Experiments for sterile neutrino searches

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International Workshop on
"RENO-50" toward Neutrino Mass Hierarchy

Seoul National University, Korea

13-14 June 2013

Three-Neutrino Mixing Paradigm

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

VLBL Reactor
$\bar{\nu}_e$ disappearance

$$\left.\begin{matrix} \left(\begin{matrix} \text{SNO, BOREXino} \\ \text{Super-Kamiokande} \\ \text{GALLEX/GNO, SAGE} \\ \text{Homestake, Kamiokande} \end{matrix}\right) \\ \\ \text{(KamLAND)} \end{matrix}\right\} \to \begin{cases} \Delta m^2_{\text{S}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 \vartheta_{\text{S}} \simeq 0.30 \end{cases}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

LBL Accelerator
ν_μ disappearance

LBL Accelerator
$\nu_\mu \to \nu_\tau$

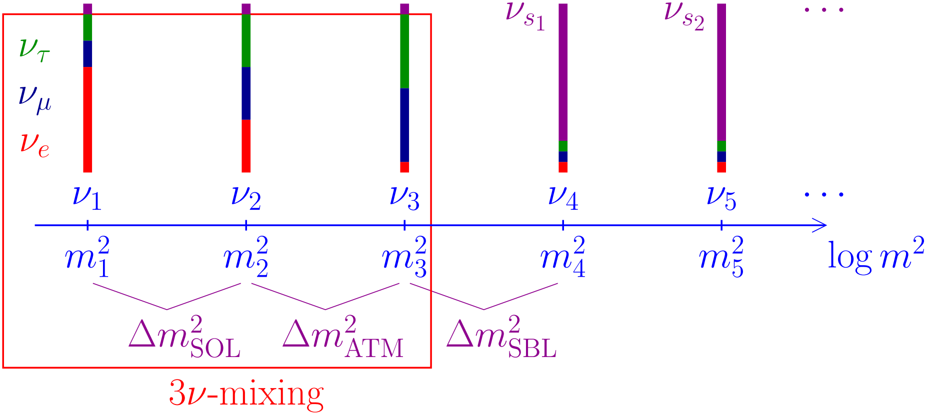
$$\left.\begin{matrix} \left(\begin{matrix} \text{Super-Kamiokande} \\ \text{Kamiokande, IMB} \\ \text{MACRO, Soudan-2} \end{matrix}\right) \\ \text{(K2K, MINOS, T2K)} \\ \\ \text{(Opera)} \end{matrix}\right\} \to \begin{cases} \Delta m^2_{\text{A}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 \vartheta_{\text{A}} \simeq 0.50 \end{cases}$$

LBL Accelerator
$\nu_\mu \to \nu_e$

LBL Reactor
$\bar{\nu}_e$ disappearance

$$\left.\begin{matrix} \text{(T2K, MINOS)} \\ \left(\begin{matrix} \text{Daya Bay, RENO} \\ \text{Double Chooz} \end{matrix}\right) \end{matrix}\right\} \to \begin{cases} \Delta m^2_{\text{A}} \\ \sin^2 \vartheta_{13} \simeq 0.023 \end{cases}$$

Beyond Three-Neutrino Mixing



Sterile Neutrinos from Physics Beyond the SM

- Neutrinos are special in the Standard Model: the only neutral fermions
- In extensions of SM neutrinos can mix with non-SM fermions
- SM: $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$ $\widetilde{\Phi} = i\sigma_2\,\Phi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$
- SM singlet $\overline{L_L}\widetilde{\Phi}$ can couple to new singlet chiral fermion field f_R related to physics beyond the SM [$Y(\overline{L_L}) = +1$, $Y(\widetilde{\Phi}) = -1$]
- Known examples: light ν_R, SUSY, new symmetries, extra dimensions, mirror world, ... [see `http://www.nu.to.infn.it/Sterile_Neutrinos/`]
- Dirac mass term $\sim \overline{L_L}\widetilde{\Phi} f_R$ $+$ Majorana mass term $\sim \overline{f_R^c} f_R$
- Diagonalization of mass matrix $\Longrightarrow$ massive Majorana neutrinos
- f_R is often called Right-Handed Neutrino: $f_R \to \nu_R$

Light Sterile Neutrinos

- Light anti-ν_R are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into light sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Short-baseline anomalies + 3ν-mixing:

$$\begin{array}{ccccc} & \Delta m^2_{21} \ll & |\Delta m^2_{31}| \ll & |\Delta m^2_{41}| \leq & \ldots \\ \nu_1 & \nu_2 & \nu_3 & \nu_4 & \ldots \\ \nu_e & \nu_\mu & \nu_\tau & \nu_{s_1} & \ldots \end{array}$$

- In this talk I consider sterile neutrinos with mass scale $\sim 1\,\text{eV}$ in light of short-baseline Reactor Anomaly, Gallium Anomaly, LSND, MiniBooNE.
- Other possibilities (not incompatible):
  - Very light sterile neutrinos with mass scale $\ll 1\,\text{eV}$: important for solar neutrino phenomenology

