Neutrinos: Recent Results, New Standard Model Paradigm and Beyond

Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino
mailto://giunti@to.infn.it
Neutrino Unbound: http://www.nu.to.infn.it

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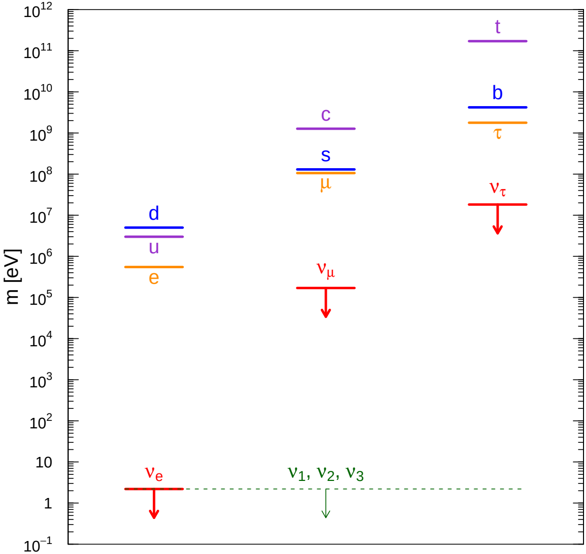
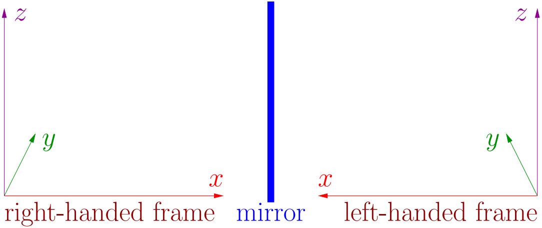
Fermion Mass Spectrum



Parity Violation

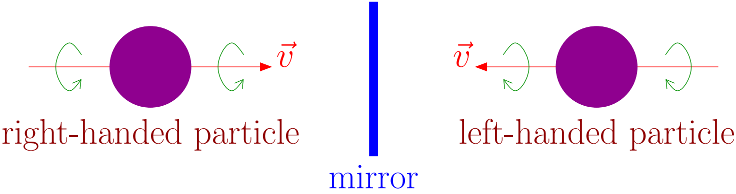
- Parity is the symmetry of space inversion (mirror transformation)



- Parity was considered to be an exact symmetry of nature
- 1956: Lee and Yang understand that Parity can be violated in Weak Interactions
- 1957: Wu et al. discover Parity violation in β-decay of polarized ^{60}Co
  Weak Interaction process: $^{60}\text{Co} \to {}^{60}\text{Ni} + e^{-} + \bar{\nu}_{e}$

Left-Handed Neutrinos

- 1957: Landau, Lee & Yang, Salam propose that neutrinos are massless and are only left-handed or right-handed



- 1958: Goldhaber, Grodzins and Sunyar measure neutrino helicity:

LEFT-HANDED

Standard Model of ElectroWeak Interactions

- Glashow (1961), Weinberg (1967) and Salam (1968)
- Left-Handed Neutrinos ν_L and Right-Handed Antineutrinos $\bar{\nu}_R$
- Parity is violated: $\nu_L \xrightarrow{P} \cancel{\nu_R}$ $\bar{\nu}_R \xrightarrow{P} \cancel{\bar{\nu}_L}$
- Particle-Antiparticle symmetry (Charge Conjugation) is violated:
  $$\nu_L \xrightarrow{C} \cancel{\bar{\nu}_L} \qquad \bar{\nu}_R \xrightarrow{C} \cancel{\nu_R}$$
- CP is conserved (one or two generations): $\nu_L \xleftrightarrow{CP} \bar{\nu}_R$
- 1964: Christenson, Cronin, Fitch and Turlay discover unexpected violation of CP in weak interactions of hadrons
- 1973: Kobayashi and Maskawa understand that CP violation requires existence of third generation: complex phase in mixing matrix

Standard Model: Three Generations

	1st Generation	2nd Generation	3rd Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} u_R \\ d_R \end{matrix} \begin{pmatrix} \bar{u}_R \\ \bar{d}_R \end{pmatrix} \begin{matrix} \bar{u}_L \\ \bar{d}_L \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{matrix} c_R \\ s_R \end{matrix} \begin{pmatrix} \bar{c}_R \\ \bar{s}_R \end{pmatrix} \begin{matrix} \bar{c}_L \\ \bar{s}_L \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \begin{matrix} t_R \\ b_R \end{matrix} \begin{pmatrix} \bar{t}_R \\ \bar{b}_R \end{pmatrix} \begin{matrix} \bar{t}_L \\ \bar{b}_L \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{matrix} \cancel{\nu_{eR}} \\ e_R \end{matrix} \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix} \begin{matrix} \cancel{\bar{\nu}_{eL}} \\ \bar{e}_L \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{matrix} \cancel{\nu_{\mu R}} \\ \mu_R \end{matrix} \begin{pmatrix} \bar{\nu}_{\mu R} \\ \bar{\mu}_R \end{pmatrix} \begin{matrix} \cancel{\bar{\nu}_{\mu L}} \\ \bar{\mu}_L \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \begin{matrix} \cancel{\nu_{\tau R}} \\ \tau_R \end{matrix} \begin{pmatrix} \bar{\nu}_{\tau R} \\ \bar{\tau}_R \end{pmatrix} \begin{matrix} \cancel{\bar{\nu}_{\tau L}} \\ \bar{\tau}_L \end{matrix}$

- No $\nu_R \implies$ No Dirac mass term $\mathcal{L}^{\mathrm{D}}_{\nu_e} \sim m^{\mathrm{D}} \nu_{eR} \nu_{eL}$
- Majorana Neutrino: $\nu = \bar{\nu} \implies \nu_R = \bar{\nu}_R$

  Majorana mass term: $\mathcal{L}^{\mathrm{M}}_{\nu_e} \sim m^{\mathrm{M}} \bar{\nu}_{eR} \nu_{eL} = m^{\mathrm{M}} \nu_{eR} \nu_{eL}$

  forbidden by Standard Model $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry!
- In Standard Model neutrinos are massless!
- Experimentally allowed until 1998, when the Super-Kamiokande atmospheric neutrino experiment obtained a model-independent proof of

