Neutrino Theory and Phenomenology

PART 3

Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino

carlo.giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

NBIA PhD School
Neutrinos underground & in the heavens

The Niels Bohr International Academy, Copenhagen, Denmark

23-27 June 2014

Neutrino Oscillations in Vacuum

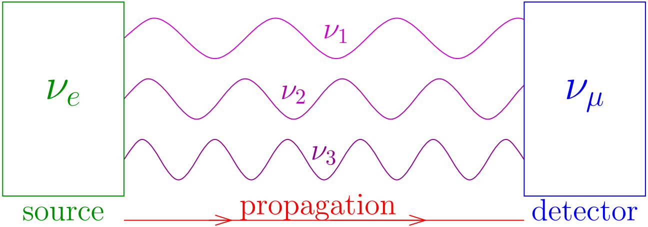
Neutrino Oscillations

- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$
\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}
$$

- U is the 3×3 unitary Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{e1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_e\rangle$$

$$E_k^2 = p^2 + m_k^2$$

at the detector there is a probability > 0 to see the neutrino as a ν_μ

Neutrino Oscillations are Flavor Transitions

$$\nu_e \to \nu_\mu \qquad \nu_e \to \nu_\tau \qquad \nu_\mu \to \nu_e \qquad \nu_\mu \to \nu_\tau$$
$$\bar{\nu}_e \to \bar{\nu}_\mu \qquad \bar{\nu}_e \to \bar{\nu}_\tau \qquad \bar{\nu}_\mu \to \bar{\nu}_e \qquad \bar{\nu}_\mu \to \bar{\nu}_\tau$$

transition probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

Early History of Neutrino Oscillations

- 1957: Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\Longrightarrow$ $\nu \leftrightarrows \bar{\nu}$
- In 1957 only one neutrino $\nu = \nu_e$ was known!
- 1958: Goldhaber, Grodzins and Sunyar measure neutrino helicity: ν_L
- Then, in weak interactions ν_L and $\bar{\nu}_R$
- Helicity conservation $\Longrightarrow$ $\nu_L \leftrightarrows \bar{\nu}_L$
- $\bar{\nu}_L$ is a sterile neutrino (Pontecorvo, 1967)
- 1962: Lederman, Schwartz and Steinberger discover ν_μ
- 1962: Maki, Nakagava, Sakata proposed a model with neutrino mixing:

$$\nu_e = \cos\vartheta\,\nu_1 + \sin\vartheta\,\nu_2$$
$$\nu_\mu = -\sin\vartheta\,\nu_1 + \cos\vartheta\,\nu_2$$

  "weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrows \nu_\mu$"
- 1967: Pontecorvo: $\nu_e \leftrightarrows \nu_\mu$ oscillations and applications (solar neutrinos)

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1\,\text{MeV}$ are detectable!

Charged-Current Processes: Threshold

$$\nu + A \to B + C$$
$$\Downarrow$$
$$s = 2 E m_A + m_A^2 \geq (m_B + m_C)^2$$
$$\Downarrow$$
$$E_{\text{th}} = \frac{(m_B + m_C)^2}{2 m_A} - \frac{m_A}{2}$$

$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$	$E_{\text{th}} = 0.233\,\text{MeV}$
$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$	$E_{\text{th}} = 0.81\,\text{MeV}$
$\bar{\nu}_e + p \to n + e^+$	$E_{\text{th}} = 1.8\,\text{MeV}$
$\nu_\mu + n \to p + \mu^-$	$E_{\text{th}} = 110\,\text{MeV}$
$\nu_\mu + e^- \to \nu_e + \mu^-$	$E_{\text{th}} \simeq \frac{m_\mu^2}{2 m_e} = 10.9\,\text{GeV}$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0 \, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO), $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

