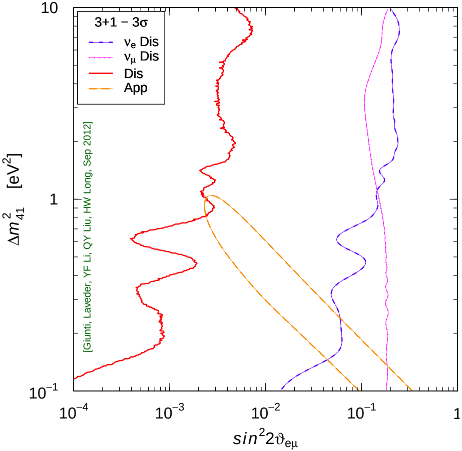
- New Reactor $\bar{\nu}_e$ Fluxes
  [Mueller et al., PRC 83 (2011) 054615]
  [Mention et al., PRD 83 (2011) 073006]
  [Huber, PRC 84 (2011) 024617]
- KARMEN + LSND
  $\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$
  [Conrad, Shaevitz, PRD 85 (2012) 013017]
  [Giunti, Laveder, PLB 706 (2011) 200]
- SUN&KamLAND + ϑ_{13}
  [Giunti, YF Li, PRD 80 (2009) 113007]
  [Palazzo, PRD 83 (2011) 113013]
  [Palazzo, PRD 85 (2012) 077301]
  [Giunti, Laveder, Li, Liu, Long, Sep 2012]

ν_μ and $\bar{\nu}_\mu$ Disappearance



- ATM constraint on $|U_{\mu 4}|^2$
  [Maltoni, Schwetz, PRD 76 (2007) 093005]
- MINOS constraint on $|U_{\mu 4}|^2$
  [Giunti, Laveder, PRD 84 (2011) 093006]
- To be included: MiniBooNE/SciBooNE limit on SBL $\bar{\nu}_\mu$ disappearance
  [arXiv:1208.0322]
- To be included: AMANDA/Ice Cube limit on SBL ν_μ and $\bar{\nu}_\mu$ disappearance
  [Esmaili, Halzen, Peres, arXiv:1206.6903]
  [Halzen @ NOW2012]

3+1



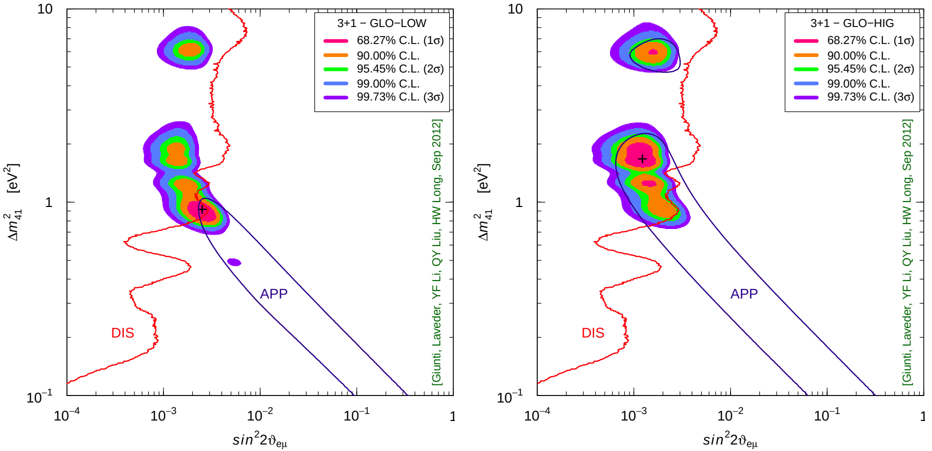
- 2012 MiniBooNE data
- GoF = 9.3%
- PGoF = 0.002%
- missing new ICARUS constraint (added later)

- 3+1 & 3+2: Appearance-Disappearance tension
- Tension reduced in 3+1+NSI [Akhmedov, Schwetz, JHEP 10 (2010) 115]
- No tension in 3+1+CPTV [Barger, Marfatia, Whisnant, PLB 576 (2003) 303] [Giunti, Laveder, PRD 82 (2010) 093016, PRD 83 (2011) 053006]

