OVERVIEW OF NEUTRINO MASSES AND MIXING

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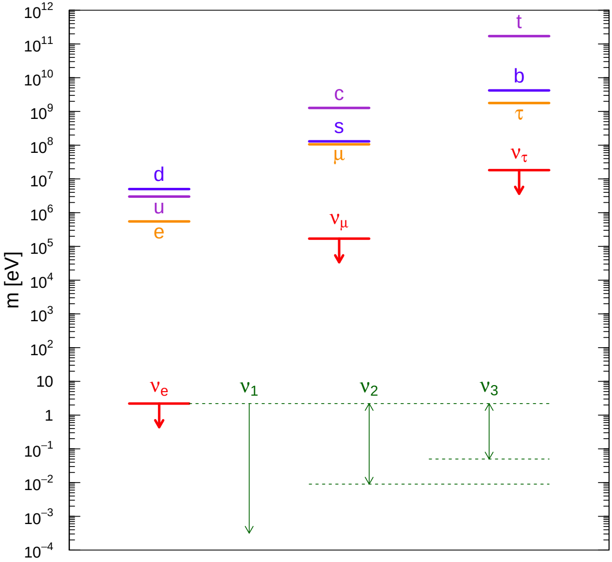
XXIIIe Congrès Général de la Société Française de Physique

Strasbourg

24-28 August 2015

Fermion Mass Spectrum



Standard Model: Massless Neutrinos

	1$^{\text{st}}$ Generation	2$^{\text{nd}}$ Generation	3$^{\text{rd}}$ Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} u_R \\ d_R \end{matrix} \begin{pmatrix} \bar{u}_R \\ \bar{d}_R \end{pmatrix} \begin{matrix} \bar{u}_L \\ \bar{d}_L \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{matrix} c_R \\ s_R \end{matrix} \begin{pmatrix} \bar{c}_R \\ \bar{s}_R \end{pmatrix} \begin{matrix} \bar{c}_L \\ \bar{s}_L \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \begin{matrix} t_R \\ b_R \end{matrix} \begin{pmatrix} \bar{t}_R \\ \bar{b}_R \end{pmatrix} \begin{matrix} \bar{t}_L \\ \bar{b}_L \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{matrix} \xcancel{\nu_{eR}} \\ e_R \end{matrix} \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix} \begin{matrix} \xcancel{\bar{\nu}_{eL}} \\ \bar{e}_L \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{matrix} \xcancel{\nu_{\mu R}} \\ \mu_R \end{matrix} \begin{pmatrix} \bar{\nu}_{\mu R} \\ \bar{\mu}_R \end{pmatrix} \begin{matrix} \xcancel{\bar{\nu}_{\mu L}} \\ \bar{\mu}_L \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \begin{matrix} \xcancel{\nu_{\tau R}} \\ \tau_R \end{matrix} \begin{pmatrix} \bar{\nu}_{\tau R} \\ \bar{\tau}_R \end{pmatrix} \begin{matrix} \xcancel{\bar{\nu}_{\tau L}} \\ \bar{\tau}_L \end{matrix}$

- No ν_R $\Longrightarrow$ No Dirac mass term $\mathcal{L}^{\text{D}}_{\nu_e} \sim m^{\text{D}} \nu_{eR} \nu_{eL}$
- Majorana Neutrino: $\nu = \bar{\nu} \Longrightarrow \nu_R = \bar{\nu}_R$

  Majorana mass term: $\mathcal{L}^{\text{M}}_{\nu_e} \sim m^{\text{M}} \bar{\nu}_{eR} \nu_{eL} = m^{\text{M}} \nu_{eR} \nu_{eL}$

  forbidden by Standard Model $\text{SU}(2)_L \times \text{U}(1)_Y$ symmetry!
- In Standard Model neutrinos are massless!
- Experimentally allowed until 1998, when the Super-Kamiokande atmospheric neutrino experiment obtained a model-independent proof of

  Neutrino Oscillations

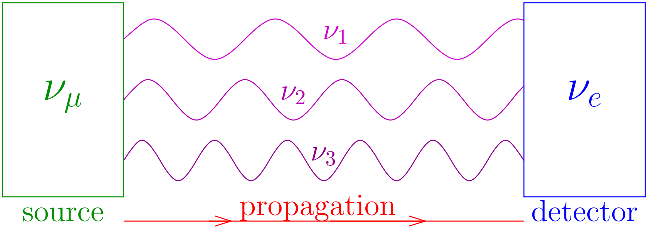
Neutrino Oscillations

- 1957: Bruno Pontecorvo proposed a form of neutrino oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955).
- Theoretical and experimental developments led to neutrino mixing [Maki, Nakagawa, Sakata, Prog. Theor. Phys. 28 (1962) 870] and the theory of neutrino oscillations as flavor transitions which oscillate with distance [Pontecorvo, Sov. Phys. JETP 26 (1968) 984; Gribov, Pontecorvo, PLB 28 (1969); Bilenky, Pontecorvo, Sov. J. Nucl. Phys. 24 (1976) 316, PLB 61 (1976) 248; Fritzsch, Minkowski, Phys. Lett. B62 (1976) 72; Eliezer, Swift, Nucl. Phys. B105 (1976) 45] .
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$
\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}
$$

