

Theory and Phenomenology of Massive Neutrinos

Part I: Theory of Neutrino Masses and Mixing

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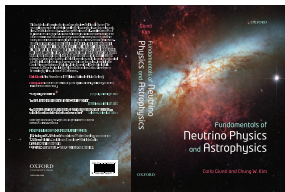
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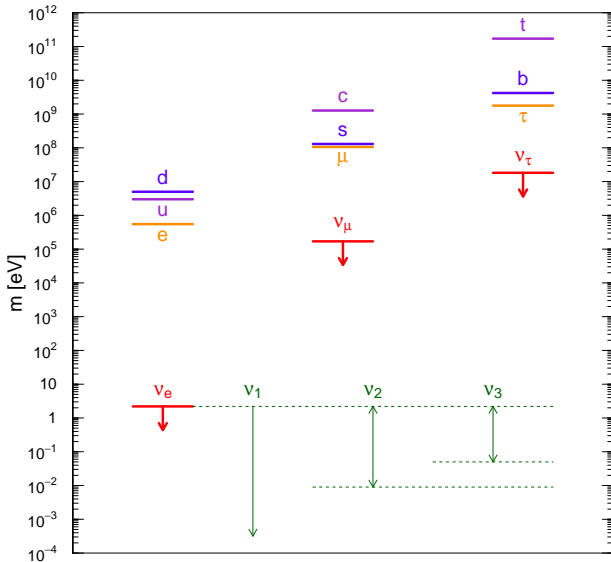


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and
Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Sterile Neutrinos

Fermion Mass Spectrum



Dirac Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
 - Higgs Mechanism in SM
 - SM Extension: Dirac Neutrino Masses
 - Three-Generations Dirac Neutrino Masses
 - Mixing
 - CP Violation
 - Lepton Numbers Violating Processes
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Sterile Neutrinos

Dirac Mass

▶ Dirac Equation: $(i\partial - m)\nu(x) = 0$ ($\partial \equiv \gamma^\mu \partial_\mu$)

▶ Dirac Lagrangian: $\mathcal{L}(x) = \bar{\nu}(x)(i\partial - m)\nu(x)$

▶ Chiral decomposition: $\nu_L \equiv P_L \nu$, $\nu_R \equiv P_R \nu$, $\nu = \nu_L + \nu_R$

Left and Right-handed Projectors: $P_L \equiv \frac{1 - \gamma^5}{2}$, $P_R \equiv \frac{1 + \gamma^5}{2}$

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = P_R P_L = 0$$

$$\mathcal{L} = \bar{\nu}_L i\partial \nu_L + \bar{\nu}_R i\partial \nu_R - m(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$$

▶ In SM only ν_L by assumption \implies no neutrino mass

Note that all the other elementary fermion fields (charged leptons and quarks) have both left and right-handed components

▶ Oscillation experiments have shown that **neutrinos are massive**

▶ Simplest and natural extension of the SM: consider also ν_R as for all the other elementary fermion fields

Higgs Mechanism in SM

▶ Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ $|\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$

▶ Higgs Lagrangian: $\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$

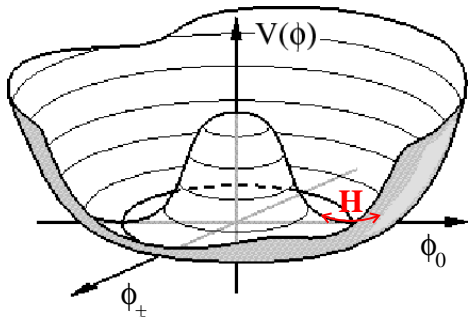
▶ Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$

▶ $\mu^2 < 0$ and $\lambda > 0 \implies V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$

$$v \equiv \sqrt{-\frac{\mu^2}{\lambda}} = (\sqrt{2} G_F)^{-1/2} \simeq 246 \text{ GeV}$$

▶ Vacuum: V_{\min} for $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

▶ Spontaneous Symmetry Breaking: $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$



▶ Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \Rightarrow |\Phi|^2 = \frac{v^2}{2} + vH + \frac{1}{2} H^2$

▶ $V = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \simeq 126 \text{ GeV}$$

$$-\mu^2 \simeq (89 \text{ GeV})^2 \quad \lambda = -\frac{\mu^2}{v^2} \simeq 0.13$$

SM Extension: Dirac Neutrino Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad \ell_R \quad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = -y^\ell \bar{L}_L \Phi \ell_R - y^\nu \bar{L}_L \tilde{\Phi} \nu_R + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{H,L} = & -\frac{y^\ell}{\sqrt{2}} (\bar{\nu}_L \quad \bar{\ell}_L) \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & -\frac{y^\nu}{\sqrt{2}} (\bar{\nu}_L \quad \bar{\ell}_L) \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \text{H.c.} \end{aligned}$$

$$\mathcal{L}_{H,L} = -y^\ell \frac{v}{\sqrt{2}} \bar{l}_L l_R - y^\nu \frac{v}{\sqrt{2}} \bar{\nu}_L \nu_R$$

$$- \frac{y^\ell}{\sqrt{2}} \bar{l}_L l_R H - \frac{y^\nu}{\sqrt{2}} \bar{\nu}_L \nu_R H + \text{H.c.}$$

$$m_\ell = y^\ell \frac{v}{\sqrt{2}}$$

$$m_\nu = y^\nu \frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^\ell}{\sqrt{2}} = \frac{m_\ell}{v}$$

$$g_{\nu H} = \frac{y^\nu}{\sqrt{2}} = \frac{m_\nu}{v}$$

$$v = \left(\sqrt{2} G_F \right)^{-1/2} = 246 \text{ GeV}$$

PROBLEM: $y^\nu \lesssim 10^{-11} \ll y^e \sim 10^{-6}$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell\alpha}_{\beta} \overline{L}'_{\alpha L} \Phi \ell'_{\beta R} + Y'^{\nu\alpha}_{\beta} \overline{L}'_{\alpha L} \tilde{\Phi} \nu'_{\beta R} \right] + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \sum_{\alpha,\beta=e,\mu,\tau} \left[Y_{\alpha\beta}^{l\ell} \overline{l'_{\alpha L}} l'_{\beta R} + Y_{\alpha\beta}^{l\nu} \overline{\nu'_{\alpha L}} \nu'_{\beta R} \right] + \text{H.c.}$$