3+2

- 3+2 is preferred to 3+1 only if there is CP-violating difference of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions
- 2010 MiniBooNE antineutrino data indicated neutrino-antineutrino difference
- in 2010 it was reasonable and useful to consider 3+2
- neutrino-antineutrino difference almost disappeared with 2012 MiniBooNE antineutrino data
- in 2012 3+2 is reduced to 3+1 by Okkam razor shaving

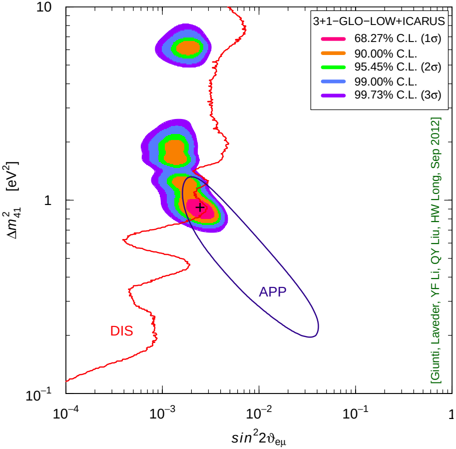
3+1 Global Fit without ICARUS

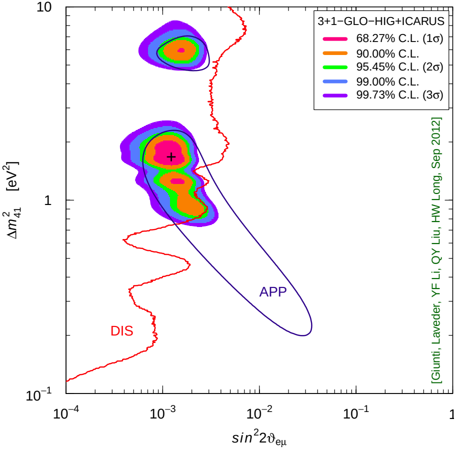


No Osc. GoF = 0.017%
3+1 GoF = 9.3%
PGoF = 0.002%

No Osc. GoF = 0.75%
3+1 GoF = 30%
PGoF = 0.6%

3+1 Global Fit with ICARUS





No Osc. GoF = 0.021%
3+1 GoF = 9.9%
PGoF = 0.01%

No Osc. GoF = 0.87%
3+1 GoF = 32%
PGoF = 0.7%

Cosmology

- N_s = number of thermalized sterile neutrinos (not necessarily integer)

  [Melchiorri, Tamborra, Saviano @ NOW2012]

- CMB and LSS in ΛCDM: $N_s = 1.3 \pm 0.9 \qquad m_s < 0.66\,\mathrm{eV}$ (95% C.L.)

  [Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301]

  $N_s = 1.61 \pm 0.92 \qquad m_s < 0.70\,\mathrm{eV}$ (95% C.L.)

  [Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, PRD 83 (2011) 115023]

- BBN:
  - $N_s = 0.22 \pm 0.59$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313]
  - $N_s = 0.64^{+0.40}_{-0.35}$ [Izotov, Thuan, ApJL 710 (2010) L67]
  - $N_s \leq 1$ at 95% C.L. [Mangano, Serpico, PLB 701 (2011) 296]