  Neutrino Oscillations

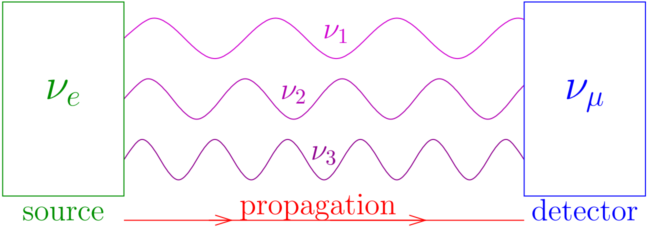
Neutrino Oscillations

- 1957: Bruno Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955)
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}$$

- U is the 3×3 unitary Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{e1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_e\rangle$$

$$E_k^2 = p^2 + m_k^2$$

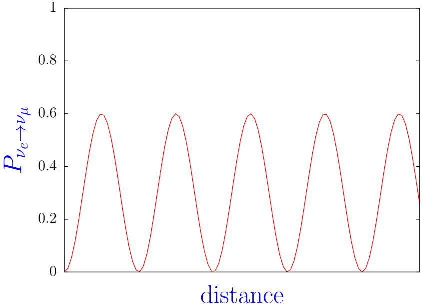
at the detector there is a probability > 0 to see the neutrino as a ν_μ

Neutrino Oscillations are Flavor Transitions

$$\begin{array}{cccc} \nu_e \to \nu_\mu & \nu_e \to \nu_\tau & \nu_\mu \to \nu_e & \nu_\mu \to \nu_\tau \\ \bar{\nu}_e \to \bar{\nu}_\mu & \bar{\nu}_e \to \bar{\nu}_\tau & \bar{\nu}_\mu \to \bar{\nu}_e & \bar{\nu}_\mu \to \bar{\nu}_\tau \end{array}$$

transition probabilities depend on U and $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$

- Neutrino Oscillations are due to interference of different phases of massive neutrinos
- Phases of massive neutrinos depend on distance $\Longrightarrow$ Oscillations depend on distance L
- Relativistic neutrinos: $\Delta E \simeq \Delta m^2/E$
- $P_{\nu_e \to \nu_\mu} = \sin^2 2\vartheta_{e\mu} \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$



- Oscillations measured without doubt for the first time by the Super-Kamiokande atmospheric neutrino experiment in 1998

Observations of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

SNO, BOREXino
Super-Kamiokande
GALLEX/GNO, SAGE
Homestake, Kamiokande

VLBL Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\rightarrow \begin{cases} \Delta m^2_{\text{S}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{S}} \simeq 0.30 \end{cases}$

Atmospheric
$\nu_\mu \to \nu_\tau$

Super-Kamiokande
Kamiokande, IMB
MACRO, Soudan-2

LBL Accelerator
ν_μ disappearance

(K2K, MINOS, T2K)

LBL Accelerator
$\nu_\mu \to \nu_\tau$

(Opera)

$\rightarrow \begin{cases} \Delta m^2_{\text{A}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{A}} \simeq 0.50 \end{cases}$

LBL Accelerator
$\nu_\mu \to \nu_e$

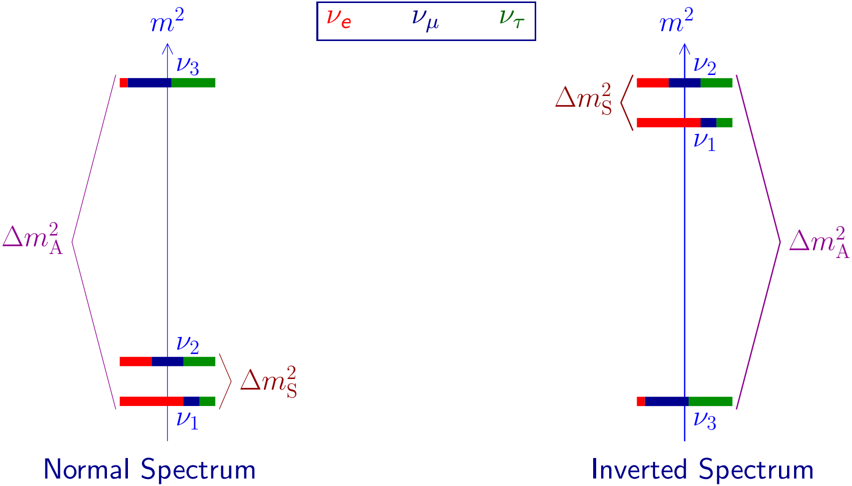
(T2K, MINOS)

LBL Reactor
$\bar{\nu}_e$ disappearance

Daya Bay, RENO
Double Chooz

$\rightarrow \begin{cases} \Delta m^2_{\text{A}} \\ \sin^2\vartheta_{13} \simeq 0.023 \end{cases}$

New SM Paradigm: Three-Neutrino Mixing



$\Delta m^2_{\rm S} = \Delta m^2_{21} = 7.50 \pm 0.20 \times 10^{-5}\,{\rm eV}^2$ uncertainty $\simeq 2.6\%$

$\Delta m^2_{\rm A} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| = 2.32^{+0.12}_{-0.08} \times 10^{-3}\,{\rm eV}^2$ uncertainty $\simeq 5\%$

New Standard Model with Massive Neutrinos

Standard Model can be extended with ν_R

Dirac neutrino mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \nu_R \nu_L \Rightarrow m^{\mathrm{D}} \lesssim 100\,\mathrm{GeV}$

surprise: Majorana neutrino mass for ν_R is allowed! $\mathcal{L}_R^{\mathrm{M}} \sim -\frac{1}{2} m_R^{\mathrm{M}} \bar{\nu}_R \nu_R$

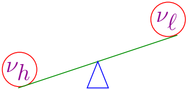
Four degrees of freedom: $\nu_L\,,\ \bar{\nu}_R\,,\ \nu_R\,,\ \bar{\nu}_L$

Total neutrino mass term: $\mathcal{L}^{\mathrm{D+M}} \sim \begin{pmatrix} \nu_L & \bar{\nu}_R \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \bar{\nu}_L \\ \nu_R \end{pmatrix}$

m_R^{M} can be arbitrarily large (not protected by SM symmetries)

$m_R^{\mathrm{M}} \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^{\mathrm{M}} \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \quad \Rightarrow \quad m_\ell \simeq \frac{(m^{\mathrm{D}})^2}{m_R^{\mathrm{M}}}, \qquad m_h \simeq m_R^{\mathrm{M}}$



see-saw mechanism

natural explanation of smallness
of light neutrino masses

massive neutrinos are Majorana!