- U is the 3×3 Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = U_{\mu 1}\,|\nu_1\rangle + U_{\mu 2}\,|\nu_2\rangle + U_{\mu 3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{\mu 1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{\mu 2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{\mu 3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\mu\rangle$$

$$E_k^2 = p^2 + m_k^2$$

$$P_{\nu_\mu\to\nu_e}(t>0) = |\langle\nu_e|\nu(t>0)\rangle|^2 \sim \sum_{k>j} \mathrm{Re}\left[U_{ek}\,U_{\mu k}^*\,U_{ej}^*\,U_{\mu j}\right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)$$

transition probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

$$\begin{array}{cccc} \nu_e \to \nu_\mu & \nu_e \to \nu_\tau & \nu_\mu \to \nu_e & \nu_\mu \to \nu_\tau \\ \bar\nu_e \to \bar\nu_\mu & \bar\nu_e \to \bar\nu_\tau & \bar\nu_\mu \to \bar\nu_e & \bar\nu_\mu \to \bar\nu_\tau \end{array}$$

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \le \vartheta_{ab} \le \frac{\pi}{2}$ $\quad 0 \le \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

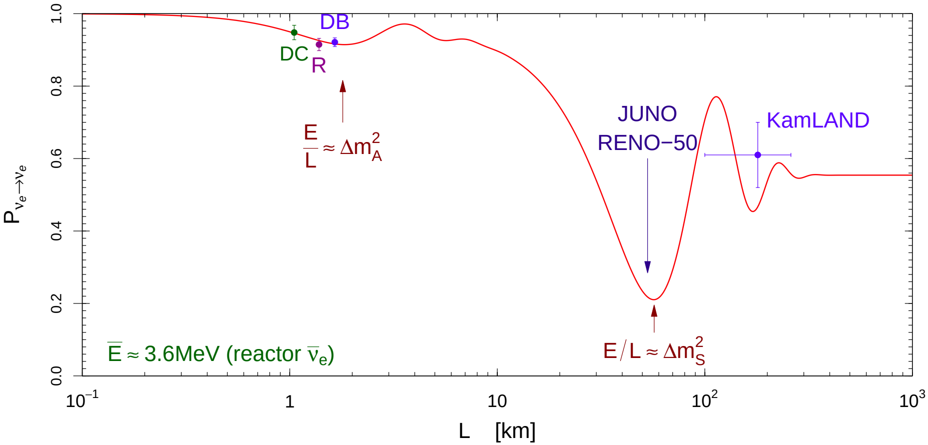
OSCILLATION PARAMETERS

$$\begin{cases} \text{3 Mixing Angles: } \vartheta_{12},\ \vartheta_{23},\ \vartheta_{13} \\ \text{1 CPV Dirac Phase: } \delta_{13} \\ \text{2 independent } \Delta m^2_{kj} \equiv m^2_k - m^2_j\text{: } \Delta m^2_{21},\ \Delta m^2_{31} \end{cases}$$

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

Experimental Evidences of Neutrino Oscillations

Solar $\nu_e \to \nu_\mu, \nu_\tau$	SNO, BOREXino Super-Kamiokande GALLEX/GNO, SAGE Homestake, Kamiokande	$\to$ $\Delta m^2_{\rm S} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$ $\sin^2\vartheta_{\rm S} = \sin^2\vartheta_{12} \simeq 0.30$
VLBL Reactor $\bar{\nu}_e$ disappearance	(KamLAND)	
Atmospheric $\nu_\mu \to \nu_\tau$	Super-Kamiokande Kamiokande, IMB MACRO, Soudan-2	$\to$ $\Delta m^2_{\rm A} = \lvert\Delta m^2_{31}\rvert \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ $\sin^2\vartheta_{\rm A} = \sin^2\vartheta_{23} \simeq 0.50$
LBL Accelerator ν_μ disappearance	K2K, MINOS T2K, NOνA	
LBL Accelerator $\nu_\mu \to \nu_\tau$	(Opera)	
LBL Accelerator $\nu_\mu \to \nu_e$	(T2K, MINOS, NOνA)	$\to$ $\Delta m^2_{\rm A} = \lvert\Delta m^2_{31}\rvert$ $\sin^2\vartheta_{13} \simeq 0.023$
LBL Reactor $\bar{\nu}_e$ disappearance	Daya Bay, RENO Double Chooz	