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \left[\overline{l'_L} Y^{l\ell} l'_R + \overline{\nu'_L} Y^{l\nu} \nu'_R \right] + \text{H.c.}$$

$$l'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \quad l'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y^{l\ell} \equiv \begin{pmatrix} Y_{ee}^{l\ell} & Y_{e\mu}^{l\ell} & Y_{e\tau}^{l\ell} \\ Y_{\mu e}^{l\ell} & Y_{\mu\mu}^{l\ell} & Y_{\mu\tau}^{l\ell} \\ Y_{\tau e}^{l\ell} & Y_{\tau\mu}^{l\ell} & Y_{\tau\tau}^{l\ell} \end{pmatrix}$$

$$Y^{l\nu} \equiv \begin{pmatrix} Y_{ee}^{l\nu} & Y_{e\mu}^{l\nu} & Y_{e\tau}^{l\nu} \\ Y_{\mu e}^{l\nu} & Y_{\mu\mu}^{l\nu} & Y_{\mu\tau}^{l\nu} \\ Y_{\tau e}^{l\nu} & Y_{\tau\mu}^{l\nu} & Y_{\tau\tau}^{l\nu} \end{pmatrix}$$

$$M^{l\ell} = \frac{v}{\sqrt{2}} Y^{l\ell}$$

$$M^{l\nu} = \frac{v}{\sqrt{2}} Y^{l\nu}$$

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \left[\overline{\ell}'_L Y^{\ell\ell} \ell'_R + \overline{\nu}'_L Y^{\nu\nu} \nu'_R \right] + \text{H.c.}$$

Diagonalization of $Y^{\ell\ell}$ and $Y^{\nu\nu}$ with **unitary** V_L^ℓ , V_R^ℓ , V_L^ν , V_R^ν

$$\ell'_L = V_L^\ell \ell_L \quad \ell'_R = V_R^\ell \ell_R \quad \nu'_L = V_L^\nu \mathbf{n}_L \quad \nu'_R = V_R^\nu \mathbf{n}_R$$

Important general remark: unitary transformations are allowed because they leave invariant the kinetic terms in the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \overline{\ell}'_L i \not{\partial} \ell'_L + \overline{\ell}'_R i \not{\partial} \ell'_R + \overline{\nu}'_L i \not{\partial} \nu'_L + \overline{\nu}'_R i \not{\partial} \nu'_R \\ &= \overline{\ell}_L V_L^{\ell\dagger} i \not{\partial} V_L^\ell \ell_L + \dots \\ &= \overline{\ell}_L i \not{\partial} \ell_L + \overline{\ell}_R i \not{\partial} \ell_R + \overline{\nu}_L i \not{\partial} \nu_L + \overline{\nu}_R i \not{\partial} \nu_R \end{aligned}$$

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \left[\overline{\ell}'_L Y'^{\ell} \ell'_R + \overline{\nu}'_L Y'^{\nu} \nu'_R \right] + \text{H.c.}$$

$$\ell'_L = V_L^{\ell} \ell_L \quad \ell'_R = V_R^{\ell} \ell_R \quad \nu'_L = V_L^{\nu} \nu_L \quad \nu'_R = V_R^{\nu} \nu_R$$

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \left[\overline{\ell}_L V_L^{\ell \dagger} Y'^{\ell} V_R^{\ell} \ell_R + \overline{\nu}_L V_L^{\nu \dagger} Y'^{\nu} V_R^{\nu} \nu_R \right] + \text{H.c.}$$

$$V_L^{\ell \dagger} Y'^{\ell} V_R^{\ell} = Y^{\ell} \quad Y_{\alpha\beta}^{\ell} = y_{\alpha}^{\ell} \delta_{\alpha\beta} \quad (\alpha, \beta = e, \mu, \tau)$$

$$V_L^{\nu \dagger} Y'^{\nu} V_R^{\nu} = Y^{\nu} \quad Y_{kj}^{\nu} = y_k^{\nu} \delta_{kj} \quad (k, j = 1, 2, 3)$$

Real and Positive $y_{\alpha}^{\ell}, y_k^{\nu}$

$$V_L^{\dagger} Y' V_R = Y \quad \iff \quad Y' = V_L \quad Y \quad V_R^{\dagger}$$

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Massive Chiral Lepton Fields

$\ell_L = V_L^{\ell\dagger} \ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\ell_R = V_R^{\ell\dagger} \ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger} \nu'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger} \nu'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{H,L} &= - \left(\frac{v+H}{\sqrt{2}} \right) \left[\overline{\ell}_L Y^\ell \ell_R + \overline{\mathbf{n}}_L Y^\nu \mathbf{n}_R \right] + \text{H.c.} \\ &= - \left(\frac{v+H}{\sqrt{2}} \right) \left[\sum_{\alpha=e,\mu,\tau} y_\alpha^\ell \overline{\ell}_{\alpha L} \ell_{\alpha R} + \sum_{k=1}^3 y_k^\nu \overline{\nu}_{kL} \nu_{kR} \right] + \text{H.c.} \end{aligned}$$

Massive Dirac Lepton Fields

$$l_\alpha \equiv l_{\alpha L} + l_{\alpha R} \quad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \quad (k = 1, 2, 3)$$

$$\mathcal{L}_{H,L} = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^l v}{\sqrt{2}} \bar{l}_\alpha l_\alpha - \sum_{k=1}^3 \frac{y_k^\nu v}{\sqrt{2}} \bar{\nu}_k \nu_k \quad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^l}{\sqrt{2}} \bar{l}_\alpha l_\alpha H - \sum_{k=1}^3 \frac{y_k^\nu}{\sqrt{2}} \bar{\nu}_k \nu_k H \quad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^l v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \quad m_k = \frac{y_k^\nu v}{\sqrt{2}} \quad (k = 1, 2, 3)$$