Flavor Transitions

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{CC} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\quad \nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad$ States

initial flavor: $\alpha = e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U_{\alpha k}^* \, e^{-iE_k t + i p_k x} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0,0) = \sum_k U_{\alpha k}^* U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t>0, x>0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha\to\nu_\beta}(t,x) = \left|\mathcal{A}_{\nu_\alpha\to\nu_\beta}(t,x)\right|^2 = \left|\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k}\right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(L,E) &= \left|\sum_k U_{\alpha k}^*\, e^{-i m_k^2 L/2E}\, U_{\beta k}\right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathrm{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \xrightarrow{\mathrm{CP}} \quad \nu_{\alpha L}^{\mathrm{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathrm{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathrm{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CPT}} \quad P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $\quad A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\quad \Longrightarrow \quad A_{\alpha\beta}^{\text{CPT}} = 0 \quad$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $\quad U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha\to\nu_\beta} \quad \xrightarrow{\text{CP}} \quad P_{\bar\nu_\alpha\to\bar\nu_\beta}$$

CP Asymmetries: $A_{\alpha\beta}^{\text{CP}} = P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta}$

$$\boxed{A_{\alpha\beta}^{\text{CP}}(L,E) = 4\sum_{k>j} \text{Im}\left[U_{\alpha k}^{*} U_{\beta k} U_{\alpha j} U_{\beta j}^{*}\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)}$$

Jarlskog rephasing invariant: $\quad \text{Im}\left[U_{\alpha k}^{*} U_{\beta k} U_{\alpha j} U_{\beta j}^{*}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$$

$$J \neq 0 \quad \Longleftrightarrow \quad \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

$$
\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A_{\alpha\beta}^{\text{CPT}} \\
&= P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\beta\to\bar\nu_\alpha} \\
&= P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta} \leftarrow A_{\alpha\beta}^{\text{CP}} \\
&+ P_{\bar\nu_\alpha\to\bar\nu_\beta} - P_{\nu_\beta\to\nu_\alpha} \leftarrow -A_{\beta\alpha}^{\text{CPT}} = 0 \\
&+ P_{\nu_\beta\to\nu_\alpha} - P_{\bar\nu_\beta\to\bar\nu_\alpha} \leftarrow A_{\beta\alpha}^{\text{CP}} \\
&= A_{\alpha\beta}^{\text{CP}} + A_{\beta\alpha}^{\text{CP}} \qquad \Longrightarrow \qquad \boxed{A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}}
\end{aligned}
$$

T Symmetry

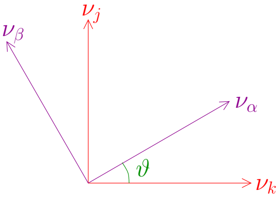
$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\mathsf{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $\quad A^{\mathsf{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$
\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\mathsf{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow A^{\mathsf{T}}_{\alpha\beta} \\
&+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \leftarrow A^{\mathsf{CP}}_{\beta\alpha} \\
&= A^{\mathsf{T}}_{\alpha\beta} + A^{\mathsf{CP}}_{\beta\alpha} \\
&= A^{\mathsf{T}}_{\alpha\beta} - A^{\mathsf{CP}}_{\alpha\beta} \qquad \Longrightarrow \qquad \boxed{A^{\mathsf{T}}_{\alpha\beta} = A^{\mathsf{CP}}_{\alpha\beta}}
\end{aligned}
$$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle$$
$$|\nu_\beta\rangle = -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m_k^2 - m_j^2$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

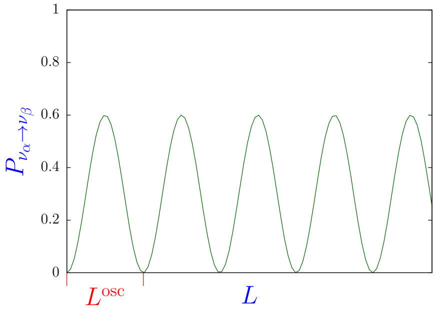
Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

oscillation phase

$$\frac{\Delta m^2 L}{4E} = 1.27\,\frac{\Delta m^2[\text{eV}^2]\,L[\text{m}]}{E[\text{MeV}]} = 1.27\,\frac{\Delta m^2[\text{eV}^2]\,L[\text{km}]}{E[\text{GeV}]}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47\,\frac{E\,[\text{MeV}]}{\Delta m^2\,[\text{eV}^2]}\,\text{m} = 2.47\,\frac{E\,[\text{GeV}]}{\Delta m^2\,[\text{eV}^2]}\,\text{km}$$



