Status of Three-Neutrino Mixing and Beyond

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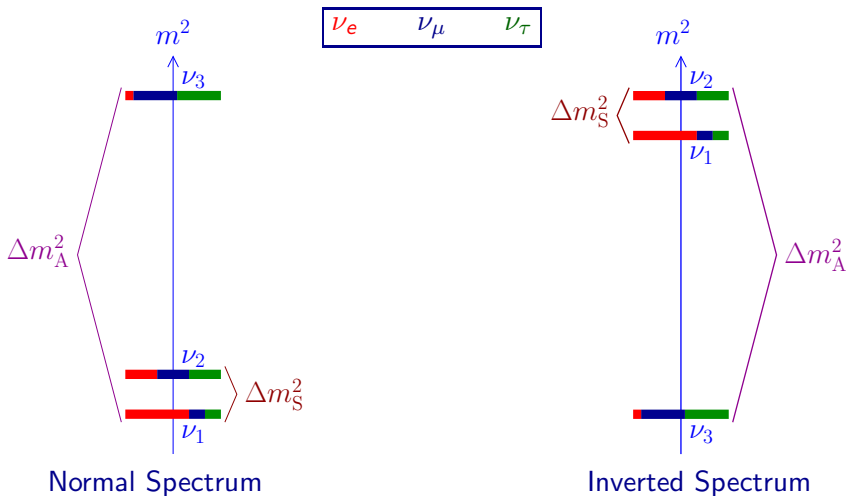
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Neutrino Unbound: http://www.nu.to.infn.it

PPC 2012
International Workshop on the Interconnection between
Particle Physics and Cosmology

5-9 November 2012, KIAS, Seoul, Korea

Three-Neutrino Mixing Paradigm

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

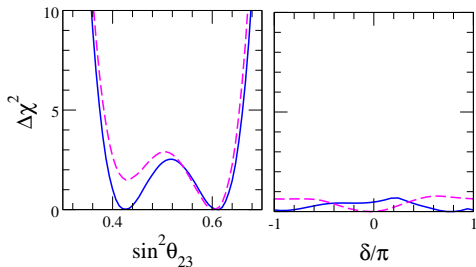| $\vartheta_{23} \simeq \vartheta_A$	Chooz, Palo Verde	$\vartheta_{12} \simeq \vartheta_S$	$\beta\beta_{0\nu}$
$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$	T2K, MINOS	$\sin^2\vartheta_{12} = 0.30 \pm 0.01$	
	Daya Bay, RENO		
	$\sin^2\vartheta_{13} = 0.023 \pm 0.002$		

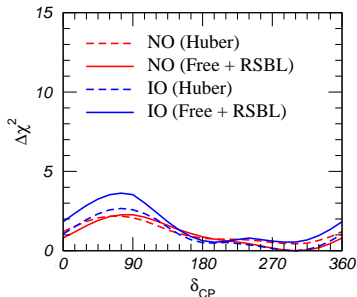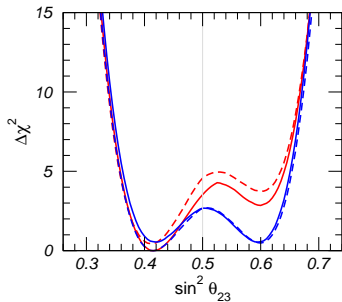$$\frac{\delta\sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \simeq 40\% \qquad \frac{\delta\sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \simeq 10\% \qquad \frac{\delta\sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \simeq 5\%$$

[Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, PRD 86 (2012) 013012]