    [Das, Pulido, Picariello, PRD 79 (2009) 073010]
    [de Holanda, Smirnov, PRD 83 (2011) 113011]

  - Heavy sterile neutrinos with mass scale $\gg 1\,\text{eV}$: could be Warm Dark Matter

    [Kusenko, Phys. Rept. 481 (2009) 1]
    [Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191]
    [Drewes, arXiv:1303.6912]

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

$$|U_{e4}|^2 \ll 1 \,, \quad |U_{\mu 4}|^2 \ll 1 \,, \quad |U_{\tau 4}|^2 \ll 1 \,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \\ U_{s1} & U_{s2} & U_{s3} \end{matrix} \; \underset{\substack{\uparrow \\ \text{SBL}}}{\boxed{\begin{matrix} U_{e4} \\ U_{\mu 4} \\ U_{\tau 4} \\ U_{s4} \end{matrix}}}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

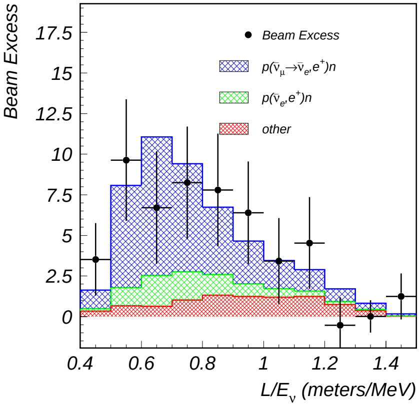
$$\Downarrow$$

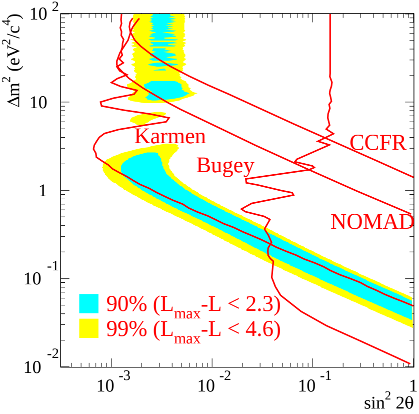
$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad L \simeq 30\,\text{m}$ $\qquad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$



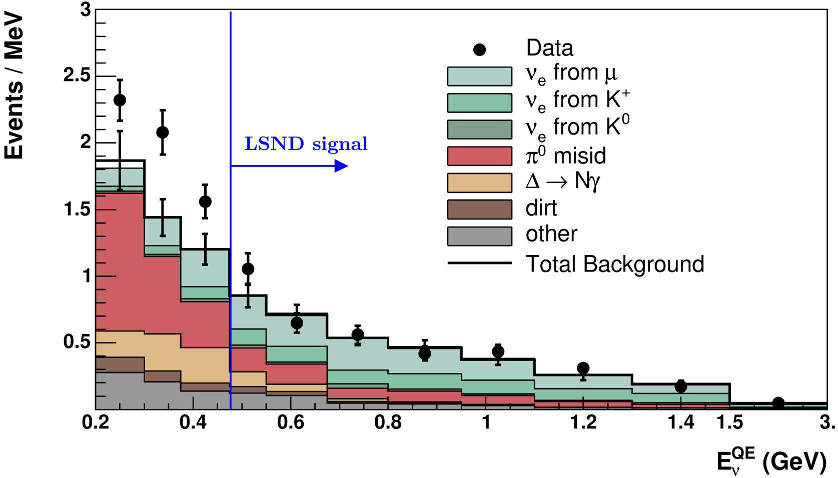


3.8σ excess $\qquad \Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2 \qquad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

MiniBooNE Neutrinos

[PRL 98 (2007) 231801; PRL 102 (2009) 101802]

$\nu_\mu \to \nu_e$ $L \simeq 541\,\text{m}$ $200\,\text{MeV} \le E \lesssim 3\,\text{GeV}$

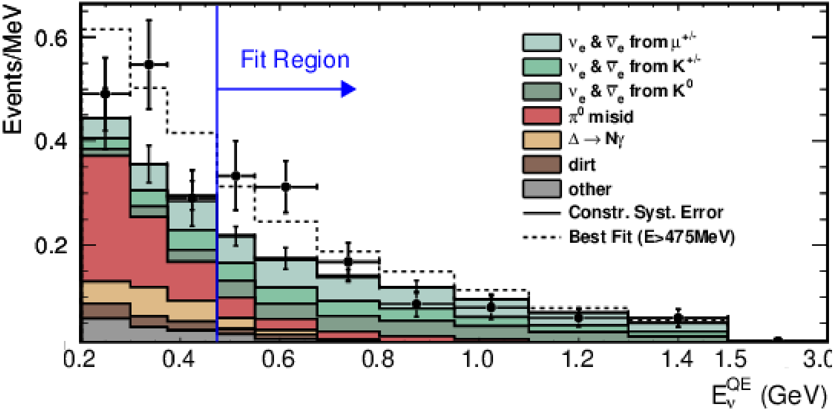


- no $\nu_\mu \to \nu_e$ signal corresponding to LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal ($E > 475\,\text{MeV}$)
- low-energy anomaly