Lepton-Higgs coupling \propto Lepton Mass

Quantization

$$\nu_k(x) = \int \frac{d^3 p}{(2\pi)^3 2E_k} \sum_{h=\pm 1} \left[a_k^{(h)}(p) u_k^{(h)}(p) e^{-ip \cdot x} + b_k^{(h)\dagger}(p) v_k^{(h)}(p) e^{ip \cdot x} \right]$$

$$p^0 = E_k = \sqrt{\vec{p}^2 + m_k^2} \quad \begin{aligned} (\not{p} - m_k) u_k^{(h)}(p) &= 0 \\ (\not{p} + m_k) v_k^{(h)}(p) &= 0 \end{aligned}$$

$$\frac{\vec{p} \cdot \vec{\Sigma}}{|\vec{p}|} u_k^{(h)}(p) = h u_k^{(h)}(p)$$

$$\frac{\vec{p} \cdot \vec{\Sigma}}{|\vec{p}|} v_k^{(h)}(p) = -h v_k^{(h)}(p)$$

$$\{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} = \{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = (2\pi)^3 2E_k \delta^3(\vec{p} - \vec{p}') \delta_{hh'}$$

$$\{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} = \{a_k^{(h)\dagger}(p), a_k^{(h')\dagger}(p')\} = 0$$

$$\{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = \{b_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$

$$\{a_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$

$$\{a_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathcal{L}_1^{(\text{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \text{H.c.}$$

Weak Charged Current: $j_W^\rho = j_{W,L}^\rho + j_{W,Q}^\rho$

Leptonic Weak Charged Current

$$j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \bar{\ell}'_{\alpha L} \gamma^\rho \nu'_{\alpha L} = 2 \bar{\ell}'_L \gamma^\rho \nu'_L$$

$$\underline{\ell}'_L = V_L^\ell \ell_L \quad \underline{\nu}'_L = V_L^\nu \mathbf{n}_L$$

$$j_{W,L}^{\rho\dagger} = 2 \bar{\ell}'_L V_L^{\ell\dagger} \gamma^\rho V_L^\nu \mathbf{n}_L = 2 \bar{\ell}'_L \gamma^\rho V_L^{\ell\dagger} V_L^\nu \mathbf{n}_L = 2 \bar{\ell}'_L \gamma^\rho U \mathbf{n}_L$$

Mixing Matrix

$$U = V_L^{\ell\dagger} V_L^\nu$$

▶ **Definition:** Left-Handed Flavor Neutrino Fields

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} V_L^\nu \mathbf{n}_L = V_L^{\ell\dagger} \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

▶ They allow us to write the **Leptonic Weak Charged Current** as in the SM:

$$j_{W,L}^{\rho\dagger} = 2 \overline{\ell}_L \gamma^\rho \nu_L = 2 \sum_{\alpha=e,\mu,\tau} \overline{\ell}_{\alpha L} \gamma^\rho \nu_{\alpha L}$$

▶ Each **left-handed flavor neutrino field** is associated with the corresponding **charged lepton field** which describes a massive charged lepton:

$$j_{W,L}^{\rho\dagger} = 2 (\overline{e}_L \gamma^\rho \nu_{eL} + \overline{\mu}_L \gamma^\rho \nu_{\mu L} + \overline{\tau}_L \gamma^\rho \nu_{\tau L})$$

▶ In practice **left-handed flavor neutrino fields** are useful for calculations in the SM approximation of massless neutrinos (**interactions**).

▶ If neutrino masses must be taken into account, it is necessary to use

$$j_{W,L}^{\rho\dagger} = 2 \overline{\ell}_L \gamma^\rho U \mathbf{n}_L = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{\ell}_{\alpha L} \gamma^\rho U_{\alpha k} \nu_{kL}$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ		L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0	(ν_e^c, e^+)	-1	0	0
(ν_μ, μ^-)	0	+1	0	(ν_μ^c, μ^+)	0	-1	0
(ν_τ, τ^-)	0	0	+1	(ν_τ^c, τ^+)	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Standard Model:

Lepton numbers are conserved

▶ L_e, L_μ, L_τ are conserved in the Standard Model with massless neutrinos

▶ Dirac mass term:

$$\mathcal{L}^D = - \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^D & m_{e\mu}^D & m_{e\tau}^D \\ m_{\mu e}^D & m_{\mu\mu}^D & m_{\mu\tau}^D \\ m_{\tau e}^D & m_{\tau\mu}^D & m_{\tau\tau}^D \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

▶ L is conserved: $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \implies |\Delta L| = 0$

- ▶ **Leptonic Weak Charged Current** is invariant under the global U(1) gauge transformations

$$\ell_{\alpha L} \rightarrow e^{i\varphi_\alpha} \ell_{\alpha L} \quad \nu_{\alpha L} \rightarrow e^{i\varphi_\alpha} \nu_{\alpha L} \quad (\alpha = e, \mu, \tau)$$

- ▶ If neutrinos are massless (SM), Noether's theorem implies that there is, for each flavor, a conserved current:

$$j_\alpha^\rho = \overline{\nu_{\alpha L}} \gamma^\rho \nu_{\alpha L} + \overline{\ell_\alpha} \gamma^\rho \ell_\alpha \quad \partial_\rho j_\alpha^\rho = 0$$

and a conserved charge:

$$L_\alpha = \int d^3x j_\alpha^0(x) \quad \partial_0 L_\alpha = 0$$

$$\begin{aligned} :L_\alpha: &= \int \frac{d^3p}{(2\pi)^3 2E} \left[a_{\nu_\alpha}^{(-)\dagger}(p) a_{\nu_\alpha}^{(-)}(p) - b_{\nu_\alpha}^{(+)\dagger}(p) b_{\nu_\alpha}^{(+)}(p) \right] \\ &+ \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a_{\ell_\alpha}^{(h)\dagger}(p) a_{\ell_\alpha}^{(h)}(p) - b_{\ell_\alpha}^{(h)\dagger}(p) b_{\ell_\alpha}^{(h)}(p) \right] \end{aligned}$$