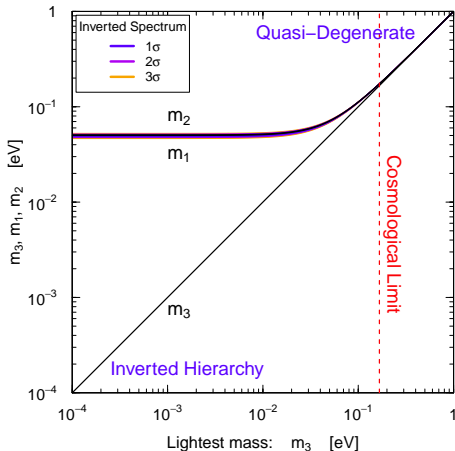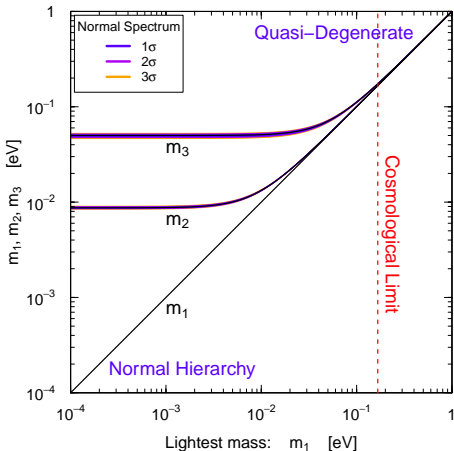[Forero, Tortola, Valle, PRD 86 (2012) 073012]

[Gonzalez-Garcia, Maltoni, Salvado, Schwetz, arXiv:1209.3023; http://www.nu-fit.org]

Open Problems

- $\vartheta_{23} < 45°$?
  - Atmospheric Neutrinos, T2K, NOνA, ...

- CP violation ?
  - NOνA, LAGUNA, CERN-GS, HyperK, . . .

- Mass Hierarchy ?
  - NOνA, Atmospheric Neutrinos, Day Bay II, Supernova Neutrinos, . . .

- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, . . .

- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, . . .

Absolute Scale of Neutrino Masses

$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}\,\text{eV}$
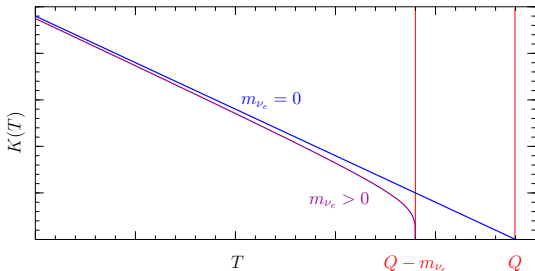
Tritium Beta-Decay

$${}^{3}\text{H} \rightarrow {}^{3}\text{He} + e^{-} + \bar{\nu}_e \qquad \frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C\, G_F)^2}{2\pi^3} |\mathcal{M}|^2 \, F(E)\, pE\, (Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{{}^{3}\text{H}} - M_{{}^{3}\text{He}} - m_e = 18.58\,\text{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C\, G_F)^2}{2\pi^3} |\mathcal{M}|^2 \, F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2} \right]^{1/2}$$

$m_{\nu_e} < 2.2\,\text{eV}$ (95% C.L.)

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

future: KATRIN
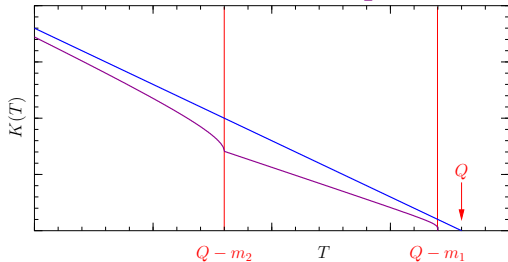[www.katrin.kit.edu]
start data taking in 2015
sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$

Neutrino Mixing $\implies K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$

analysis of data is different from the no-mixing case:
$2N - 1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

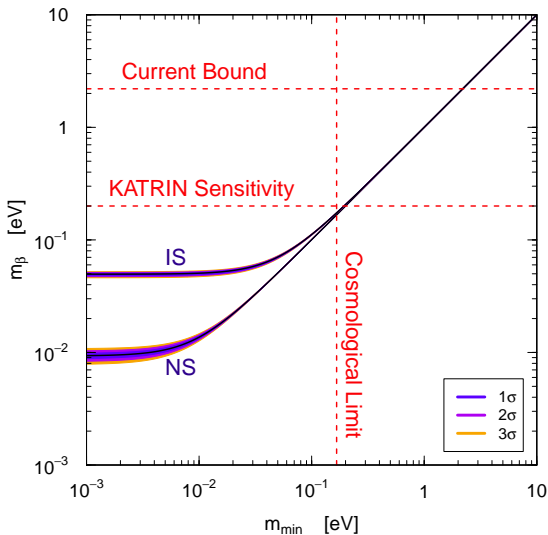if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$