MiniBooNE Antineutrinos - 2009-2010

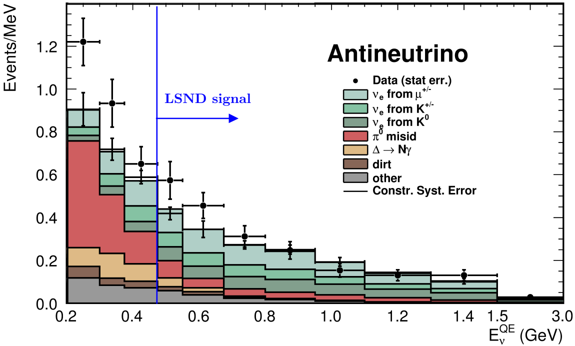
[PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ $L \simeq 541\,\text{m}$ $200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$



- agreement with LSND $\bar{\nu}_{\mu} \to \bar{\nu}_{e}$ signal ($E > 475\,\text{MeV}$)
- similar L/E but different L and E $\Longrightarrow$ oscillations
- CP violation?

MiniBooNE $\bar{\nu}$ - Neutrino 2012 - 6 June



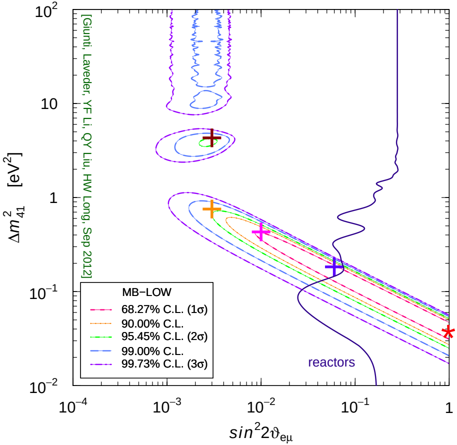
		1st half			2nd half	
	data	mc	excess	data	mc	excess
200-475	119	100.5±14.3	18.5 (1.3s)	138	100.0±14.1	38 (2.7s)
475-1250	120	99.1±14.0	20.9 (1.5s)	101	103.1±14.4	-2.2 (-0.2s)

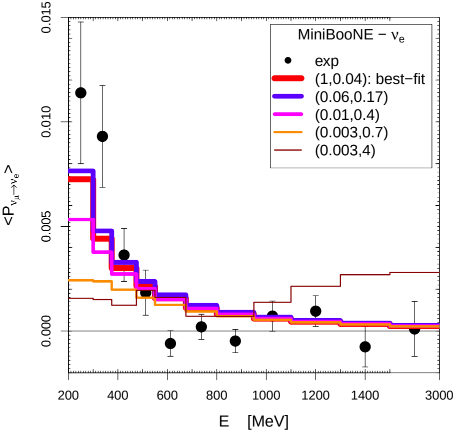
? agreement with LSND signal ? CP violation ?

? nevertheless, claim of evidence of oscillations of ν and $\bar{\nu}$?

[arXiv:1207.4809, duplicated in arXiv:1303.2588 → PRL 110 (2013) 161801]

MiniBooNE ν and $\bar{\nu}$ - arXiv:1207.4809





- Fit of low-energy excess is marginal
- It requires $\Delta m^2_{41} \lesssim 0.4 \text{eV}^2$
- Neutrino energy reconstruction problem?

[Martini, Ericson, Chanfray, PRD 85 (2012) 093012]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

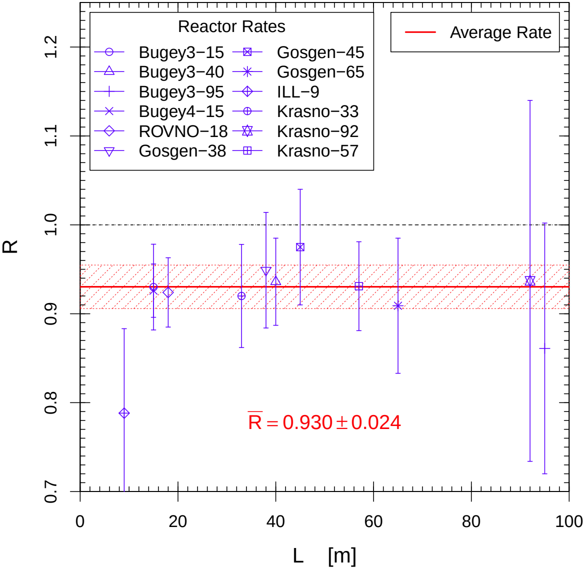
[update in White Paper, arXiv:1204.5379]

new reactor $\bar{\nu}_e$ fluxes

[Mueller et al, PRC 83 (2011) 054615]

[Huber, PRC 84 (2011) 024617]

