

Neutrino Theory and Phenomenology

PART 1

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Neutrino Unbound: <http://www.nu.to.infn.it>

NBIA PhD School

Neutrinos underground & in the heavens

The Niels Bohr International Academy, Copenhagen, Denmark

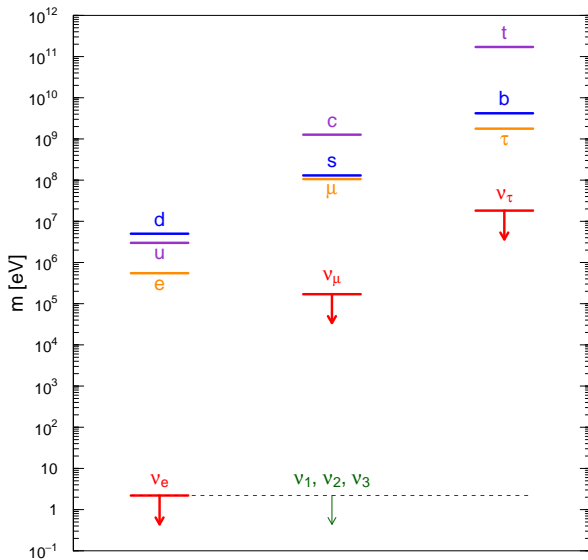
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Neutrino Theory and Phenomenology

- Neutrino Masses and Mixing
- Neutrino Oscillations in Vacuum
- Neutrino Oscillations in Matter

Neutrino Masses and Mixing

Fermion Mass Spectrum



Dirac Mass

▶ Dirac Equation: $(i\partial - m)\nu(x) = 0$ ($\partial \equiv \gamma^\mu \partial_\mu$)

▶ Dirac Lagrangian: $\mathcal{L}(x) = \bar{\nu}(x)(i\partial - m)\nu(x)$

▶ Chiral decomposition: $\nu_L \equiv P_L \nu$, $\nu_R \equiv P_R \nu$, $\nu = \nu_L + \nu_R$

Left and Right-handed Projectors: $P_L \equiv \frac{1 - \gamma^5}{2}$, $P_R \equiv \frac{1 + \gamma^5}{2}$

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = P_R P_L = 0$$

$$\mathcal{L} = \bar{\nu}_L i\partial \nu_L + \bar{\nu}_R i\partial \nu_R - m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$$

▶ In SM only ν_L by assumption \implies no neutrino mass

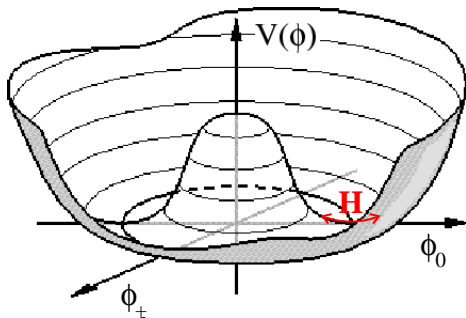
Note that all the other elementary fermion fields (charged leptons and quarks) have both left and right-handed components

▶ Oscillation experiments have shown that **neutrinos are massive**

▶ Simplest and natural extension of the SM: consider also ν_R as for all the other elementary fermion fields

Higgs Mechanism in SM

- ▶ Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ $|\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$
- ▶ Higgs Lagrangian: $\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$
- ▶ Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$
- ▶ $\mu^2 < 0$ and $\lambda > 0 \implies V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$
$$v \equiv \sqrt{-\frac{\mu^2}{\lambda}} = (\sqrt{2} G_F) \simeq 246 \text{ GeV}$$
- ▶ Vacuum: V_{min} for $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- ▶ Spontaneous Symmetry Breaking: $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$



► Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \Rightarrow |\Phi|^2 = \frac{v^2}{2} + vH + \frac{1}{2} H^2$

► $V = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \simeq 126 \text{ GeV}$$

$$-\mu^2 \simeq (89 \text{ GeV})^2 \quad \lambda = -\frac{\mu^2}{v^2} \simeq 0.13$$