3-GEN $\Rightarrow$ effective low-energy 3-ν mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k} \nu_k \qquad (\alpha = e, \mu, \tau)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} = \vartheta_A} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} = \vartheta_S} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$\vartheta_{23} = \vartheta_A$

$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$

Chooz, Palo Verde

T2K, MINOS

Daya Bay, RENO

$\sin^2\vartheta_{13} = 0.023 \pm 0.002$

$\vartheta_{12} = \vartheta_S$

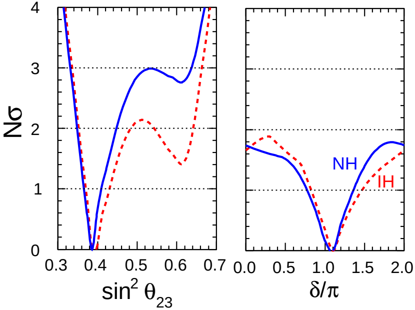
$\sin^2\vartheta_{12} = 0.30 \pm 0.01$

$\beta\beta_{0\nu}$

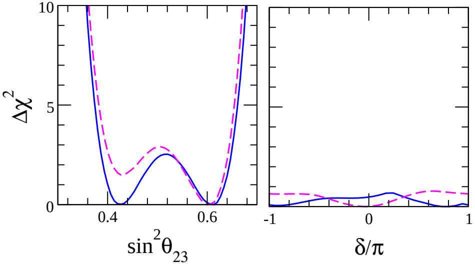
$$\frac{\delta \sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \simeq 40\% \qquad \frac{\delta \sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \simeq 10\% \qquad \frac{\delta \sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \simeq 5\%$$

$\delta_{13} \neq 0, \pi \quad \Longrightarrow$ CP violation in ν osc.

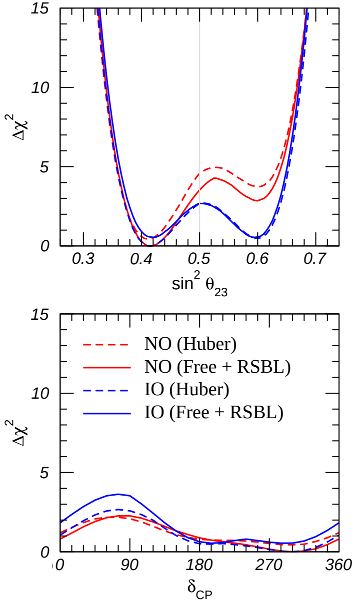
$$P_{\nu_\alpha\to\nu_\beta} \neq P_{\bar\nu_\alpha\to\bar\nu_\beta} \qquad (\alpha \neq \beta)$$



[Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, PRD 86 (2012) 013012]



[Forero, Tortola, Valle, PRD 86 (2012) 073012]

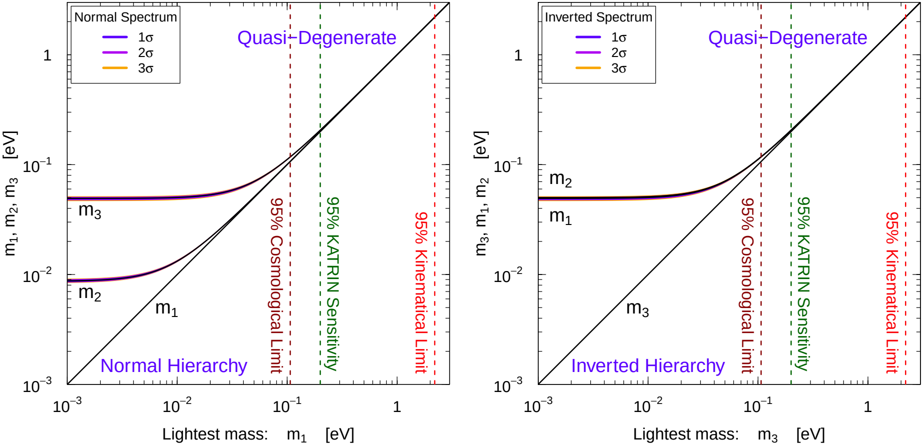


[Gonzalez-Garcia, Maltoni, Salvado, Schwetz, JHEP 12 (2012) 123; `http://www.nu-fit.org`]

Open Problems

- $\vartheta_{23} < 45^\circ$?
  - Atmospheric ν, T2K, NOνA, ...
- Mass Hierarchy ?
  - NOνA, Atmospheric ν, Day Bay II, RENO-50, Supernova ν, ...
- CP violation ?
  - NOνA, LAGUNA-LBNO, LBNE (USA), HyperK, ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Absolute Scale of Neutrino Masses



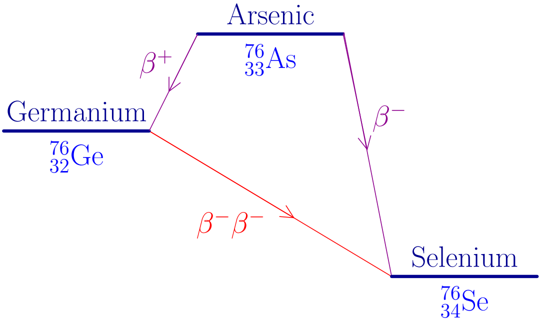
$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{S}}^2 \qquad m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{A}}^2$$

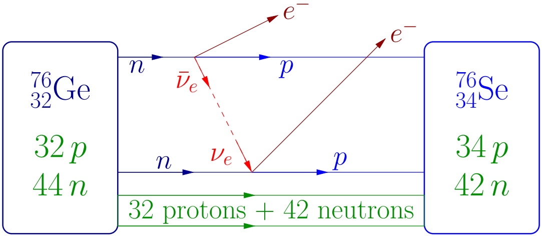
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{A}}^2 \qquad m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{A}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_{\mathrm{A}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

95% Cosmological Limit: Planck + WMAP9 + highL + BAO [arXiv:1303.5076]