- ▶ Lepton-Higgs Yukawa Lagrangian:

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \left[\sum_{\alpha=e,\mu,\tau} y_{\alpha}^{\ell} \overline{l_{\alpha L}} l_{\alpha R} + \sum_{k=1}^3 y_k^{\nu} \overline{\nu_{kL}} \nu_{kR} \right] + \text{H.c.}$$

- ▶ Mixing: $\nu_{\alpha L} = \sum_{k=1}^3 U_{\alpha k} \nu_{kL} \iff \nu_{kL} = \sum_{\alpha=e,\mu,\tau} U_{\alpha k}^* \nu_{\alpha L}$

$$\mathcal{L}_{H,L} = - \left(\frac{v+H}{\sqrt{2}} \right) \sum_{\alpha=e,\mu,\tau} \left[y_{\alpha}^{\ell} \overline{l_{\alpha L}} l_{\alpha R} + \overline{\nu_{\alpha L}} \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR} \right] + \text{H.c.}$$

- ▶ Invariant for

$$l_{\alpha L} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha L}, \quad \nu_{\alpha L} \rightarrow e^{i\varphi_{\alpha}} \nu_{\alpha L}$$

$$l_{\alpha R} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha R}, \quad \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR} \rightarrow e^{i\varphi_{\alpha}} \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR}$$

- ▶ But kinetic part of neutrino Lagrangian is not invariant

$$\mathcal{L}_{\text{kinetic}}^{(\nu)} = \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} i \not{\partial} \nu_{\alpha L} + \sum_{k=1}^3 \overline{\nu_{kR}} i \not{\partial} \nu_{kR}$$

because $\sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR}$ is not a unitary combination of the ν_{kR} 's

Total Lepton Number

- ▶ Dirac neutrino masses violate conservation of Flavor Lepton Numbers
- ▶ Total Lepton Number is conserved, because Lagrangian is invariant under the global U(1) gauge transformations

$$\begin{aligned} \nu_{kL} &\rightarrow e^{i\varphi} \nu_{kL}, & \nu_{kR} &\rightarrow e^{i\varphi} \nu_{kR} & (k = 1, 2, 3) \\ l_{\alpha L} &\rightarrow e^{i\varphi} l_{\alpha L}, & l_{\alpha R} &\rightarrow e^{i\varphi} l_{\alpha R} & (\alpha = e, \mu, \tau) \end{aligned}$$

- ▶ From Noether's theorem:

$$j^\rho = \sum_{k=1}^3 \bar{\nu}_k \gamma^\rho \nu_k + \sum_{\alpha=e,\mu,\tau} \bar{l}_\alpha \gamma^\rho l_\alpha \quad \partial_\rho j^\rho = 0$$

$$\text{Conserved charge: } L_\alpha = \int d^3x j_\alpha^0(x) \quad \partial_0 L_\alpha = 0$$

$$\begin{aligned} :L: &= \sum_{k=1}^3 \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a_{\nu_k}^{(h)\dagger}(p) a_{\nu_k}^{(h)}(p) - b_{\nu_k}^{(h)\dagger}(p) b_{\nu_k}^{(h)}(p) \right] \\ &+ \sum_{\alpha=e,\mu,\tau} \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a_{l_\alpha}^{(h)\dagger}(p) a_{l_\alpha}^{(h)}(p) - b_{l_\alpha}^{(h)\dagger}(p) b_{l_\alpha}^{(h)}(p) \right] \end{aligned}$$

Mixing Matrix

$$\blacktriangleright U = V_L^{\ell\dagger} V_L^\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

- ▶ Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \quad \Rightarrow \quad \frac{N(N-1)}{2} = 3 \quad \text{Mixing Angles}$$
$$\frac{N(N+1)}{2} = 6 \quad \text{Phases}$$

- ▶ Not all phases are physical observables
- ▶ Neutrino Lagrangian: kinetic terms + mass terms + weak interactions
- ▶ Mixing is due to the diagonalization of the mass terms
- ▶ The kinetic terms are invariant under unitary transformations of the fermion fields
- ▶ What is the effect of mixing in weak interactions?

▶ Weak Charged Current:
$$j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{\ell_{\alpha L}} \gamma^\rho U_{\alpha k} \nu_{kL}$$

- ▶ Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)

$$\ell_\alpha \rightarrow e^{i\varphi_\alpha} \ell_\alpha \quad (\alpha = e, \mu, \tau), \quad \nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3)$$

- ▶ Performing this transformation, the Weak Charged Current becomes

$$j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{\ell_{\alpha L}} e^{-i\varphi_\alpha} \gamma^\rho U_{\alpha k} e^{i\varphi_k} \nu_{kL}$$

$$j_{W,L}^{\rho\dagger} = 2 \underbrace{e^{-i(\varphi_e - \varphi_1)}}_1 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{\ell_{\alpha L}} \underbrace{e^{-i(\varphi_\alpha - \varphi_e)}}_2 \gamma^\rho U_{\alpha k} \underbrace{e^{i(\varphi_k - \varphi_1)}}_2 \nu_{kL}$$

- ▶ There are 5 independent combinations of the phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- ▶ 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the lepton fields leaves the Weak Charged Current invariant \iff conservation of Total Lepton Number.

