Neutrino Theory and Phenomenology

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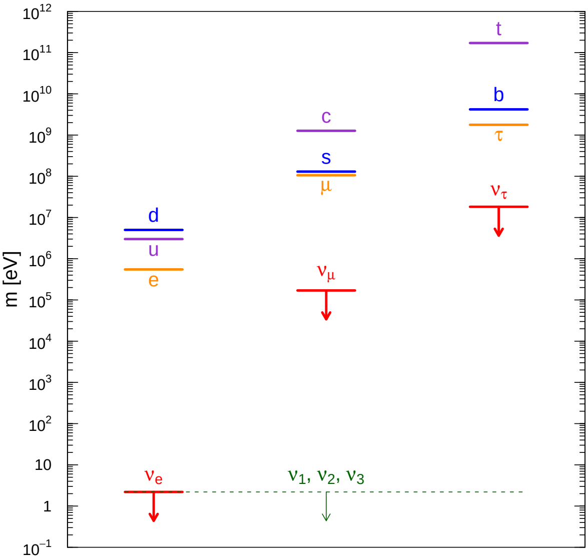
Neutrino Theory and Phenomenology

- Neutrino Masses and Mixing
- Neutrino Oscillations in Vacuum
- Neutrino Oscillations in Matter
- Neutrino Mixing Phenomenology

Neutrino Masses and Mixing

Fermion Mass Spectrum



Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0 \qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$
- Dirac Lagrangian: $\mathscr{L}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$
- Chiral decomposition: $\nu_L \equiv P_L \nu\,, \qquad \nu_R \equiv P_R \nu\,, \qquad \nu = \nu_L + \nu_R$

  Left and Right-handed Projectors: $\quad P_L \equiv \dfrac{1-\gamma^5}{2}\,, \quad P_R \equiv \dfrac{1+\gamma^5}{2}$

  $$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L + P_R = 1\,, \quad P_L P_R = P_R P_L = 0$$

  $$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

- In SM only ν_L by assumption $\Longrightarrow$ no neutrino mass
  Note that all the other elementary fermion fields (charged leptons and quarks) have both left and right-handed components
- Oscillation experiments have shown that neutrinos are massive
- Simplest and natural extension of the SM: consider also ν_R
  as for all the other elementary fermion fields

Higgs Mechanism in SM

- Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ $\qquad |\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$

- Higgs Lagrangian: $\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$

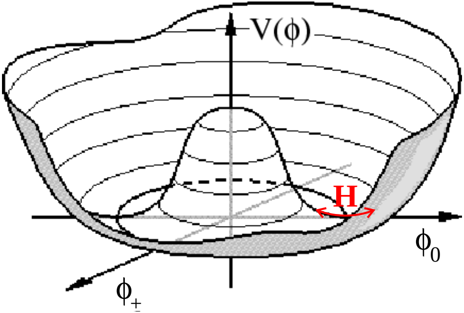
- Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$

- $\mu^2 < 0$ and $\lambda > 0 \implies V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$

$$v \equiv \sqrt{-\frac{\mu^2}{\lambda}} = \left(\sqrt{2} G_{\text{F}}\right) \simeq 246\,\text{GeV}$$

- Vacuum: V_{min} for $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

- Spontaneous Symmetry Breaking: $\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_Q$



- Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \Longrightarrow |\Phi|^2 = \frac{v^2}{2} + vH + \frac{1}{2} H^2$

- $V = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \simeq 126\,\mathrm{GeV}$$

$$-\mu^2 \simeq (89\,\mathrm{GeV})^2 \qquad \lambda = -\frac{\mu^2}{v^2} \simeq 0.13$$

SM Extension: Dirac ν Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -y^{\ell}\,\overline{L_L}\,\Phi\,\ell_R - y^{\nu}\,\overline{L_L}\,\widetilde{\Phi}\,\nu_R + \mathrm{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2\,\Phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} = & -\frac{y^{\ell}}{\sqrt{2}}\begin{pmatrix}\overline{\nu_L} & \overline{\ell_L}\end{pmatrix}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}\ell_R \\ & -\frac{y^{\nu}}{\sqrt{2}}\begin{pmatrix}\overline{\nu_L} & \overline{\ell_L}\end{pmatrix}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}\nu_R + \mathrm{H.c.} \end{aligned}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\, y^{\ell}\, \frac{v}{\sqrt{2}}\, \overline{\ell_L}\, \ell_R - y^{\nu}\, \frac{v}{\sqrt{2}}\, \overline{\nu_L}\, \nu_R$$

$$- \frac{y^{\ell}}{\sqrt{2}}\, \overline{\ell_L}\, \ell_R\, H - \frac{y^{\nu}}{\sqrt{2}}\, \overline{\nu_L}\, \nu_R\, H + \mathrm{H.c.}$$

$$m_{\ell} = y^{\ell}\, \frac{v}{\sqrt{2}} \qquad\qquad m_{\nu} = y^{\nu}\, \frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_{\ell}}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_{\nu}}{v}$$

$$v = \left(\sqrt{2} G_{\mathrm{F}}\right)^{1/2} = 246\,\mathrm{GeV}$$

PROBLEM: $\quad y^{\nu} \lesssim 10^{-11} \lll y^{e} \sim 10^{-6}$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}^{\mathrm{D}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\mathscr{L}^{\mathrm{D}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[\frac{v}{\sqrt{2}} Y'^{\ell}_{\alpha\beta} \, \overline{\ell'_{\alpha L}} \, \ell'_{\beta R} + \frac{v}{\sqrt{2}} Y'^{\nu}_{\alpha\beta} \, \overline{\nu'_{\alpha L}} \, \nu'_{\beta R} \right] + \text{H.c.}$$

$$\mathscr{L}^{\mathrm{D}} = - \left[\overline{\boldsymbol{\ell}'_L} \, M'^{\ell} \, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L} \, M'^{\nu} \, \boldsymbol{\nu}'_R \right] + \text{H.c.}$$

$$\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \boldsymbol{\ell}'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$M'^{\ell} = \frac{v}{\sqrt{2}} \, Y'^{\ell} \qquad\qquad M'^{\nu} = \frac{v}{\sqrt{2}} \, Y'^{\nu}$$

$$M'^{\ell} \equiv \begin{pmatrix} M'^{\ell}_{ee} & M'^{\ell}_{e\mu} & M'^{\ell}_{e\tau} \\ M'^{\ell}_{\mu e} & M'^{\ell}_{\mu\mu} & M'^{\ell}_{\mu\tau} \\ M'^{\ell}_{\tau e} & M'^{\ell}_{\tau\mu} & M'^{\ell}_{\tau\tau} \end{pmatrix} \qquad\qquad M'^{\nu} \equiv \begin{pmatrix} M'^{\nu}_{ee} & M'^{\nu}_{e\mu} & M'^{\nu}_{e\tau} \\ M'^{\nu}_{\mu e} & M'^{\nu}_{\mu\mu} & M'^{\nu}_{\mu\tau} \\ M'^{\nu}_{\tau e} & M'^{\nu}_{\tau\mu} & M'^{\nu}_{\tau\tau} \end{pmatrix}$$

$$\mathscr{L}^{\mathrm{D}} = -\overline{\boldsymbol{\ell}'_L}\, M'^{\ell}\, \boldsymbol{\ell}'_R - \overline{\boldsymbol{\nu}'_L}\, M'^{\nu}\, \boldsymbol{\nu}'_R + \text{H.c.}$$

Diagonalization of M'^{ℓ} and M'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \mathbf{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \mathbf{n}_R$$

Important general remark: unitary transformations are allowed because they leave invariant the kinetic terms in the Lagrangian

$$\begin{aligned}
\mathscr{L}_{\text{kin}} &= \overline{\boldsymbol{\ell}'_L} i\partial\!\!\!/ \boldsymbol{\ell}'_L + \overline{\boldsymbol{\ell}'_R} i\partial\!\!\!/ \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L} i\partial\!\!\!/ \boldsymbol{\nu}'_L + \overline{\boldsymbol{\nu}'_R} i\partial\!\!\!/ \boldsymbol{\nu}'_R \\
&= \overline{\boldsymbol{\ell}_L} V^{\ell\dagger}_L i\partial\!\!\!/ V^{\ell}_L \boldsymbol{\ell}_L + \ldots \\
&= \overline{\boldsymbol{\ell}_L} i\partial\!\!\!/ \boldsymbol{\ell}_L + \overline{\boldsymbol{\ell}_R} i\partial\!\!\!/ \boldsymbol{\ell}_R + \overline{\boldsymbol{\nu}_L} i\partial\!\!\!/ \boldsymbol{\nu}_L + \overline{\boldsymbol{\nu}_R} i\partial\!\!\!/ \boldsymbol{\nu}_R
\end{aligned}$$

$$\mathscr{L}^{\mathrm{D}} = -\overline{\boldsymbol{\ell}'_L}\, M'^{\ell}\, \boldsymbol{\ell}'_R - \overline{\boldsymbol{\nu}'_L}\, M'^{\nu}\, \boldsymbol{\nu}'_R + \mathrm{H.c.}$$

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \mathbf{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \mathbf{n}_R$$

$$\mathscr{L}^{\mathrm{D}} = -\overline{\boldsymbol{\ell}_L}\, V^{\ell\dagger}_L M'^{\ell} V^{\ell}_R \boldsymbol{\ell}_R - \overline{\boldsymbol{\nu}_L}\, V^{\nu\dagger}_L M'^{\nu} V^{\nu}_R \boldsymbol{\nu}_R + \mathrm{H.c.}$$

$$V^{\ell\dagger}_L\ M'^{\ell}\ V^{\ell}_R = M^{\ell} \qquad M^{\ell}_{\alpha\beta} = m^{\ell}_{\alpha}\,\delta_{\alpha\beta} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$V^{\nu\dagger}_L\ M'^{\nu}\ V^{\nu}_R = M^{\nu} \qquad M^{\nu}_{kj} = m^{\nu}_{k}\,\delta_{kj} \qquad (k, j = 1, 2, 3)$$

Real and Positive m^{ℓ}_{α}, m^{ν}_{k}

Massive Chiral Lepton Fields

$\boldsymbol{\ell}_L = V_L^{\ell\dagger}\,\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\boldsymbol{\ell}_R = V_R^{\ell\dagger}\,\boldsymbol{\ell}'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger}\,\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger}\,\boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}^{\mathrm{D}} &= -\,\overline{\boldsymbol{\ell}_L}\,M^{\ell}\,\boldsymbol{\ell}_R - \overline{\mathbf{n}_L}\,M^{\nu}\,n_R + \text{H.c.} \\ &= -\sum_{\alpha=e,\mu,\tau} m_\alpha^{\ell}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} - \sum_{k=1}^{3} m_k^{\nu}\,\overline{\nu_{kL}}\,\nu_{kR} + \text{H.c.} \end{aligned}$$

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{CC})} = -\frac{g}{2\sqrt{2}}\, j_W^{\rho}\, W_{\rho} + \mathrm{H.c.}$$

Weak Charged Current: $\quad j_W^{\rho} = j_{W,\mathrm{L}}^{\rho} + j_{W,\mathrm{Q}}^{\rho}$

Leptonic Weak Charged Current

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\ell'_{\alpha L}}\, \gamma^{\rho}\, \nu'_{\alpha L} = 2\, \overline{\boldsymbol{\ell}'_L}\, \gamma^{\rho}\, \boldsymbol{\nu}'_L$$

$$\underline{\boldsymbol{\ell}'_L = V_L^{\ell}\, \boldsymbol{\ell}_L} \qquad \underline{\boldsymbol{\nu}'_L = V_L^{\nu}\, \mathbf{n}_L}$$

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2\, \overline{\boldsymbol{\ell}_L}\, V_L^{\ell\dagger}\, \gamma^{\rho}\, V_L^{\nu}\, \mathbf{n}_L = 2\, \overline{\boldsymbol{\ell}_L}\, \gamma^{\rho}\, V_L^{\ell\dagger}\, V_L^{\nu}\, \mathbf{n}_L = 2\, \overline{\boldsymbol{\ell}_L}\, \gamma^{\rho}\, U\, \mathbf{n}_L$$

Mixing Matrix

$$\boxed{U = V_L^{\ell\dagger}\, V_L^{\nu}}$$

- Definition: Left-Handed Flavor Neutrino Fields

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,V_L^{\nu}\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\end{pmatrix}$$

- They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,\boldsymbol{\nu}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\ell_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L}$$

- Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton:

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2\left(\overline{e_L}\,\gamma^\rho\,\nu_{eL} + \overline{\mu_L}\,\gamma^\rho\,\nu_{\mu L} + \overline{\tau_L}\,\gamma^\rho\,\nu_{\tau L}\right)$$

- In practice left-handed flavor neutrino fields are useful for calculations in the SM approximation of massless neutrinos (interactions).

- If neutrino masses must be taken into account, it is necessary to use

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,U\,\mathbf{n}_L = 2\sum_{\alpha=e,\mu,\tau}\sum_{k=1}^{3}\overline{\ell_{\alpha L}}\,\gamma^\rho\,U_{\alpha k}\,\nu_{kL}$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Standard Model: Lepton numbers are conserved

- L_e, L_μ, L_τ are conserved in the Standard Model with massless neutrinos

- Mass term:

$$\mathscr{L}^{\mathrm{D}} = -\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \mathrm{H.c.}$$

L_e, L_μ, L_τ are not conserved

- L is conserved: $\quad L(\nu_{\alpha R}) = L(\nu_{\beta L}) \implies |\Delta L| = 0$

Mixing Matrix

- $U = V_L^{\ell\dagger} \, V_L^{\nu} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

- Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \qquad \Longrightarrow \qquad \begin{array}{ll} \dfrac{N(N-1)}{2} = 3 & \quad \text{Mixing Angles} \\ \dfrac{N(N+1)}{2} = 6 & \quad \text{Phases} \end{array}$$

- Not all phases are physical observables

- Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)
$$\ell_{\alpha} \to e^{i\varphi_{\alpha}} \, \ell_{\alpha} \quad (\alpha = e, \mu, \tau) \, , \qquad \nu_k \to e^{i\varphi_k} \, \nu_k \quad (k = 1, 2, 3)$$
- Performing this transformation, the Charged Current becomes

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, e^{-i\varphi_{\alpha}} \, \gamma^{\rho} \, U_{\alpha k} \, e^{i\varphi_k} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \, \underbrace{e^{-i(\varphi_e - \varphi_1)}}_{1} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_{\alpha} - \varphi_e)}}_{2} \, \gamma^{\rho} \, U_{\alpha k} \, \underbrace{e^{i(\varphi_k - \varphi_1)}}_{2} \, \nu_{kL}$$

- There are 5 independent combinations of the phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad$ $s_{ab} \equiv \sin\vartheta_{ab}$ $\quad$ $0 \le \vartheta_{ab} \le \frac{\pi}{2}$ $\quad$ $0 \le \delta_{13} < 2\pi$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} e^{i\delta_{23}} \\ 0 & -s_{23} e^{-i\delta_{23}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12} e^{-i\delta'_{12}} & 0 \\ -s'_{12} e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

Jarlskog Rephasing Invariant

- Simplest rephasing invariants: $|U_{\alpha k}|^2 = U_{\alpha k} U_{\alpha k}^*$, $U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$

$$\Im\mathfrak{m}\left[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}\right] = \pm J$$

$$J = \Im\mathfrak{m}\left[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}\right] = \Im\mathfrak{m} \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- In standard parameterization:

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos\vartheta_{13} \sin 2\vartheta_{13} \sin\delta_{13} \end{aligned}$$

- Jarlskog invariant is useful for quantifying CP violation due to $U \neq U^*$ in a parameterization-independent way.
- All measurable CP-violation effects depend on J.