- Quasi-Degenerate:
$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

- Inverted Hierarchy:
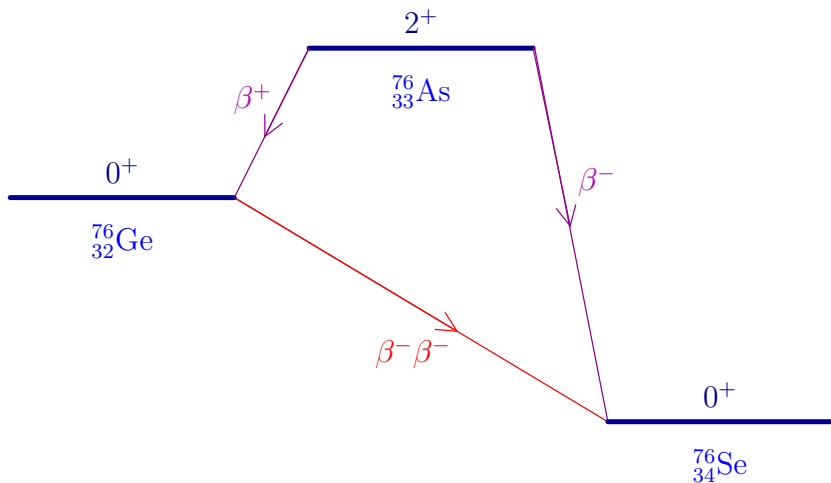$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$

- Normal Hierarchy:
$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$
$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\mathrm{eV}^2$

- $m_\beta \lesssim 4 \times 10^{-2}\,\mathrm{eV}$
$\Downarrow$
Normal Spectrum

Neutrinoless Double-Beta Decay
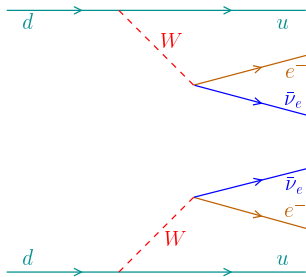

Effective Majorana Neutrino Mass: $\quad m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
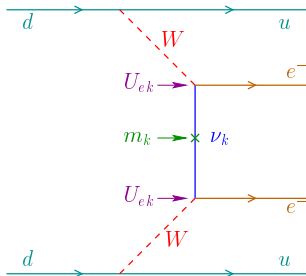in the Standard Model

Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |\mathcal{M}_{0\nu}|^2 |m_{\beta\beta}|^2$$

effective
Majorana $\quad |m_{\beta\beta}| = \left| \sum_k U_{ek}^2 \, m_k \right|$
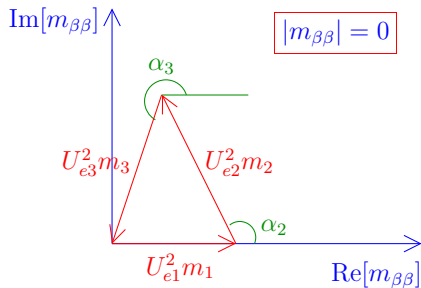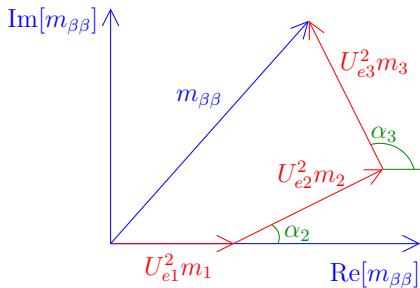mass

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$

Experimental Bounds

EXO (^{136}Xe) [PRL 109 (2012) 032505]

$$T_{1/2}^{0\nu} > 1.6 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.14 - 0.38\,\text{eV}$$