Types of Experiments

transitions due to Δm^2 observable only if $\dfrac{\Delta m^2 L}{E} \gtrsim 1 \Leftrightarrow \Delta m^2 \gtrsim \left(\dfrac{L}{E}\right)^{-1}$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$\dfrac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL

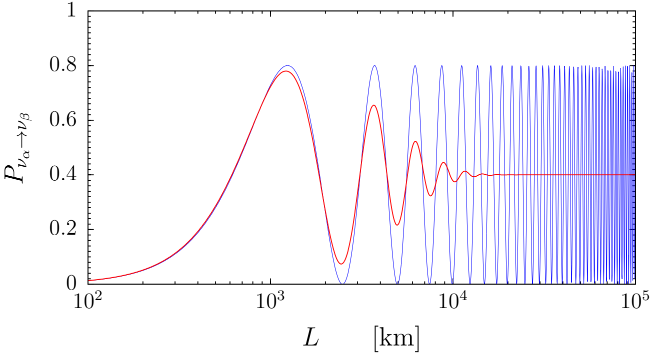
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$

KamLAND

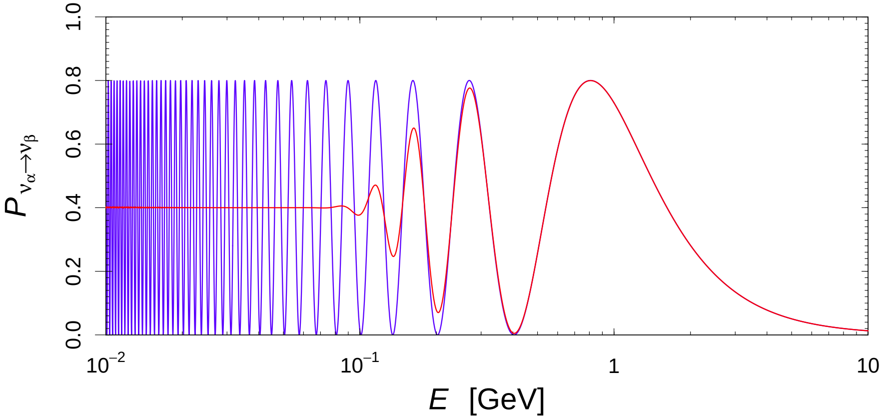
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$



$\Delta m^2 = 10^{-3}\,\text{eV}$ $\quad \sin^2 2\vartheta = 0.8 \quad \langle E\rangle = 1\,\text{GeV} \quad \sigma_E = 0.1\,\text{GeV}$

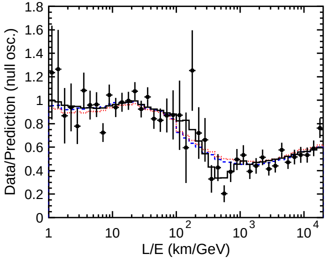
$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha\neq\beta)$$



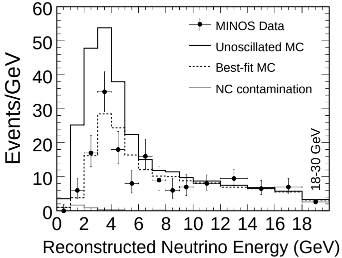
$\Delta m^2 = 10^{-3}\,\mathrm{eV}$ $\quad \sin^2 2\vartheta = 0.8$ $\quad L = 10^3\,\mathrm{km}$ $\quad \sigma_E = 0.01\,\mathrm{GeV}$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha \neq \beta)$$

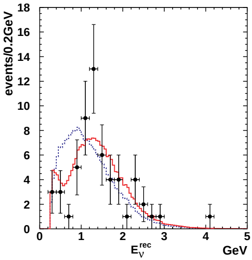
Observations of Neutrino Oscillations



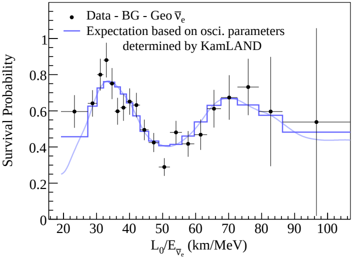
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]



[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]