CP Violation

- $U \neq U^* \quad \Longrightarrow \quad$ CP Violation

- General conditions for CP violation (14 conditions):
  1. No two charged leptons or two neutrinos are degenerate in mass (6 conditions)
  2. No mixing angle is equal to 0 or $\pi/2$ (6 conditions)
  3. The physical phase is different from 0 or π (2 conditions)

- These 14 conditions are combined into the single condition $\det C \neq 0$

$$C = -i \left[M'^{\nu} \, M'^{\nu\dagger} \, , \, M'^{\ell} \, M'^{\ell\dagger} \right]$$

$$\det C = -2 \, J \left(m_{\nu_2}^2 - m_{\nu_1}^2 \right) \left(m_{\nu_3}^2 - m_{\nu_1}^2 \right) \left(m_{\nu_3}^2 - m_{\nu_2}^2 \right) \left(m_{\mu}^2 - m_{e}^2 \right) \left(m_{\tau}^2 - m_{e}^2 \right) \left(m_{\tau}^2 - m_{\mu}^2 \right)$$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunietz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

Maximal CP Violation

- Maximal CP violation is defined as the case in which $|J|$ has its maximum possible value

$$|J|_{\text{max}} = \frac{1}{6\sqrt{3}}$$

- In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4\,, \quad s_{13} = 1/\sqrt{3}\,, \quad \sin\delta_{13} = \pm 1$$

- This case is called Trimaximal Mixing. All the absolute values of the elements of the mixing matrix are equal to $1/\sqrt{3}$:

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp\frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- The unitarity of V_L^ℓ, V_R^ℓ and V_L^ν implies that the expression of the neutral weak current in terms of the lepton fields with definite masses is the same as that in terms of the primed lepton fields:

$$\begin{aligned} j^\rho_{Z,\mathrm{L}} =& 2\, g_L^\nu\, \overline{\boldsymbol{\nu}'_L}\, \gamma^\rho\, \boldsymbol{\nu}'_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}'_L}\, \gamma^\rho \boldsymbol{\ell}'_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}'_R}\, \gamma^\rho \boldsymbol{\ell}'_R \\ =& 2\, g_L^\nu\, \overline{\mathbf{n}_L}\, V_L^{\nu\dagger}\, \gamma^\rho\, V_L^\nu\, \mathbf{n}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, V_L^{\ell\dagger}\, \gamma^\rho\, V_L^\ell\, \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, V_R^{\ell\dagger}\, \gamma^\rho\, V_R^\ell\, \boldsymbol{\ell}_R \\ =& 2\, g_L^\nu\, \overline{\mathbf{n}_L}\, \gamma^\rho\, \mathbf{n}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \end{aligned}$$

- The unitarity of U implies the same expression for the neutral weak current in terms of the flavor neutrino fields $\boldsymbol{\nu}_L = U\, \mathbf{n}_L$:

$$\begin{aligned} j^\rho_{Z,\mathrm{L}} &= 2\, g_L^\nu\, \overline{\boldsymbol{\nu}_L}\, U\, \gamma^\rho\, U^\dagger\, \boldsymbol{\nu}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \\ &= 2\, g_L^\nu\, \overline{\boldsymbol{\nu}_L}\, \gamma^\rho\, \boldsymbol{\nu}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \end{aligned}$$

Lepton Numbers Violating Processes

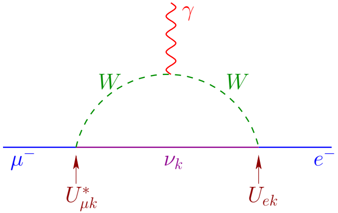
Dirac mass term allows L_e, L_μ, L_τ violating processes

Example: $\mu^\pm \to e^\pm + \gamma \,, \qquad \mu^\pm \to e^\pm + e^+ + e^-$

$$\boxed{\mu^- \to e^- + \gamma}$$

$$\sum_k U_{\mu k}^* U_{ek} = 0 \Longrightarrow \text{only part of } \nu_k \text{ propagator} \propto m_k \text{ contributes}$$

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi} \left| \sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$



Suppression factor: $\dfrac{m_k}{m_W} \lesssim 10^{-11}$ for $m_k \lesssim 1\,\text{eV}$

$(\text{BR})_{\text{the}} \lesssim 10^{-47} \qquad (\text{BR})_{\text{exp}} \lesssim 10^{-11}$

Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- Dirac Equation: $(i\gamma^{\mu}\partial_{\mu} - m)\,\psi = 0$
- Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$
\begin{aligned}
i\gamma^{\mu}\partial_{\mu}\psi_L &= m\,\psi_R \\
i\gamma^{\mu}\partial_{\mu}\psi_R &= m\,\psi_L
\end{aligned}
$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$
\begin{aligned}
i\gamma^{\mu}\partial_{\mu}\psi_L &= 0 \\
i\gamma^{\mu}\partial_{\mu}\psi_R &= 0
\end{aligned}
$$

- A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor).

- ψ_L and ψ_R have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \qquad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to parity violation ($\psi_L \overset{P}{\rightleftharpoons} \psi_R$)

- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields $\Longrightarrow$ Two-component Theory of a Massless Neutrino (1957)

- $V - A$ Charged-Current Weak Interactions $\Longrightarrow \nu_L$

- In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of ν_R

Majorana Equation

- Can a two-component spinor describe a massive fermion?

Yes! (E. Majorana, 1937)

- Trick: ψ_R and ψ_L are not independent: $\boxed{\psi_R = \psi_L^c = \mathcal{C}\,\overline{\psi_L}^T}$

charge-conjugation matrix: $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$

- ψ_L^c is right-handed: $P_R \psi_L^c = \psi_L^c \qquad P_L \psi_L^c = 0$

- $i\gamma^\mu \partial_\mu \psi_L = m\,\psi_R \quad \to \quad \boxed{i\gamma^\mu \partial_\mu \psi_L = m\,\psi_L^c}$ Majorana Equation

- Majorana Field: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^c$

$\boxed{\psi = \psi^c}$ Majorana Condition

- $\psi = \psi^c$ implies the equality of particle and antiparticle

- Only neutral fermions can be Majorana particles

- For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi^c}\gamma^\mu\psi^c = -\psi^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\overline{\psi}^T = \overline{\psi}\mathcal{C}\gamma^{\mu T}\mathcal{C}^\dagger\psi = -\overline{\psi}\gamma^\mu\psi = 0$$

- Only two independent components: $\psi = \begin{pmatrix} i\sigma^2\chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

Majorana Lagrangian

Dirac Lagrangian

$$
\begin{aligned}
\mathscr{L}^{\mathrm{D}} &= \overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu \\
&= \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R)
\end{aligned}
$$

$$
\nu_R \quad \to \quad \nu_L^c = \mathcal{C}\,\overline{\nu_L}^T
$$

$$
\frac{1}{2}\,\mathscr{L}^{\mathrm{D}} \quad \to \quad \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right)
$$

Majorana Lagrangian

$$
\begin{aligned}
\mathscr{L}^{\mathrm{M}} &= \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right) \\
&= \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)
\end{aligned}
$$

Lepton Number

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu = \nu^c} \quad \longrightarrow \quad \cancel{L = -1}$$

$$\nu_L \implies L = +1 \qquad\qquad \nu_L^c \implies L = -1$$

$$\mathscr{L}^{\text{M}} = -\frac{m}{2}\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta^-_{0\nu})$$
$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta^+_{0\nu})$$

No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T\,\mathcal{C}^\dagger\,\nu_L - \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix}\phi_+(x) \\ \phi_0(x)\end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T\,\mathcal{C}^\dagger\,\nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed $Y = 2$ Higgs triplet ($I = 1$, $I_3 = -1$)
- Compare with Dirac Mass Term $\propto \overline{\nu_R}\nu_L$ with $I_3 = 1/2$ and $Y = -1$ balanced by $\phi_0 \to v$ with $I_3 = -1/2$ and $Y = +1$

Confusing Majorana Antineutrino Terminology

- A Majorana neutrino is the same as a Majorana antineutrino
- Neutrino interactions are described by the CC and NC Lagrangians

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_L} \, \gamma^\mu \, \ell_L \, W_\mu + \overline{\ell_L} \, \gamma^\mu \, \nu_L \, W_\mu^\dagger \right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2 \cos \vartheta_{\mathrm{W}}} \, \overline{\nu_L} \, \gamma^\mu \, \nu_L \, Z_\mu$$

- Dirac: $\nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{cases}$
- Majorana: $\nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{cases}$
- Common implicit definitions:

left-handed Majorana neutrino $\equiv$ neutrino
right-handed Majorana neutrino $\equiv$ antineutrino

Mixing of Three Majorana Neutrinos

- $\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}^{\mathrm{M}} &= \frac{1}{2} \boldsymbol{\nu}'^{T}_{L} \, \mathcal{C}^{\dagger} \, M^{L} \, \boldsymbol{\nu}'_{L} + \mathrm{H.c.} \\ &= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} + \mathrm{H.c.} \end{aligned}$$

- In general, the matrix M^L is a complex symmetric matrix

$$\begin{aligned} \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} &= \sum_{\alpha,\beta} \left(\nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} \right)^{T} \\ &= - \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, M^{L}_{\alpha\beta} \, (\mathcal{C}^{\dagger})^{T} \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\alpha L} \\ &= \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\beta\alpha} \, \nu'_{\beta L} \end{aligned}$$

$$M^{L}_{\alpha\beta} = M^{L}_{\beta\alpha} \quad \Longleftrightarrow \quad M^{L} = M^{L\,T}$$

Diagonalization of Majorana Mass Matrix

- $\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \boldsymbol{\nu}_L'^T \, \mathcal{C}^\dagger \, M^L \, \boldsymbol{\nu}_L' + \text{H.c.}$

- $\boldsymbol{\nu}_L' = V_L^\nu \, \mathbf{n}_L \qquad \Longrightarrow \qquad \mathscr{L}^{\mathrm{M}} = \frac{1}{2} \boldsymbol{\nu}_L'^T \, (V_L^\nu)^T \, \mathcal{C}^\dagger \, M^L \, V_L^\nu \, \boldsymbol{\nu}_L' + \text{H.c.}$

- $(V_L^\nu)^T \, M^L \, V_L^\nu = M \,, \qquad M_{kj} = m_k \, \delta_{kj} \qquad (k, j = 1, 2, 3)$

- Neutrino fields with definite mass: $\quad \mathbf{n}_L = V_L^{\nu\dagger} \, \boldsymbol{\nu}_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k \left(\nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu_{kL}^T \right)$$

Mixing Matrix

- Leptonic Weak Charged Current:

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,U\,\mathbf{n}_L \qquad \text{with} \qquad U = V^{\ell\dagger}_L\,V^\nu_L$$

- As in the Dirac case, we define the left-handed flavor neutrino fields as

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V^{\ell\dagger}_L\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- In this way, as in the Dirac case, the Leptonic Weak Charged Current has the SM form

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,\boldsymbol{\nu}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\ell_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- Majorana Mass Term $\mathscr{L}^{\mathrm{M}} = \dfrac{1}{2} \sum_{k=1}^{3} m_k \, \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + \text{H.c.}$ is not invariant under the global U(1) gauge transformations

$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \quad (k = 1, 2, 3)$$

- For eliminating some of the 6 phases of the unitary mixing matrix we can use only the global phase transformations (3 arbitrary phases)

$$\ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$
- Performing the transformation $\ell_{\alpha} \to e^{i\varphi_{\alpha}} \, \ell_{\alpha}$ we obtain

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, e^{-i\varphi_{\alpha}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \, \underbrace{e^{-i\varphi_{e}}}_{1} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_{\alpha}-\varphi_{e})}}_{2} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$$

- We can eliminate 3 phases of the mixing matrix: one overall phase and two phases which can be factorized on the left.
- In the Dirac case we could eliminate also two phases which can be factorized on the right.

- In the Majorana case there are two additional physical Majorana phases which can be factorized on the right of the mixing matrixd:

$$\boxed{U = U^{\mathrm{D}}\, D^{\mathrm{M}}} \qquad D^{\mathrm{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- U^{D} is a Dirac mixing matrix, with one Dirac phase

- Standard parameterization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

One-Generation Dirac-Majorana Mass Term

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}^{\text{D+M}} = \mathscr{L}^{\text{D}} + \cancel{\mathscr{L}^{L}} + \mathscr{L}^{R}$$

$$\mathscr{L}^{\text{D}} = -m_{\text{D}}\,\overline{\nu_R}\,\nu_L + \text{H.c.} \qquad \text{Dirac Mass Term}$$

$$\mathscr{L}^{L} = \frac{1}{2}\,m_L\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.}$$

ν_L Majorana Mass Term forbidden by SM Symmetries

$$\mathscr{L}^{R} = \frac{1}{2}\,m_R\,\nu_R^T\,\mathcal{C}^\dagger\,\nu_R + \text{H.c.}$$

New ν_R Majorana Mass Term allowed by SM Symmetries!

Seesaw Mechanism

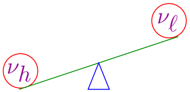
$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \mathrm{H.c.}$$

m_R can be arbitrarily large (not protected by SM symmetries)

$m_R \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R \gg m_{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} \Longrightarrow \quad m_\ell \simeq \frac{m_{\mathrm{D}}^2}{m_R} \qquad m_h \simeq m_R$

natural explanation of smallness
of light neutrino masses



seesaw mechanism

massive neutrinos are Majorana!

$$\nu_\ell \simeq -i\,(\nu_L - \nu_L^c) \qquad \nu_h \simeq \nu_R + \nu_R^c$$

3-GEN $\Rightarrow$ effective low-energy 3-ν mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Effective Majorana Mass from New Physics BSM

- Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- Dimensionless action: $I = \int \mathrm{d}^4x \, \mathscr{L}(x) \Longrightarrow \mathscr{L}(x) \sim [E]^4$
- Kinetic terms: $\overline{\psi} i \partial\!\!\!/ \psi \sim [E]^4, \quad (\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- Mass terms: $m \overline{\psi} \psi \sim [E]^4, \quad m^2 \phi^\dagger \phi \sim [E]^4$
- CC weak interaction: $g \, \overline{\boldsymbol{\nu}_L} \gamma^\rho \boldsymbol{\ell}_L W_\rho \sim [E]^4$
- Yukawa couplings: $y \, \overline{L_L} \Phi \ell_R \sim [E]^4$
- Product of fields $\mathscr{O}_d$ with energy dimension $d \equiv$ dim-d operator
- $\mathscr{L}_{(\mathscr{O}_d)} = C_{(\mathscr{O}_d)} \mathscr{O}_d \quad \Longrightarrow \quad C_{(\mathscr{O}_d)} \sim [E]^{4-d}$
- $\mathscr{O}_{d>4}$ are not renormalizable

- SM Lagrangian includes all $\mathscr{O}_{d\leq 4}$ invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- SM cannot be considered as the final theory of everything
- SM is an effective low-energy theory
- It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- It is plausible that at low-energy there are effective non-renormalizable $\mathscr{O}_{d>4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- All $\mathscr{O}_d$ must respect $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- $\mathscr{O}_{d>4}$ is suppressed by a coefficient $\mathcal{M}^{4-d}$, where $\mathcal{M}$ is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$

- Analogy with $\quad \mathscr{L}_{\text{eff}}^{(\text{CC})} \propto G_{\text{F}}\,(\overline{\nu_{eL}}\gamma^\rho e_L)\,(\overline{e_L}\gamma_\rho \nu_{eL}) + \ldots$

$$\mathscr{O}_6 \to (\overline{\nu_{eL}}\gamma^\rho e_L)\,(\overline{e_L}\gamma_\rho \nu_{eL}) + \ldots \qquad\qquad \frac{g_6}{\mathcal{M}^2} \to \frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

- $\mathcal{M}^{4-d}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM

- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality

- $\mathscr{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)

- $\mathscr{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)

- Only one dim-5 operator:

$$\mathscr{O}_5 = (L_L^T\,\sigma_2\,\Phi)\,\mathcal{C}^\dagger\,(\Phi^T\,\sigma_2\,L_L) + \text{H.c.}$$
$$= \frac{1}{2}\,(L_L^T\,\mathcal{C}^\dagger\,\sigma_2\,\vec{\sigma}\,L_L)\cdot(\Phi^T\,\sigma_2\,\vec{\sigma}\,\Phi) + \text{H.c.}$$

$$\mathscr{L}_5 = \frac{g_5}{2\mathcal{M}}\,(L_L^T\,\mathcal{C}^\dagger\,\sigma_2\,\vec{\sigma}\,L_L)\cdot(\Phi^T\,\sigma_2\,\vec{\sigma}\,\Phi) + \text{H.c.}$$

- Electroweak Symmetry Breaking: $\Phi = \begin{pmatrix}\phi_+\\ \phi_0\end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathscr{L}_{\text{mass}}^{\text{M}} = \frac{1}{2}\,\frac{g_5\,v^2}{\mathcal{M}}\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.} \quad \Longrightarrow \quad \boxed{m = \frac{g_5\,v^2}{\mathcal{M}}}$

- The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM

- $m \propto \dfrac{v^2}{\mathcal{M}} \propto \dfrac{m_{\mathrm{D}}^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses (special case: Seesaw Mechanism)

- Example: $m_{\mathrm{D}} \sim v \sim 10^2\,\mathrm{GeV}$ and $\mathcal{M} \sim 10^{15}\,\mathrm{GeV} \implies m \sim 10^{-2}\,\mathrm{eV}$

Seesaw Mechanism from Effective Lagrangian

- Dirac–Majorana neutrino mass term with $m_L = 0$:

$$\mathscr{L}^{\text{D+M}} = -m_{\text{D}} \left(\overline{\nu_R} \, \nu_L + \overline{\nu_L} \, \nu_R \right) + \frac{1}{2} \, m_R \left(\nu_R^T \, \mathcal{C}^\dagger \, \nu_R + \nu_R^\dagger \, \mathcal{C} \, \nu_R^* \right)$$

- Above the electroweak symmetry-breaking scale:

$$\mathscr{L}^{\text{D+M}} = -y^\nu \left(\overline{\nu_R} \, \widetilde{\Phi}^\dagger \, L_L + \overline{L_L} \, \widetilde{\Phi} \, \nu_R \right) + \frac{1}{2} \, m_R \left(\nu_R^T \, \mathcal{C}^\dagger \, \nu_R + \nu_R^\dagger \, \mathcal{C} \, \nu_R^* \right)$$

- If $m_R \gg v \Longrightarrow \nu_R$ is static $\Longrightarrow$ kinetic term in equation of motion can be neglected:

$$0 \simeq \frac{\partial \mathscr{L}^{\text{D+M}}}{\partial \nu_R} = m_R \, \nu_R^T \, \mathcal{C}^\dagger - y^\nu \, \overline{L_L} \, \widetilde{\Phi}$$

$$\nu_R \simeq -\frac{y^\nu}{m_R} \, \widetilde{\Phi}^T \, \mathcal{C} \, \overline{L_L}^T$$

$$\mathscr{L}^{\text{D+M}} \rightarrow \mathscr{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \, \frac{(y^\nu)^2}{m_R} (L_L^T \, \sigma_2 \, \Phi) \, \mathcal{C}^\dagger \, (\Phi^T \, \sigma_2 \, L_L) + \text{H.c.}$$

$$\mathscr{L}_5 = \frac{g}{\mathcal{M}} \left(L_L^T \, \sigma_2 \, \Phi \right) \mathcal{C}^\dagger \left(\Phi^T \, \sigma_2 \, L_L \right) + \text{H.c.}$$

$$\mathscr{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \, \frac{(y^\nu)^2}{m_R} \left(L_L^T \, \sigma_2 \, \Phi \right) \mathcal{C}^\dagger \left(\Phi^T \, \sigma_2 \, L_L \right) + \text{H.c.}$$

$$g = -\frac{(y^\nu)^2}{2} \qquad\qquad \mathcal{M} = m_R$$

- Seesaw mechanism is a particular case of the effective Lagrangian approach.
- Seesaw mechanism is obtained when dimension-five operator is generated only by the presence of ν_R with $m_R \sim \mathcal{M}$.
- In general, other terms can contribute to $\mathscr{L}_5$.