2.8σ anomaly



Gallium Anomaly

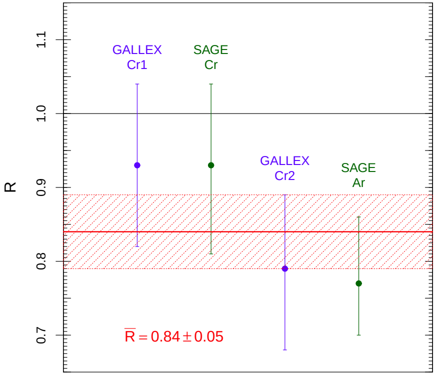
Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

Anomaly supported by new ${}^{71}\text{Ga}({}^{3}\text{He}, {}^{3}\text{H}){}^{71}\text{Ge}$ cross section measurement

[Frekers et al., PLB 706 (2011) 134]



$E \sim 0.7\,\text{MeV}$

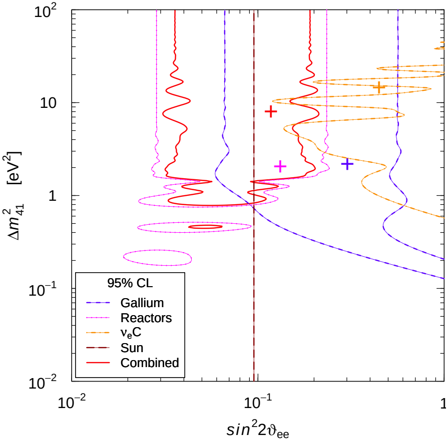
$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$

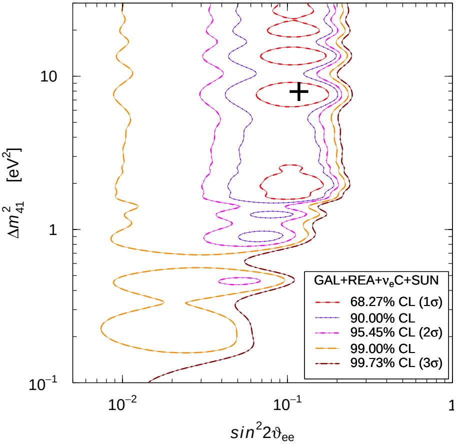
$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

2.9σ anomaly

Global ν_e and $\bar{\nu}_e$ Disappearance

[Giunti, Laveder, Y.F. Li, Q.Y. Liu, H.W. Long, PRD 86 (2012) 113014]





$\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$

KARMEN + LSND

[Conrad, Shaevitz, PRD 85 (2012) 013017]

[Giunti, Laveder, PLB 706 (2011) 200]

solar ν_e + KamLAND $\bar{\nu}_e$ + ϑ_{13}

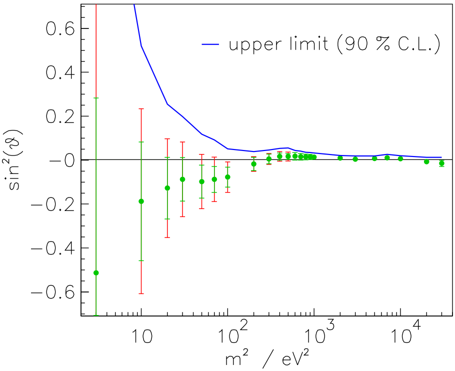
[Giunti, Li, PRD 80 (2009) 113007]

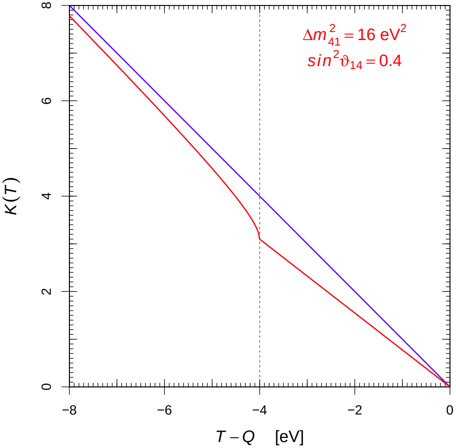
[Palazzo, PRD 83 (2011) 113013]

[Palazzo, PRD 85 (2012) 077301]

Mainz Limit on m_4^2

[Kraus, Singer, Valerius, Weinheimer, arXiv:1210.4194]