Majorana ν: Neutrinoless Double-Beta Decay





$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,m^2_{\beta\beta}$$

Effective Majorana Mass

$$m_{\beta\beta} = \left|\sum_{k=1}^{3} U^2_{ek}\,m_k\right|$$

KamLAND-Zen + EXO

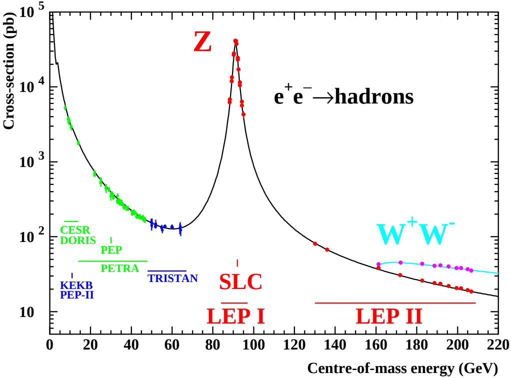
$$^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba} + e^- + e^-$$

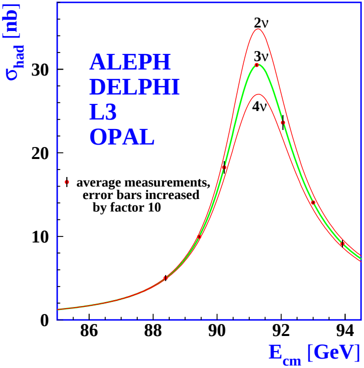
[PRL 109 (2012) 032505; arXiv:1211.3863]

$$T^{0\nu}_{1/2} > 1.9\times10^{25}\,\mathrm{y}\,(90\%\mathrm{C.L.})$$

$$|m_{\beta\beta}| \lesssim 0.12 - 0.25\,\mathrm{eV}$$

Three Active Neutrinos ⇔ Three Generations



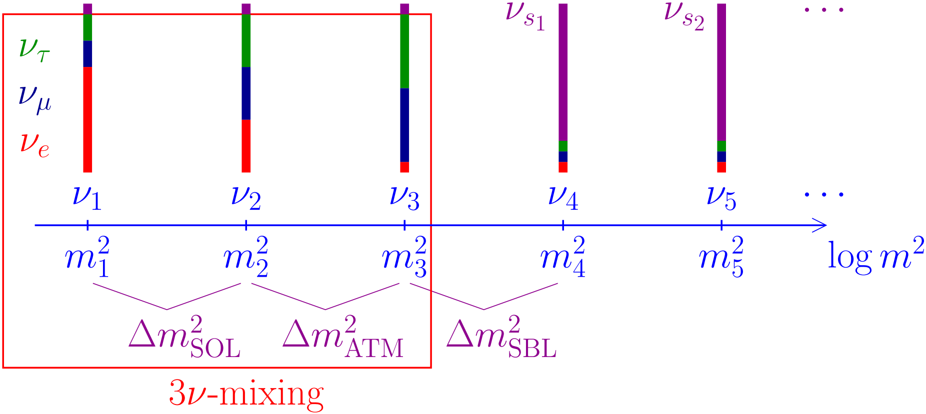


[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_{\nu_a}\,\Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_{\nu_a} = 2.9840 \pm 0.0082}$$

Beyond 3ν Mixing $\Longrightarrow$ Sterile Neutrinos



Light Sterile Neutrinos

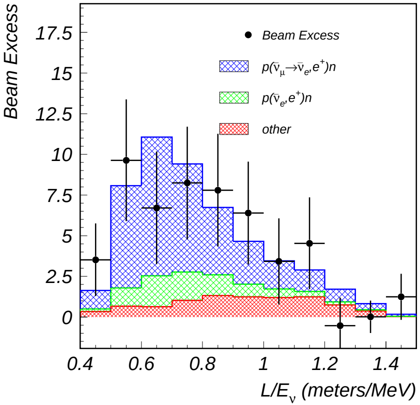
- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into light sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Short-baseline anomalies + 3ν-mixing:

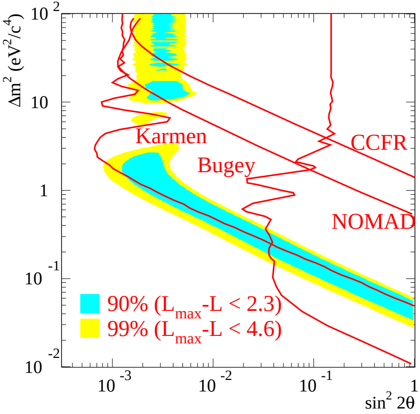
$$\begin{array}{ccccc} \multicolumn{5}{c}{\Delta m^2_{21} \ll |\Delta m^2_{31}| \ll |\Delta m^2_{41}| \le \ldots} \\ \nu_1 & \nu_2 & \nu_3 & \nu_4 & \ldots \\ \nu_e & \nu_\mu & \nu_\tau & \nu_{s_1} & \ldots \end{array}$$

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e \qquad L \simeq 30\,\text{m} \qquad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$



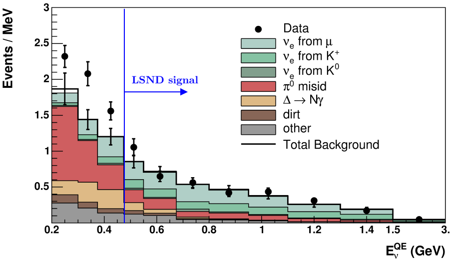


3.8σ excess $\qquad \Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2 \qquad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

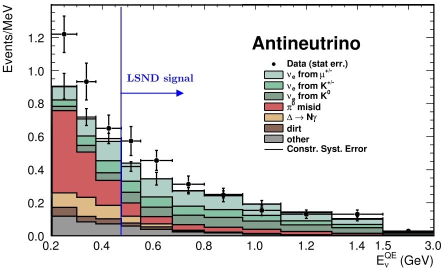
MiniBooNE

$L \simeq 541\,\mathrm{m} \qquad 200\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



- Agreement with LSND signal?
- CP violation?
- Low-energy anomaly!
  Neutrino energy reconstruction problem? [Martini, Ericson, Chanfray, PRD 85 (2012) 093012]

Reactor Electron Antineutrino Anomaly

[Mention, Fechner, Lasserre, Mueller,
Lhuillier, Cribier, Letourneau,
PRD 83 (2011) 073006]