Generalized Seesaw

- General effective Dirac-Majorana mass matrix:

$$M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$$

- m_L generated by dim-5 operator:

$$m_L \ll m_{\text{D}} \ll m_R$$

- Eigenvalues:

$$\begin{vmatrix} m_L - \mu & m_{\text{D}} \\ m_{\text{D}} & m_R - \mu \end{vmatrix} = 0$$

$$\mu^2 - (\cancel{m_L} + m_R)\,\mu + m_L m_R - m_{\text{D}}^2 = 0$$

$$\mu = \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 - 4\left(m_L m_R - m_{\text{D}}^2\right)}\right]$$

$$\begin{aligned}\mu &= \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 - 4\left(m_L m_R - m_{\text{D}}^2\right)}\right] \\ &= \frac{1}{2}\left[m_R \pm m_R\left(1 - 4\frac{m_L m_R - m_{\text{D}}^2}{m_R^2}\right)^{1/2}\right] \\ &\simeq \frac{1}{2}\left[m_R \pm m_R\left(1 - 2\frac{m_L m_R - m_{\text{D}}^2}{m_R^2}\right)\right]\end{aligned}$$

$$+ \quad \to \quad m_{\text{heavy}} \simeq m_R$$

$$- \quad \to \quad m_{\text{light}} \simeq m_L - \frac{m_{\text{D}}^2}{m_R}$$

Type I seesaw: $m_L \ll \frac{m_{\text{D}}^2}{m_R} \quad \Longrightarrow \quad m_{\text{light}} \simeq \frac{m_{\text{D}}^2}{m_R}$

Type II seesaw: $m_L \gg \frac{m_{\text{D}}^2}{m_R} \quad \Longrightarrow \quad m_{\text{light}} \simeq m_L$

Right-Handed Neutrino Mass Term

Majorana mass term for ν_R respects the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ Standard Model Symmetry!

$$\mathcal{L}_R^{\mathrm{M}} = -\frac{1}{2}\, m \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

Majorana mass term for ν_R breaks Lepton number conservation!

Three possibilities:

- Lepton number can be explicitly broken
- Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\mathcal{L}_\Phi = -y_d \left(\overline{L_L}\, \Phi\, \nu_R + \overline{\nu_R}\, \Phi^\dagger\, L_L \right) \xrightarrow[\langle\Phi\rangle \neq 0]{} -m_\mathrm{D} \left(\overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right)$$

$$\mathcal{L}_\eta = -y_s \left(\eta\, \overline{\nu_R^c}\, \nu_R + \eta^\dagger\, \overline{\nu_R}\, \nu_R^c \right) \xrightarrow[\langle\eta\rangle \neq 0]{} -\frac{1}{2}\, m_R \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$\eta = 2^{-1/2} \left(\langle\eta\rangle + \rho + i\,\chi \right) \qquad \mathcal{L}_\text{mass} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_\mathrm{D} \\ m_\mathrm{D} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\underset{\text{scale of } L \text{ violation}}{m_R} \gg \underset{\text{EW scale}}{m_\mathrm{D}} \Longrightarrow \text{Seesaw: } \boxed{m_\ell \simeq \frac{m_\mathrm{D}^2}{m_R}}$$

$\rho =$ massive scalar, $\chi =$ Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{i y_s}{\sqrt{2}}\, \chi \left[\overline{\nu_h} \gamma^5 \nu_h - \frac{m_\mathrm{D}}{m_R} \left(\overline{\nu_h} \gamma^5 \nu_\ell + \overline{\nu_\ell} \gamma^5 \nu_h \right) + \left(\frac{m_\mathrm{D}}{m_R} \right)^2 \overline{\nu_\ell} \gamma^5 \nu_\ell \right]$$

Three-Generation Mixing

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \mathscr{L}^{\text{D}}_{\text{mass}} + \mathscr{L}^{L}_{\text{mass}} + \mathscr{L}^{R}_{\text{mass}}$$

$$\mathscr{L}^{\text{D}}_{\text{mass}} = - \sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}} \, M^{\text{D}}_{s\alpha} \, \nu'_{\alpha L} + \text{H.c.}$$

$$\mathscr{L}^{L}_{\text{mass}} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} + \text{H.c.}$$

$$\mathscr{L}^{R}_{\text{mass}} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'^{T}_{sR} \, \mathcal{C}^{\dagger} \, M^{R}_{ss'} \, \nu'_{s'R} + \text{H.c.}$$

$$\mathbf{N}'_L \equiv \begin{pmatrix} \boldsymbol{\nu}'_L \\ \boldsymbol{\nu}'^{C}_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'^{C}_R \equiv \begin{pmatrix} \nu'^{C}_{1R} \\ \vdots \\ \nu'^{C}_{N_S R} \end{pmatrix}$$

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \frac{1}{2} \, \mathbf{N}'^{T}_L \, \mathcal{C}^{\dagger} \, M^{\text{D+M}} \, \mathbf{N}'_L + \text{H.c.} \qquad M^{\text{D+M}} = \begin{pmatrix} M^L & M^{\text{D}\,T} \\ M^{\text{D}} & M^R \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term $\Longrightarrow$ massive Majorana neutrinos

- Seesaw Mechanism $\Longrightarrow$ right-handed neutrinos have large Majorana masses and are decoupled from the low-energy phenomenology.

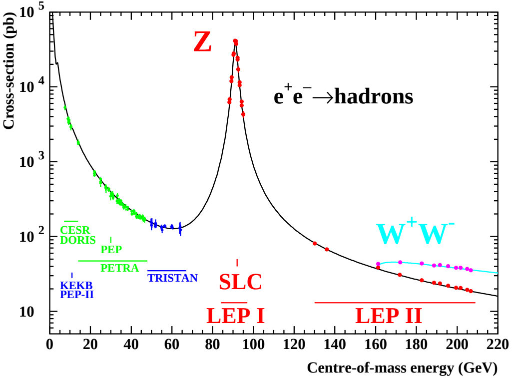
- If all right-handed neutrinos have large Majorana masses, at low energy we have an effective mixing of three Majorana neutrinos.

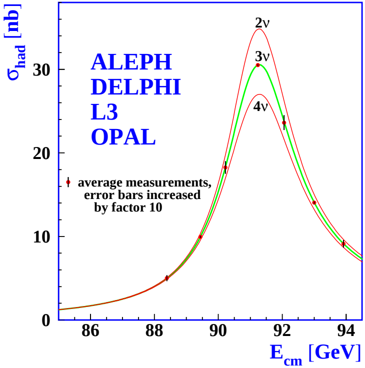
- It is possible that not all right-handed neutrinos have large Majorana masses: some right-handed neutrinos may correspond to low-energy Majorana particles which belong to new physics beyond the Standard Model.

- Light anti-ν_R are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

$$e^+ e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \; \nu_\mu \; \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots}$$

Sterile Neutrinos

- Sterile means no standard model interactions
- Obviously no electromagnetic interactions as normal active neutrinos
- Thus sterile means no standard weak interactions
- But sterile neutrinos are not absolutely sterile:
  - Gravitational Interactions
  - New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos
  - Indirect evidence through combined fit of data
- Powerful window on new physics beyond the Standard Model

Neutrino Oscillations in Vacuum

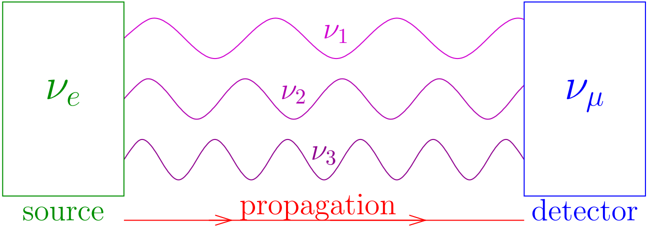
Neutrino Oscillations

- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$
\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}
$$

- U is the 3×3 unitary Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{e1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_e\rangle$$

$$E_k^2 = p^2 + m_k^2$$

at the detector there is a probability > 0 to see the neutrino as a ν_μ

Neutrino Oscillations are Flavor Transitions

$$\begin{array}{cccc} \nu_e \to \nu_\mu & \nu_e \to \nu_\tau & \nu_\mu \to \nu_e & \nu_\mu \to \nu_\tau \\ \bar{\nu}_e \to \bar{\nu}_\mu & \bar{\nu}_e \to \bar{\nu}_\tau & \bar{\nu}_\mu \to \bar{\nu}_e & \bar{\nu}_\mu \to \bar{\nu}_\tau \end{array}$$

transition probabilities depend on U and $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$

Early History of Neutrino Oscillations

- 1957: Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\Longrightarrow$ $\nu \leftrightarrows \bar{\nu}$
- In 1957 only one neutrino $\nu = \nu_e$ was known!
- 1958: Goldhaber, Grodzins and Sunyar measure neutrino helicity: ν_L
- Then, in weak interactions ν_L and $\bar{\nu}_R$
- Helicity conservation $\Longrightarrow$ $\nu_L \leftrightarrows \bar{\nu}_L$
- $\bar{\nu}_L$ is a sterile neutrino (Pontecorvo, 1967)
- 1962: Lederman, Schwartz and Steinberger discover ν_μ
- 1962: Maki, Nakagava, Sakata proposed a model with neutrino mixing:

$$\begin{aligned} \nu_e &= \cos\vartheta\,\nu_1 + \sin\vartheta\,\nu_2 \\ \nu_\mu &= -\sin\vartheta\,\nu_1 + \cos\vartheta\,\nu_2 \end{aligned}$$

  "weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrows \nu_\mu$"
- 1967: Pontecorvo: $\nu_e \leftrightarrows \nu_\mu$ oscillations and applications (solar neutrinos)

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1\,\text{MeV}$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{array}{c} \nu + A \to B + C \\ \Downarrow \\ s = 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \dfrac{(m_B + m_C)^2}{2m_A} - \dfrac{m_A}{2} \end{array}}$$

$$\begin{array}{ll} \nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\ \nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\ \bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\ \nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\ \nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV} \end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO) , $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

Flavor Transitions

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{CC} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\quad \nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad$ States

initial flavor: $\alpha = e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U_{\alpha k}^* \, e^{-iE_k t + i p_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0,0) = \sum_k U_{\alpha k}^* U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t>0, x>0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha\to\nu_\beta}(t,x) = \left|\mathcal{A}_{\nu_\alpha\to\nu_\beta}(t,x)\right|^2 = \left|\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k}\right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \left|\sum_k U_{\alpha k}^*\, e^{-i m_k^2 L/2E}\, U_{\beta k}\right|^2 \qquad \Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

$$= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathsf{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\mathsf{CP}} \nu_{\alpha L}^{\mathsf{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathsf{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathsf{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\Longrightarrow$ $A_{\alpha\beta}^{\text{CPT}} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $U \leftrightarrows U^*$ $\alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$\boxed{A^{\text{CP}}_{\alpha\beta}(L, E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

$$J \neq 0 \iff \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

$$
\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A_{\alpha\beta}^{\text{CPT}} \\
&= P_{\nu_\alpha\to\nu_\beta} - P_{\bar{\nu}_\beta\to\bar{\nu}_\alpha} \\
&= P_{\nu_\alpha\to\nu_\beta} - P_{\bar{\nu}_\alpha\to\bar{\nu}_\beta} \leftarrow A_{\alpha\beta}^{\text{CP}} \\
&+ P_{\bar{\nu}_\alpha\to\bar{\nu}_\beta} - P_{\nu_\beta\to\nu_\alpha} \leftarrow -A_{\beta\alpha}^{\text{CPT}} = 0 \\
&+ P_{\nu_\beta\to\nu_\alpha} - P_{\bar{\nu}_\beta\to\bar{\nu}_\alpha} \leftarrow A_{\beta\alpha}^{\text{CP}} \\
&= A_{\alpha\beta}^{\text{CP}} + A_{\beta\alpha}^{\text{CP}} \qquad \Longrightarrow \qquad \boxed{A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}}
\end{aligned}
$$

T Symmetry

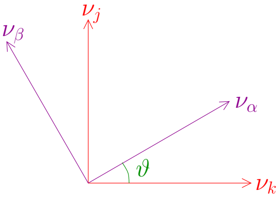
$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $\quad A^{\text{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow A^{\text{T}}_{\alpha\beta} \\
&+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{T}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{T}}_{\alpha\beta} - A^{\text{CP}}_{\alpha\beta} \qquad \Longrightarrow \qquad \boxed{A^{\text{T}}_{\alpha\beta} = A^{\text{CP}}_{\alpha\beta}}
\end{aligned}$$

Two-Neutrino Mixing and Oscillations

$$\boxed{\begin{aligned} |\nu_\alpha\rangle &= \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle \\ |\nu_\beta\rangle &= -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle \end{aligned}}$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m_k^2 - m_j^2$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

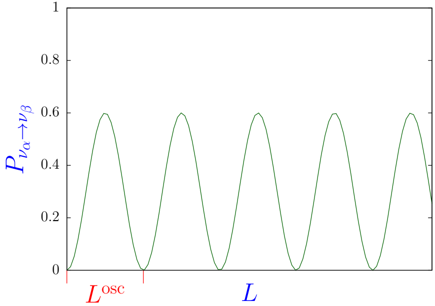
Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

oscillation phase

$$\frac{\Delta m^2 L}{4E} = 1.27\,\frac{\Delta m^2[\text{eV}^2]\,L[\text{m}]}{E[\text{MeV}]} = 1.27\,\frac{\Delta m^2[\text{eV}^2]\,L[\text{km}]}{E[\text{GeV}]}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47\,\frac{E\,[\text{MeV}]}{\Delta m^2\,[\text{eV}^2]}\,\text{m} = 2.47\,\frac{E\,[\text{GeV}]}{\Delta m^2\,[\text{eV}^2]}\,\text{km}$$



Types of Experiments

transitions due to Δm^2 observable only if $\dfrac{\Delta m^2 L}{E} \gtrsim 1 \Leftrightarrow \Delta m^2 \gtrsim \left(\dfrac{L}{E}\right)^{-1}$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$\dfrac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL

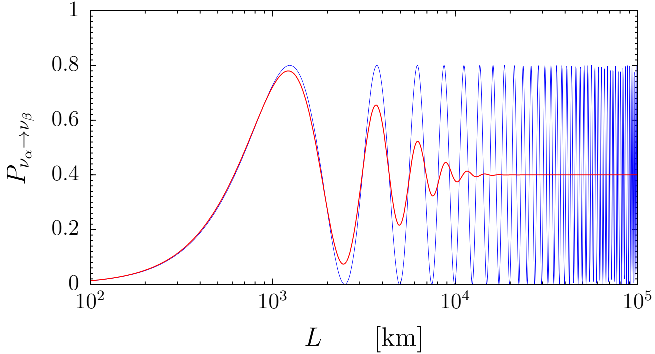
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$