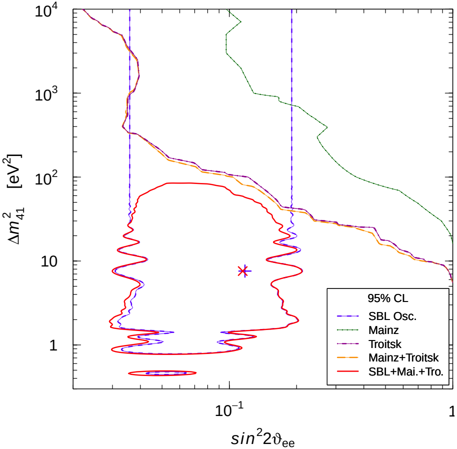


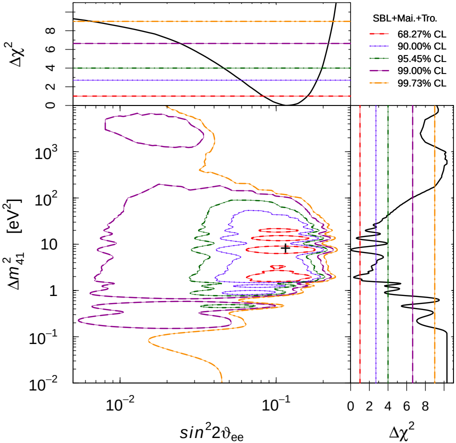


$$m_4 \gg m_1, m_2, m_3 \quad \Longrightarrow \quad \Delta m^2_{41} \equiv m_4^2 - m_1^2 \simeq m_4^2$$

Troitsk: Surprising Much Better Limit on m_4^2

[Belesev, Berlev, Geraskin, Golubev, Likhovid, Nozik, Pantuev, Parfenov, Skasyrskaya, arXiv:1211.7193]

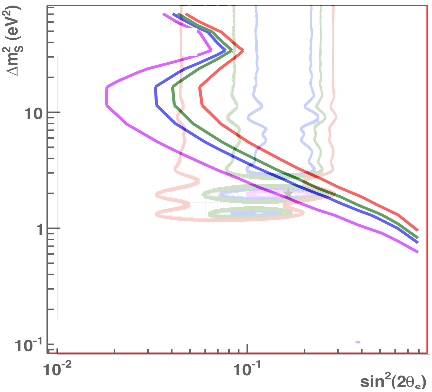




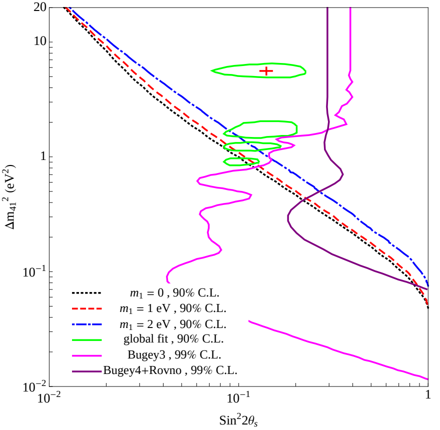
$$2\sigma : \quad 0.85 \lesssim \Delta m^2_{41} \lesssim 43\,\text{eV}^2 \quad \Longrightarrow \quad 6\,\text{cm} \lesssim \frac{L^{\text{OSC}}_{41}}{E\,[\text{MeV}]} \lesssim 3\,\text{m}$$

[Giunti, Laveder, Y.F. Li, H.W. Long, PRD 87 (2013) 013004]

KATRIN Sensitivity



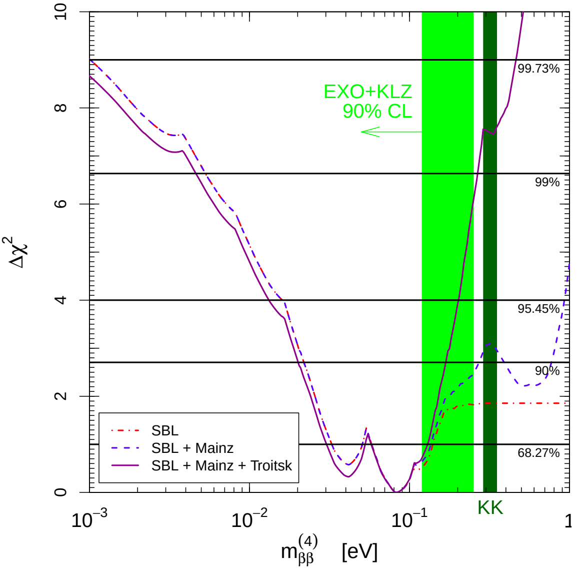
[Formaggio, Barrett, PLB 706 (2011) 68]



[Esmaili, Peres, PRD 85 (2012) 117301]

[see also Sejersen Riis, Hannestad, JCAP (2011) 1475; Sejersen Riis, Hannestad, Weinheimer, PRC 84 (2011) 045503]

Neutrinoless Double-β Decay



$$|m_{\beta\beta}| = \left|\sum_{k=1}^{4} U_{ek}^2\, m_k\right|$$

$$m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

caveat:

possible cancellation

with $m_{\beta\beta}^{(3\nu-IH)}$

[Barry et al, JHEP 07 (2011) 091]

[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, JPG 39 (2012) 124008]

Cancellation with $m_{\beta\beta}^{(\text{light})}$?