1.0
0.8
0.6
0.4
0.2
0.0
$P_{\nu_e \to \nu_e}$
10^{-1}
1
10
10^2
10^3
L [km]
DB
DC
R
$\frac{E}{L} \approx \Delta m^2_A$
JUNO
RENO−50
KamLAND
$E/L \approx \Delta m^2_S$
$\overline{E} \approx 3.6\mathrm{MeV}$ (reactor $\overline{\nu}_e$)

Recent Global Fits

- Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo
  Status of three-neutrino oscillation parameters, circa 2013
  Phys.Rev. D89 (2014) 093018, arXiv:1312.2878
- Forero, Tortola, Valle
  Neutrino oscillations refitted
  Phys.Rev. D90 (2014) 093006, arXiv:1405.7540
- Gonzalez-Garcia, Maltoni, Schwetz
  Updated fit to three neutrino mixing: status of leptonic CP violation
  JHEP 1411 (2014) 052, arXiv:1409.5439
- Bergstrom, Gonzalez-Garcia, Maltoni, Schwetz
  Bayesian global analysis of neutrino oscillation data
  arXiv:1507.04366

νfit
www.nu-fit.org

$$\Delta m^2_{\rm S} = \Delta m^2_{21} \simeq 7.5 \pm 0.3 \times 10^{-5}\,{\rm eV}^2 \qquad \text{uncertainty} \simeq 3\%$$

$$\Delta m^2_{\rm A} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \pm 0.1 \times 10^{-3}\,{\rm eV}^2 \qquad \text{uncertainty} \simeq 4\%$$

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} = \vartheta_{\rm A}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\text{Daya Bay, RENO}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} = \vartheta_{\rm S}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$

$P_{\rm osc} \propto \sin^2 2\vartheta_{23}$

maximal and flat

at $\vartheta_{23} = 45^\circ$

Double Chooz

T2K, MINOS

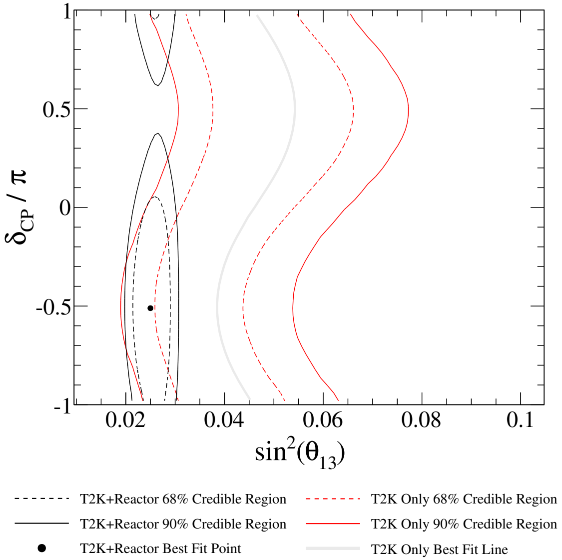
$\sin^2\vartheta_{13} \simeq 0.023 \pm 0.002$

$\boxed{\delta_{13} \approx 3\pi/2?}$

$\sin^2\vartheta_{12} \simeq 0.30 \pm 0.01$

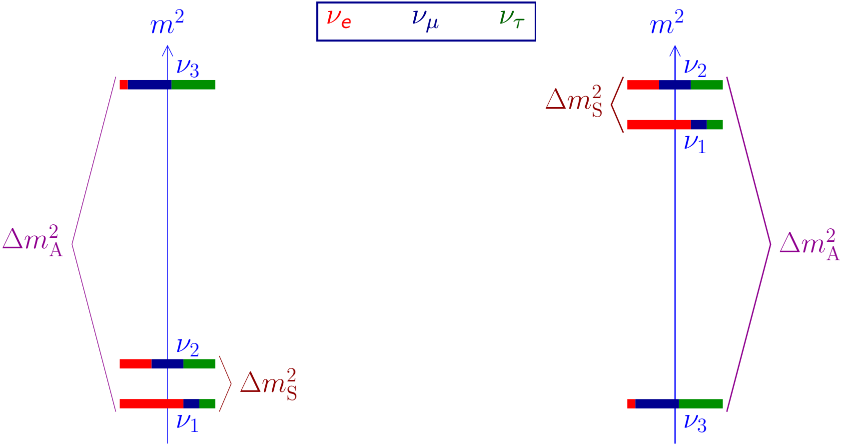
$$\frac{\delta \sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \approx 40\% \qquad \frac{\delta \sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \approx 10\% \qquad \frac{\delta \sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \approx 5\%$$

Maximal CP Violation?