KamLAND

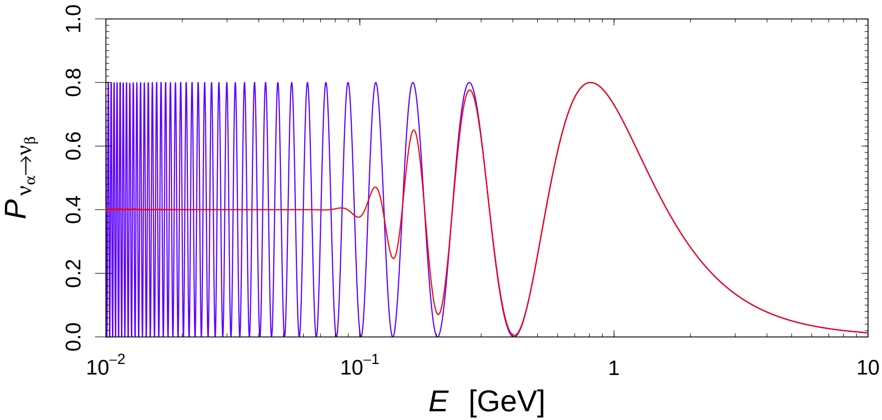
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$



$\Delta m^2 = 10^{-3}\,\mathrm{eV}$ $\qquad$ $\sin^2 2\vartheta = 0.8$ $\qquad$ $\langle E \rangle = 1\,\mathrm{GeV}$ $\qquad$ $\sigma_E = 0.1\,\mathrm{GeV}$

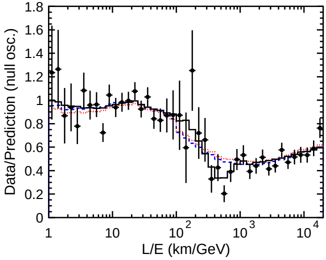
$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\, \mathrm{d}E\right] \qquad (\alpha \neq \beta)$$



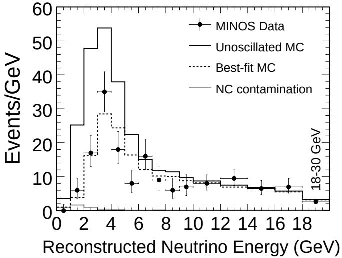
$\Delta m^2 = 10^{-3}\,\text{eV} \qquad \sin^2 2\vartheta = 0.8 \qquad L = 10^3\,\text{km} \qquad \sigma_E = 0.01\,\text{GeV}$

$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha \neq \beta)$$

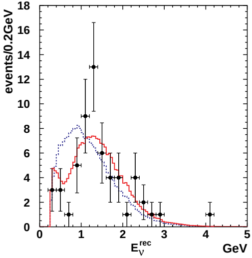
Observations of Neutrino Oscillations



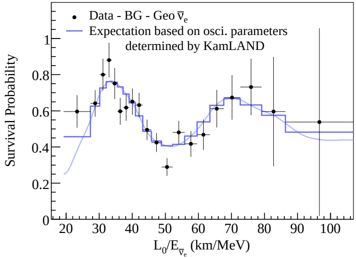
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]



[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]

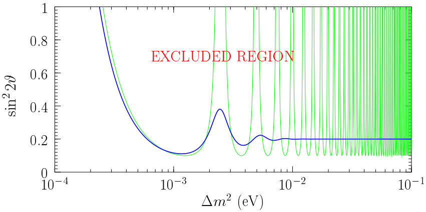


[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]

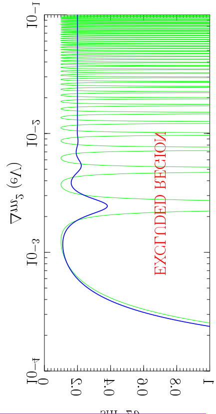
Exclusion Curves

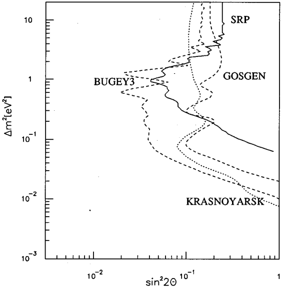
$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E) \, \mathrm{d}E \right] \qquad (\alpha \neq \beta)$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle \leq P^{\text{max}}_{\nu_\alpha \to \nu_\beta} \quad \Longrightarrow \quad \sin^2 2\vartheta \leq \frac{2 \, P^{\text{max}}_{\nu_\alpha \to \nu_\beta}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E) \, \mathrm{d}E}$$

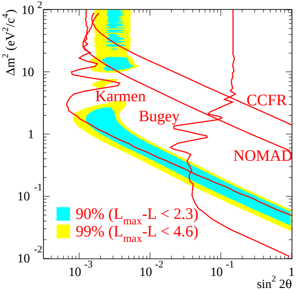


rotate
and
mirror

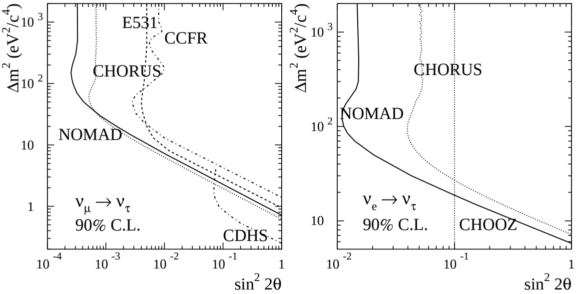




Reactor SBL Experiments: $\bar{\nu}_e \to \bar{\nu}_e$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{\tau}$ and $\overset{(-)}{\nu}_{e} \to \overset{(-)}{\nu}_{\tau}$

Experimental Evidences of Neutrino Oscillations

Source	Experiments	Result
Solar $\nu_e \to \nu_\mu, \nu_\tau$	(SNO, BOREXino, Super-Kamiokande, GALLEX/GNO, SAGE, Homestake, Kamiokande)	$\Delta m^2_{\text{S}} \simeq 7.5 \times 10^{-5}\,\text{eV}^2$, $\sin^2\vartheta_{\text{S}} \simeq 0.30$
VLBL Reactor $\bar{\nu}_e$ disappearance	(KamLAND)	
Atmospheric $\nu_\mu \to \nu_\tau$	(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)	$\Delta m^2_{\text{A}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$, $\sin^2\vartheta_{\text{A}} \simeq 0.50$
LBL Accelerator ν_μ disappearance	(K2K, MINOS, T2K)	
LBL Accelerator $\nu_\mu \to \nu_\tau$	(OPERA)	
LBL Accelerator $\nu_\mu \to \nu_e$	(T2K, MINOS)	Δm^2_{A}, $\sin^2\vartheta_{13} \simeq 0.023$
LBL Reactor $\bar{\nu}_e$ disappearance	(Daya Bay, RENO, Double Chooz)	

Three-Neutrino Mixing

- $\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$

- three left-handed flavor fields: ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$

- three left-handed massive fields: ν_{1L}, ν_{2L}, ν_{3L}

- right-handed components are not needed

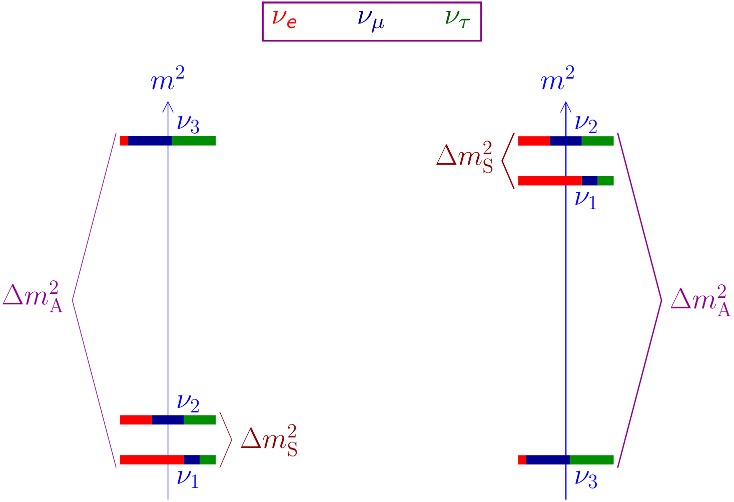
- in neutrino oscillations Dirac = Majorana

- only two independent Δm^2

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

- $\Delta m^2_{\text{S}} = \Delta m^2_{21} = 7.5 \pm 0.2 \times 10^{-5}\, \text{eV}^2$ uncertainty $\simeq 3\%$

- $\Delta m^2_{\text{A}} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| = 2.4 \pm 0.1 \times 10^{-3}\, \text{eV}^2$ uncertainty $\simeq 4\%$



Normal Spectrum
$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Spectrum
$\Delta m^2_{32} < \Delta m^2_{31} < 0$

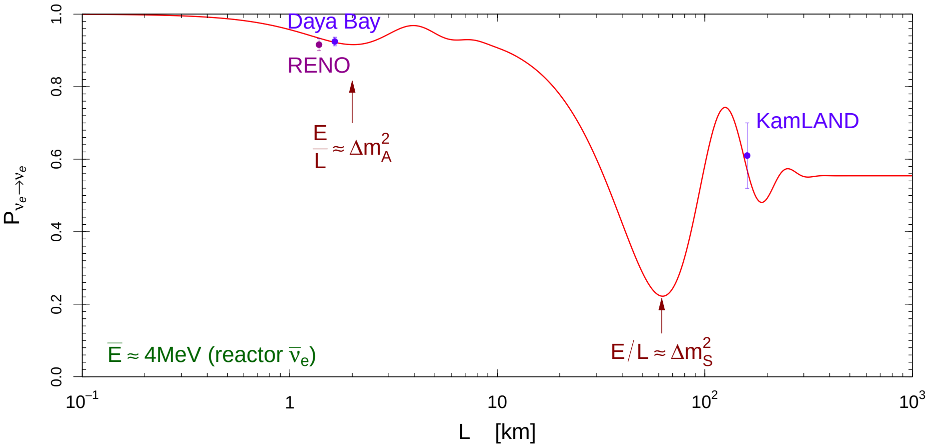
absolute scale is not determined by neutrino oscillation data

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\vartheta_{23} = \vartheta_{\rm A}$	Daya Bay, RENO	$\vartheta_{12} = \vartheta_{\rm S}$
$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$	Double Chooz	$\sin^2\vartheta_{12} \simeq 0.30 \pm 0.01$
$P_{\rm osc} \propto \sin^2 2\vartheta_{23}$	T2K, MINOS	
maximal and flat	$\sin^2\vartheta_{13} \simeq 0.023 \pm 0.002$	
at $\vartheta_{23} = 45^\circ$		

$$\frac{\delta\sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \simeq 40\% \qquad \frac{\delta\sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \simeq 10\% \qquad \frac{\delta\sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \simeq 5\%$$



Daya Bay
RENO
KamLAND
E/L ≈ Δm²_A
E/L ≈ Δm²_S
Ē ≈ 4MeV (reactor ν̄e)
P νe→νe
L [km]
0.0
0.2
0.4
0.6
0.8
1.0
10⁻¹
1
10
10²
10³

Effective VLBL ν_e Survival Probability

$$P_{\nu_e \to \nu_e} = \left| \sum_{k=1}^{3} |U_{ek}|^2 e^{-i m_k^2 L/2E} \right|^2$$

$$|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2 \quad \Longrightarrow \quad |U_{e1}|^2 \simeq \cos^2\vartheta_{12} \,, \; |U_{e2}|^2 \simeq \sin^2\vartheta_{12}$$

$$\begin{aligned}
P_{\nu_e \to \nu_e} &\simeq \left| \sum_{k=1}^{2} |U_{ek}|^2 e^{-i m_k^2 L/2E} \right|^2 \\
&\simeq \left| \cos^2\vartheta_{12} e^{-i m_1^2 L/2E} + \sin^2\vartheta_{12} e^{-i m_2^2 L/2E} \right|^2 \\
&= \cos^4\vartheta_{12} + \sin^4\vartheta_{12} + 2\cos^2\vartheta_{12}\cos^2\vartheta_{12}\cos\left(\frac{\Delta m_{21}^2 L}{2E}\right) \\
&= 1 - \sin^2 2\vartheta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)
\end{aligned}$$

Effective ATM and LBL Oscillation Probabilities

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i m_k^2 L / 2E} \right|^2 * \left| e^{i m_1^2 L / 2E} \right|^2$$

$$= \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}$$

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta} &= \left|\delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}\left[1-\exp\left(-i\frac{\Delta m^2_{31}L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 3}|^2|U_{\beta 3}|^2\left(2-2\cos\frac{\Delta m^2_{31}L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha 3}|^2\left(1-\cos\frac{\Delta m^2_{31}L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2\left(\delta_{\alpha\beta}-|U_{\beta 3}|^2\right)\left(1-\cos\frac{\Delta m^2_{31}L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2\left(\delta_{\alpha\beta}-|U_{\beta 3}|^2\right)\sin^2\frac{\Delta m^2_{31}L}{4E}
\end{aligned}
$$

$$
\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 3}|^2|U_{\beta 3}|^2\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)
$$

$$
\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha 3}|^2\left(1-|U_{\alpha 3}|^2\right)\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)
$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \quad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

LBL

$$\sin^2 2\vartheta_{ee} \ll 1$$

$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

Effective ATM and LBL Oscillation Amplitudes

- ν_e disappearance: Chooz, Palo Verde, Daya Bay, RENO, Double Chooz

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) = 4 s_{13}^2 c_{13}^2 = \sin^2 2\vartheta_{13} \simeq 0.09$$

- ν_μ disappearance: K2K, MINOS, T2K

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right) = 4 c_{13}^2 s_{23}^2 \left(1 - c_{13}^2 s_{23}^2\right)$$

$$\simeq 4 s_{23}^2 \left(1 - s_{23}^2\right) = \sin^2 2\vartheta_{23} \simeq 1$$

- $\nu_\mu \to \nu_e$: T2K, MINOS

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2 |U_{\mu 3}|^2 = 4 s_{13}^2 c_{13}^2 s_{23}^2 = \sin^2 2\vartheta_{13} \sin^2 \vartheta_{23}$$

$$\simeq \frac{1}{2} \sin^2 2\vartheta_{13} \simeq 0.045$$

- $\nu_\mu \to \nu_\tau$: OPERA

$$\sin^2 2\vartheta_{\mu\tau} = 4|U_{\mu 3}|^2 |U_{\tau 3}|^2 = 4 c_{13}^4 s_{23} c_{23} = c_{13}^4 \sin^2 2\vartheta_{23} \simeq \sin^2 2\vartheta_{23} \simeq 1$$

CP Violation?

- In this approximation there is no observable CP-violation effect!

- CP-violation can be observed only with sensitivity to Δm^2_{21}: in vacuum

$$
\begin{aligned}
A^{\mathrm{CP}}_{\alpha\beta} &= P_{\nu_\alpha\to\nu_\beta} - P_{\bar{\nu}_\alpha\to\bar{\nu}_\beta} \\
&= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)
\end{aligned}
$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

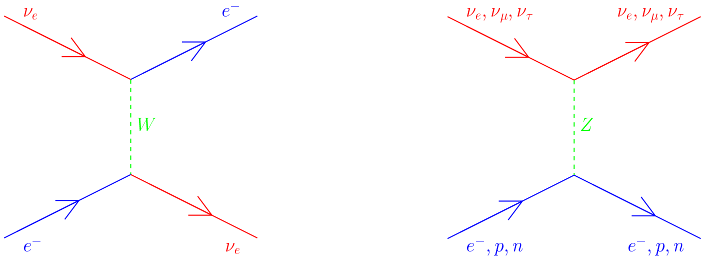
$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

- Necessary conditions for observation of CP violation:
  - Sensitivity to all mixing angles, including small ϑ_{13}
  - Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

Neutrino Oscillations in Matter

Effective Potentials in Matter

coherent interactions with medium: forward elastic CC and NC scattering



$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

only $V_{\text{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\text{CC}} = -V_{\text{CC}} \qquad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

Evolution of Neutrino Flavors in Matter

- Flavor neutrino ν_α with momentum p: $\quad |\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* \, |\nu_k(p)\rangle$
- Evolution is determined by Hamiltonian
- Hamiltonian in vacuum: $\mathcal{H} = \mathcal{H}_0$

$$\mathcal{H}_0 \, |\nu_k(p)\rangle = E_k \, |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