- ▶ The mixing matrix contains 1 Physical Phase.
- ▶ It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos \vartheta_{ab} \quad s_{ab} \equiv \sin \vartheta_{ab} \quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \quad 0 \leq \delta_{13} < 2\pi$$

3 Mixing Angles ϑ_{12} , ϑ_{23} , ϑ_{13} and 1 Phase δ_{13}

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23}e^{i\delta_{23}} \\ 0 & -s_{23}e^{-i\delta_{13}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12}e^{-i\delta'_{12}} & 0 \\ -s'_{12}e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

CP Violation

- ▶ $U \neq U^* \implies$ CP Violation (CPV)
- ▶ General conditions for CP violation (14 conditions):
 1. No charged leptons or neutrinos are degenerate in mass (6 conditions)
 2. No mixing angle is equal to 0 or $\pi/2$ (6 conditions)
 3. The physical phase is different from 0 or π (2 conditions)

- ▶ These 14 conditions are combined into the single condition

$$\det C \neq 0 \quad \text{with} \quad C = -i [M^{\nu\mu} M^{\nu\mu\dagger}, M^{\ell e} M^{\ell e\dagger}]$$

$$\det C = -2 J (m_{\nu_2}^2 - m_{\nu_1}^2) (m_{\nu_3}^2 - m_{\nu_1}^2) (m_{\nu_3}^2 - m_{\nu_2}^2) \\ (m_\mu^2 - m_e^2) (m_\tau^2 - m_e^2) (m_\tau^2 - m_\mu^2) \neq 0$$

- ▶ Jarlskog rephasing invariant: $J = \Im [U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}]$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunietz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

Example: $\vartheta_{12} = 0$

$$U = R_{23}R_{13}W_{12}$$

$$W_{12} = \begin{pmatrix} \cos \vartheta_{12} & \sin \vartheta_{12} e^{-i\delta_{12}} & 0 \\ -\sin \vartheta_{12} e^{-i\delta_{12}} & \cos \vartheta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vartheta_{12} = 0 \quad \Rightarrow \quad W_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{1}$$

real mixing matrix

$$U = R_{23}R_{13}$$

Example: $\vartheta_{13} = \pi/2$

$$U = R_{23} W_{13} R_{12}$$

$$W_{13} = \begin{pmatrix} \cos \vartheta_{13} & 0 & \sin \vartheta_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -\sin \vartheta_{13} e^{i\delta_{13}} & 0 & \cos \vartheta_{13} \end{pmatrix}$$

$$\vartheta_{13} = \pi/2 \quad \Rightarrow \quad W_{13} = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -e^{i\delta_{13}} & 0 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & 0 \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta_{13}} & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix}$$

$$\lambda_{\mu 1} - \lambda_{\mu 2} = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi \qquad \lambda_{\tau 1} - \lambda_{\mu 1} = \lambda_{\tau 2} - \lambda_{\mu 2} \pm \pi$$

$$\nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3), \qquad \ell_\alpha \rightarrow e^{i\varphi_\alpha} \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

$$U \rightarrow \begin{pmatrix} e^{-i\varphi_e} & 0 & 0 \\ 0 & e^{-i\varphi_\mu} & 0 \\ 0 & 0 & e^{-i\varphi_\tau} \end{pmatrix} \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{i(-\delta_{13}-\varphi_e+\varphi_3)} \\ |U_{\mu 1}| e^{i(\lambda_{\mu 1}-\varphi_\mu+\varphi_1)} & |U_{\mu 2}| e^{i(\lambda_{\mu 2}-\varphi_\mu+\varphi_2)} & 0 \\ |U_{\tau 1}| e^{i(\lambda_{\tau 1}-\varphi_\tau+\varphi_1)} & |U_{\tau 2}| e^{i(\lambda_{\tau 2}-\varphi_\tau+\varphi_2)} & 0 \end{pmatrix}$$

$$\varphi_1 = 0 \qquad \varphi_\mu = \lambda_{\mu 1} \qquad \varphi_\tau = \lambda_{\tau 1} \qquad \varphi_2 = \varphi_\mu - \lambda_{\mu 2} = \lambda_{\mu 1} - \lambda_{\mu 2}$$

$$\varphi_2 = \varphi_\tau - \lambda_{\tau 2} \pm \pi = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi = \lambda_{\mu 1} - \lambda_{\mu 2} \qquad \text{OK!}$$

$$U = \begin{pmatrix} 0 & 0 & \pm 1 \\ |U_{\mu 1}| & |U_{\mu 2}| & 0 \\ |U_{\tau 1}| & -|U_{\tau 2}| & 0 \end{pmatrix}$$

Example: $m_{\nu_2} = m_{\nu_3}$

$$j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

$$U = R_{12} R_{13} W_{23} \quad \Rightarrow \quad j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L W_{23}^\dagger R_{13}^\dagger R_{12}^\dagger \gamma^\rho \ell_L$$

$$W_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \vartheta_{23} & \sin \vartheta_{23} e^{-i\delta_{23}} \\ 0 & -\sin \vartheta_{23} e^{-i\delta_{23}} & \cos \vartheta_{23} \end{pmatrix}$$

$$W_{23} \mathbf{n}_L = \mathbf{n}'_L \quad R_{12} R_{13} = U' \quad \Rightarrow \quad j_{W,L}^\rho = 2 \overline{\mathbf{n}'_L} U'^\dagger \gamma^\rho \ell_L$$

ν_2 and ν_3 are indistinguishable

$$\text{drop the prime} \quad \Rightarrow \quad j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

$$\text{real mixing matrix} \quad U = R_{12} R_{13}$$

Jarlskog Rephasing Invariant

- ▶ Simplest rephasing invariants: $|U_{\alpha k}|^2 = U_{\alpha k} U_{\alpha k}^*$, $U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$
- ▶ Simplest CPV rephasing invariants: $\Im[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}] = \pm J$

$$J = \Im[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}] = \Im \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- ▶ In standard parameterization:

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos \vartheta_{13} \sin 2\vartheta_{13} \sin \delta_{13} \end{aligned}$$

- ▶ Jarlskog invariant is useful for quantifying CP violation due to $U \neq U^*$ in a parameterization-independent way.
- ▶ All measurable CP-violation effects depend on J .