SM Extension: Dirac ν Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad \ell_R \quad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = -y^\ell \overline{L}_L \Phi \ell_R - y^\nu \overline{L}_L \tilde{\Phi} \nu_R + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{H,L} = & -\frac{y^\ell}{\sqrt{2}} (\overline{\nu}_L \quad \overline{\ell}_L) \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & -\frac{y^\nu}{\sqrt{2}} (\overline{\nu}_L \quad \overline{\ell}_L) \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \text{H.c.} \end{aligned}$$

$$\mathcal{L}_{H,L} = -y^\ell \frac{v}{\sqrt{2}} \bar{\ell}_L \ell_R - y^\nu \frac{v}{\sqrt{2}} \bar{\nu}_L \nu_R$$

$$- \frac{y^\ell}{\sqrt{2}} \bar{\ell}_L \ell_R H - \frac{y^\nu}{\sqrt{2}} \bar{\nu}_L \nu_R H + \text{H.c.}$$

$$m_\ell = y^\ell \frac{v}{\sqrt{2}}$$

$$m_\nu = y^\nu \frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^\ell}{\sqrt{2}} = \frac{m_\ell}{v}$$

$$g_{\nu H} = \frac{y^\nu}{\sqrt{2}} = \frac{m_\nu}{v}$$

$$v = \left(\sqrt{2} G_F \right)^{1/2} = 246 \text{ GeV}$$

PROBLEM: $y^\nu \lesssim 10^{-11} \ll y^e \sim 10^{-6}$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}^D = - \sum_{\alpha, \beta=e, \mu, \tau} \left[Y_{\alpha\beta}^{le} \overline{L'_{\alpha L}} \Phi \ell'_{\beta R} + Y_{\alpha\beta}^{l\nu} \overline{L'_{\alpha L}} \tilde{\Phi} \nu'_{\beta R} \right] + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\mathcal{L}^D = - \sum_{\alpha, \beta = e, \mu, \tau} \left[\frac{v}{\sqrt{2}} Y_{\alpha\beta}^{i\ell} \overline{\ell'_{\alpha L}} \ell'_{\beta R} + \frac{v}{\sqrt{2}} Y_{\alpha\beta}^{i\nu} \overline{\nu'_{\alpha L}} \nu'_{\beta R} \right] + \text{H.c.}$$

$$\mathcal{L}^D = - \left[\overline{\ell'_L} M^{i\ell} \ell'_R + \overline{\nu'_L} M^{i\nu} \nu'_R \right] + \text{H.c.}$$

$$\ell'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \quad \ell'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$M^{i\ell} = \frac{v}{\sqrt{2}} Y^{i\ell}$$

$$M^{i\nu} = \frac{v}{\sqrt{2}} Y^{i\nu}$$

$$M^{i\ell} \equiv \begin{pmatrix} M^{i\ell}_{ee} & M^{i\ell}_{e\mu} & M^{i\ell}_{e\tau} \\ M^{i\ell}_{\mu e} & M^{i\ell}_{\mu\mu} & M^{i\ell}_{\mu\tau} \\ M^{i\ell}_{\tau e} & M^{i\ell}_{\tau\mu} & M^{i\ell}_{\tau\tau} \end{pmatrix}$$

$$M^{i\nu} \equiv \begin{pmatrix} M^{i\nu}_{ee} & M^{i\nu}_{e\mu} & M^{i\nu}_{e\tau} \\ M^{i\nu}_{\mu e} & M^{i\nu}_{\mu\mu} & M^{i\nu}_{\mu\tau} \\ M^{i\nu}_{\tau e} & M^{i\nu}_{\tau\mu} & M^{i\nu}_{\tau\tau} \end{pmatrix}$$

$$\mathcal{L}^D = -\overline{\ell'_L} M^{\ell\ell} \ell'_R - \overline{\nu'_L} M^{\nu\nu} \nu'_R + \text{H.c.}$$

Diagonalization of $M^{\ell\ell}$ and $M^{\nu\nu}$ with **unitary** V_L^ℓ , V_R^ℓ , V_L^ν , V_R^ν

$$\ell'_L = V_L^\ell \ell_L \quad \ell'_R = V_R^\ell \ell_R \quad \nu'_L = V_L^\nu \mathbf{n}_L \quad \nu'_R = V_R^\nu \mathbf{n}_R$$