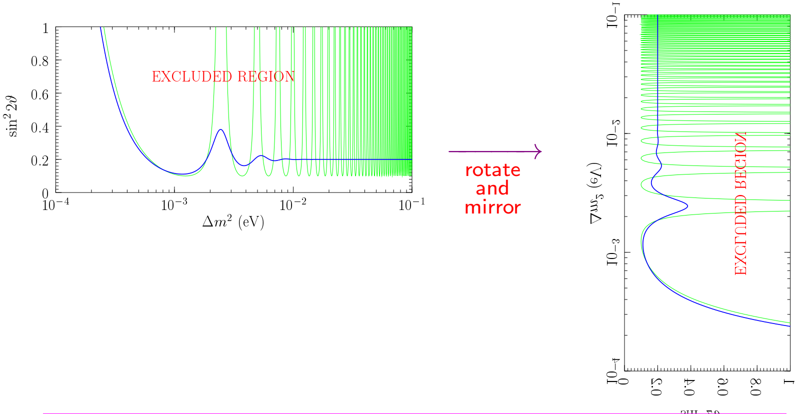


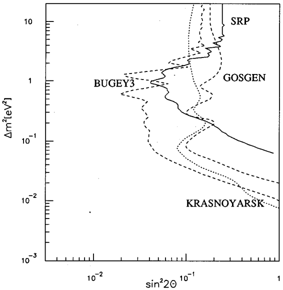
[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]

Exclusion Curves

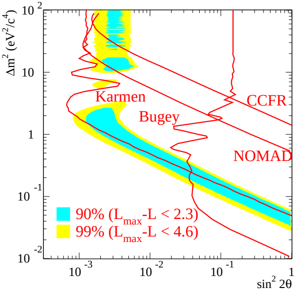
$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,dE\right] \qquad (\alpha\neq\beta)$$

$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle \leq P^{\max}_{\nu_\alpha\to\nu_\beta} \quad \Longrightarrow \quad \sin^2 2\vartheta \leq \frac{2\,P^{\max}_{\nu_\alpha\to\nu_\beta}}{1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,dE}$$

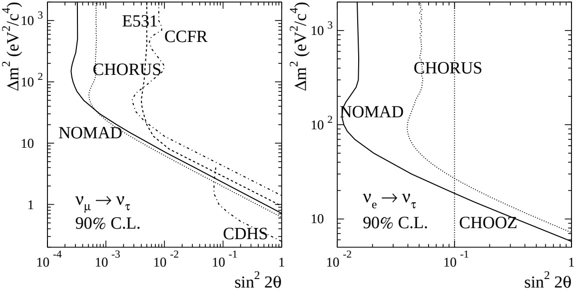




Reactor SBL Experiments: $\bar{\nu}_e \to \bar{\nu}_e$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\tau$ and $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\tau$

Experimental Evidences of Neutrino Oscillations

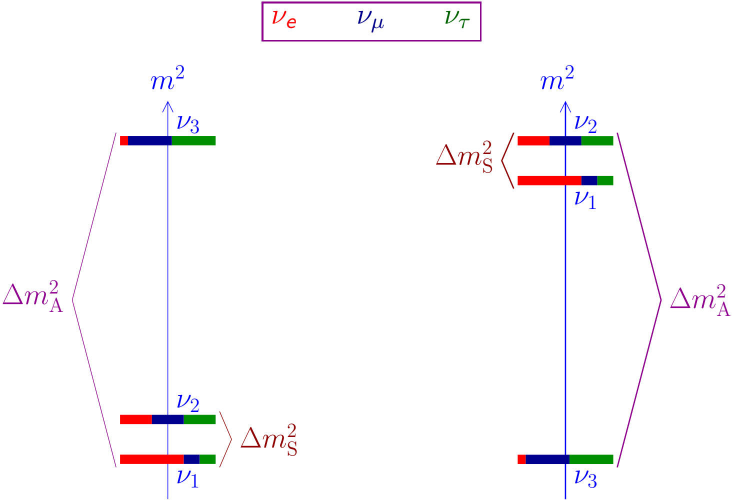
Channel	Experiments		Result
Solar $\nu_e \to \nu_\mu, \nu_\tau$	SNO, BOREXino; Super-Kamiokande; GALLEX/GNO, SAGE; Homestake, Kamiokande	$\to$	$\Delta m^2_{\text{S}} \simeq 7.5 \times 10^{-5}\,\text{eV}^2$, $\sin^2\vartheta_{\text{S}} \simeq 0.30$
VLBL Reactor $\bar{\nu}_e$ disappearance	(KamLAND)		
Atmospheric $\nu_\mu \to \nu_\tau$	Super-Kamiokande; Kamiokande, IMB; MACRO, Soudan-2	$\to$	$\Delta m^2_{\text{A}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$, $\sin^2\vartheta_{\text{A}} \simeq 0.50$
LBL Accelerator ν_μ disappearance	(K2K, MINOS, T2K)		
LBL Accelerator $\nu_\mu \to \nu_\tau$	(OPERA)		
LBL Accelerator $\nu_\mu \to \nu_e$	(T2K, MINOS)	$\to$	Δm^2_{A}, $\sin^2\vartheta_{13} \simeq 0.023$
LBL Reactor $\bar{\nu}_e$ disappearance	Daya Bay, RENO; Double Chooz		