CUORICINO (^{130}Te) [AP 34 (2011) 822]

$$T_{1/2}^{0\nu} > 2.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.3 - 0.7\,\text{eV}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$T_{1/2}^{0\nu} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$T_{1/2}^{0\nu} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$T_{1/2}^{0\nu} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.}) \implies |m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}$$
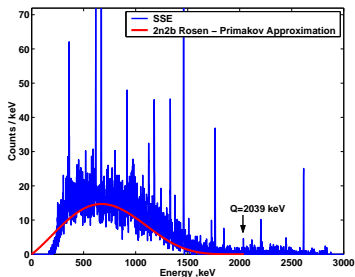
Experimental Positive Indication

$$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25} \, \text{y}$$ 6.5σ evidence
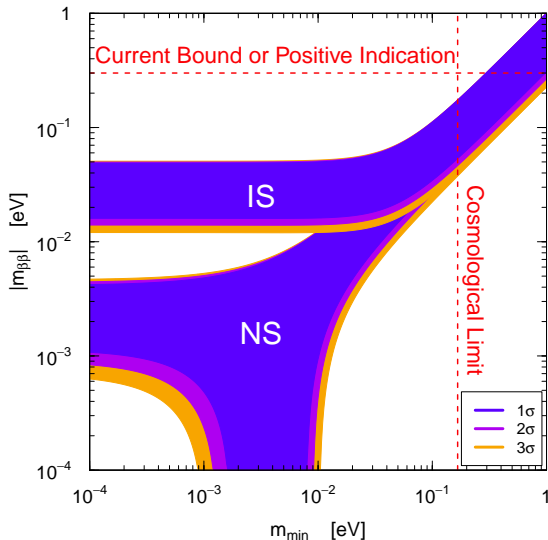
$$|m_{\beta\beta}| = 0.32 \pm 0.03 \, \text{eV}$$

very exciting: Majorana ν and large mass scale

partially excluded by EXO and CUORICINO

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$

- ► Positive indication: tension with cosmology

- ► Quasi-Degenerate:

  $$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2}$$

- ► Inverted Hierarchy:

  $$|m_{\beta\beta}| \simeq \sqrt{\Delta m_A^2 (1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2)}$$
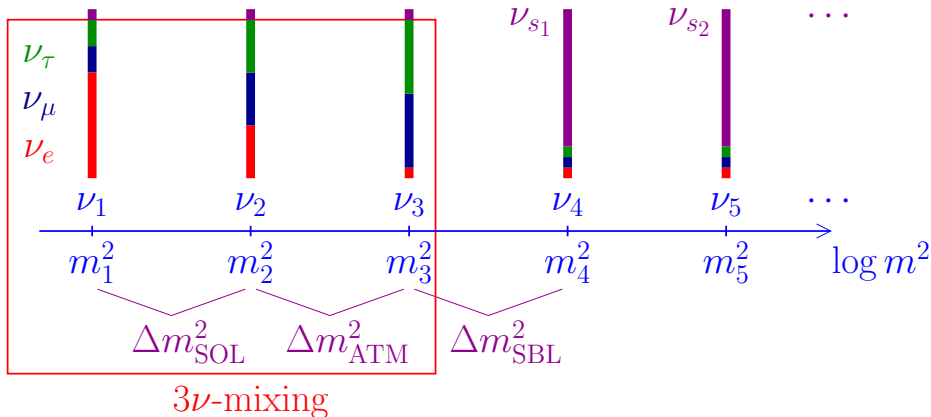
- ► Normal Hierarchy:

  $$|m_{\beta\beta}| \simeq |s_{12}^2 \sqrt{\Delta m_S^2} + e^{i\alpha} s_{13}^2 \sqrt{\Delta m_A^2}$$

  $$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3} \, \text{eV}$$

  $m_1 \gtrsim 10^{-3} \, \text{eV} \Rightarrow$ cancellation?

$|m_{\beta\beta}| \lesssim 10^{-2} \, \text{eV} \implies$ Normal Spectrum

Beyond Three-Neutrino Mixing

<u>Sterile Neutrinos</u>

- Sterile means no standard model interactions (e.g. $\nu_R^c = \nu_{sL}$)

- Oscillation observables:

  - Disappearance of active neutrinos (neutral current deficit)

  - Indirect evidence through combined fit of data (current indication)

- Short-baseline anomalies $+ 3\nu$-mixing:

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2| \ll |\Delta m_{41}^2| \leq \dots$$

ν_1	ν_2	ν_3	ν_4	$\dots$
ν_e	ν_μ	ν_τ	ν_{s_1}	$\dots$

- Neutrino number and mass observable:

  - Number of thermalized relativistic particles in early Universe (BBN, CMB, BAO)

  - m_4 effects in cosmology (CMB, LLS), direct β-decay neutrino mass measurements and neutrinoless double-β decay (if Majorana)

Reactor Electron Antineutrino Anomaly
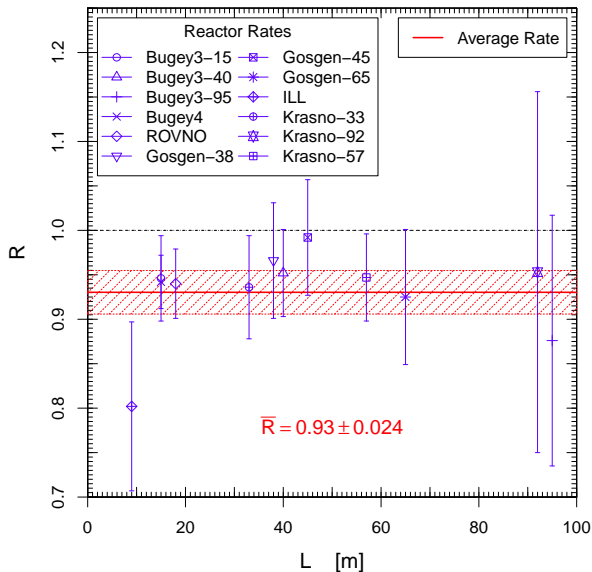

[Mention et al, PRD 83 (2011) 073006]

[update in White Paper, arXiv:1204.5379

new reactor $\bar{\nu}_e$ fluxes

[Mueller et al, PRC 83 (2011) 054615]

[Huber, PRC 84 (2011) 024617]
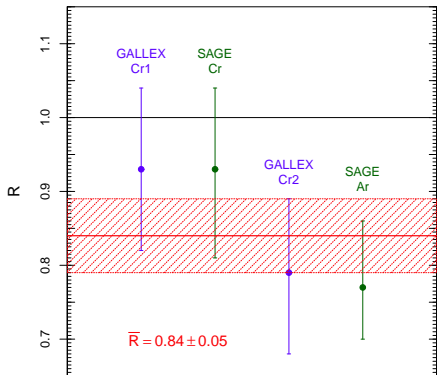
2.8σ anomaly

Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \rightarrow {}^{51}\text{V} + \nu_e$ $\qquad$ $e^- + {}^{37}\text{Ar} \rightarrow {}^{37}\text{Cl} + \nu_e$

Anomaly supported by new ${}^{71}\text{Ga}({}^{3}\text{He}, {}^{3}\text{H}){}^{71}\text{Ge}$ cross section measurement

[Frekers et al., PLB 706 (2011) 134]

$E \sim 0.7\,\text{MeV}$

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$

$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

2.9σ anomaly

3+1 SBL ν_e and $\bar{\nu}_e$ Survival Probability

$$P_{\overset{(-)}{\nu_e} \to \overset{(-)}{\nu_e}} = 1 - \sin^2 2\vartheta_{ee} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

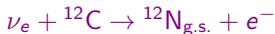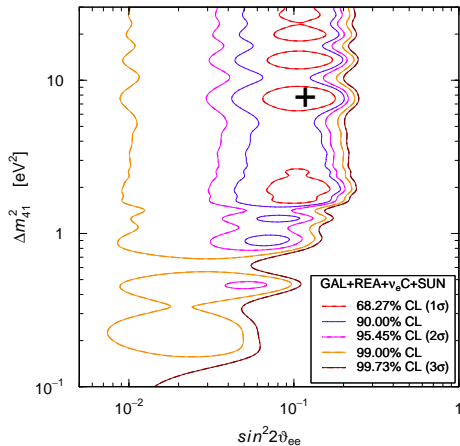$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right)$$
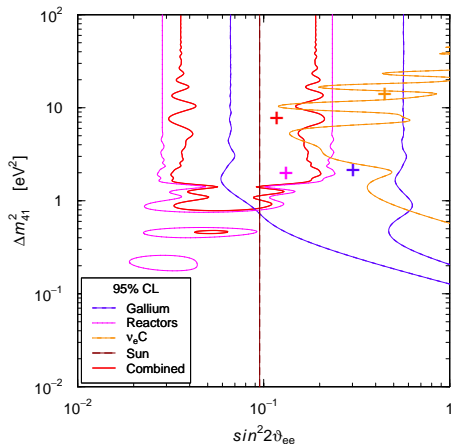
standard parameterization