[Barry, Rodejohann, Zhang, JHEP 07 (2011) 091]; Li, Liu, PLB 706 (2012) 406; Rodejohann, JPG 39 (2012) 124008]

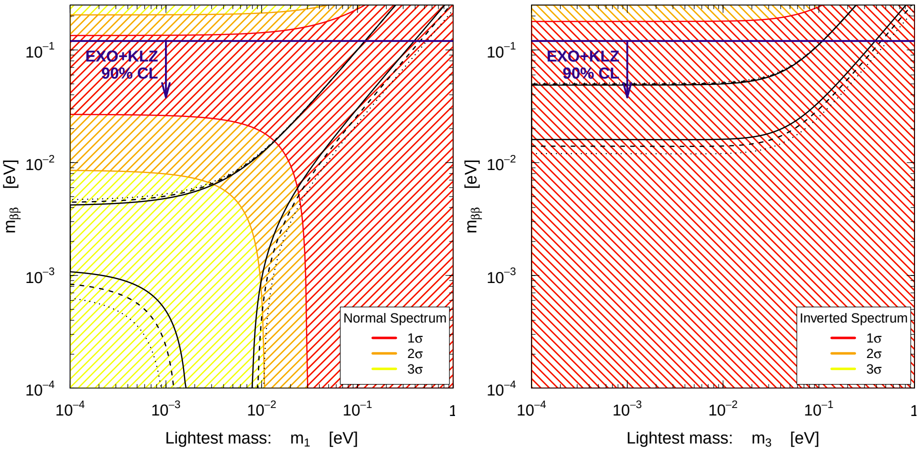
$$m_{\beta\beta}^{(\text{light})} = \left| \sum_{k=1}^{3} U_{ek}^2 \, m_k \right| \qquad m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

$$m_{\beta\beta} = m_{\beta\beta}^{(\text{light})} + e^{i\alpha_4} m_{\beta\beta}^{(4)} \qquad m_{\beta\beta}^{(4)} \gtrsim 10^{-2}\,\text{eV}$$

- **Normal Hierarchy:** $m_{\beta\beta}^{(\text{light})} \lesssim 4.5 \times 10^{-3}\,\text{eV}$ (95% CL)
  no cancellation is possible

- **Inverted Hierarchy:** $1.4 \times 10^{-2} \lesssim m_{\beta\beta}^{(\text{light})} \lesssim 5.0 \times 10^{-2}\,\text{eV}$ (95% CL)
  cancellation is possible

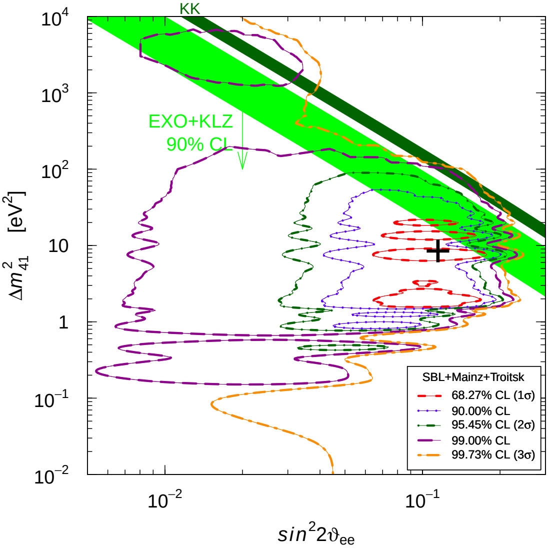
- **Quasi-Degenerate:** $m_{\beta\beta}^{(\text{light})} \gtrsim 5.0 \times 10^{-2}\,\text{eV}$ cancellation is possible


EXO+KLZ
90% CL
$m_{\beta\beta}$ [eV]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
Lightest mass: m_1 [eV]
Normal Spectrum
1σ
2σ
3σ
EXO+KLZ
90% CL
$m_{\beta\beta}$ [eV]
Lightest mass: m_3 [eV]
Inverted Spectrum
1σ
2σ
3σ

Assumption: no cancellation

$$m_{\beta\beta} \geq m_{\beta\beta}^{(4)}$$
$$= |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

$$\Delta m_{41}^2 = \left(\frac{m_{\beta\beta}^{(4)}}{|U_{e4}|^2}\right)^2$$
$$\leq \left(\frac{m_{\beta\beta}}{|U_{e4}|^2}\right)^2$$



3+1: Appearance vs Disappearance

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$

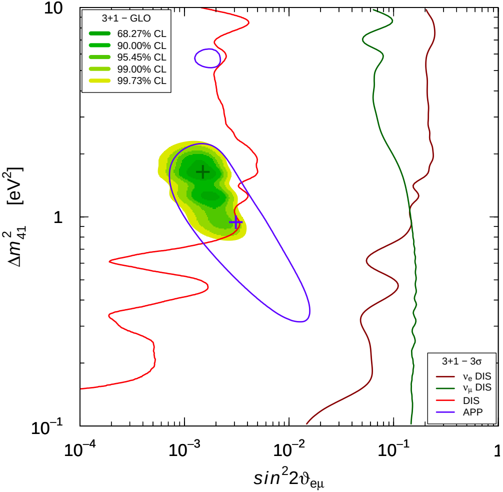
- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247]