Three-Neutrino Mixing

- $\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$
- three left-handed flavor fields: ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$
- three left-handed massive fields: ν_{1L}, ν_{2L}, ν_{3L}
- right-handed components are not needed
- in neutrino oscillations Dirac = Majorana
- only two independent Δm^2

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

- $\Delta m^2_{\text{S}} = \Delta m^2_{21} = 7.5 \pm 0.2 \times 10^{-5} \, \text{eV}^2$ uncertainty $\simeq 3\%$
- $\Delta m^2_{\text{A}} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| = 2.4 \pm 0.1 \times 10^{-3} \, \text{eV}^2$ uncertainty $\simeq 4\%$



Normal Spectrum

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Spectrum

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

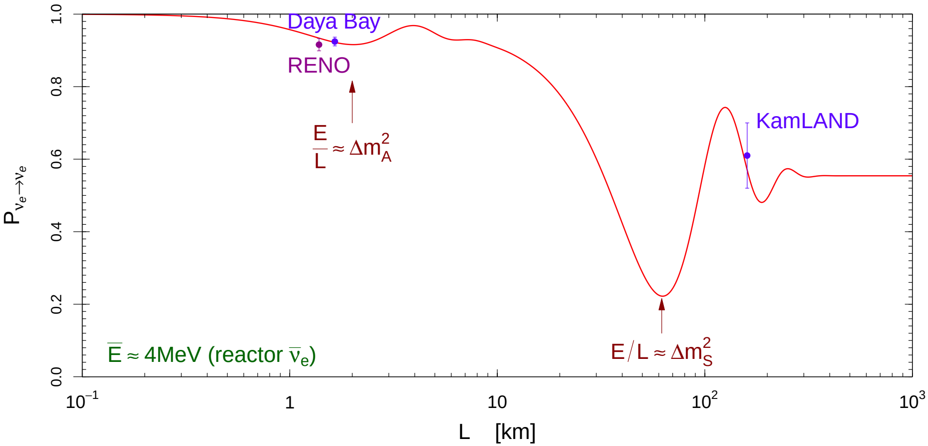
absolute scale is not determined by neutrino oscillation data

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\vartheta_{23} = \vartheta_{A}$	Daya Bay, RENO	$\vartheta_{12} = \vartheta_{S}$
$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$	Double Chooz	$\sin^2\vartheta_{12} \simeq 0.30 \pm 0.01$
$P_{\text{osc}} \propto \sin^2 2\vartheta_{23}$	T2K, MINOS	
maximal and flat	$\sin^2\vartheta_{13} \simeq 0.023 \pm 0.002$	
at $\vartheta_{23} = 45^\circ$		

$$\frac{\delta \sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \simeq 40\% \qquad \frac{\delta \sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \simeq 10\% \qquad \frac{\delta \sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \simeq 5\%$$



Daya Bay
RENO
KamLAND
E/L ≈ Δm²A
E/L ≈ Δm²S
Ē ≈ 4MeV (reactor ν̄e)
P νe→νe
L [km]
0.0
0.2
0.4
0.6
0.8
1.0
10⁻¹
1
10
10²
10³