$$U_{e1} = c_{12}c_{13}c_{14} \quad U_{e2} = s_{12}c_{13}c_{14} \quad U_{e3} = s_{13}c_{14}e^{-i\delta_{13}} \quad U_{e4} = s_{14}e^{-i\delta_{14}}$$

$$\sin^2 2\vartheta_{ee} = \sin^2 2\vartheta_{14}$$

Global ν_e and $\bar{\nu}_e$ Disappearance

$\nu_e + {}^{12}\text{C} \rightarrow {}^{12}\text{N}_{\text{g.s.}} + e^-$

KARMEN + LSND

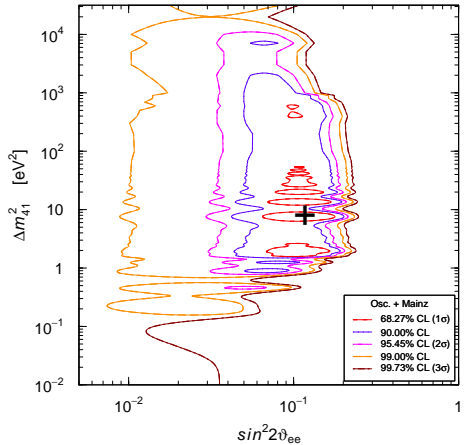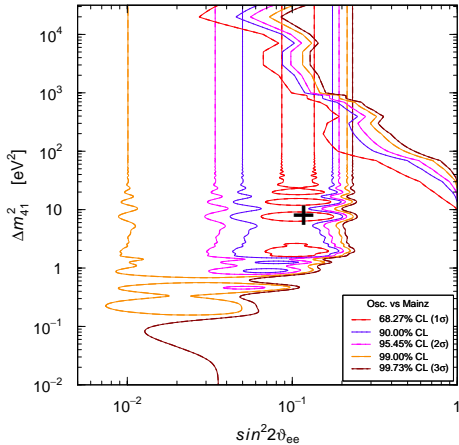solar ν_e + KamLAND $\bar{\nu}_e$ + ϑ_{13}

Mainz Limit on m_4^2

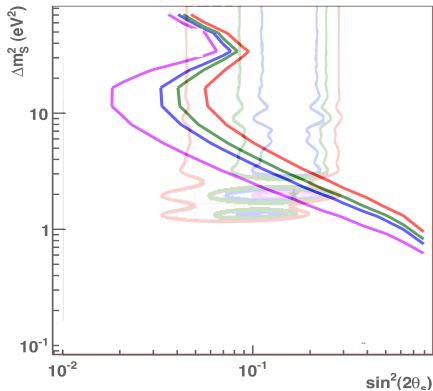[Kraus, Singer, Valerius, Weinheimer, arXiv:1210.4194]

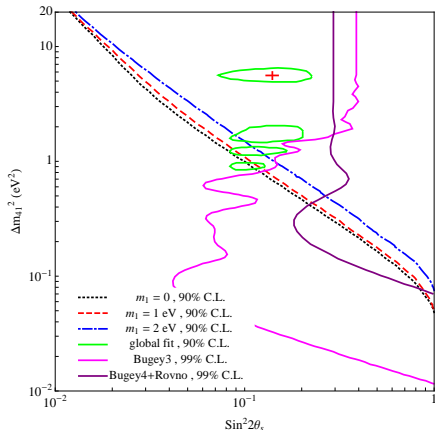$$m_4 \gg m_1, m_2, m_3 \implies \Delta m_{41}^2 \equiv m_4^2 - m_1^2 \simeq m_4^2$$