T2K, Phys.Rev. D91 (2015) 072010, arXiv:1502.01550

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

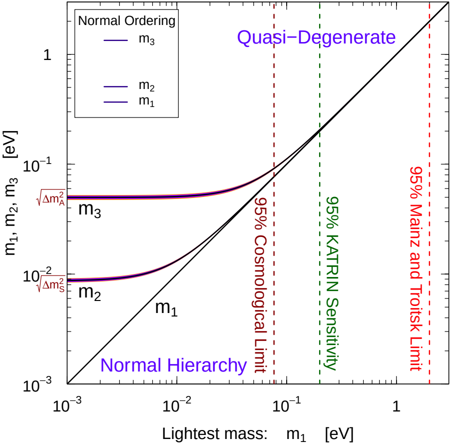
$\Delta m^2_{32} < \Delta m^2_{31} < 0$

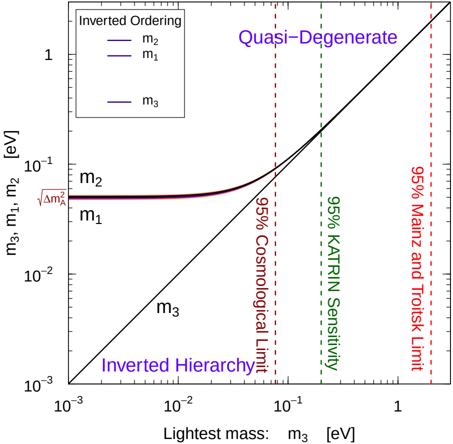
absolute scale is not determined by neutrino oscillation data

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), PINGU (Antarctica), ORCA (EU), INO (India), ...
- Mass Ordering (Hierarchy) ?
  - NOνA (USA), JUNO (China), RENO-50 (Korea), PINGU (Antarctica), ORCA (EU), INO (India), ...
- CP violation ? $\delta_{13} \approx 3\pi/2$?
  - T2K (Japan), NOνA (USA), DUNE (USA), HyperK (Japan), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Absolute Scale of Neutrino Masses





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}\,\text{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

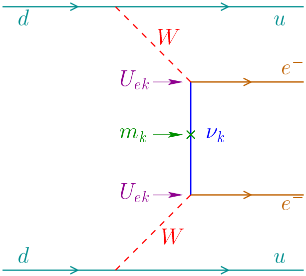
Neutrinoless Double-β Decay

$$\Delta L = 2$$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

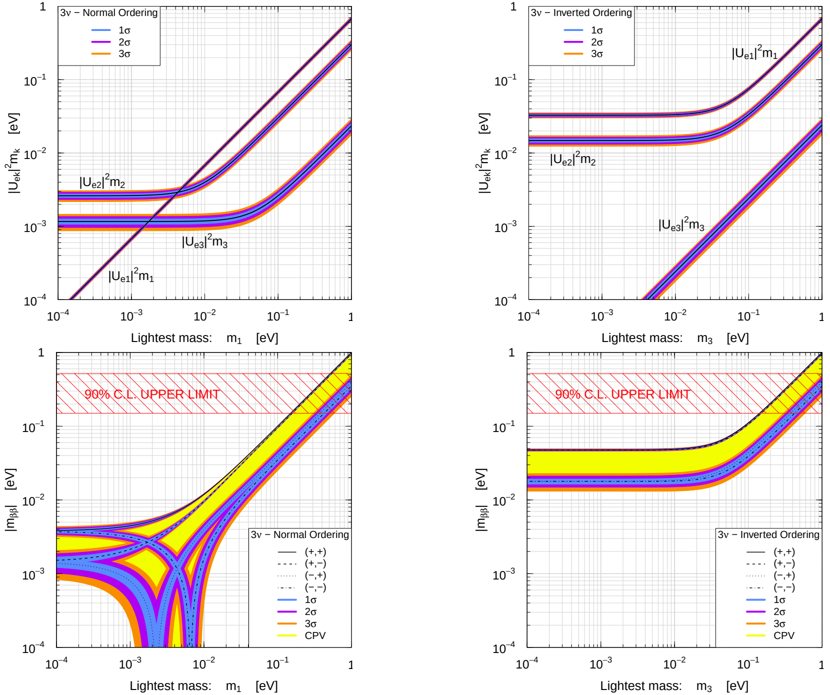


Effective Majorana Neutrino Mass

$$m_{\beta\beta} = |U_{e1}|^2\,m_1 + |U_{e2}|^2\,e^{i\alpha_{21}}\,m_2 + |U_{e3}|^2\,e^{i\alpha_{31}}\,m_3$$

$$\alpha_{21} = 2\lambda_{21} \qquad \alpha_{31} = 2\,(\lambda_{31} - \delta_{13})$$