- Hamiltonian in matter: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \, |\nu_\alpha(p)\rangle = V_\alpha \, |\nu_\alpha(p)\rangle$
- Schrödinger evolution equation: $i \, \dfrac{\mathrm{d}}{\mathrm{d}t} \, |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle$
- Initial condition: $|\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$
- For $t > 0$ the state $|\nu(p,t)\rangle$ is a superposition of all flavors:

$$|\nu(p,t)\rangle = \sum_\beta \varphi_\beta(p,t) |\nu_\beta(p)\rangle$$

- Transition probability: $P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2$

evolution equation of states

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle \,, \qquad |\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes

$$\varphi_\beta(p,t) = \langle \nu_\beta(p) | \nu(p,t) \rangle \,, \qquad \varphi_\beta(p,0) = \delta_{\alpha\beta}$$

evolution of flavor transition amplitudes

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H} | \nu(p,t) \rangle$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t) \rangle$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_\beta(p,t) = \langle \nu_\beta(p)|\mathcal{H}_0|\nu(p,t)\rangle + \langle \nu_\beta(p)|\mathcal{H}_I|\nu(p,t)\rangle$$

$$\langle \nu_\beta(p)|\mathcal{H}_0|\nu(p,t)\rangle =$$

$$\sum_\rho \sum_{k,j} \underbrace{\langle \nu_\beta(p)|\nu_k(p)\rangle}_{U_{\beta k}} \underbrace{\langle \nu_k(p)|\mathcal{H}_0|\nu_j(p)\rangle}_{\delta_{kj}E_k} \underbrace{\langle \nu_j(p)|\nu_\rho(p)\rangle}_{U^*_{\rho j}} \underbrace{\langle \nu_\rho(p)|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho \sum_k U_{\beta k} E_k U^*_{\rho k}\,\varphi_\rho(p,t)$$

$$\langle \nu_\beta(p)|\mathcal{H}_I|\nu(p,t)\rangle = \sum_\rho \underbrace{\langle \nu_\beta(p)|\mathcal{H}_I|\nu_\rho(p)\rangle}_{\delta_{\beta\rho}V_\beta} \underbrace{\langle \nu_\rho(p)|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho \delta_{\beta\rho} V_\beta\,\varphi_\rho(p,t)$$

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k}\, E_k\, U^*_{\rho k} + \delta_{\beta\rho}\, V_\beta \right) \varphi_\rho}$$

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $\quad E = p \quad t = x$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \varphi_\beta(p,x) = (p + V_{\text{NC}}) \varphi_\beta(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{\text{CC}} \right) \varphi_\rho(p,x)$$

$$\psi_\beta(p,x) = \varphi_\beta(p,x)\, e^{ipx + i\int_0^x V_{\text{NC}}(x')\,\mathrm{d}x'}$$

$$\Downarrow$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \psi_\beta = e^{ipx + i\int_0^x V_{\text{NC}}(x')\,\mathrm{d}x'} \left(-p - V_{\text{NC}} + i \frac{\mathrm{d}}{\mathrm{d}x} \right) \varphi_\beta$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \psi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{\text{CC}} \right) \psi_\rho}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2}$$

evolution of flavor transition amplitudes in matrix form

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\,\Psi_\alpha = \frac{1}{2E}\left(U\,\mathbb{M}^2\,U^\dagger + \mathbb{A}\right)\Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2EV_{\mathrm{CC}} = 2\sqrt{2}EG_{\mathrm{F}}N_e$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U\,\mathbb{M}^2\,U^\dagger \xrightarrow{\text{matter}} U\,\mathbb{M}^2\,U^\dagger + 2\,E\,\underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

potential due to coherent forward elastic scattering

Two-Neutrino Mixing

$$\nu_e \to \nu_\mu \text{ transitions with } U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$U \, \mathbb{M}^2 \, U^\dagger = \begin{pmatrix} \cos^2\vartheta m_1^2 + \sin^2\vartheta m_2^2 & \cos\vartheta \sin\vartheta \left(m_2^2 - m_1^2 \right) \\ \cos\vartheta \sin\vartheta \left(m_2^2 - m_1^2 \right) & \sin^2\vartheta m_1^2 + \cos^2\vartheta m_2^2 \end{pmatrix}$$

$$= \frac{1}{2} \Sigma m^2 + \frac{1}{2} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}$$

$\uparrow$

irrelevant common phase

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad\qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}}$$

initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_e(0)\\\psi_\mu(0)\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$

$$\boxed{\begin{array}{l}P_{\nu_e\to\nu_\mu}(x)=|\psi_\mu(x)|^2\\ P_{\nu_e\to\nu_e}(x)=|\psi_e(x)|^2=1-P_{\nu_e\to\nu_\mu}(x)\end{array}}$$

Constant Matter Density

$$\boxed{i\frac{d}{dx}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{CC} & \Delta m^2\sin 2\vartheta\\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

$$\frac{dA_{CC}}{dx}=0$$

diagonalization of effective hamiltonian: $\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_M & \sin\vartheta_M\\ -\sin\vartheta_M & \cos\vartheta_M\end{pmatrix}\begin{pmatrix}\psi_1^M\\ \psi_2^M\end{pmatrix}$

$$\begin{pmatrix}\cos\vartheta_M & -\sin\vartheta_M\\ \sin\vartheta_M & \cos\vartheta_M\end{pmatrix}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{CC} & \Delta m^2\sin 2\vartheta\\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_M & \sin\vartheta_M\\ -\sin\vartheta_M & \cos\vartheta_M\end{pmatrix}=$$

$$=\begin{pmatrix}A_{CC}-\Delta m_M^2 & 0\\ 0 & A_{CC}+\Delta m_M^2\end{pmatrix}$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_1^M\\ \psi_2^M\end{pmatrix}=\left[\underset{\uparrow}{\frac{A_{CC}}{4E}}+\frac{1}{4E}\begin{pmatrix}-\Delta m_M^2 & 0\\ 0 & \Delta m_M^2\end{pmatrix}\right]\begin{pmatrix}\psi_1^M\\ \psi_2^M\end{pmatrix}$$

irrelevant common phase

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\mathrm{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A_{\mathrm{CC}}^{\mathrm{R}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N_e^{\mathrm{R}} = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\mathrm{F}}}$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}$$

$$\nu_e\to\nu_\mu \quad \Longrightarrow \quad \begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}\psi_1^{\mathrm{M}}(0)\\ \psi_2^{\mathrm{M}}(0)\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}}\end{pmatrix}$$

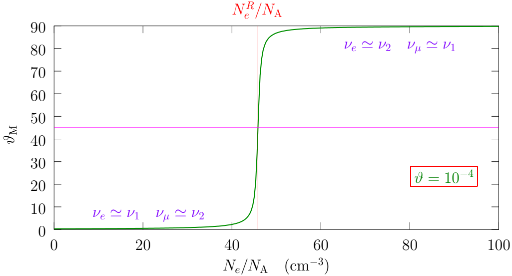
$$\psi_1^{\mathrm{M}}(x) = \cos\vartheta_{\mathrm{M}}\exp\left(i\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)$$

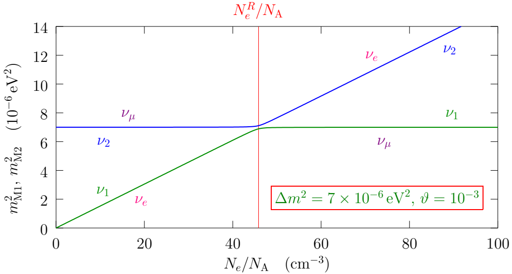
$$\psi_2^{\mathrm{M}}(x) = \sin\vartheta_{\mathrm{M}}\exp\left(-i\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x)=|\psi_\mu(x)|^2=\left|-\sin\vartheta_{\mathrm{M}}\psi_1^{\mathrm{M}}(x)+\cos\vartheta_{\mathrm{M}}\psi_2^{\mathrm{M}}(x)\right|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x)=\sin^2 2\vartheta_{\mathrm{M}}\ \sin^2\left(\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)





$$\nu_e = \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta\\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

tentative diagonalization: $\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} =$$

$$= \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta\\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

if matter densitity is not constant $\mathrm{d}\vartheta_{\mathrm{M}}/\mathrm{d}x \neq 0$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} = \left[\underset{\substack{\uparrow\\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix} + \begin{pmatrix}0 & -i\dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x}\\ i\dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} = \Bigg[\underbrace{\frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}}_{\uparrow\ \text{adiabatic}} + \underbrace{\begin{pmatrix}0 & -i\,\frac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x}\\ i\,\frac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0\end{pmatrix}}_{\substack{\uparrow\\ \text{non-adiabatic}\\ \text{maximum at resonance}}}\Bigg]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

initial conditions:

$$\begin{pmatrix}\psi_1^{\mathrm{M}}(0)\\ \psi_2^{\mathrm{M}}(0)\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{\mathrm{M}} & -\sin\vartheta^0_{\mathrm{M}}\\ \sin\vartheta^0_{\mathrm{M}} & \cos\vartheta^0_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{\mathrm{M}}\\ \sin\vartheta^0_{\mathrm{M}}\end{pmatrix}$$

solution approximating all non-adiabatic $\nu_1^{\mathrm{M}} \leftrightarrows \nu_2^{\mathrm{M}}$ transitions in resonance

$$\begin{aligned}
\psi_1^{\mathrm{M}}(x) \simeq\ & \left[\cos\vartheta^0_{\mathrm{M}}\exp\left(i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{11} + \sin\vartheta^0_{\mathrm{M}}\exp\left(-i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{21}\right]\\
& \times\exp\left(i\int_{x_{\mathrm{R}}}^{x}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\\
\psi_2^{\mathrm{M}}(x) \simeq\ & \left[\cos\vartheta^0_{\mathrm{M}}\exp\left(i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{12} + \sin\vartheta^0_{\mathrm{M}}\exp\left(-i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{22}\right]\\
& \times\exp\left(-i\int_{x_{\mathrm{R}}}^{x}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)
\end{aligned}$$

Averaged ν_e Survival Probability on Earth

$$\psi_e(x) = \cos\vartheta\,\psi_1^{\mathrm{M}}(x) + \sin\vartheta\,\psi_2^{\mathrm{M}}(x)$$

neglect interference (averaged over energy spectrum)

$$\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = \cos^2\vartheta\,\cos^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{11}^{\mathrm{R}}|^2 + \cos^2\vartheta\,\sin^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{21}^{\mathrm{R}}|^2 + \sin^2\vartheta\,\cos^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{12}^{\mathrm{R}}|^2 + \sin^2\vartheta\,\sin^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{22}^{\mathrm{R}}|^2$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^{\mathrm{R}}|^2 = |\mathcal{A}_{21}^{\mathrm{R}}|^2 = P_{\mathrm{c}} \qquad |\mathcal{A}_{11}^{\mathrm{R}}|^2 = |\mathcal{A}_{22}^{\mathrm{R}}|^2 = 1 - P_{\mathrm{c}}$$

$P_{\mathrm{c}} \equiv$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{c}}\right)\cos 2\vartheta_{\mathrm{M}}^0\,\cos 2\vartheta}$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: $\gamma = \left.\frac{\Delta m^2_{\mathrm{M}}/2E}{2|d\vartheta_{\mathrm{M}}/dx|}\right|_{\mathrm{R}} = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A_{\mathrm{CC}}}{dx}\right|_{\mathrm{R}}}$

$A \propto x$ $\quad F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $\quad F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ $\quad F = 1 - \tan^2\vartheta$

[Pizzochero, PRD 36 (1987) 2293]

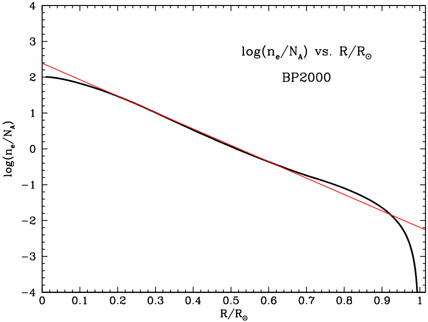
[Toshev, PLB 196 (1987) 170]

[Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

Solar Neutrinos

SUN: $N_e(x) \simeq N_e^{\text{c}} \exp\left(-\frac{x}{x_0}\right)$ $N_e^{\text{c}} = 245\, N_{\text{A}}/\text{cm}^3$ $x_0 = \frac{R_\odot}{10.54}$



$$\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right) \cos 2\vartheta^0_{\text{M}} \, \cos 2\vartheta$$

$$P_{\text{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left|\frac{\text{d} \ln A_{\text{CC}}}{\text{d}x}\right|_{\text{R}}}$$

$$F = 1 - \tan^2\vartheta$$

$$A_{\text{CC}} = 2\sqrt{2} E G_{\text{F}} N_e$$

practical prescription:

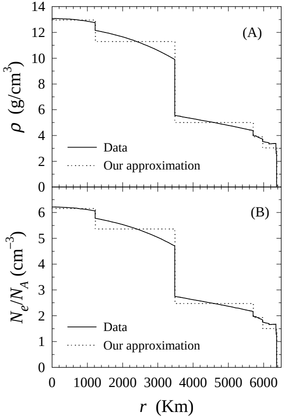
[Lisi et al., PRD 63 (2001) 093002]

$$\begin{cases} \text{numerical } \left|\text{d} \ln A_{\text{CC}}/\text{d}x\right|_{\text{R}} & \text{for } x \leq 0.904 R_\odot \\ \left|\text{d} \ln A_{\text{CC}}/\text{d}x\right|_{\text{R}} \to \dfrac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{cases}$$

Electron Neutrino Regeneration in the Earth

$$P^{\text{sun+earth}}_{\nu_e\to\nu_e} = \overline{P}^{\text{sun}}_{\nu_e\to\nu_e} + \frac{\left(1 - 2\overline{P}^{\text{sun}}_{\nu_e\to\nu_e}\right)\left(P^{\text{earth}}_{\nu_2\to\nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



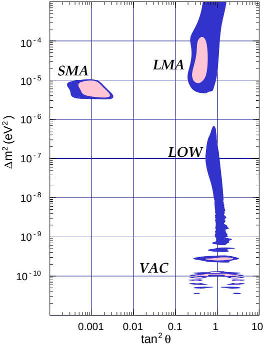
$P^{\text{earth}}_{\nu_2\to\nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

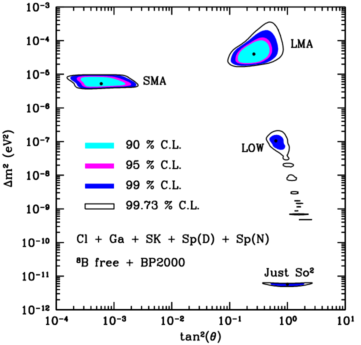
Wave functions of flavor neutrinos are joined at the boundaries of steps.

Solar Neutrino Oscillations

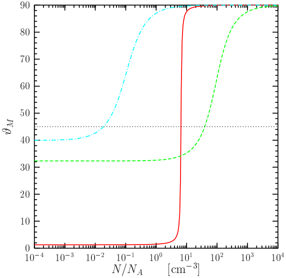
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



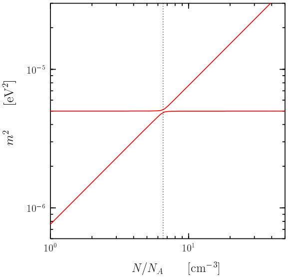
[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



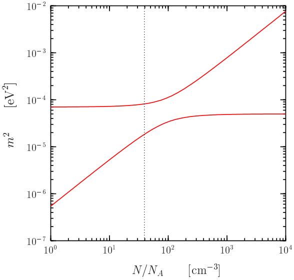
solid line: $\Delta m^2 = 5 \times 10^{-6}\,\mathrm{eV}^2$
(typical SMA) $\tan^2\vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\mathrm{eV}^2$
(typical LMA) $\tan^2\vartheta = 0.4$

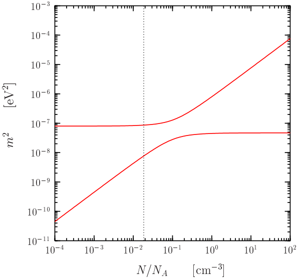
dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\mathrm{eV}^2$
(typical LOW) $\tan^2\vartheta = 0.7$



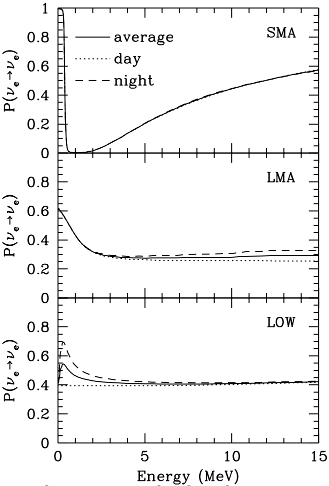
typical SMA



typical LMA



typical LOW

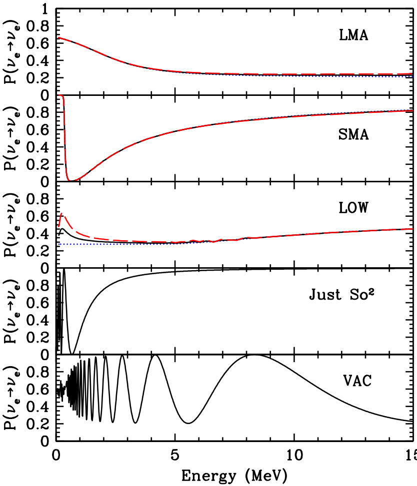


[Bahcall, Krastev, Smirnov, PRD 58 (1998) 096016]

SMA: $\Delta m^2 = 5.0 \times 10^{-6}\ \text{eV}^2$ $\sin^2 2\vartheta = 3.5 \times 10^{-3}$