Δm_{41}^2 **Upper Bound**

KATRIN Sensitivity

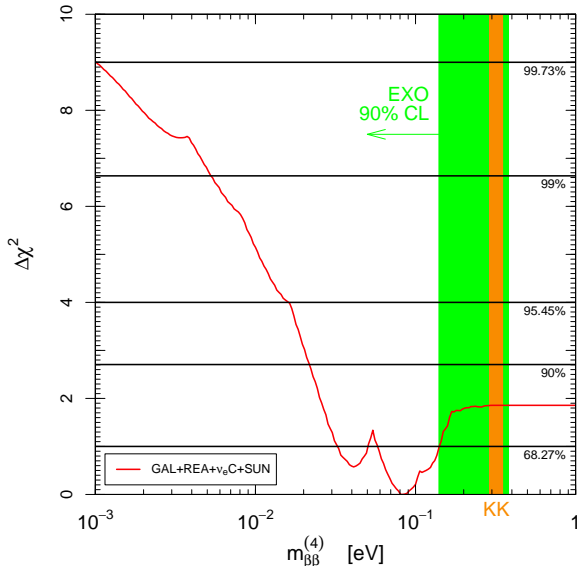[Formaggio, Barrett, PLB 706 (2011) 68]

[Esmaili, Peres, PRD 85 (2012) 117301]

[see also Sejersen Riis, Hannestad, JCAP (2011) 1475; Sejersen Riis, Hannestad, Weinheimer, PRC 84 (2011) 045503]

Neutrinoless Double-β Decay

$$|m_{\beta\beta}| = \left| \sum_{k=1}^{4} U_{ek}^2 \, m_k \right|$$

$$m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

caveat:
possible cancellation
with $m_{\beta\beta}^{(3\nu-IH)}$

[Barry et al, JHEP 07 (2011) 091]

[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, arXiv:1206.2560]
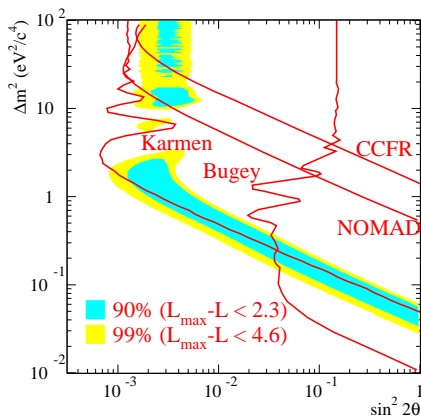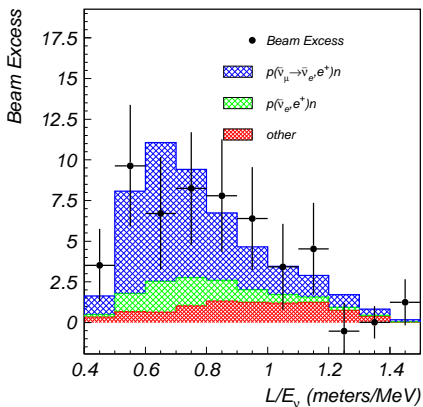
LSND

$\bar{\nu}_\mu \to \bar{\nu}_e$ $L \simeq 30\,\text{m}$ $20\,\text{MeV} \leq E \leq 200\,\text{MeV}$

3.8σ excess $\Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2$ $(\gg \Delta m^2_A \gg \Delta m^2_S)$

3+1: Appearance vs Disappearance

▶ ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

▶ ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

▶ $\nu_\mu \to \nu_e$ experiments:
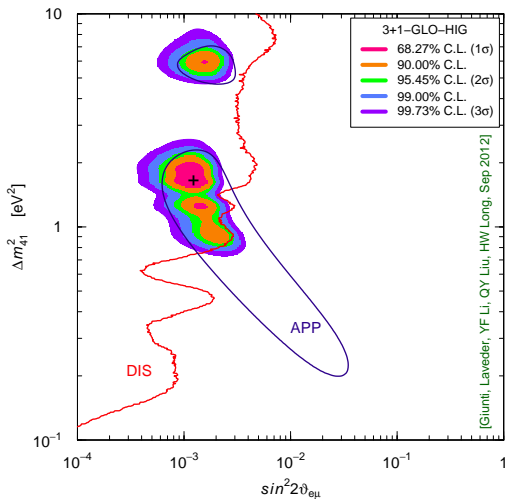
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2 |U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