possible cancellations between the three mass contributions

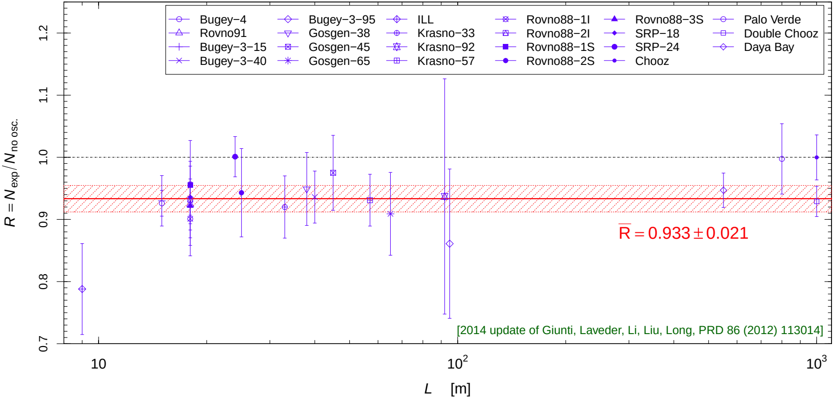

3ν − Normal Ordering
1σ
2σ
3σ
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
$|U_{e1}|^2m_1$
$|U_{ek}|^2m_k$ [eV]
Lightest mass: m_1 [eV]
3ν − Inverted Ordering
$|U_{e1}|^2m_1$
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
Lightest mass: m_3 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
(+,+)
(+,−)
(−,+)
(−,−)
CPV

Indications of SBL Oscillations Beyond 3ν

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006; update in White Paper, arXiv:1204.5379]

New reactor $\bar{\nu}_e$ fluxes [Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]



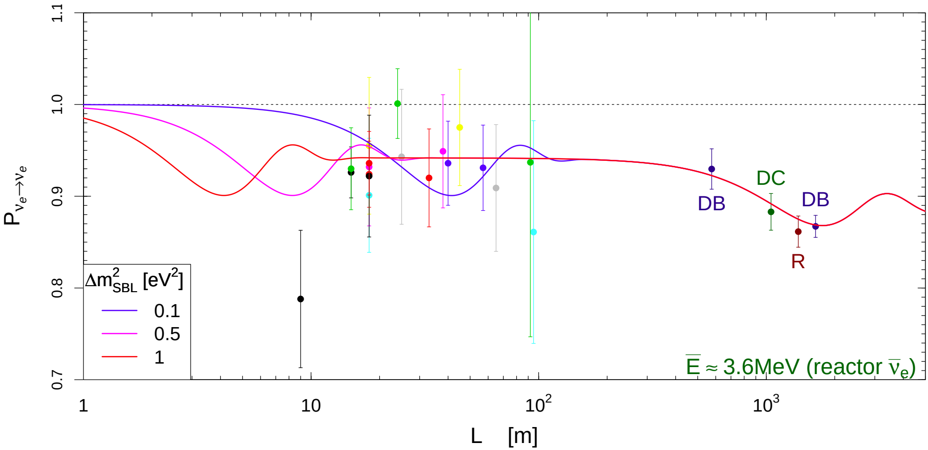
$\bar{\nu}_e \to \bar{\nu}_e$ $\quad L \sim 10 - 100\,\text{m}$ $\quad E \sim 4\,\text{MeV}$

Nominal $\approx 3.1\sigma$ deficit $\quad \Delta m^2 \gtrsim 0.5\,\text{eV}^2 \quad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

[see also: Sinev, arXiv:1103.2452; Ciuffoli, Evslin, Li, JHEP 12 (2012) 110; Zhang, Qian, Vogel, PRD 87 (2013) 073018; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Ivanov et al, PRC 88 (2013) 055501]

Problem: unknown $\bar{\nu}_e$ flux uncertainties?

[Hayes, Friar, Garvey, Jonkmans, PRL 112 (2014) 202501; Dwyer, Langford, PRL 114 (2015) 012502]

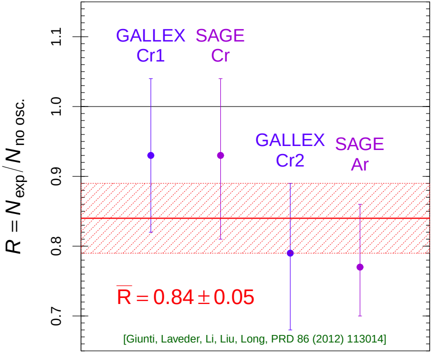

$P_{\nu_e \to \nu_e}$
1.1
1.0
0.9
0.8
0.7
1
10
10^2
10^3
L [m]
Δm^2_{SBL} [eV^2]
0.1
0.5
1
DB
DC
DB
R
$\overline{E} \approx 3.6$MeV (reactor $\overline{\nu}_e$)

Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$



$\bar{\nu}_e \to \bar{\nu}_e \qquad E \sim 0.7\,\mathrm{MeV}$

$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$

$\langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$

Nominal $\approx 2.9\sigma$ anomaly

$\Delta m^2 \gtrsim 1\,\mathrm{eV}^2 \qquad (\gg \Delta m^2_{\mathrm{A}} \gg \Delta m^2_{\mathrm{S}})$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807]

[Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344; MPLA 22 (2007) 2499; PRD 78 (2008) 073009; PRC 83 (2011) 065504]

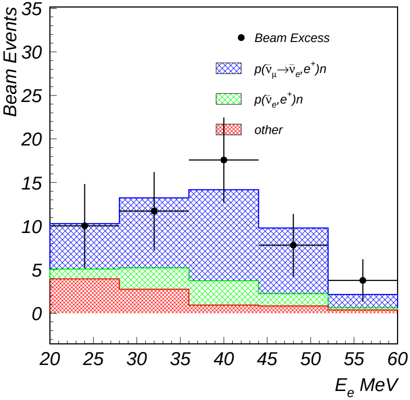
[Mention et al, PRD 83 (2011) 073006]

- ${}^{3}\mathrm{He} + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + {}^{3}\mathrm{H}$ cross section measurement [Frekers et al., PLB 706 (2011) 134]
- $E_{\mathrm{th}}(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-) = 233.5 \pm 1.2\,\mathrm{keV}$ [Frekers et al., PLB 722 (2013) 233]

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 30\,\text{m}$ $\quad 20\,\text{MeV} \leq E \leq 60\,\text{MeV}$



- Well known source of $\bar{\nu}_\mu$:

$$\mu^+ \text{ at rest} \to e^+ + \nu_e + \bar{\nu}_\mu$$

- $\bar{\nu}_\mu \xrightarrow[L \simeq 30\,\text{m}]{} \bar{\nu}_e$
- Well known detection process of $\bar{\nu}_e$:

$$\bar{\nu}_e + p \to n + e^+$$

- But signal not seen by KARMEN with same method at $L \simeq 18\,\text{m}$

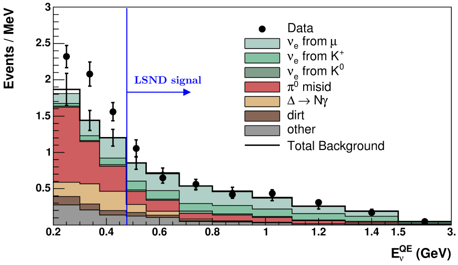
[PRD 65 (2002) 112001]

Nominal $\approx 3.8\sigma$ excess $\quad \Delta m^2 \gtrsim 0.2\,\text{eV}^2 \quad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

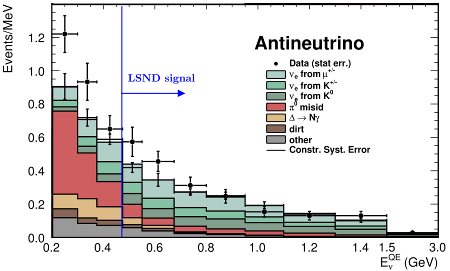
MiniBooNE

$L \simeq 541\,\mathrm{m}$ $\qquad$ $200\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]

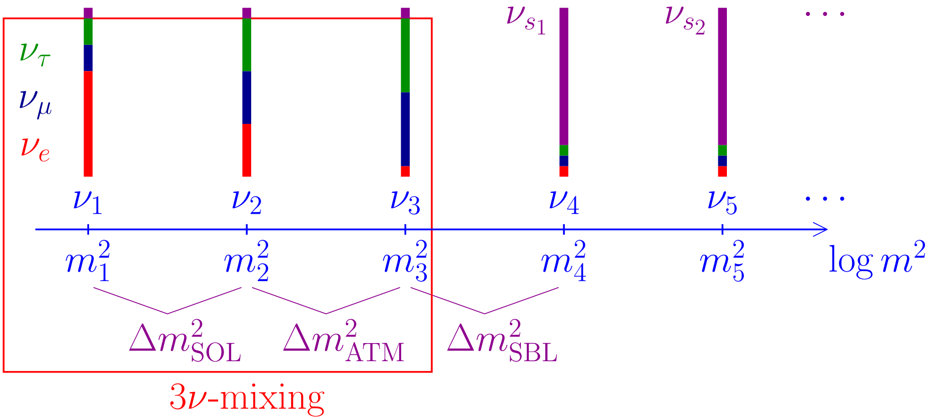


$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



- Purpose: check LSND signal.
- Different L and E.
- Similar L/E (oscillations).
- No money, no Near Detector.
- LSND signal: $E > 475\,\mathrm{MeV}$.
- Agreement with LSND signal?
- CP violation?
- Low-energy anomaly!