LMA: $\Delta m^2 = 1.6 \times 10^{-5}\ \text{eV}^2$ $\sin^2 2\vartheta = 0.57$

LOW: $\Delta m^2 = 7.9 \times 10^{-8}\ \text{eV}^2$ $\sin^2 2\vartheta = 0.95$



[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]

LMA: $\Delta m^2 = 4.2 \times 10^{-5}\ \text{eV}^2$ $\tan^2\vartheta = 0.26$

SMA: $\Delta m^2 = 5.2 \times 10^{-6}\ \text{eV}^2$ $\tan^2\vartheta = 5.5 \times 10^{-4}$

LOW: $\Delta m^2 = 7.6 \times 10^{-8}\ \text{eV}^2$ $\tan^2\vartheta = 0.72$

Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}\ \text{eV}^2$ $\tan^2\vartheta = 1.0$

VAC: $\Delta m^2 = 1.4 \times 10^{-10}\ \text{eV}^2$ $\tan^2\vartheta = 0.38$

SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of D_2O, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.

$$
\begin{array}{ll}
\text{CC:} & \nu_e + d \to p + p + e^- \\
\text{NC:} & \nu + d \to p + n + \nu \\
\text{ES:} & \nu + e^- \to \nu + e^-
\end{array}
$$

$$
\left.
\begin{array}{l}
\text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2\,\text{MeV} \\
\text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2\,\text{MeV} \\
\text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0\,\text{MeV}
\end{array}
\right\} \Longrightarrow {}^8\text{B}, \; hep
$$

D_2O phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02$$
$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.01 \pm 0.13$$
$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

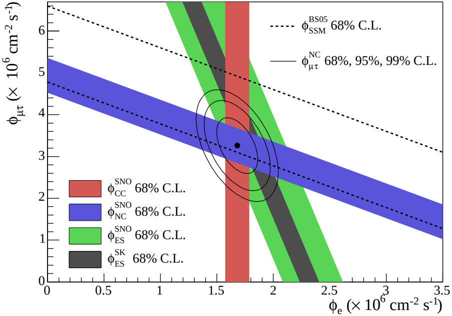
NaCl phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.31 \pm 0.02$$
$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.03 \pm 0.09$$
$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]

$$\Phi^{\rm SNO}_{\nu_e} = 1.76 \pm 0.11 \times 10^6 \, \mathrm{cm}^{-2} \, \mathrm{s}^{-1}$$

$$\Phi^{\rm SNO}_{\nu_\mu,\nu_\tau} = 5.41 \pm 0.66 \times 10^6 \, \mathrm{cm}^{-2} \, \mathrm{s}^{-1}$$



SNO solved
solar neutrino problem

$\Downarrow$

Neutrino Physics

(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

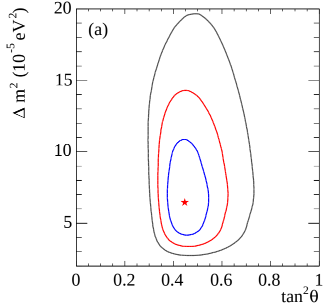
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5} \, \mathrm{eV}^2$$

$$\tan^2 \vartheta \simeq 0.45$$



[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

average distance from reactors: 180 km
- 6.7% of flux from one reactor at 88 km
- 79% of flux from 26 reactors at 138–214 km
- 14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

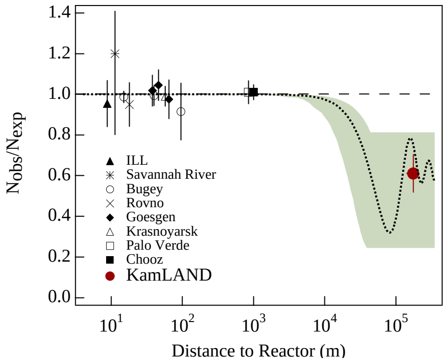
data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$
expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$
observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

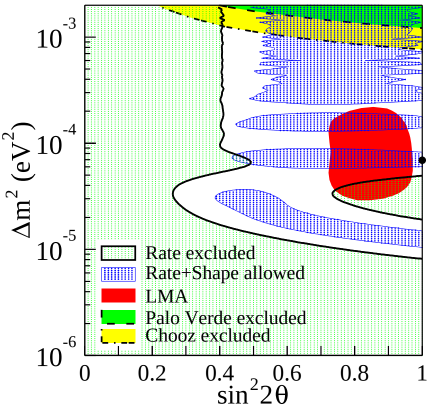
99.95% C.L. evidence of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



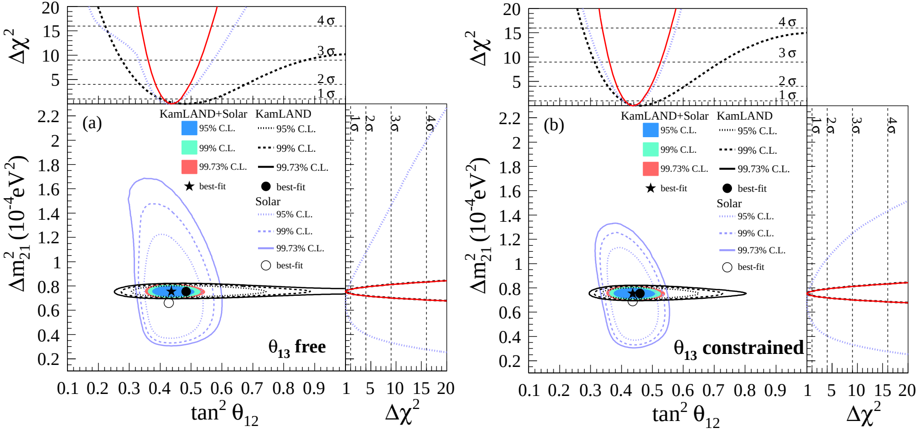
Shade: 95% C.L. LMA

Curve: $\left\{ \begin{array}{l} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{array} \right.$



95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]



$$\Delta m^2_{21} = 7.53^{+0.19}_{-0.18} \times 10^{-5}\ \text{eV}^2 \qquad \Delta m^2_{21} = 7.53 \pm 0.18 \times 10^{-5}\ \text{eV}^2$$

$$\tan^2\vartheta_{12} = 0.437^{+0.029}_{-0.026} \qquad \tan^2\vartheta_{12} = 0.436^{+0.029}_{-0.025}$$

$$\sin^2\vartheta_{13} = 0.023 \pm 0.015 \qquad \sin^2\vartheta_{13} = 0.023 \pm 0.002$$

[KamLAND, arXiv:1303.4667]

LMA Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2 \quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right)\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$P_{\text{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta\left|\frac{d\ln A}{dx}\right|_{\text{R}}} \qquad F = 1 - \tan^2\vartheta$$

$$A_{\text{CC}} \simeq 2\sqrt{2}EG_{\text{F}}N^{\text{c}}_e\exp\left(-\frac{x}{x_0}\right) \quad \Longrightarrow \quad \left|\frac{d\ln A}{dx}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3\times 10^{-15}\,\text{eV}$$

$$\tan^2\vartheta \simeq 0.4 \quad \Longrightarrow \quad \sin^2 2\vartheta \simeq 0.82\,,\ \cos 2\vartheta \simeq 0.43 \qquad\qquad \gamma \simeq 2\times 10^4\left(\frac{E}{\text{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \quad \Longrightarrow \quad P_{\text{c}} \ll 1 \quad \Longrightarrow \quad \boxed{\overline{P}^{\text{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2}\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta_{\mathrm{M}}^{0} = \frac{\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter [Bahcall, Peña-Garay, hep-ph/0305159]

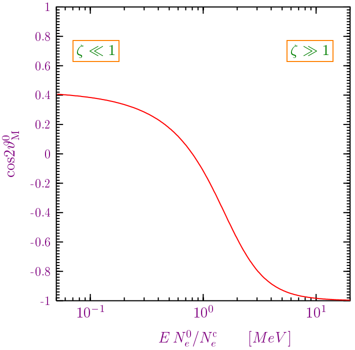
$$\zeta = \frac{A_{\mathrm{CC}}^{0}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}}\right) \left(\frac{N_e^0}{N_e^{\mathrm{c}}}\right)$$

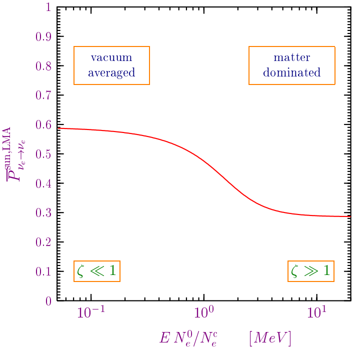
$$\zeta \ll 1 \Longrightarrow \vartheta_{\mathrm{M}}^{0} \simeq \vartheta \Longrightarrow \overline{P}^{\mathrm{sun}}_{\nu_e \to \nu_e} \simeq 1 - \tfrac{1}{2} \sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \Longrightarrow \vartheta_{\mathrm{M}}^{0} \simeq \pi/2 \Longrightarrow \overline{P}^{\mathrm{sun}}_{\nu_e \to \nu_e} \simeq \sin^2 \vartheta$$

matter dominated survival probability

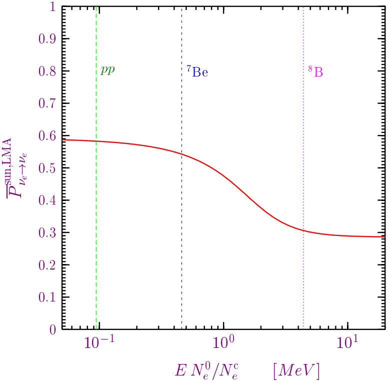




$\langle E \rangle_{pp} \simeq 0.27\,\text{MeV}\,,\ \langle r_0 \rangle_{pp} \simeq 0.1\,R_\odot \quad \Longrightarrow \quad \langle E\,N_e^0/N_e^c \rangle_{pp} \simeq 0.094\,\text{MeV}$

$E_{^7\text{Be}} \simeq 0.86\,\text{MeV}\,,\ \langle r_0 \rangle_{^7\text{Be}} \simeq 0.06\,R_\odot \quad \Longrightarrow \quad \langle E\,N_e^0/N_e^c \rangle_{^7\text{Be}} \simeq 0.46\,\text{MeV}$

$\langle E \rangle_{^8\text{B}} \simeq 6.7\,\text{MeV}\,,\ \langle r_0 \rangle_{^8\text{B}} \simeq 0.04\,R_\odot \quad \Longrightarrow \quad \langle E\,N_e^0/N_e^c \rangle_{^8\text{B}} \simeq 4.4\,\text{MeV}$



each neutrino experiment is mainly sensitive to one flux

each neutrino experiment is mainly sensitive to ϑ

accurate *pp* experiment can improve determination of ϑ

[Bahcall, Peña-Garay, hep-ph/0305159]

Mass Hierarchy

1. Matter Effect (Atmospheric, Long-Baseline, Supernova Experiments):
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} > 0$ NH
   - $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} < 0$ IH
2. Phase Difference (Reactor $\bar{\nu}_e \to \bar{\nu}_e$):

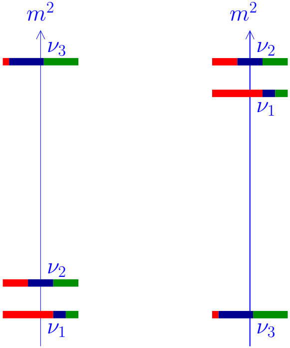
Normal Hierarchy

$|\Delta m^2_{31}|$

$||$

$|\Delta m^2_{32}| + |\Delta m^2_{21}|$

$|\Delta m^2_{31}| > |\Delta m^2_{32}|$



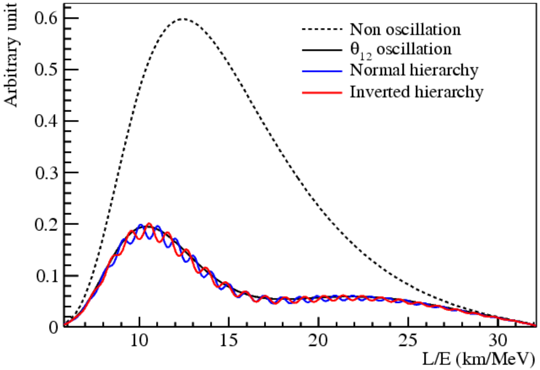
Inverted Hierarchy

$|\Delta m^2_{31}|$

$||$

$|\Delta m^2_{32}| - |\Delta m^2_{21}|$

$|\Delta m^2_{31}| < |\Delta m^2_{32}|$



S.T. Petcov et al., PLB533(2002)94
S.Choubey et al., PRD68(2003)113006
J. Learned et al., hep-ex/0612022

L. Zhan, Y. Wang, J. Cao, L. Wen,
PRD78:111103, 2008
PRD79:073007, 2009

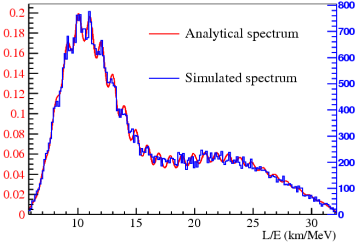
Precision energy spectrum measurement: Looking for interference between P_{31} and P_{32}

➔ relative measurement

$$F(L/E) = \phi(E)\sigma(E)P_{ee}(L/E)$$

$$\begin{aligned} P_{ee}(L/E) &= 1 - P_{21} - P_{31} - P_{32} \\ P_{21} &= \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21}) \\ P_{31} &= \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31}) \\ P_{32} &= \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32}) \end{aligned}$$

$$\Delta_{21} << \Delta_{31} \approx \Delta_{32}$$



[Miao He, NuFact 2013]

In Neutrino Oscillations Dirac = Majorana

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495; Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Evolution of Amplitudes: $$i\,\frac{\mathrm{d}\psi_\alpha}{\mathrm{d}x} = \frac{1}{2E}\sum_\beta \left(UM^2U^\dagger + 2EV\right)_{\alpha\beta}\psi_\beta$$

difference: $$\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})}D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2D \quad \Longrightarrow \quad DM^2D^\dagger = M^2$$

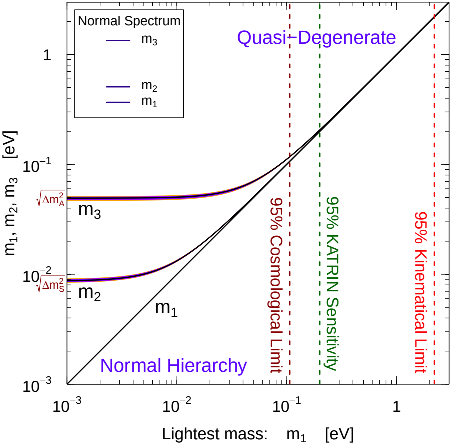
$$U^{(\mathrm{M})}M^2(U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})}DM^2D^\dagger(U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})}M^2(U^{(\mathrm{D})})^\dagger$$

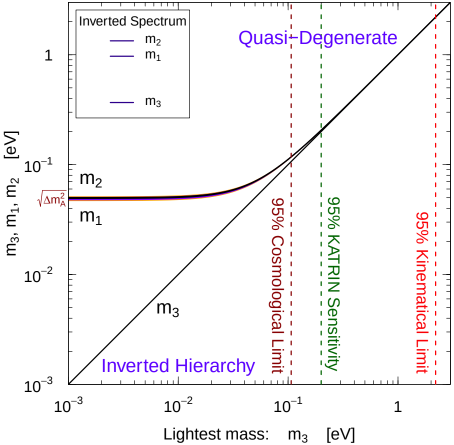
Neutrino Mixing Phenomenology

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), IceCube-PINGU, INO (India), ...
- Mass Hierarchy ?
  - NOνA (USA), JUNO (China), RENO-50 (Korea), IceCube-PINGU, INO (India), ...
- CP violation ?
  - NOνA (USA), LBNE (USA), LAGUNA-LBNO (EU), HyperK (Japan), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Absolute Scale of Neutrino Masses





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{S}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{A}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{A}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{A}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_{\mathrm{A}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

95% Cosmological Limit: Planck + WMAP9 + highL + BAO [arXiv:1303.5076]

Tritium Beta-Decay

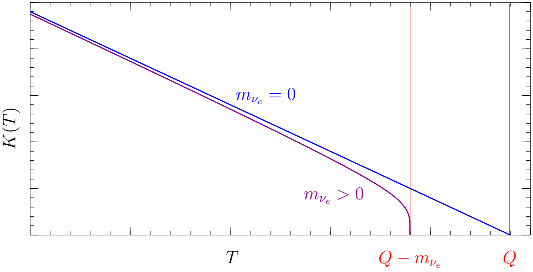
$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

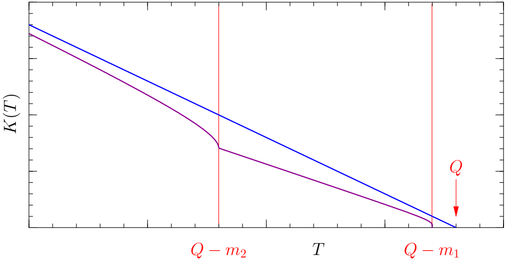
future: KATRIN

[www.katrin.kit.edu]

start data taking in 2015

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses ($m_k \ll Q - T$)