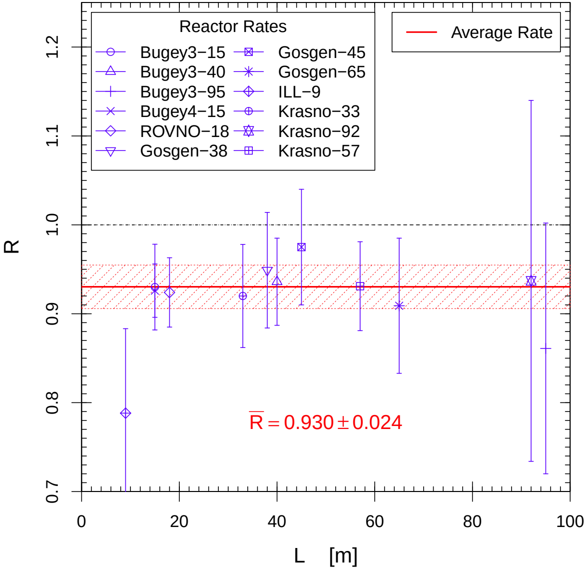
[update in White Paper, arXiv:1204.5379]

new reactor $\bar{\nu}_e$ fluxes

[Mueller, Lhuillier, Fallot, Letourneau,
Cormon, Fechner, Giot, Lasserre, Martino,
Mention, Porta, Yermia,
PRC 83 (2011) 054615]

[Huber, PRC 84 (2011) 024617]

2.8σ anomaly



Gallium Anomaly

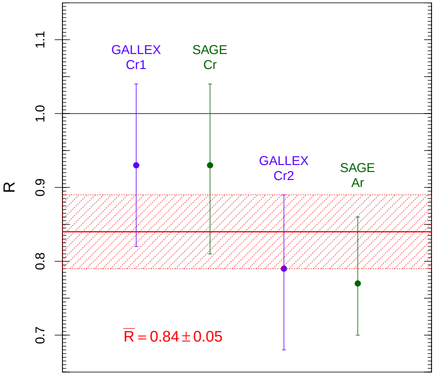
Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

Anomaly supported by new ${}^{71}\text{Ga}({}^{3}\text{He}, {}^{3}\text{H}){}^{71}\text{Ge}$ cross section measurement

[Frekers et al., PLB 706 (2011) 134]



$E \sim 0.7\,\text{MeV}$

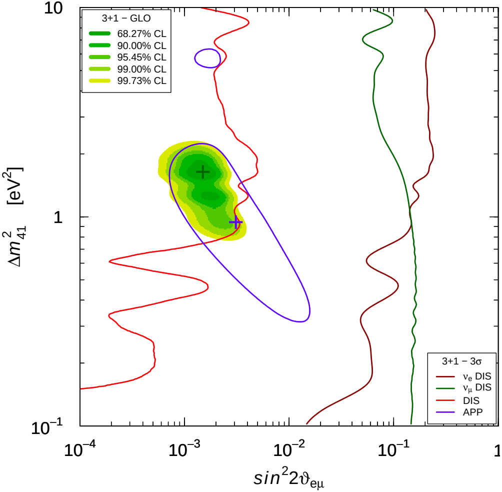
$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$

$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

2.9σ anomaly

3+1 Global Fit

[Giunti, Laveder, Y.F. Li, H.W. Long, in preparation (2013)]



- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND (Y), MiniBooNE (?), OPERA (N), ICARUS (N), KARMEN (N), NOMAD (N), BNL-E776 (N)
- DIS ν_e & $\bar{\nu}_e$: Reactors (Y), Gallium (Y), ν_eC (N), Solar (N)
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW (N), MINOS (N), Atmospheric (N), MiniBooNE/SciBooNE (N)

No Osc. GoF = 1%

3+1 GoF = 33%

PGoF = 10%

[see also Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

Conclusions

- Robust New Standard Model Paradigm: Three-Neutrino Mixing
  Experimental problems: $\vartheta_{23} < 45^\circ$?, CP Violation, Mass Hierarchy, Absolute Mass Scale, Dirac or Majorana?
  Theoretical problems: Why lepton mixing is so different from quark mixing? Is it due to Majorana nature of ν's?
  Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

- Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance:
  - Reactor $\bar{\nu}_e$ and Gallium ν_e anomalies
  - Many promising projects to test short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources
  - Independent tests through effect of m_4 in β-decay and $(\beta\beta)_{0\nu}$-decay

- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - MiniBooNE experiment has been inconclusive
  - If $|U_{e4}| > 0$ why not $|U_{\mu4}| > 0$? $\Longrightarrow$ Maybe LSND luckily observed a fluctuation of a small $\bar{\nu}_\mu \to \bar{\nu}_e$ transition probability with amplitude $\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2$, not seen by other appearance experiments
  - Better experiments are needed to check LSND signal

Backup Slides

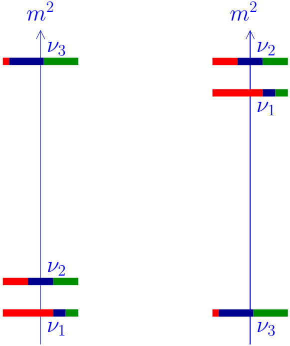
Mass Hierarchy

1. Matter Effect (Atmospheric, Long-Baseline, Supernova Experiments):
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} > 0$ NH
   - $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} < 0$ IH
2. Phase Difference (Reactor $\bar{\nu}_e \to \bar{\nu}_e$):

Normal Hierarchy

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$$

$$|\Delta m^2_{31}| > |\Delta m^2_{32}|$$



Inverted Hierarchy

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$$

$$|\Delta m^2_{31}| < |\Delta m^2_{32}|$$

CP Violation

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

Necessary conditions for observation of CP violation:

- Sensitivity to all mixing angles, including small ϑ_{13}
- Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

Tritium Beta-Decay

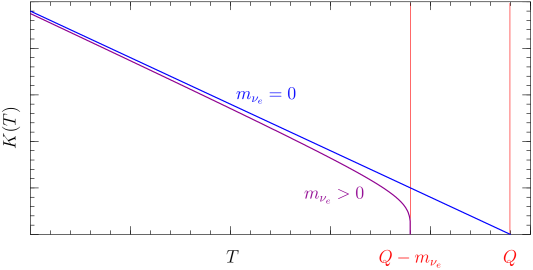
$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$$\boxed{m_{\nu_e} < 2.2\,\text{eV} \quad (95\%\ \text{C.L.})}$$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