Important general remark: unitary transformations are allowed because they leave invariant the kinetic terms in the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \overline{\ell'_L} i \not{\partial} \ell'_L + \overline{\ell'_R} i \not{\partial} \ell'_R + \overline{\nu'_L} i \not{\partial} \nu'_L + \overline{\nu'_R} i \not{\partial} \nu'_R \\ &= \overline{\ell_L} V_L^{\ell\dagger} i \not{\partial} V_L^\ell \ell_L + \dots \\ &= \overline{\ell_L} i \not{\partial} \ell_L + \overline{\ell_R} i \not{\partial} \ell_R + \overline{\nu_L} i \not{\partial} \nu_L + \overline{\nu_R} i \not{\partial} \nu_R \end{aligned}$$

$$\mathcal{L}^D = -\overline{\ell'_L} M'^{\ell} \ell'_R - \overline{\nu'_L} M'^{\nu} \nu'_R + \text{H.c.}$$

$$\ell'_L = V_L^{\ell} \ell_L \quad \ell'_R = V_R^{\ell} \ell_R \quad \nu'_L = V_L^{\nu} \mathbf{n}_L \quad \nu'_R = V_R^{\nu} \mathbf{n}_R$$

$$\mathcal{L}^D = -\overline{\ell_L} V_L^{\ell\dagger} M'^{\ell} V_R^{\ell} \ell_R - \overline{\nu_L} V_L^{\nu\dagger} M'^{\nu} V_R^{\nu} \nu_R + \text{H.c.}$$

$$V_L^{\ell\dagger} M'^{\ell} V_R^{\ell} = M^{\ell} \quad M_{\alpha\beta}^{\ell} = m_{\alpha}^{\ell} \delta_{\alpha\beta} \quad (\alpha, \beta = e, \mu, \tau)$$

$$V_L^{\nu\dagger} M'^{\nu} V_R^{\nu} = M^{\nu} \quad M_{kj}^{\nu} = m_k^{\nu} \delta_{kj} \quad (k, j = 1, 2, 3)$$

Real and Positive m_{α}^{ℓ} , m_k^{ν}

Massive Chiral Lepton Fields

$\ell_L = V_L^{\ell\dagger} \ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\ell_R = V_R^{\ell\dagger} \ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger} \nu'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger} \nu'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned}
 \mathcal{L}^D &= -\bar{\ell}_L M^\ell \ell_R - \bar{\mathbf{n}}_L M^\nu \mathbf{n}_R + \text{H.c.} \\
 &= - \sum_{\alpha=e,\mu,\tau} m_\alpha^\ell \bar{\ell}_{\alpha L} \ell_{\alpha R} - \sum_{k=1}^3 m_k^\nu \bar{\nu}_{kL} \nu_{kR} + \text{H.c.}
 \end{aligned}$$

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathcal{L}_1^{(\text{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \text{H.c.}$$

Weak Charged Current: $j_W^\rho = j_{W,L}^\rho + j_{W,Q}^\rho$

Leptonic Weak Charged Current

$$j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \bar{\ell}'_{\alpha L} \gamma^\rho \nu'_{\alpha L} = 2 \bar{\ell}'_L \gamma^\rho \nu'_L$$

$$\underline{\ell}'_L = V_L^\ell \ell_L \quad \underline{\nu}'_L = V_L^\nu \mathbf{n}_L$$

$$j_{W,L}^{\rho\dagger} = 2 \bar{\ell}'_L V_L^{\ell\dagger} \gamma^\rho V_L^\nu \mathbf{n}_L = 2 \bar{\ell}'_L \gamma^\rho V_L^{\ell\dagger} V_L^\nu \mathbf{n}_L = 2 \bar{\ell}'_L \gamma^\rho U \mathbf{n}_L$$

Mixing Matrix

$$U = V_L^{\ell\dagger} V_L^\nu$$

- ▶ **Definition:** Left-Handed Flavor Neutrino Fields

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} V_L^\nu \mathbf{n}_L = V_L^{\ell\dagger} \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- ▶ They allow us to write the **Leptonic Weak Charged Current** as in the SM:

$$j_{W,L}^{\rho\dagger} = 2 \bar{\ell}_L \gamma^\rho \nu_L = 2 \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha L} \gamma^\rho \nu_{\alpha L}$$

- ▶ Each **left-handed flavor neutrino field** is associated with the corresponding **charged lepton field** which describes a massive charged lepton:

$$j_{W,L}^{\rho\dagger} = 2 (\bar{e}_L \gamma^\rho \nu_{eL} + \bar{\mu}_L \gamma^\rho \nu_{\mu L} + \bar{\tau}_L \gamma^\rho \nu_{\tau L})$$