Maximal CP Violation

- ▶ Maximal CP violation is defined as the case in which $|J|$ has its maximum possible value

$$|J|_{\max} = \frac{1}{6\sqrt{3}}$$

- ▶ In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4, \quad s_{13} = 1/\sqrt{3}, \quad \sin \delta_{13} = \pm 1$$

- ▶ This case is called **Trimaximal Mixing**. All the absolute values of the elements of the mixing matrix are equal to $1/\sqrt{3}$:

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp \frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- ▶ Neutral-Current Weak Interaction Lagrangian:

$$\mathcal{L}_1^{(\text{NC})} = -\frac{g}{2 \cos \vartheta_W} j_Z^\rho Z_\rho \quad j_Z^\rho = j_{Z,L}^\rho + j_{Z,Q}^\rho$$

- ▶ Leptonic Weak Neutral Current: ($g_L^\nu = \frac{1}{2}$, $g_L^\ell = -\frac{1}{2} + \sin^2 \vartheta_W$, $g_R^\ell = \sin^2 \vartheta_W$)

$$j_{Z,L}^\rho = 2g_L^\nu \bar{\nu}_L' \gamma^\rho \nu_L' + 2g_L^\ell \bar{\ell}_L' \gamma^\rho \ell_L' + 2g_R^\ell \bar{\ell}_R' \gamma^\rho \ell_R'$$

- ▶ Invariant under mixing transformations with unitarity V_L^ℓ , V_R^ℓ , V_L^ν :

$$\begin{aligned} j_{Z,L}^\rho &= 2g_L^\nu \bar{\mathbf{n}}_L V_L^{\nu\dagger} \gamma^\rho V_L^\nu \mathbf{n}_L + 2g_L^\ell \bar{\ell}_L V_L^{\ell\dagger} \gamma^\rho V_L^\ell \ell_L + 2g_R^\ell \bar{\ell}_R V_R^{\ell\dagger} \gamma^\rho V_R^\ell \ell_R \\ &= 2g_L^\nu \bar{\mathbf{n}}_L \gamma^\rho \mathbf{n}_L + 2g_L^\ell \bar{\ell}_L \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R \gamma^\rho \ell_R \end{aligned}$$

- ▶ Invariant also under the mixing transformation $\nu_L = U \mathbf{n}_L$ which defines the flavor neutrino fields:

$$\begin{aligned} j_{Z,L}^\rho &= 2g_L^\nu \bar{\nu}_L U \gamma^\rho U^\dagger \nu_L + 2g_L^\ell \bar{\ell}_L \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R \gamma^\rho \ell_R \\ &= 2g_L^\nu \bar{\nu}_L \gamma^\rho \nu_L + 2g_L^\ell \bar{\ell}_L \gamma^\rho \ell_L + 2g_R^\ell \bar{\ell}_R \gamma^\rho \ell_R \end{aligned}$$

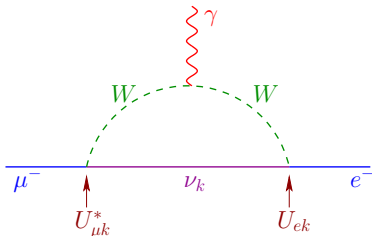
- ▶ Mixing has no effect in neutral-current weak interactions.

Lepton Numbers Violating Processes

Dirac mass term allows L_e, L_μ, L_τ violating processes

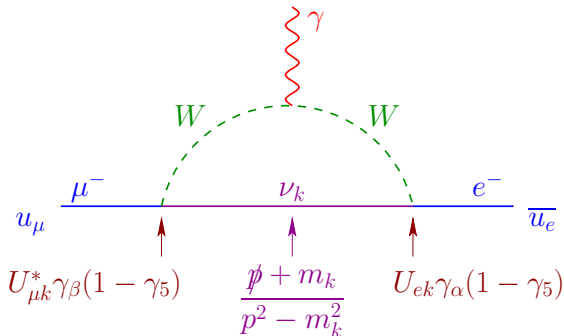
Example: $\mu^\pm \rightarrow e^\pm + \gamma, \quad \mu^\pm \rightarrow e^\pm + e^+ + e^-$

$$\mu^- \rightarrow e^- + \gamma$$



$$\sum_k U_{\mu k}^* U_{ek} = 0 \quad \Rightarrow \quad \text{GIM suppression:} \quad A \propto \sum_k U_{\mu k}^* U_{ek} f(m_k)$$

$$\begin{aligned} \mathcal{L}_1^{(CC)} &= -\frac{g}{2\sqrt{2}} W^\alpha [\bar{\nu}_e \gamma_\alpha (1 - \gamma_5) e + \bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu + \dots] \\ &= -\frac{g}{2\sqrt{2}} W^\alpha \sum_k [\bar{\nu}_k U_{ek}^* \gamma_\alpha (1 - \gamma_5) e + \bar{\nu}_k U_{\mu k}^* \gamma_\alpha (1 - \gamma_5) \mu + \dots] \end{aligned}$$



$$A \propto \sum_k \bar{u}_e U_{ek} \gamma_\alpha (1 - \gamma_5) \frac{\not{p} + m_k}{p^2 - m_k^2} U_{\mu k}^* \gamma_\beta (1 - \gamma_5) u_\mu$$

$$\frac{1}{p^2 - m_k^2} = p^{-2} \left(1 - \frac{m_k^2}{p^2}\right)^{-1} \simeq p^{-2} \left(1 + \frac{m_k^2}{p^2}\right)$$

$$A \propto \sum_k U_{ek} U_{\mu k}^* \left(1 + \frac{m_k^2}{p^2}\right) = \sum_k U_{ek} U_{\mu k}^* \frac{m_k^2}{p^2} \rightarrow \sum_k U_{ek} U_{\mu k}^* \frac{m_k^2}{m_W^2}$$

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi} \left| \sum_k U_{ek} U_{\mu k}^* \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$

[Petcov, Sov. J. Nucl. Phys. 25 (1977) 340; Bilenky, Petcov, Pontecorvo, PLB 67 (1977) 309; Lee, Shrock, PRD 16 (1977) 1444]

$$\text{Suppression factor: } \frac{m_k}{m_W} \lesssim 10^{-11} \quad \text{for } m_k \lesssim 1 \text{ eV}$$

$$(\text{BR})_{\text{the}} \lesssim 10^{-47}$$

$$(\text{BR})_{\text{exp}} \lesssim 10^{-11}$$

Majorana Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
 - Two-Component Theory of a Massless Neutrino
 - Majorana Equation
 - CP Symmetry
 - Effective Majorana Mass
 - Mixing of Three Majorana Neutrinos
- Dirac-Majorana Mass Term
- Sterile Neutrinos

Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127; T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671; A. Salam, Nuovo Cim. 5 (1957) 299]

▶ Dirac Equation: $(i\gamma^\mu \partial_\mu - m)\psi = 0$

▶ Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$

▶ Equations for the Chiral components are coupled by mass:

$$i\gamma^\mu \partial_\mu \psi_L = m \psi_R$$

$$i\gamma^\mu \partial_\mu \psi_R = m \psi_L$$

▶ They are decoupled for a massless fermion: **Weyl Equations** (1929)

$$i\gamma^\mu \partial_\mu \psi_L = 0$$

$$i\gamma^\mu \partial_\mu \psi_R = 0$$

▶ A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor).

- ▶ Chiral representation of γ matrices:

$$\gamma^0 = \begin{pmatrix} 0 & -\mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$$

$$P_L = \frac{1 - \gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{pmatrix} \quad P_R = \frac{1 + \gamma^5}{2} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix}$$

- ▶ Four-components Dirac spinor: $\psi_L = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

- ▶ The Weyl spinors ψ_L and ψ_R have only two components:

$$\psi_L = P_L \psi = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \quad \psi_R = P_R \psi = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- ▶ The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to **parity violation** ($\psi_L \xrightarrow{P} \psi_R$)
- ▶ The discovery of **parity violation** in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields \implies **Two-component Theory of a Massless Neutrino** (1957)
- ▶ 1958: Goldhaber, Grodzins and Sunyar measured the polarization of the neutrino in the electron capture $e^- + {}^{152}\text{Eu} \rightarrow {}^{152}\text{Sm}^* + \nu_e$, with the subsequent decay ${}^{152}\text{Sm}^* \rightarrow {}^{152}\text{Sm} + \gamma \implies$ **neutrinos are left-handed** $\implies \nu_L$ [PR 109 (1958) 1015]
- ▶ 1958: **V – A theory of Charged-Current Weak Interactions** by Feynman and Gell-Mann [PR 109 (1958) 193], Sudarshan and Marshak [PR 109 (1958) 1860], J.J. Sakurai [NC 7 (1958) 649]
- ▶ In the 1960s, the **Two-component Theory of a Massless Neutrino** was incorporated in the **Standard Model of Electroweak Interactions** of Glashow [NP 22 (1961) 579], Weinberg [PRL 19 (1967) 1264], Salam [Proc. of the 8th Nobel Symposium, 1968] through the **assumption of the absence of ν_R**

Majorana Equation

- ▶ Can a two-component spinor describe a massive fermion?

Yes! (E. Majorana, 1937)

- ▶ Trick: ψ_R and ψ_L are not independent:

$$\psi_R = \psi_L^c = C \overline{\psi_L}^T$$

charge-conjugation matrix: $C \gamma_\mu^T C^{-1} = -\gamma_\mu$

- ▶ ψ_L^c is right-handed: $P_R \psi_L^c = \psi_L^c$ $P_L \psi_L^c = 0$

- ▶ $i\gamma^\mu \partial_\mu \psi_L = m \psi_R$ \rightarrow $i\gamma^\mu \partial_\mu \psi_L = m \psi_L^c$ Majorana equation

- ▶ Majorana field: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^c$

$$\psi = \psi^c$$

Majorana condition

- ▶ $\psi = \psi^c$ implies the equality of particle and antiparticle
- ▶ Only neutral fermions can be Majorana particles
- ▶ For a Majorana field, the electromagnetic current vanishes identically:

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}^c\gamma^\mu\psi^c = -\psi^T C^\dagger \gamma^\mu C \bar{\psi}^T = \bar{\psi} C \gamma^\mu T C^\dagger \psi = -\bar{\psi}\gamma^\mu\psi = 0$$

- ▶ Only two independent components:
$$\psi = \begin{pmatrix} i\sigma^2 \chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$$

Majorana Lagrangian

Dirac Lagrangian

$$\begin{aligned}\mathcal{L}^D &= \bar{\nu}(i\partial - m)\nu \\ &= \bar{\nu}_L i\partial \nu_L + \bar{\nu}_R i\partial \nu_R - m(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)\end{aligned}$$

$$\nu_R \rightarrow \nu_L^c = C \bar{\nu}_L^T$$

$$\frac{1}{2} \mathcal{L}^D \rightarrow \bar{\nu}_L i\partial \nu_L - \frac{m}{2} \left(-\nu_L^T C^\dagger \nu_L + \bar{\nu}_L C \bar{\nu}_L^T \right)$$

Majorana Lagrangian

$$\begin{aligned}\mathcal{L}^M &= \bar{\nu}_L i\partial \nu_L - \frac{m}{2} \left(-\nu_L^T C^\dagger \nu_L + \bar{\nu}_L C \bar{\nu}_L^T \right) \\ &= \bar{\nu}_L i\partial \nu_L - \frac{m}{2} (\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c)\end{aligned}$$

- ▶ Majorana field: $\nu = \nu_L + \nu_L^c$
such that it satisfies the Majorana condition $\nu^c = \nu$

- ▶ Majorana Lagrangian: $\mathcal{L}^M = \frac{1}{2} \bar{\nu} (i\not{\partial} - m) \nu|_{\nu=\nu^c}$

- ▶ Quantized Dirac Neutrino Field:

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a^{(h)}(p) u^{(h)}(p) e^{-ip \cdot x} + b^{(h)\dagger}(p) v^{(h)}(p) e^{ip \cdot x} \right]$$

- ▶ Quantized Majorana Neutrino Field [$b^{(h)}(p) = a^{(h)}(p)$]

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a^{(h)}(p) u^{(h)}(p) e^{-ip \cdot x} + a^{(h)\dagger}(p) v^{(h)}(p) e^{ip \cdot x} \right]$$