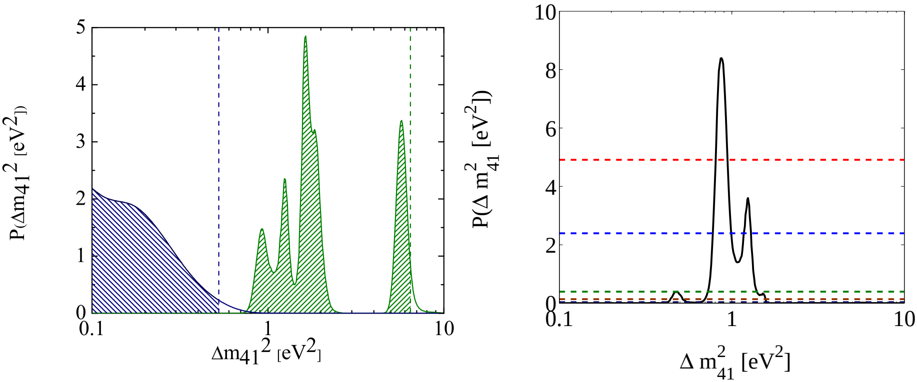
- CMB+LSS+BBN: $N_s = 0.85^{+0.39}_{-0.56}$ (95% C.L.)

  [Hamann, Hannestad, Raffelt, Wong, JCAP 1109 (2011) 034]

- Standard ΛCDM: 3+1 allowed, 3+2 disfavored

Combined Oscillation and Cosmology Fit

[Archidiacono, Fornengo, Giunti, Melchiorri, arXiv:1207.6515]



- Mass Hierarchy: $m_4 \gg m_3, m_2, m_1 \quad \Longrightarrow \quad m_4 \simeq \sqrt{\Delta m^2_{41}}$
- Cosmology: $m_4 < 0.73\,\text{eV}^2$ (95% Bayesian CL)
- Oscillation + Cosmology: $0.85 < m_4 < 1.18\,\text{eV}^2$ (95% Bayesian CL)

Conclusions

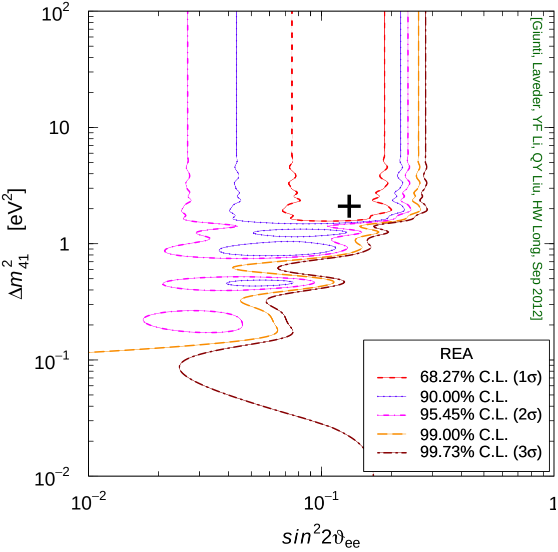
- Short-baseline neutrino oscillation anomalies $\Longrightarrow$ sterile neutrinos

- Short-Baseline $\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ Signal is not feeling well:
  - MiniBooNE 2012 antineutrino data are similar to neutrino data (LSND signal diminished and low-energy anomaly appeared)
  - Probably there is no CP violation $\Longrightarrow$ no need of 3+2
  - The decrease of MiniBooNE-LSND agreement is discouraging
  - Better experiments are needed to clarify situation [C.Rubbia @ NOW2012]

- Short-Baseline ν_{e} and $\bar{\nu}_{e}$ Disappearance is in good health:
  - Reactor $\bar{\nu}_{e}$ anomaly is alive and exciting
  - Gallium ν_{e} anomaly strengthened by new cross-section measurements
  - Many promising projects to test short-baseline ν_{e} and $\bar{\nu}_{e}$ disappearance in a few years with reactors, radioactive sources and accelerators [Ianni, Link, Gaffiot @ NOW2012]
  - Independent tests through effects of m_4 in β-decay and $(\beta\beta)_{0\nu}$-decay