- ▶ In practice **left-handed flavor neutrino fields** are useful for calculations in the SM approximation of massless neutrinos (**interactions**).
- ▶ If neutrino masses must be taken into account, it is necessary to use

$$j_{W,L}^{\rho\dagger} = 2 \bar{\ell}_L \gamma^\rho U \mathbf{n}_L = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \bar{\ell}_{\alpha L} \gamma^\rho U_{\alpha k} \nu_{kL}$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ		L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0	(ν_e^c, e^+)	-1	0	0
(ν_μ, μ^-)	0	+1	0	(ν_μ^c, μ^+)	0	-1	0
(ν_τ, τ^-)	0	0	+1	(ν_τ^c, τ^+)	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Standard Model:

Lepton numbers are conserved

▶ L_e, L_μ, L_τ are conserved in the Standard Model with massless neutrinos

▶ Mass term:

$$\mathcal{L}^D = - \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^D & m_{e\mu}^D & m_{e\tau}^D \\ m_{\mu e}^D & m_{\mu\mu}^D & m_{\mu\tau}^D \\ m_{\tau e}^D & m_{\tau\mu}^D & m_{\tau\tau}^D \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

▶ L is conserved: $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \implies |\Delta L| = 0$

Mixing Matrix

$$\blacktriangleright U = V_L^{\ell\dagger} V_L^\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

- ▶ Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \quad \Longrightarrow \quad \begin{array}{ll} \frac{N(N-1)}{2} = 3 & \text{Mixing Angles} \\ \frac{N(N+1)}{2} = 6 & \text{Phases} \end{array}$$

- ▶ Not all phases are physical observables
- ▶ Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- ▶ Weak Charged Current: $j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{l_{\alpha L}} \gamma^\rho U_{\alpha k} \nu_{kL}$
- ▶ Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)

$$l_\alpha \rightarrow e^{i\varphi_\alpha} l_\alpha \quad (\alpha = e, \mu, \tau), \quad \nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3)$$
- ▶ Performing this transformation, the Charged Current becomes

$$j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{l_{\alpha L}} e^{-i\varphi_\alpha} \gamma^\rho U_{\alpha k} e^{i\varphi_k} \nu_{kL}$$

$$j_{W,L}^{\rho\dagger} = 2 \underbrace{e^{-i(\varphi_e - \varphi_1)}}_1 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{l_{\alpha L}} \underbrace{e^{-i(\varphi_\alpha - \varphi_e)}}_2 \gamma^\rho U_{\alpha k} \underbrace{e^{i(\varphi_k - \varphi_1)}}_2 \nu_{kL}$$

- ▶ There are 5 independent combinations of the phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- ▶ It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos \vartheta_{ab} \quad s_{ab} \equiv \sin \vartheta_{ab} \quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \quad 0 \leq \delta_{13} < 2\pi$$

3 Mixing Angles ϑ_{12} , ϑ_{23} , ϑ_{13} and 1 Phase δ_{13}

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23}e^{i\delta_{23}} \\ 0 & -s_{23}e^{-i\delta_{23}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12}e^{-i\delta'_{12}} & 0 \\ -s'_{12}e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

Jarlskog Rephasing Invariant

- ▶ Simplest rephasing invariants: $|U_{\alpha k}|^2 = U_{\alpha k} U_{\alpha k}^*$, $U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$

$$\Im[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}] = \pm J$$

$$J = \Im[U_{e2} U_{e3}^* U_{\mu2}^* U_{\mu3}] = \Im \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- ▶ In standard parameterization:

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos \vartheta_{13} \sin 2\vartheta_{13} \sin \delta_{13} \end{aligned}$$

- ▶ Jarlskog invariant is useful for quantifying CP violation due to $U \neq U^*$ in a parameterization-independent way.
- ▶ All measurable CP-violation effects depend on J .