Effective VLBL ν_e Survival Probability

$$P_{\nu_e\to\nu_e} = \left| \sum_{k=1}^{3} |U_{ek}|^2 e^{-im_k^2 L/2E} \right|^2$$

$$|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2 \quad \Longrightarrow \quad |U_{e1}|^2 \simeq \cos^2\vartheta_{12} \,, \; |U_{e2}|^2 \simeq \sin^2\vartheta_{12}$$

$$\begin{aligned}
P_{\nu_e\to\nu_e} &\simeq \left| \sum_{k=1}^{2} |U_{ek}|^2 e^{-im_k^2 L/2E} \right|^2 \\
&\simeq \left| \cos^2\vartheta_{12} e^{-im_1^2 L/2E} + \sin^2\vartheta_{12} e^{-im_2^2 L/2E} \right|^2 \\
&= \cos^4\vartheta_{12} + \sin^4\vartheta_{12} + 2\cos^2\vartheta_{12}\cos^2\vartheta_{12}\cos\left(\frac{\Delta m_{21}^2 L}{2E}\right) \\
&= 1 - \sin^2 2\vartheta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)
\end{aligned}$$

Effective ATM and LBL Oscillation Probabilities

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{3} U_{\alpha k}^* U_{\beta k} e^{-im_k^2 L/2E} \right|^2 * \left| e^{im_1^2 L/2E} \right|^2$$

$$= \left| \sum_{k=1}^{3} U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^* U_{\beta 3}$$

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta} &= \left|\delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}\left[1 - \exp\left(-i\frac{\Delta m_{31}^2 L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \left(2 - 2\cos\frac{\Delta m_{31}^2 L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha 3}|^2 \left(1 - \cos\frac{\Delta m_{31}^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2\right)\left(1 - \cos\frac{\Delta m_{31}^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2\right)\sin^2\frac{\Delta m_{31}^2 L}{4E}
\end{aligned}
$$

$$
\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)
$$

$$
\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)
$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \quad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & \boxed{U_{e3}} \\ U_{\mu 1} & U_{\mu 2} & \boxed{U_{\mu 3}} \\ U_{\tau 1} & U_{\tau 2} & \boxed{U_{\tau 3}} \end{pmatrix}$$

LBL

$$\sin^2 2\vartheta_{ee} \ll 1$$

$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

Effective ATM and LBL Oscillation Amplitudes

- ν_e disappearance: Chooz, Palo Verde, Daya Bay, RENO, Double Chooz

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) = 4 s_{13}^2 c_{13}^2 = \sin^2 2\vartheta_{13} \simeq 0.09$$

- ν_μ disappearance: K2K, MINOS, T2K

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right) = 4 c_{13}^2 s_{23}^2 \left(1 - c_{13}^2 s_{23}^2\right)$$

$$\simeq 4 s_{23}^2 \left(1 - s_{23}^2\right) = \sin^2 2\vartheta_{23} \simeq 1$$

- $\nu_\mu \to \nu_e$: T2K, MINOS

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2 |U_{\mu 3}|^2 = 4 s_{13}^2 c_{13}^2 s_{23}^2 = \sin^2 2\vartheta_{13} \sin^2 \vartheta_{23}$$

$$\simeq \frac{1}{2} \sin^2 2\vartheta_{13} \simeq 0.045$$

- $\nu_\mu \to \nu_\tau$: OPERA

$$\sin^2 2\vartheta_{\mu\tau} = 4|U_{\mu 3}|^2 |U_{\tau 3}|^2 = 4 c_{13}^4 s_{23} c_{23} = c_{13}^4 \sin^2 2\vartheta_{23} \simeq \sin^2 2\vartheta_{23} \simeq 1$$

CP Violation?

- In this approximation there is no observable CP-violation effect!
- CP-violation can be observed only with sensitivity to Δm^2_{21}: in vacuum

$$
\begin{aligned}
A^{\text{CP}}_{\alpha\beta} &= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \\
&= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)
\end{aligned}
$$

$$J_{\alpha\beta} = \text{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

- Necessary conditions for observation of CP violation:
  - Sensitivity to all mixing angles, including small ϑ_{13}
  - Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}