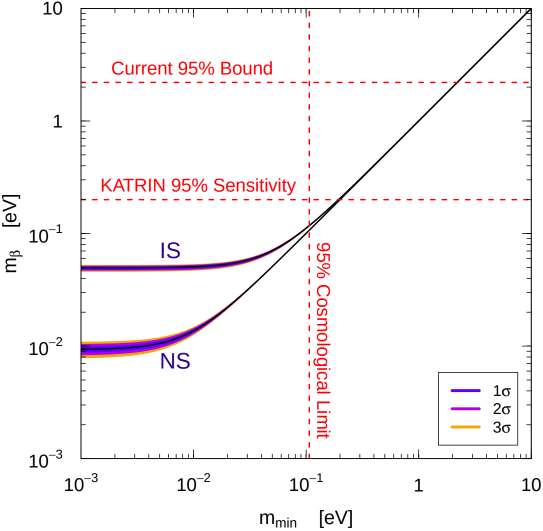
effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$



- Quasi-Degenerate:

  $$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- Inverted Hierarchy:

  $$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_{\mathrm{A}}^2 \simeq \Delta m_{\mathrm{A}}^2$$

- Normal Hierarchy:

  $$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_{\mathrm{S}}^2 + s_{13}^2 \Delta m_{\mathrm{A}}^2$$

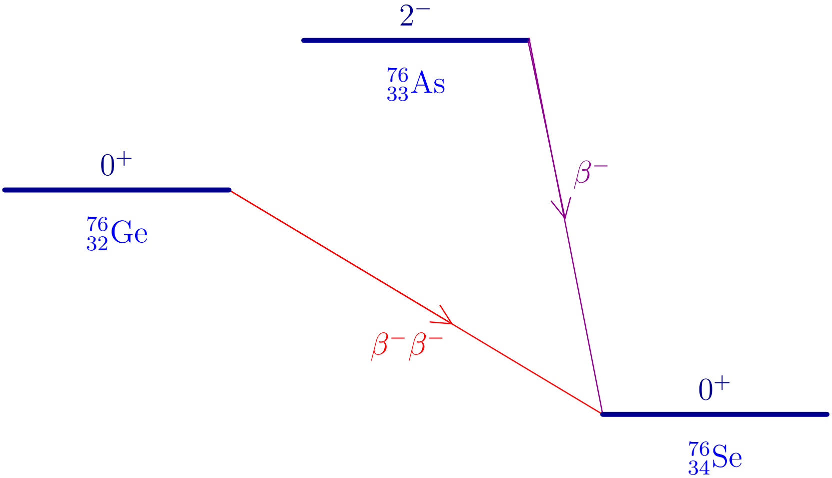
  $$\simeq 2 \times 10^{-5} + 6 \times 10^{-5} \, \mathrm{eV}^2$$

- $m_\beta \lesssim 4 \times 10^{-2} \, \mathrm{eV}$

  $\Downarrow$

  Normal Spectrum

Neutrinoless Double-Beta Decay



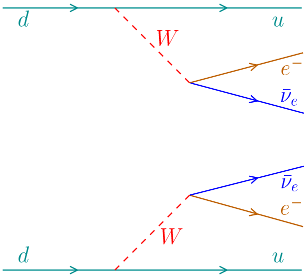
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



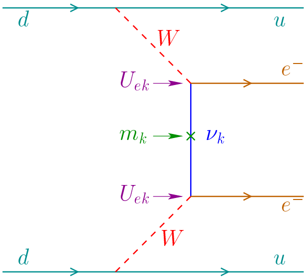
Neutrinoless Double-β Decay: $\Delta L = 2$

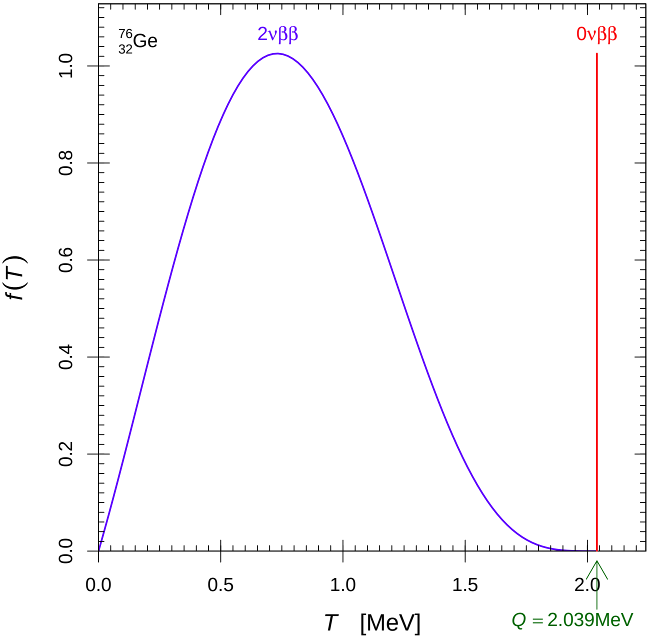
$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass

$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$$





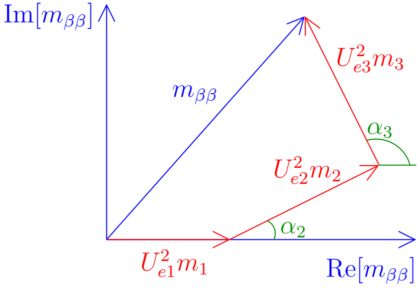
$^{76}_{32}$Ge
2νββ
0νββ
$f(T)$
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
T [MeV]
$Q = 2.039$MeV

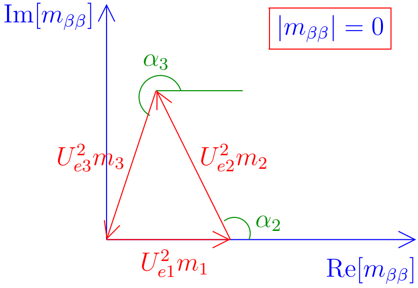
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

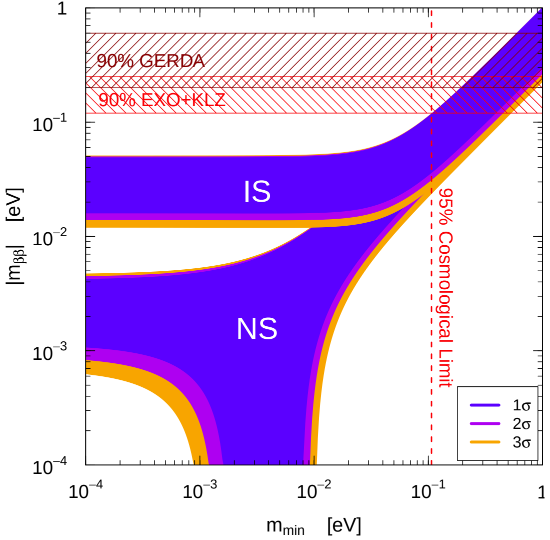
$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$





Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}}(1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12}\sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13}\sqrt{\Delta m^2_{\text{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\,\text{eV}$$

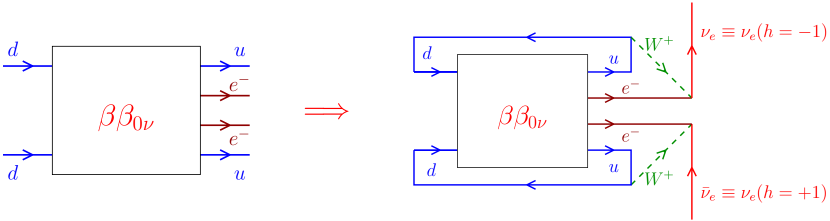
$m_1 \gtrsim 10^{-3}\,\text{eV} \Rightarrow$ cancellation?

$|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ Normal Spectrum

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

$|m_{\beta\beta}|$ can vanish because of unfortunate cancellations among m_1, m_2, m_3 contributions or because neutrinos are Dirac

$\beta\beta_{0\nu}$ decay can be generated by another mechanism beyond SM



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term: $\boxed{\mathcal{L}^{\mathrm{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right)}$

four-loop diagram calculation: $m_{ee} \sim 10^{-24}\,\mathrm{eV}$ [Duerr, Lindner, Merle, JHEP 06 (2011) 091]

- In any case finding $\beta\beta_{0\nu}$ decay is important information to solve the Dirac-Majorana question in favor of Majorana
- On the other hand, it is not possible to prove experimentally that neutrinos are Dirac.
  A Dirac neutrino is equivalent to 2 Majorana neutrinos with the same mass.
  Impossible to prove experimentally that mass splitting is exactly zero.

Sterile Neutrinos

- I consider sterile neutrinos with mass scale $\sim 1\,\mathrm{eV}$ in light of short-baseline Reactor Anomaly, Gallium Anomaly, LSND.
- Other possibilities (not incompatible):
  - Very light sterile neutrinos with mass scale $\ll 1\,\mathrm{eV}$: important for solar neutrino phenomenology

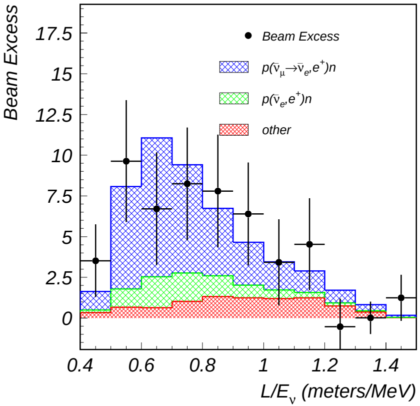
    [Das, Pulido, Picariello, PRD 79 (2009) 073010]
    [de Holanda, Smirnov, PRD 83 (2011) 113011]
  - Heavy sterile neutrinos with mass scale $\gg 1\,\mathrm{eV}$: could be Warm Dark Matter

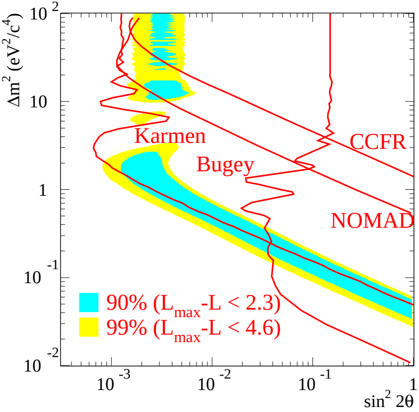
    [Kusenko, Phys. Rept. 481 (2009) 1]
    [Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191]
    [Drewes, IJMPE, 22 (2013) 1330019]

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e \qquad L \simeq 30\,\text{m} \qquad 20\,\text{MeV} \leq E \leq 200\,\text{MeV}$



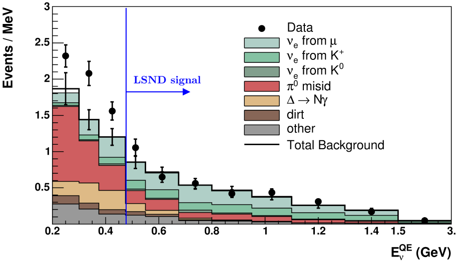


3.8σ excess $\qquad \Delta m^2_{\text{LSND}} \gtrsim 0.2\,\text{eV}^2 \qquad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

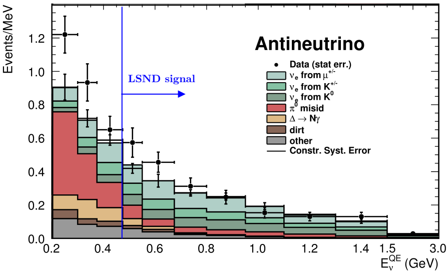
MiniBooNE

$L \simeq 541\,\text{m} \qquad 200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]

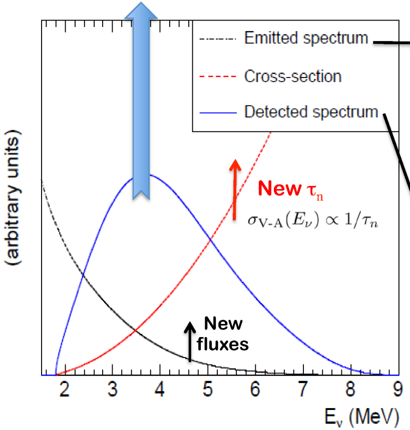


- Purpose: check LSND signal.
- Different L and E.
- Similar L/E (oscillations).
- LSND signal: $E > 475\,\text{MeV}$.

- Agreement with LSND signal?
- CP violation?
- Low-energy anomaly!

New Reactor $\bar{\nu}_e$ Fluxes

Increased prediction of detected flux by 6.5%



i) **Neutrino Emission:**

- **Improved reactor neutrino spectra $\to$ +3.5%**
- **Accounting for long-lived isotopes in reactors $\to$ +1%**

ii) **Neutrino Detection:**

- **Reevaluation of $\sigma_{\text{IBD}} \to$ +1.5% (evolution of the neutron life time)**
- **Reanalysis of all SBL experiments**

[T. Lasserre, TAUP 2013]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

[update in White Paper, arXiv:1204.5379]

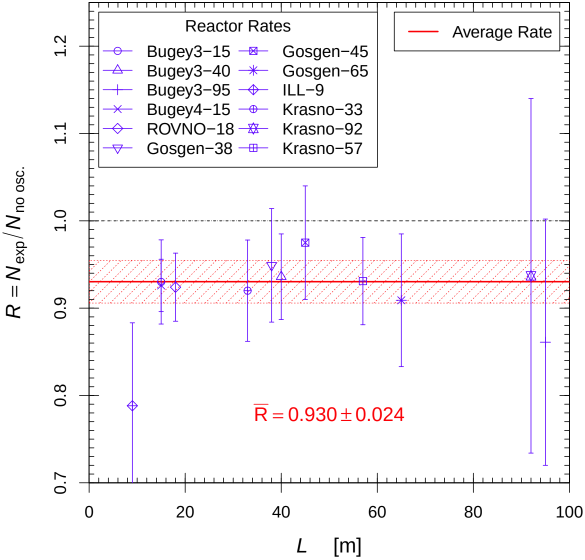
new reactor $\bar{\nu}_e$ fluxes

[Mueller et al, PRC 83 (2011) 054615]

[Huber, PRC 84 (2011) 024617]

$\sim 2.8\sigma$ anomaly

[see also:
Sinev, arXiv:1103.2452;
Ciuffoli, Evslin, Li, JHEP 12 (2012) 110;
Zhang, Qian, Vogel, PRD 87 (2013) 073018;
Ivanov et al, PRC 88 (2013) 055501]



Gallium Anomaly

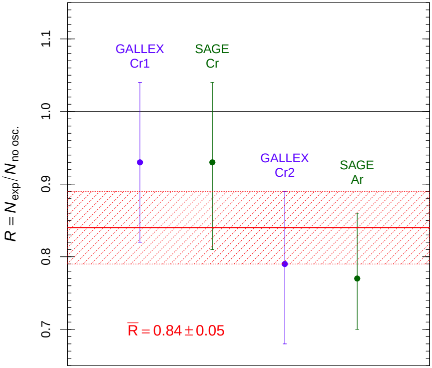
Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\qquad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

Anomaly supported by new ${}^{71}\text{Ga}({}^{3}\text{He}, {}^{3}\text{H}){}^{71}\text{Ge}$ cross section measurement

[Frekers et al., PLB 706 (2011) 134]



$E \sim 0.7\,\text{MeV}$

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$

$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

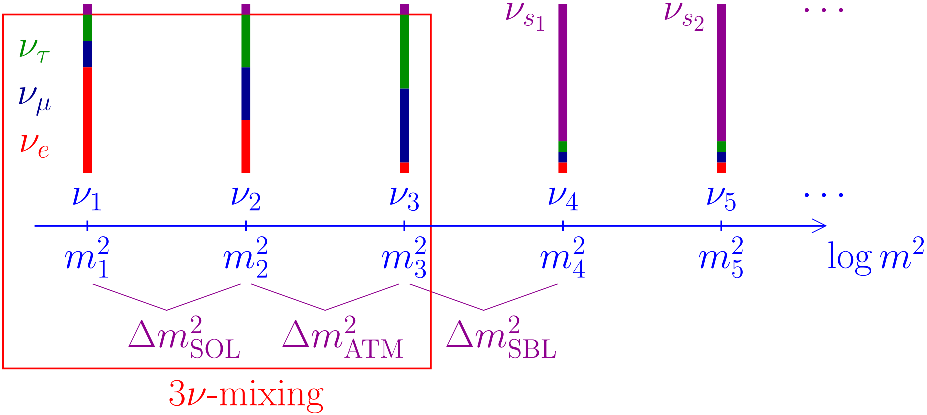
$\sim 2.9\sigma$ anomaly

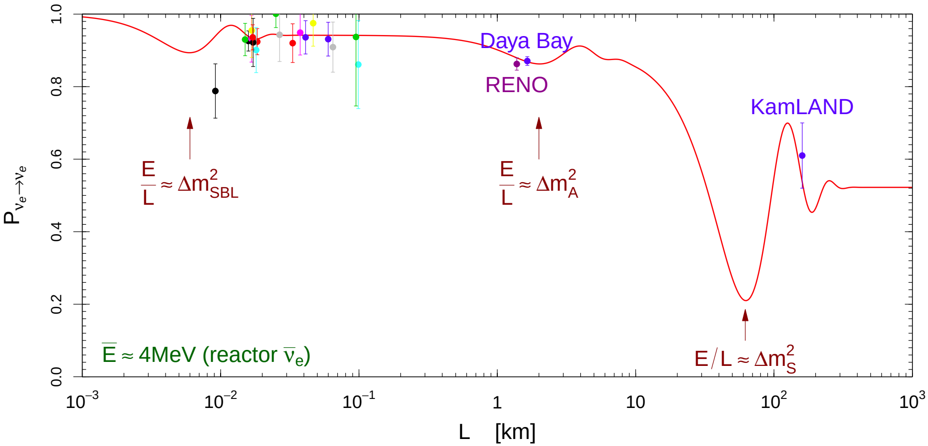
[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807]

[Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344; MPLA 22 (2007) 2499; PRD 78 (2008) 073009; PRC 83 (2011) 065504; PRD 86 (2012) 113014]

[Mention et al, PRD 83 (2011) 073006]

Beyond Three-Neutrino Mixing: Sterile Neutrinos





$P_{\nu_e \to \nu_e}$
1.0
0.8
0.6
0.4
0.2
0.0
Daya Bay
RENO
KamLAND
$\frac{E}{L} \approx \Delta m^2_{SBL}$
$\frac{E}{L} \approx \Delta m^2_{A}$
$E/L \approx \Delta m^2_{S}$
$\overline{E} \approx 4\text{MeV}$ (reactor $\overline{\nu}_e$)
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
10^{3}
L [km]

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

$$P_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\alpha}} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing: $|U_{e4}|^2 \ll 1\,,\ |U_{\mu 4}|^2 \ll 1\,,\ |U_{\tau 4}|^2 \ll 1\,,\ |U_{s4}|^2 \simeq 1$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

but CP violation is not observable in SBL experiments!