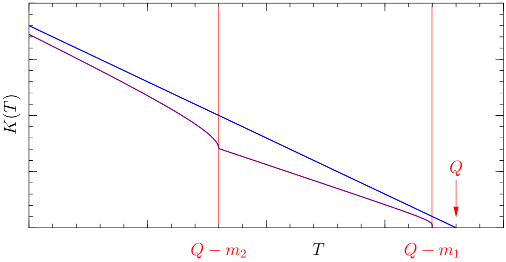
future: KATRIN

[`www.katrin.kit.edu`]

start data taking in 2015

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

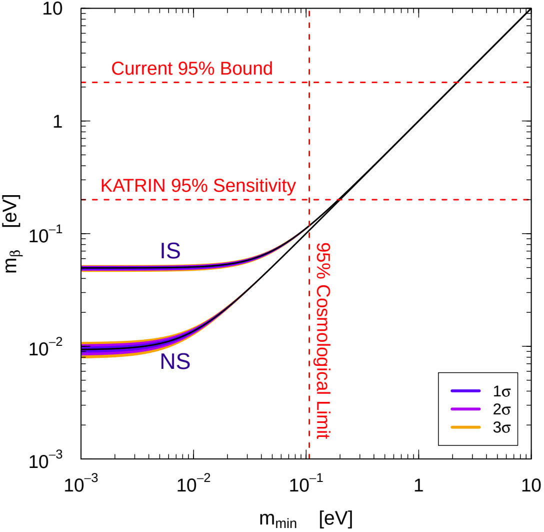
if experiment is not sensitive to masses $(m_k \ll Q - T)$

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$\begin{aligned} K^2 &= (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right] \\ &= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2} \end{aligned}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



- Quasi-Degenerate:

$$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- Inverted Hierarchy:

$$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_{\text{A}}^2 \simeq \Delta m_{\text{A}}^2$$

- Normal Hierarchy:

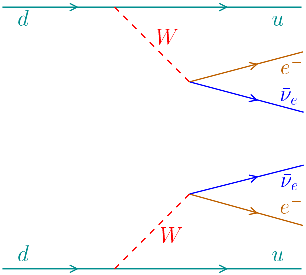
$$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_{\text{S}}^2 + s_{13}^2 \Delta m_{\text{A}}^2$$
$$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\text{eV}^2$$

- $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV}$

$$\Downarrow$$

Normal Spectrum

Two-Neutrino Double-β Decay: $\Delta L = 0$

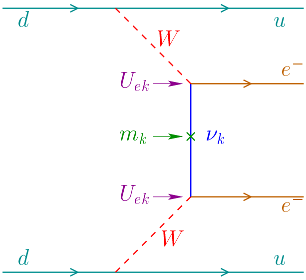


$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

Neutrinoless Double-β Decay: $\Delta L = 2$



$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass

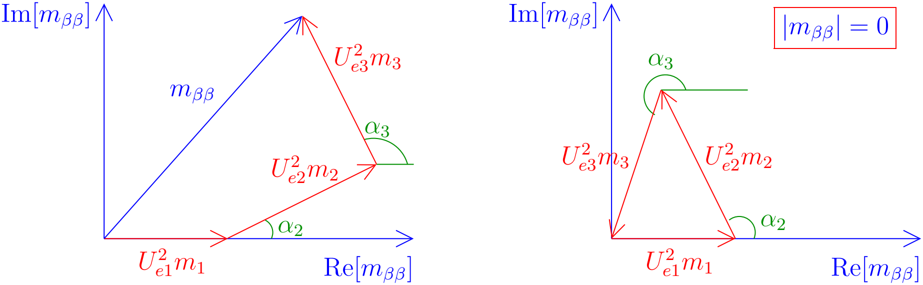
$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$$

Effective Majorana Neutrino Mass

$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$ complex U_{ek} ⇒ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$



Experimental Bounds

KamLAND-Zen (^{136}Xe) [arXiv:1211.3863]

$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y}\,(90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.12 - 0.25\,\text{eV}}$ (KLZ+EXO)

EXO (^{136}Xe) [PRL 109 (2012) 032505]

$\boxed{T^{0\nu}_{1/2} > 1.6 \times 10^{25}\,\text{y}\,(90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.14 - 0.38\,\text{eV}}$

CUORICINO (^{130}Te) [AP 34 (2011) 822]

$\boxed{T^{0\nu}_{1/2} > 2.8 \times 10^{24}\,\text{y}\,(90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.3 - 0.7\,\text{eV}}$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y}\,(90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y}\,(90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$\boxed{T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y}\,(90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$

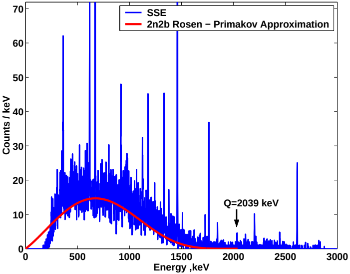
Experimental Positive Indication of $\beta\beta_{0\nu}$-Decay

[Klapdor et al., MPLA 16 (2001) 2409]

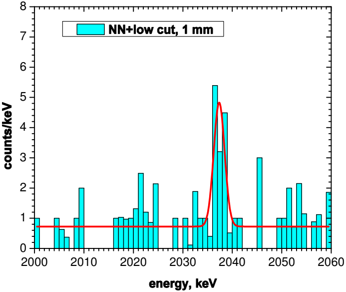
$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}$ y