3+1 Global Fit

[Giunti, Laveder, Y.F. Li, H.W. Long (2013) in preparation]



No Osc. GoF = 1%

3+1 GoF = 33%

PGoF = 10%

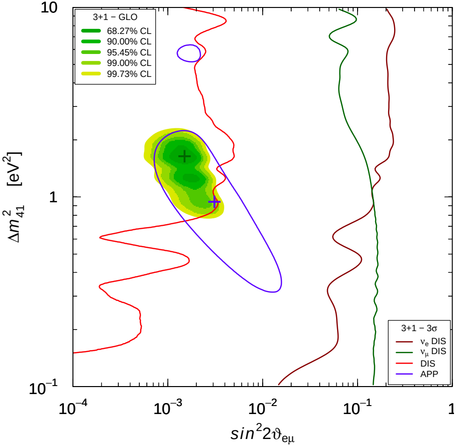
- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND (Y), MiniBooNE (?), OPERA (N), ICARUS (N), KARMEN (N), NOMAD (N), BNL-E776 (N)
- DIS ν_e & $\bar{\nu}_e$: Reactors (Y), Gallium (Y), ν_eC (N), Solar (N)
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW (N), MINOS (N), Atmospheric (N), MiniBooNE/SciBooNE (N)

[see also Kopp, Machado,

Maltoni, Schwetz, JHEP 1305 (2013) 050]

MiniBooNE Impact on SBL Oscillations?

with MiniBooNE

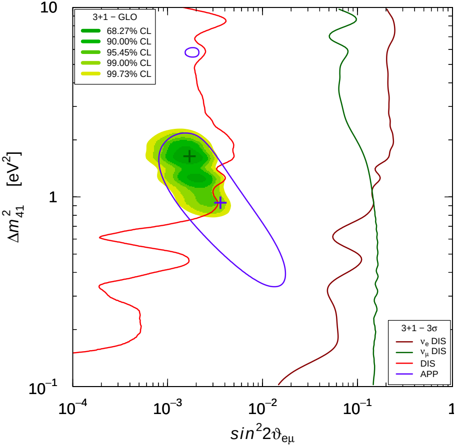


No Osc. GoF = 1%

3+1 GoF = 33%

PGoF = 10%

without MiniBooNE



No Osc. GoF = 0.4%

3+1 GoF = 22%

PGoF = 5%

Cosmology

- N_s = number of thermalized sterile neutrinos (not necessarily integer)

- CMB+LSS in ΛCDM: $N_s = 1.3 \pm 0.9 \quad m_s < 0.66\,\text{eV}\ (95\%\ \text{C.L.})$

  [Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301]

  $$N_s = 1.61 \pm 0.92 \qquad m_s < 0.70\,\text{eV} \quad (95\%\ \text{C.L.})$$

  [Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, PRD 83 (2011) 115023]

- BBN: $\begin{cases} N_s \leq 1 \text{ at } 95\%\ \text{C.L.} & \text{[Mangano, Serpico, PLB 701 (2011) 296]} \\ N_s = 0.0 \pm 0.5 & \text{[Pettini, Cooke, arXiv:1205.3785]} \end{cases}$

- CMB+LSS+BBN in ΛCDM: $N_s = 0.85^{+0.39}_{-0.56} \quad (95\%\ \text{C.L.})$

  [Hamann, Hannestad, Raffelt, Wong, JCAP 1109 (2011) 034]

- Standard ΛCDM in 2012: 3+1 allowed, 3+2 disfavored

Recent CMB Measurements

- highL South Pole Telescope (SPT) [arXiv:1212.6267]

  $N_{\text{eff}} = 3.62 \pm 0.48$ (WMAP7+SPT)

  $N_{\text{eff}} = 3.71 \pm 0.35$ (WMAP7+SPT+BAO+HST)

- highL Atacama Cosmology Telescope (ACT) [arXiv:1301.0824]

  $N_{\text{eff}} = 2.79 \pm 0.56$ (WMAP7+ACT)

  $N_{\text{eff}} = 3.50 \pm 0.42$ (WMAP7+ACT+BAO+HST)

- Planck [arXiv:1303.5076]

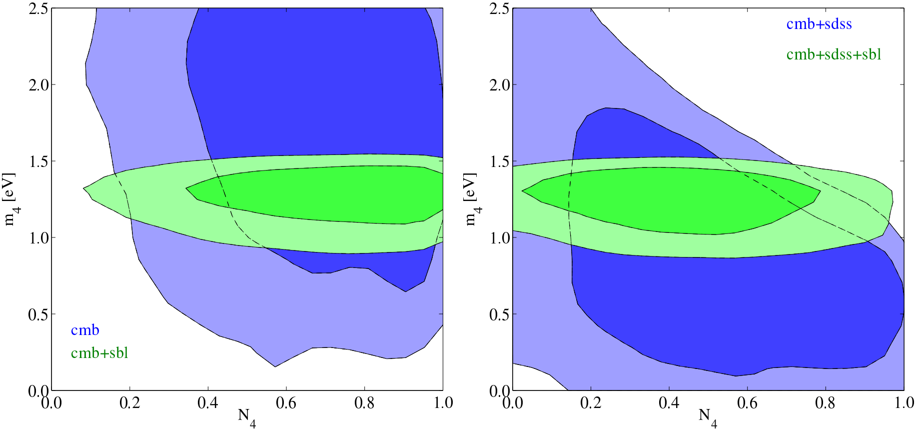
  $N_{\text{eff}} = 3.36^{+0.68}_{-0.64}$ (95%; Planck+WMAP9+highL)