CP Violation

- ▶ $U \neq U^* \implies$ CP Violation
- ▶ General conditions for CP violation (14 conditions):
 1. No two charged leptons or two neutrinos are degenerate in mass (6 conditions)
 2. No mixing angle is equal to 0 or $\pi/2$ (6 conditions)
 3. The physical phase is different from 0 or π (2 conditions)
- ▶ These 14 conditions are combined into the single condition $\det C \neq 0$

$$C = -i [M'^{\nu} M'^{\nu\dagger}, M'^{\ell} M'^{\ell\dagger}]$$

$$\det C = -2 J (m_{\nu_2}^2 - m_{\nu_1}^2) (m_{\nu_3}^2 - m_{\nu_1}^2) (m_{\nu_3}^2 - m_{\nu_2}^2) \\ (m_{\mu}^2 - m_e^2) (m_{\tau}^2 - m_e^2) (m_{\tau}^2 - m_{\mu}^2)$$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunitz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

Maximal CP Violation

- ▶ Maximal CP violation is defined as the case in which $|J|$ has its maximum possible value

$$|J|_{\max} = \frac{1}{6\sqrt{3}}$$

- ▶ In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4, \quad s_{13} = 1/\sqrt{3}, \quad \sin \delta_{13} = \pm 1$$

- ▶ This case is called **Trimaximal Mixing**. All the absolute values of the elements of the mixing matrix are equal to $1/\sqrt{3}$:

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp \frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- ▶ The unitarity of V_L^ℓ , V_R^ℓ and V_L^ν implies that the expression of the neutral weak current in terms of the lepton fields with definite masses is the same as that in terms of the primed lepton fields:

$$\begin{aligned}j_{Z,L}^\rho &= 2 g_L^\nu \overline{\nu_L'} \gamma^\rho \nu_L' + 2 g_L^\ell \overline{\ell_L'} \gamma^\rho \ell_L' + 2 g_R^\ell \overline{\ell_R'} \gamma^\rho \ell_R' \\ &= 2 g_L^\nu \overline{\mathbf{n}_L} V_L^{\nu\dagger} \gamma^\rho V_L^\nu \mathbf{n}_L + 2 g_L^\ell \overline{\ell_L} V_L^{\ell\dagger} \gamma^\rho V_L^\ell \ell_L + 2 g_R^\ell \overline{\ell_R} V_R^{\ell\dagger} \gamma^\rho V_R^\ell \ell_R \\ &= 2 g_L^\nu \overline{\mathbf{n}_L} \gamma^\rho \mathbf{n}_L + 2 g_L^\ell \overline{\ell_L} \gamma^\rho \ell_L + 2 g_R^\ell \overline{\ell_R} \gamma^\rho \ell_R\end{aligned}$$

- ▶ The unitarity of U implies the same expression for the neutral weak current in terms of the flavor neutrino fields $\nu_L = U \mathbf{n}_L$:

$$\begin{aligned}j_{Z,L}^\rho &= 2 g_L^\nu \overline{\nu_L} U \gamma^\rho U^\dagger \nu_L + 2 g_L^\ell \overline{\ell_L} \gamma^\rho \ell_L + 2 g_R^\ell \overline{\ell_R} \gamma^\rho \ell_R \\ &= 2 g_L^\nu \overline{\nu_L} \gamma^\rho \nu_L + 2 g_L^\ell \overline{\ell_L} \gamma^\rho \ell_L + 2 g_R^\ell \overline{\ell_R} \gamma^\rho \ell_R\end{aligned}$$

Lepton Numbers Violating Processes

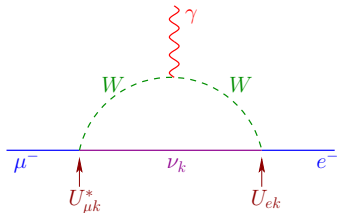
Dirac mass term allows L_e, L_μ, L_τ violating processes

Example: $\mu^\pm \rightarrow e^\pm + \gamma, \quad \mu^\pm \rightarrow e^\pm + e^+ + e^-$

$$\mu^- \rightarrow e^- + \gamma$$

$\sum_k U_{\mu k}^* U_{ek} = 0 \implies$ only part of ν_k propagator $\propto m_k$ contributes

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \frac{3\alpha}{32\pi} \underbrace{\left| \sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$



Suppression factor: $\frac{m_k}{m_W} \lesssim 10^{-11}$ for $m_k \lesssim 1 \text{ eV}$

$$(\text{BR})_{\text{the}} \lesssim 10^{-47}$$

$$(\text{BR})_{\text{exp}} \lesssim 10^{-11}$$