▶ Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu} \Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247]

3+1 Global Fit

- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND, MiniBooNE, ICARUS, KARMEN, NOMAD

- DIS ν_e & $\bar{\nu}_e$: Reactors, Gallium, $\nu_e C$, Solar

- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW, MINOS, Atmospheric

No Osc. GoF = 0.87%
3+1 GoF = 32%
PGoF = 0.7%

Cosmology

▸ N_s = number of thermalized sterile neutrinos (not necessarily integer)

▸ CMB and LSS in ΛCDM: $\quad N_s = 1.3 \pm 0.9 \quad m_s < 0.66$ eV (95% C.L.)

[Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301]

$$N_s = 1.61 \pm 0.92 \qquad m_s < 0.70 \, \text{eV} \quad (95\% \text{ C.L.})$$

[Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, PRD 83 (2011) 115023]

▸ BBN: $\begin{cases} N_s \le 1 \text{ at 95\% C.L.} & \text{[Mangano, Serpico, PLB 701 (2011) 296]} \\ N_s = 0.0 \pm 0.5 & \text{[Pettini, Cooke, arXiv:1205.3785]} \end{cases}$
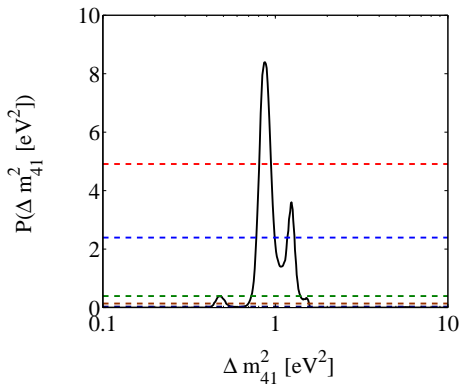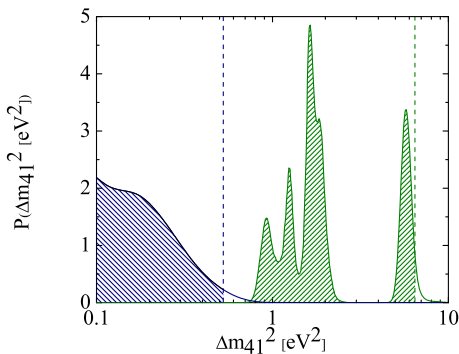
▸ CMB+LSS+BBN: $\quad N_s = 0.85^{+0.39}_{-0.56} \quad (95\% \text{ C.L.})$

[Hamann, Hannestad, Raffelt, Wong, JCAP 1109 (2011) 034]

▸ Standard ΛCDM: 3+1 allowed, 3+2 disfavored

Combined Oscillation and Cosmology Fit

[Archidiacono, Fornengo, Giunti, Melchiorri, PRD 86 (2012) 065028]

- Mass Hierarchy: $m_4 \gg m_3, m_2, m_1 \quad\Longrightarrow\quad m_4 \simeq \sqrt{\Delta m_{41}^2}$

- Cosmology: $m_4 < 0.73\,\text{eV}^2$ (95% Bayesian CL)

- Oscillation + Cosmology: $0.85 < m_4 < 1.18\,\text{eV}^2$ (95% Bayesian CL)

Conclusions

- Robust Three-Neutrino Mixing Paradigm. Open problems: $\vartheta_{23} < 45°$?, CP Violation, Mass Hierarchy, Absolute Mass Scale, Dirac or Majorana?

- Short-Baseline ν_e and $\bar{\nu}_e$ 3+1 Disappearance:
  - Reactor $\bar{\nu}_e$ anomaly is alive and exciting
  - Gallium ν_e anomaly strengthened by new cross-section measurements
  - Many promising projects to test short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources
  - Independent tests through effect of m_4 in β-decay and $(\beta\beta)_{0\nu}$-decay

- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - MiniBooNE experiment has been inconclusive
  - Better experiments are needed to check LSND signal
  - If LSND signal is confirmed $m_4 \sim 1\,\text{eV}$, marginally compatible with ΛCDM

- Cosmology:
  - Very powerful probe of neutrino number and masses
  - Model dependent. Thermalization loophole
  - Bright future (Plank, ...)