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing: $|U_{e4}|^2 \ll 1\,,\ |U_{\mu 4}|^2 \ll 1\,,\ |U_{\tau 4}|^2 \ll 1\,,\ |U_{s4}|^2 \simeq 1$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \boxed{U_{e4}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \boxed{U_{\mu 4}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \boxed{U_{\tau 4}} \\ U_{s1} & U_{s2} & U_{s3} & \boxed{U_{s4}} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases
- But CP violation is not observable in current SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea, Kelly, Kobach, PRD 91 (2015) 053005; Klop, Palazzo, PRD 91 (2015) 073017; Berryman, de Gouvea, Kelly, Kobach, arXiv:1507.03986] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, Giunti, PRD 87, 113004 (2013) 113004]

3+1: Appearance vs Disappearance

- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$

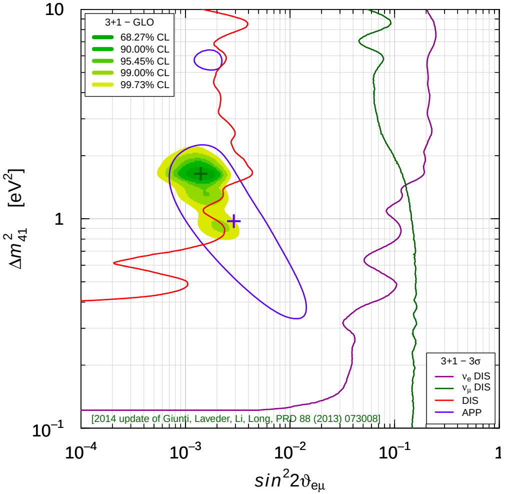
- Upper bounds on ν_e and ν_μ disappearance $\Rightarrow$ strong limit on $\nu_\mu \to \nu_e$

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, Giunti, Grimus, EPJC 1 (1998) 247]

- Similar constraint in 3+2, 3+3, ..., 3+N_s! [Giunti, Zavanin, arXiv:1508.03172]

Global 3+1 Fit

[Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008]



MiniBooNE $E > 475\,\text{MeV}$

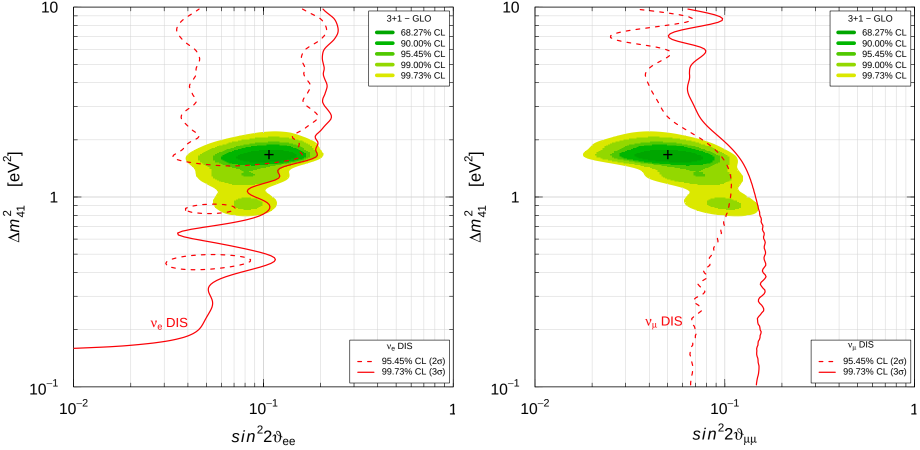
GoF $= 26\%$ PGoF $= 7\%$

- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND (ν_s), MiniBooNE (?), OPERA $(\cancel{\nu_s})$, ICARUS $(\cancel{\nu_s})$, KARMEN $(\cancel{\nu_s})$, NOMAD $(\cancel{\nu_s})$, BNL-E776 $(\cancel{\nu_s})$
- DIS ν_e & $\bar{\nu}_e$: Reactors (ν_s), Gallium (ν_s), ν_eC $(\cancel{\nu_s})$, Solar $(\cancel{\nu_s})$
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW $(\cancel{\nu_s})$, MINOS $(\cancel{\nu_s})$, Atmospheric $(\cancel{\nu_s})$, MiniBooNE/SciBooNE $(\cancel{\nu_s})$

No Osc. nominally disfavored at $\approx 6.3\sigma$

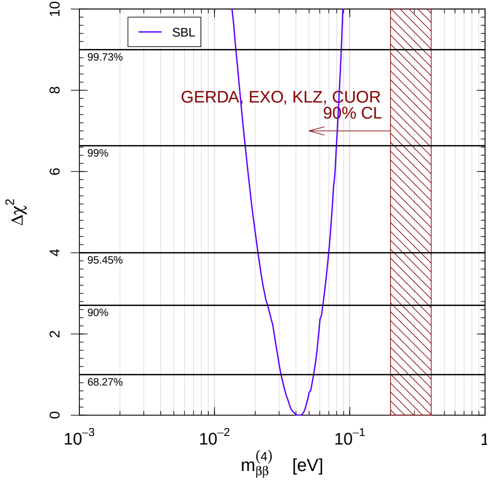
$\Delta\chi^2/\text{NDF} = 47.7/3$

ν_e and ν_μ Disappearance



Neutrinoless Double-β Decay

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3 + |U_{e4}|^2 \, e^{i\alpha_{41}} \, m_4$$