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m^2_{kj} L / 4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\phi_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\phi_{51}$$
$$+ 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin\phi_{41} \sin\phi_{51} \cos(\phi_{54} \overset{(+)}{-} \eta)$$

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2\phi_{41} + |U_{\alpha 5}|^2 \sin^2\phi_{51})$$
$$- 4|U_{\alpha 4}|^2|U_{\alpha 5}|^2 \sin^2\phi_{54}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Archidiacono et al, PRD 86 (2012) 065028; Conrad et al, AHEP 2013 (2013) 163897; Archidiacono et al, PRD 87 (2013) 125034; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008; Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

- Good: CP violation
- Bad: Two massive sterile neutrinos at the eV scale!

  4 more parameters: $\underbrace{\Delta m^2_{41}, |U_{e4}|^2, |U_{\mu 4}|^2}_{3+1}, \Delta m^2_{51}, |U_{e5}|^2, |U_{\mu 5}|^2, \eta$

3+1: Appearance vs Disappearance

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2 \left(1 - |U_{\mu 4}|^2\right) \simeq 4|U_{\mu 4}|^2$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

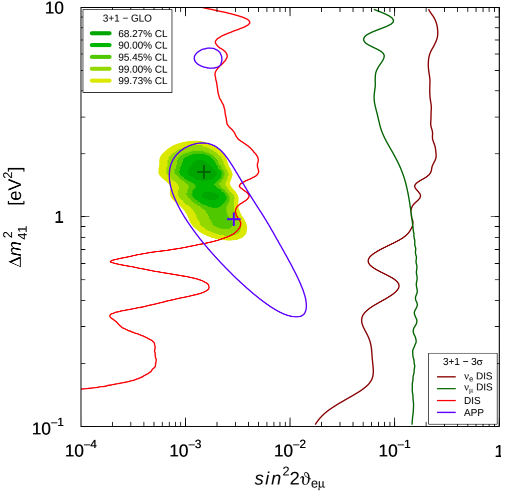
- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, IJMPA 12 (1997) 3669-3694]

[Bilenky, Giunti, Grimus, EPJC 1 (1998) 247]

3+1 Global Fit

[Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008]



MiniBooNE $E > 475\,\text{MeV}$

GoF = 29%   PGoF = 9%

- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND (Y), MiniBooNE (?), OPERA (N), ICARUS (N), KARMEN (N), NOMAD (N), BNL-E776 (N)
- DIS ν_e & $\bar{\nu}_e$: Reactors (Y), Gallium (Y), ν_eC (N), Solar (N)
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW (N), MINOS (N), Atmospheric (N), MiniBooNE/SciBooNE (N)

No Osc. excluded at 6.2σ

$\Delta\chi^2/\text{NDF} = 46.2/3$

[different approach and conclusions: Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- $\chi^2_{\rm min}$ has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\rm D} - N_{\rm P}$$

$N_{\rm D}$ = Number of Data $\qquad$ $N_{\rm P}$ = Number of Fitted Parameters

- $\langle \chi^2_{\rm min} \rangle = \text{NDF}$ $\qquad$ $\text{Var}(\chi^2_{\rm min}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\rm min}}^{\infty} p_{\chi^2}(z, \text{NDF})\, dz$ $\qquad$ $p_{\chi^2}(z,n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

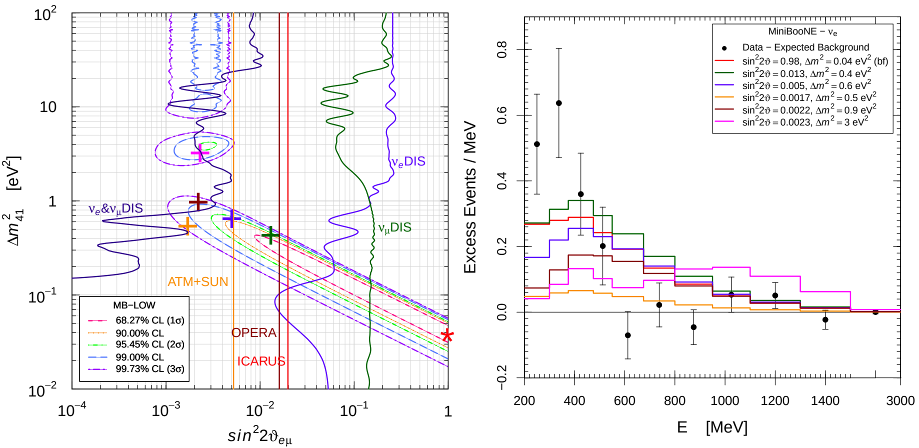
Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\rm PGoF} = (\chi^2_{\rm min})_{\rm A+B} - [(\chi^2_{\rm min})_{\rm A} + (\chi^2_{\rm min})_{\rm B}]$
- $\chi^2_{\rm PGoF}$ has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\rm PGoF} = N_{\rm P}^{\rm A} + N_{\rm P}^{\rm B} - N_{\rm P}^{\rm A+B}$$

- $\text{PGoF} = \int_{\chi^2_{\rm PGoF}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\rm PGoF})\, dz$

MiniBooNE Low-Energy Excess?

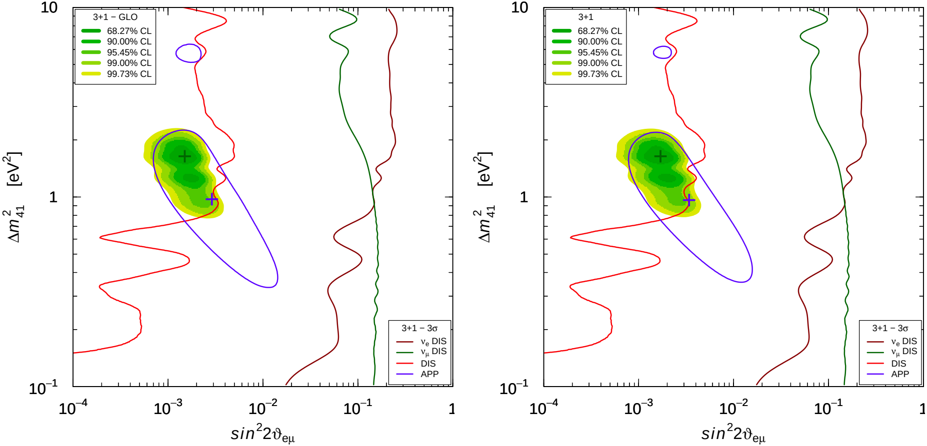


- No fit of low-energy excess for realistic $\sin^2 2\vartheta_{e\mu} \lesssim 5 \times 10^{-3}$
- APP-DIS PGoF $= 0.1\%$
- Neutrino energy reconstruction problem?

[Martini, Ericson, Chanfray, PRD 85 (2012) 093012; PRD 87 (2013) 013009]

MiniBooNE Impact on SBL Oscillations?

with MiniBooNE | without MiniBooNE



$\mathrm{GoF} = 29\%$ $\mathrm{PGoF} = 9\%$

No Osc. excluded at 6.2σ

$\Delta\chi^2/\mathrm{NDF} = 46.2/3$

$\mathrm{GoF} = 19\%$ $\mathrm{PGoF} = 8\%$

No Osc. excluded at 6.3σ

$\Delta\chi^2/\mathrm{NDF} = 47.1/3$

Without LSND: No Osc. excluded only at 2.1σ $(\Delta\chi^2/\mathrm{NDF} = 8.3/3)$

3+2

- 3+2 should be preferred to 3+1 only if
  - there is evidence of two peaks of the probability corresponding to two Δm^2's

    or
  - there is CP-violating difference of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions

- 2008 ν + 2010 $\bar{\nu}$ MiniBooNE data indicated ν–$\bar{\nu}$ difference

  $\Downarrow$

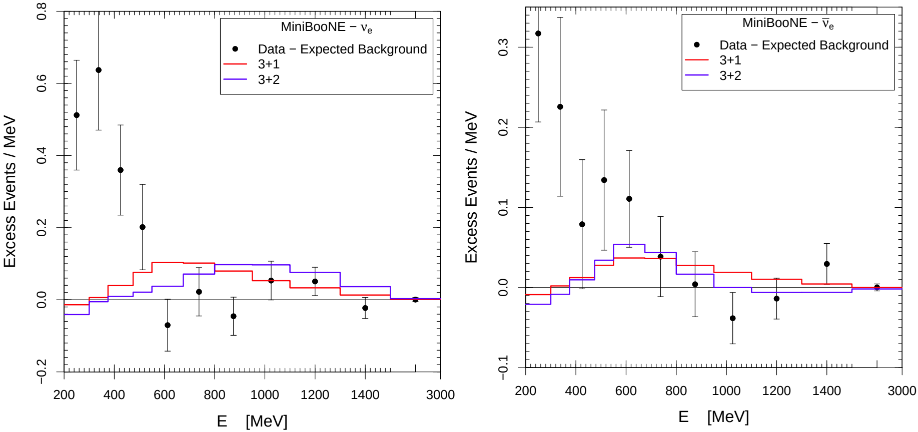
  reasonable and useful to consider 3+2

- ν–$\bar{\nu}$ difference almost disappeared with 2012 $\bar{\nu}$ data

- Okkam razor: 3+1 is enough!

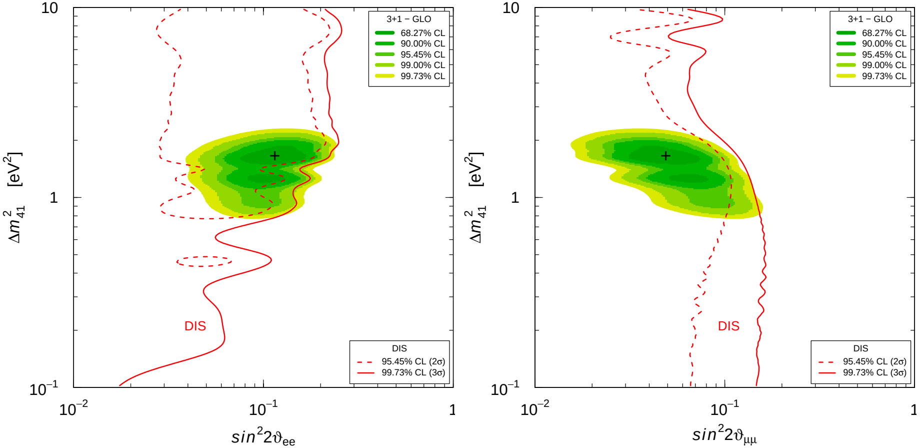
- Different approach and conclusions:
  - Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050:
    Use all MiniBooNE data. No 3+1 global fit. 3+2 slightly preferred? Small allowed region.
  - Conrad, Ignarra, Karagiorgi, Shaevitz, Spitz, AHEP 2013 (2013) 163897:
    Use all MiniBooNE data. 3+2 strongly preferred. Very small allowed regions.

MiniBooNE Low-Energy Excess?



- 3+1: GoF = 6% PGoF = 0.2%
- 3+2: GoF = 8% PGoF = 0.1%

ν_e and ν_μ Disappearance



Many Exciting New Experiments and Projects

- Reactor $\bar{\nu}_e$ Disappearance:
  - Nucifer (OSIRIS, Saclay), Stereo (ILL, Grenoble) [arXiv:1204.5379]
  - DANSS (Kalinin Nuclear Power Plant, Russia) [arXiv:1304.3696], POSEIDON (PIK, Gatchina, Russia) [arXiv:1204.2449]
  - SCRAAM (San Onofre, California) [arXiv:1204.5379]
  - CARR (China Advanced Research Reactor) [arXiv:1303.0607]
  - Neutrino-4 (SM-3, Dimitrovgrad, Russia), SOLID (BR2, Belgium), Hanaro (Korea) [D. Lhuillier, EPSHEP 2013]
- Radioactive Source ν_e and $\bar{\nu}_e$ Disappearance:
  - SOX (Borexino, Gran Sasso, Italy) [arXiv:1304.7721]
  - CeLAND (^{144}Ce@KamLAND, Japan) [arXiv:1107.2335]
  - SAGE (Baksan, Russia) [arXiv:1006.2103]
  - IsoDAR (DAEδALUS, USA) [arXiv:1210.4454, arXiv:1307.2949]
  - SNO+, Daya Bay, RENO [T. Lasserre, Neutrino 2012]
- Accelerator $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$ Appearance:
  - ICARUS/NESSIE (CERN) [arXiv:1304.2047, arXiv:1306.3455]
  - nuSTORM [arXiv:1308.0494]
  - OscSNS (Oak Ridge, USA) [arXiv:1305.4189, arXiv:1307.7097]

Effects of light sterile neutrinos can be also seen in:

- Solar neutrinos

  [Dooling et al, PRD 61 (2000) 073011, Gonzalez-Garcia et al, PRD 62 (2000) 013005;
  Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301;
  Li et al, PRD 80 (2009) 113007, PRD 87, 113004 (2013), JHEP 1308 (2013) 056;
  Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

- Atmospheric neutrinos

  [Goswami, PRD 55 (1997) 2931;
  Bilenky, Giunti, Grimus, Schwetz, PRD 60 (1999) 073007;
  Maltoni, Schwetz, Tortola, Valle, NPB 643 (2002) 321, PRD 67 (2003) 013011;
  Choubey, JHEP 12 (2007) 014;
  Razzaque, Smirnov, JHEP 07 (2011) 084, PRD 85 (2012) 093010;
  Gandhi, Ghoshal, PRD 86 (2012) 037301;
  Esmaili, Halzen, Peres, JCAP 1211 (2012) 041;
  Esmaili, Smirnov, arXiv:1307.6824]

- Supernova neutrinos

  [Caldwell, Fuller, Qian, PRD 61 (2000) 123005;
  Peres, Smirnov, NPB 599 (2001);
  Sorel, Conrad, PRD 66 (2002) 033009;
  Tamborra, Raffelt, Huedepohl, Janka, JCAP 1201 (2012) 013;
  Wu, Fischer, Martinez-Pinedo, Qian, arXiv:1305.2382]

Conclusions

- Robust Three-Neutrino Mixing Paradigm.
  Open problems: $\vartheta_{23} \lesseqgtr 45^\circ$?, Mass Hierarchy, CP Violation, Absolute Mass Scale, Dirac or Majorana?

- Very interesting indications of light sterile neutrinos with $m_s \approx 1\,\text{eV}$:
  - LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal.
  - Reactor $\bar{\nu}_e$ disappearance.
  - Gallium ν_e disappearance.

- Many promising projects to test in a few years short-baseline ν_e and $\bar{\nu}_e$ disappearance with reactors and radioactive sources.

- More difficult (expensive) projects to check the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal are under discussion.

- Cosmology:
  - Important effects of sterile neutrinos.
  - Implications depend on theoretical framework and considered data set.
  - Cosmological indications must be checked by laboratory experiments.