6.5σ evidence

[MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



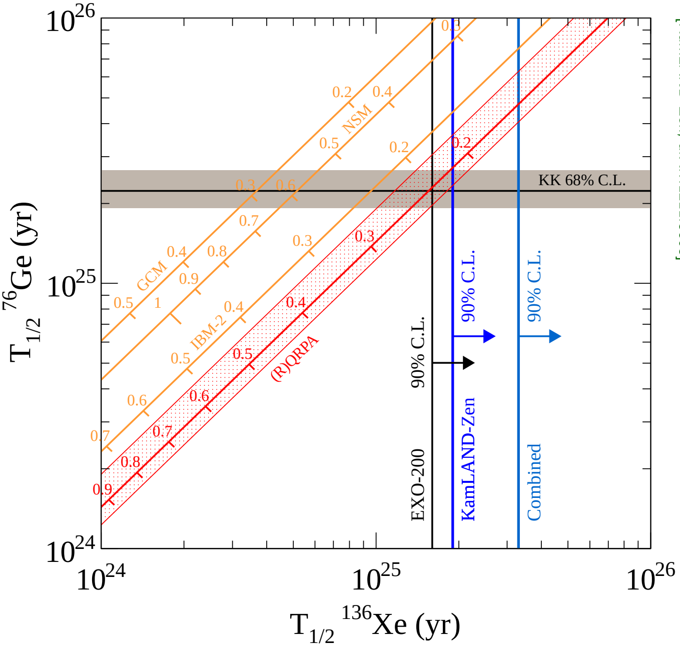
[MPLA 21 (2006) 1547]

$|m_{\beta\beta}| = 0.32 \pm 0.03\,\text{eV}$

[MPLA 21 (2006) 1547]

very exciting: Majorana ν and large mass scale

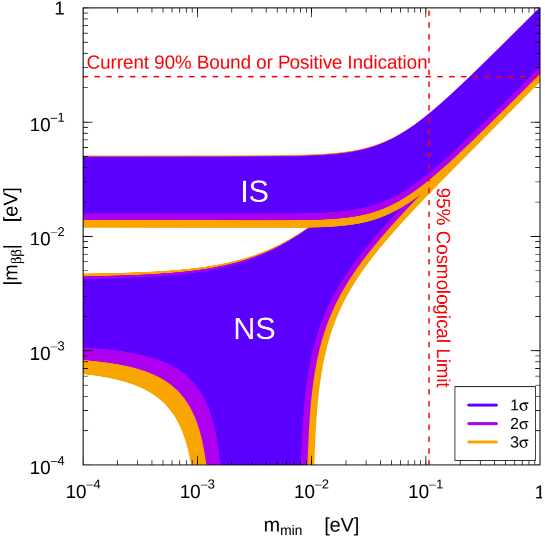
partially excluded by KamLAND-Zen, EXO and CUORICINO



[KamLAND-Zen, arXiv:1211.3863]

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$



- Positive indication: tension with cosmology
- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\text{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3} \, \text{eV}$$

$m_1 \gtrsim 10^{-3}$ eV ⇒ cancellation?

$|m_{\beta\beta}| \lesssim 10^{-2}$ eV $\Longrightarrow$ Normal Spectrum

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

$$|U_{e4}|^2 \ll 1\,, \quad |U_{\mu 4}|^2 \ll 1\,, \quad |U_{\tau 4}|^2 \ll 1\,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{array}{ccc|c|} \cline{4-4} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \\ \cline{4-4} \end{array}\right)$$

$\uparrow$

SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

Cosmology

- Relativistic energy density before photon decoupling:

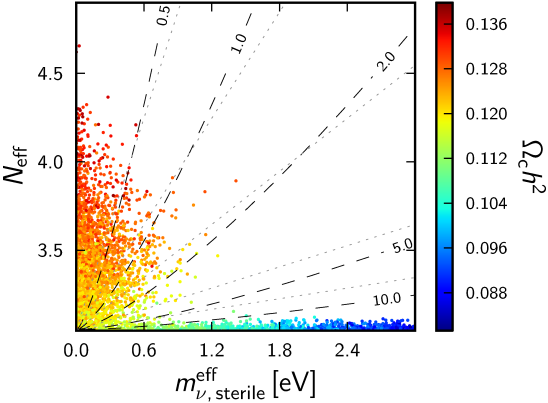
$$\rho_R = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma$$

- $N_{\text{eff}} =$ effective neutrino number
- $N_{\text{eff}} = 3.046 + N_s$
- $N_s =$ effective number of sterile neutrinos (not necessarily integer)

Planck

[arXiv:1303.5076]

$N_{\text{eff}} < 3.80 \qquad m_{\nu,\text{sterile}}^{\text{eff}} < 0.42 \qquad (95\%;\,\text{CMB}+\text{BAO})$



- $m_{\nu,\text{sterile}}^{\text{eff}} \equiv 94.1\omega_{\nu_4}$ eV
- Thermally distributed:

$$f_s(E) = \frac{1}{e^{E/T_s}+1}$$

$$m_{\nu,\text{sterile}}^{\text{eff}} = \left(\frac{T_s}{T_\nu}\right)^3 m_4 = (\Delta N_{\text{eff}})^{3/4} m_4$$

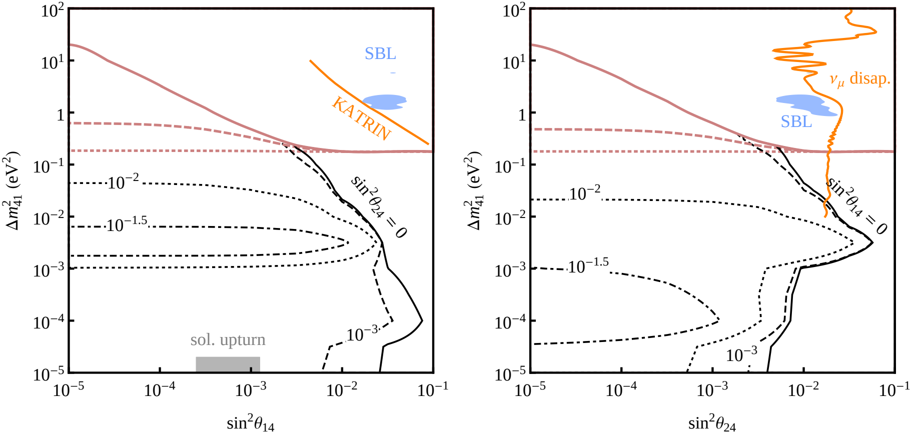
- Dodelson-Widrow:

$$f_s(E) = \frac{\chi}{e^{E/T_\nu}+1}$$

$$m_{\nu,\text{sterile}}^{\text{eff}} = \chi_s m_4$$

Standard Cosmological Scenario Mixing Bounds

[Mirizzi, Mangano, Saviano, Borriello, Giunti, Miele, Pisanti, arXiv:1303.5368]



Non-standard mechanism for partial thermalization of ν_s is needed

Large primordial neutrino asymmetry?

[Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009;

Saviano, Mirizzi, Pisanti, Serpico, Mangano, Miele, PRD 87 (2013) 073006]

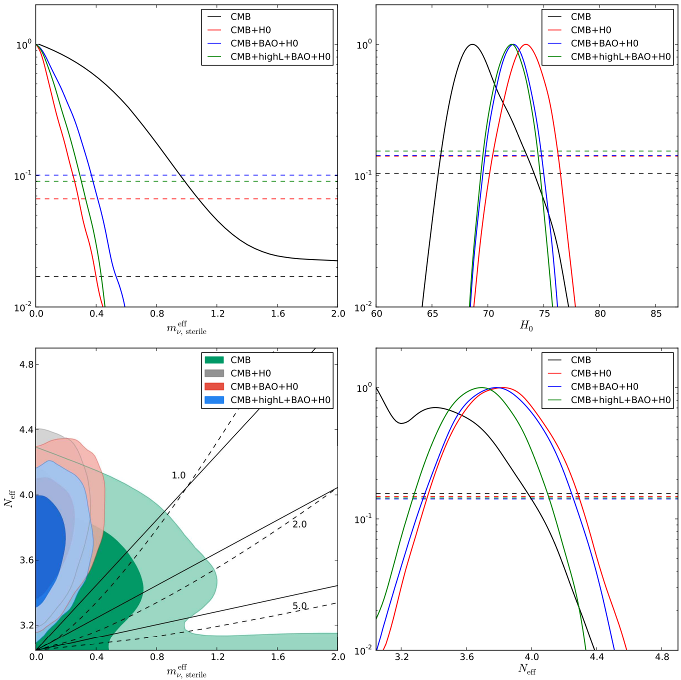
CMB + H_0

[Gariazzo, Giunti, Laveder, in preparation (2013)]

$$H_0 = \left\{ \begin{array}{ll} 67.4 \pm 1.4 & \text{Planck} \\ 70.0 \pm 2.2 & \text{WMAP-9} \\ 73.8 \pm 2.4 & \text{Cepheids+SNIa} \\ 74.3 \pm 2.6 & \text{Carnegie HP} \\ 78.7 \pm 4.5 & \text{COSMOGRAIL} \end{array} \right\} [\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}]$$

Gaussian Prior: $H_0 = 74.7 \pm 1.6\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$

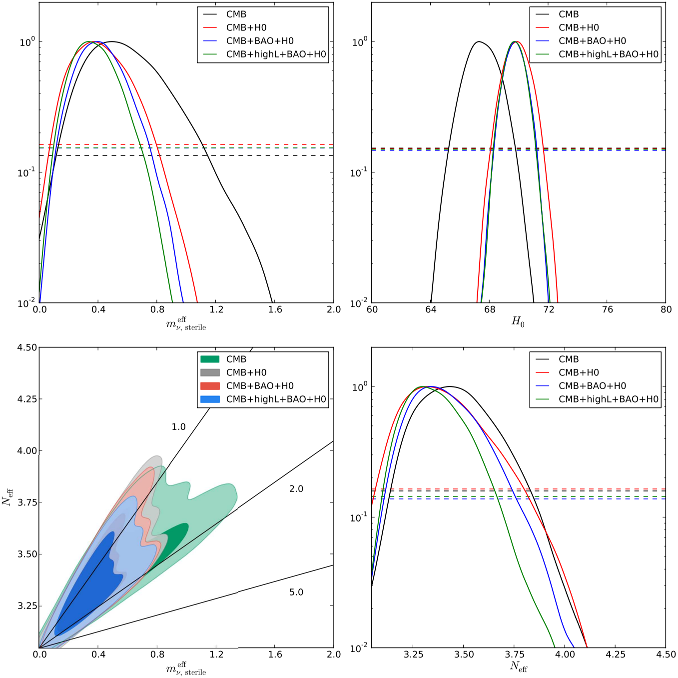
weighted average of Cepheids+SNIa, Carnegie HP, COSMOGRAIL



$3.16 < N_{\rm eff} < 4.24$ (99%)

$m_{\nu,\rm sterile}^{\rm eff} < 0.41\,\mathrm{eV}$ (99%)

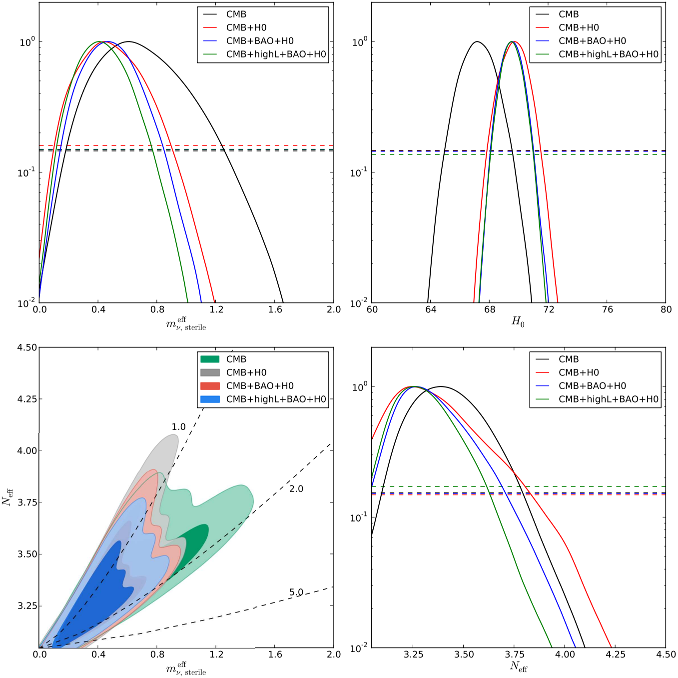
SBL Prior - Dodelson-Widrow



$N_{\text{eff}} < 3.80$ (99%)

$0.042 < m_{\nu,\text{sterile}}^{\text{eff}} < 0.81\,\text{eV}$ (99%)

SBL Prior - Thermal



$N_{\text{eff}} < 3.79$ (99%)

$0.049 < m_{\nu,\text{sterile}}^{\text{eff}} < 0.90\,\text{eV}$ (99%)