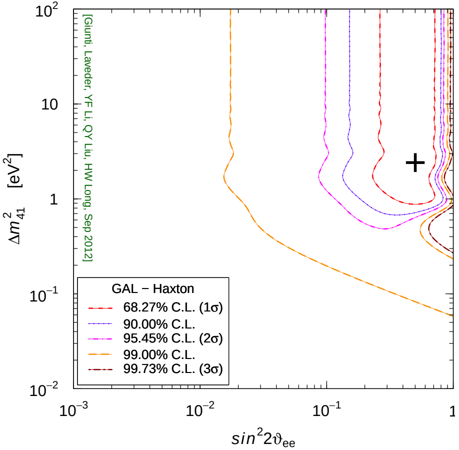
Backup Slides

Reactor $\bar{\nu}_e$ Disappearance

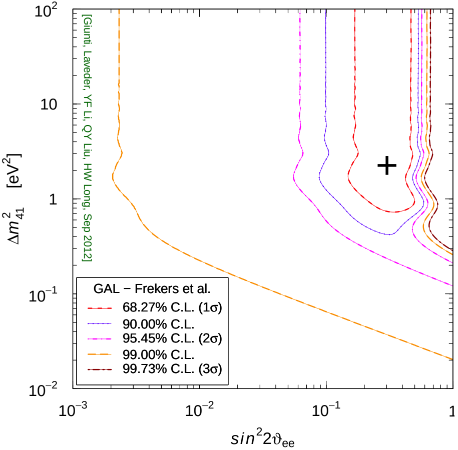


[Giunti, Laveder, YF Li, QY Liu, HW Long, Sep 2012]

Gallium ν_e Disappearance

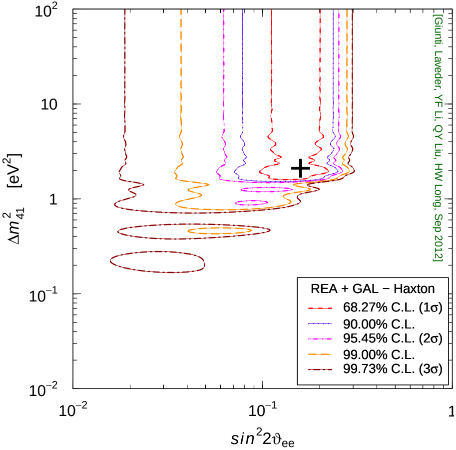


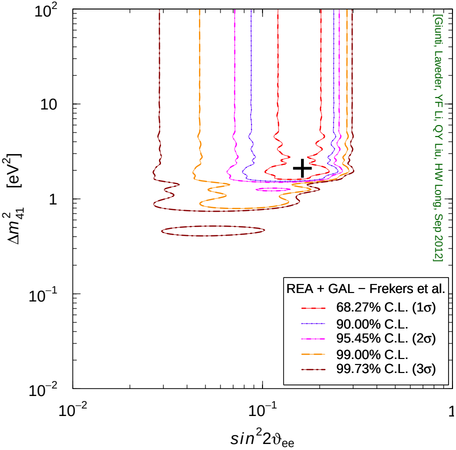
No Osc.	$\chi^2_{\min}$	14.8
	NDF	4
	GoF	0.5 %
3+1	$\chi^2_{\min}$	4.8
	NDF	2
	GoF	9.1 %
	$\Delta m^2_{41}[\text{eV}^2]$	2.24
	$\sin^2 2\vartheta_{ee}$	0.50



No Osc.	$\chi^2_{\min}$	17.2
	NDF	4
	GoF	0.2 %
3+1	$\chi^2_{\min}$	7.9
	NDF	2
	GoF	1.9 %
	$\Delta m^2_{41}[\text{eV}^2]$	2.09
	$\sin^2 2\vartheta_{ee}$	0.30

Reactor $\bar{\nu}_e$ and Gallium ν_e Disappearance





No Osc.	χ^2_{min}	45.6
	NDF	42
	GoF	32.3 %
3+1	χ^2_{min}	30.8
	NDF	40
	GoF	85 %
	Δm^2_{41}[eV^2]	1.95
	$\sin^2 2\vartheta_{ee}$	0.16

No Osc.	χ^2_{min}	48.9
	NDF	42
	GoF	21.5 %
3+1	χ^2_{min}	32.2
	NDF	40
	GoF	80 %
	Δm^2_{41}[eV^2]	1.95
	$\sin^2 2\vartheta_{ee}$	0.16