  $N_{\text{eff}} = 3.30^{+0.54}_{-0.51}$ (95%; Planck+WMAP9+highL+BAO)

  $N_{\text{eff}} = 3.52^{+0.48}_{-0.45}$ (95%; Planck+WMAP9+highL+BAO+HST)

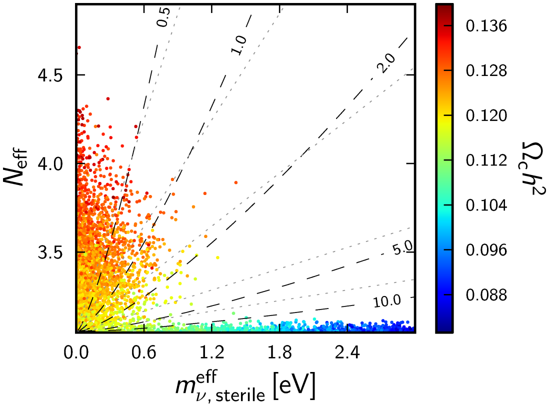
Pre-Planck Oscillation + Cosmology Fit

[Archidiacono, Fornengo, Giunti, Hannestad, Melchiorri, arXiv:1302.6720]



- Mass Hierarchy: $m_4 \gg m_3, m_2, m_1$ $\Longrightarrow$ $m_4 \simeq \sqrt{\Delta m^2_{41}}$
- $m_4 = 1.23 \pm 0.13\,\text{eV}^2$
- $N_4 < 0.83$ (95% Bayesian CL)

Planck



[arXiv:1303.5076]

- $m_{\nu,\rm sterile}^{\rm eff} \equiv 94.1\omega_{\nu_4}$ eV
- Thermally distributed:

$$m_{\nu,\rm sterile}^{\rm eff} = \left(\frac{T_s}{T_\nu}\right)^3 m_4 = (\Delta N_{\rm eff})^{3/4} m_4$$

- Dodelson-Widrow:

$$m_{\nu,\rm sterile}^{\rm eff} = \chi_s m_4$$

$$\text{Caveat: } H_0 = \left\{\begin{array}{ll} 67.4 \pm 1.4 & \text{Planck} \\ 70.0 \pm 2.2 & \text{WMAP-9} \\ 73.8 \pm 2.4 & \text{Cepheids+SNIa} \\ 74.3 \pm 2.6 & \text{Carnegie HP} \\ 78.7 \pm 4.5 & \text{COSMOGRAIL} \end{array}\right\} [\text{kms}^{-1}\text{Mpc}^{-1}]$$

Conclusions

- Short-Baseline ν_e and $\bar{\nu}_e$ 3+1 Disappearance:
  - Reactor $\bar{\nu}_e$ anomaly is alive and exciting
  - Gallium ν_e anomaly strengthened by new cross-section measurements
  - Many promising projects to test short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources
  - Independent tests through effect of m_4 in β-decay and $(\beta\beta)_{0\nu}$-decay
- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - MiniBooNE experiment has been inconclusive
  - Better experiments are needed to check LSND signal
- Light Sterile Neutrinos:
  - First new particle beyond the Standard Model?
  - Strongest hint from Reactor and Gallium Anomalies $\Longrightarrow |U_{e4}| > 0$
  - If $|U_{e4}| > 0$ why not $|U_{\mu 4}| > 0$? $\Longrightarrow$ Maybe LSND luckily observed a fluctuation of a small $\bar{\nu}_\mu \to \bar{\nu}_e$ transition probability with amplitude $\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2$, which has not been seen by other appearance experiments
  - I have great hopes in near-future experiments!

Backup Slides

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m^2_{kj} L / 4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 \phi_{41} + 4|U_{e5}|^2 |U_{\mu 5}|^2 \sin^2 \phi_{51} + 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54} \overset{(+)}{-} \eta)$$

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2 \phi_{41} + |U_{\alpha 5}|^2 \sin^2 \phi_{51}) - 4|U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \sin^2 \phi_{54}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Conrad, Ignarra, Karagiorgi, Shaevitz, Spitz, AHEP 2013 (2013) 163897; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

- Good: CP violation
- Bad: 4 more parameters: $\underbrace{\Delta m^2_{41}, |U_{e4}|^2, |U_{\mu 4}|^2}_{3+1}, \Delta m^2_{51}, |U_{e5}|^2, |U_{\mu 5}|^2, \eta$