3+1 Fit

[Giunti, Laveder, Li, Long, 2014]

$$m_{\beta\beta}^{(k)} = |U_{ek}|^2 m_k$$

$$m_1 \ll m_4$$

$$\Downarrow$$

$$m_{\beta\beta}^{(4)} \simeq |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

surprise:
possible cancellation
with $m_{\beta\beta}^{(3\nu)}$

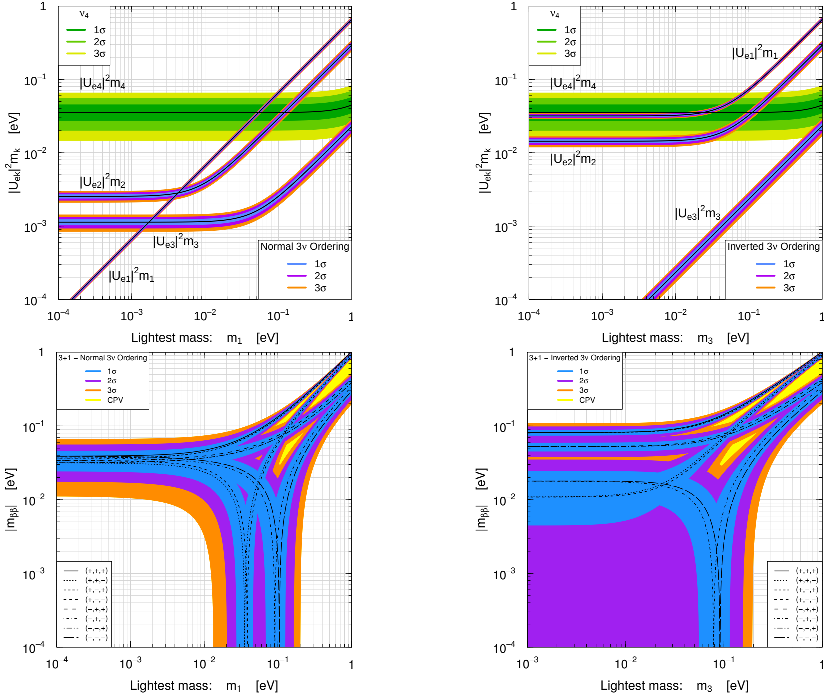
[Barry et al, JHEP 07 (2011) 091]

[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, JPG 39 (2012) 124008]

[Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

[Giunti, Zavanin, JHEP 07 (2015) 171]


ν4
1σ
2σ
3σ
|Ue4|2m4
|Ue2|2m2
|Ue3|2m3
|Ue1|2m1
Normal 3ν Ordering
1σ
2σ
3σ
|Uek|2mk [eV]
Lightest mass: m1 [eV]
ν4
1σ
2σ
3σ
|Ue1|2m1
|Ue4|2m4
|Ue2|2m2
|Ue3|2m3
Inverted 3ν Ordering
1σ
2σ
3σ
|Uek|2mk [eV]
Lightest mass: m3 [eV]
3+1 – Normal 3ν Ordering
1σ
2σ
3σ
CPV
(+,+,+)
(+,+,–)
(+,–,+)
(+,–,–)
(–,+,+)
(–,+,–)
(–,–,+)
(–,–,–)
|mββ| [eV]
Lightest mass: m1 [eV]
3+1 – Inverted 3ν Ordering
1σ
2σ
3σ
CPV
(+,+,+)
(+,+,–)
(+,–,+)
(+,–,–)
(–,+,+)
(–,+,–)
(–,–,+)
(–,–,–)
|mββ| [eV]
Lightest mass: m3 [eV]

Conclusions

- Robust Three-Neutrino Mixing Paradigm.
  Open problems with exciting experimental program: $\vartheta_{23} \lesseqgtr 45^\circ$?, Mass Ordering, CP Violation, Absolute Mass Scale, Dirac or Majorana?
  Determination of Mass Ordering is very important!
- Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance:
  - Experimental data agree on Reactor $\bar{\nu}_e$ and Gallium ν_e anomalies.
  - Problem: unknown systematic uncertainties (Reactor $\bar{\nu}_e$ flux).
  - Many promising projects to test unambiguously short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources.
  - Independent tests through effect of m_4 in β-decay and $\beta\beta_{0\nu}$-decay.
- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - Not seen by other SBL $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ experiments.
  - MiniBooNE experiment has been inconclusive.
  - Experiments with near detector are needed to check LSND signal!
  - Promising Fermilab program aimed at a conclusive solution of the mystery: a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-WA104), all Liquid Argon Time Projection Chambers.