- ▶ A Majorana field has half the degrees of freedom of a Dirac field

Lepton Number

$$\cancel{L = +1} \leftarrow \boxed{\nu = \nu^c} \rightarrow \cancel{L = -1}$$

$$\nu_L \implies L = +1$$

$$\nu_L^c \implies L = -1$$

$$\mathcal{L}^M = \bar{\nu}_L i \not{\partial} \nu_L - \frac{m}{2} (\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c)$$

Total Lepton Number is not conserved:

$$\boxed{\Delta L = \pm 2}$$

Best process to find violation of Total Lepton Number:

Neutrinoless Double- β Decay

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + 2e^- + \cancel{2\nu_e} \quad (\beta\beta_{0\nu}^-)$$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z - 2) + 2e^+ + \cancel{2\nu_e} \quad (\beta\beta_{0\nu}^+)$$

CP Symmetry

- ▶ Under a CP transformation

$$U_{\text{CP}} \nu_L(x) U_{\text{CP}}^{-1} = \xi_\nu^{\text{CP}} \gamma^0 \nu_L^c(x_P)$$

$$U_{\text{CP}} \nu_L^c(x) U_{\text{CP}}^{-1} = -\xi_\nu^{\text{CP}*} \gamma^0 \nu_L(x_P)$$

$$U_{\text{CP}} \overline{\nu_L}(x) U_{\text{CP}}^{-1} = \xi_\nu^{\text{CP}*} \overline{\nu_L^c}(x_P) \gamma^0$$

$$U_{\text{CP}} \overline{\nu_L^c}(x) U_{\text{CP}}^{-1} = -\xi_\nu^{\text{CP}} \overline{\nu_L}(x_P) \gamma^0$$

with $|\xi_\nu^{\text{CP}}|^2 = 1$, $x^\mu = (x^0, \vec{x})$, and $x_P^\mu = (x^0, -\vec{x})$

- ▶ The theory is CP-symmetric if there are values of the phase ξ_ν^{CP} such that the Lagrangian transforms as

$$U_{\text{CP}} \mathcal{L}(x) U_{\text{CP}}^{-1} = \mathcal{L}(x_P)$$

in order to keep invariant the action $I = \int d^4x \mathcal{L}(x)$

► The Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{M}}(x) = -\frac{1}{2} m \left[\overline{\nu_L^c}(x) \nu_L(x) + \overline{\nu_L}(x) \nu_L^c(x) \right]$$

transforms as

$$\begin{aligned} U_{\text{CP}} \mathcal{L}_{\text{mass}}^{\text{M}}(x) U_{\text{CP}}^{-1} = & -\frac{1}{2} m \left[-(\xi_\nu^{\text{CP}})^2 \overline{\nu_L}(x_{\text{P}}) \nu_L^c(x_{\text{P}}) \right. \\ & \left. -(\xi_\nu^{\text{CP}*})^2 \overline{\nu_L^c}(x_{\text{P}}) \nu_L(x_{\text{P}}) \right] \end{aligned}$$

► $U_{\text{CP}} \mathcal{L}_{\text{mass}}^{\text{M}}(x) U_{\text{CP}}^{-1} = \mathcal{L}_{\text{mass}}^{\text{M}}(x_{\text{P}})$ for $\xi_\nu^{\text{CP}} = \pm i$

► The one-generation Majorana theory is CP-symmetric

► The Majorana case is different from the Dirac case, in which the CP phase ξ_ν^{CP} is arbitrary

No Majorana Neutrino Mass in the SM

- ▶ Majorana Mass Term $\propto \left[\nu_L^T C^\dagger \nu_L - \overline{\nu}_L C \overline{\nu}_L^T \right]$ involves only the neutrino left-handed chiral field ν_L , which is present in the SM
- ▶ Eigenvalues of the weak isospin I , of its third component I_3 , of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	1/2	1/2 -1/2	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	1/2	1/2 -1/2	+1	1 0

- ▶ $\nu_L^T C^\dagger \nu_L$ has $I_3 = 1$ and $Y = -2 \implies$ needed $Y = 2$ Higgs triplet ($I = 1, I_3 = -1$)
- ▶ Compare with Dirac Mass Term $\propto \overline{\nu}_R \nu_L$ with $I_3 = 1/2$ and $Y = -1$ balanced by $\phi_0 \rightarrow \nu$ with $I_3 = -1/2$ and $Y = +1$

Confusing Majorana Antineutrino Terminology

- ▶ A Majorana neutrino is the same as a Majorana antineutrino
- ▶ Neutrino interactions are described by the CC and NC Lagrangians

$$\mathcal{L}_{1,L}^{\text{CC}} = -\frac{g}{\sqrt{2}} \left(\bar{\nu}_L \gamma^\mu \ell_L W_\mu + \bar{\ell}_L \gamma^\mu \nu_L W_\mu^\dagger \right)$$

$$\mathcal{L}_{1,\nu}^{\text{NC}} = -\frac{g}{2 \cos \vartheta_W} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

- ▶ Dirac: ν_L $\left\{ \begin{array}{l} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{array} \right.$
- ▶ Majorana: ν_L $\left\{ \begin{array}{l} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{array} \right.$
- ▶ Common implicit definitions:
 - left-handed Majorana neutrino \equiv neutrino
 - right-handed Majorana neutrino \equiv antineutrino

Effective Majorana Mass

- ▶ Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- ▶ Dimensionless action: $I = \int d^4x \mathcal{L}(x) \implies \mathcal{L}(x) \sim [E]^4$
- ▶ Kinetic terms: $\bar{\psi} i \not{\partial} \psi \sim [E]^4$, $(\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- ▶ Mass terms: $m \bar{\psi} \psi \sim [E]^4$, $m^2 \phi^\dagger \phi \sim [E]^4$
- ▶ CC weak interaction: $g \bar{\nu}_L \gamma^\rho \ell_L W_\rho \sim [E]^4$
- ▶ Yukawa couplings: $y \bar{L}_L \Phi \ell_R \sim [E]^4$
- ▶ Product of fields \mathcal{O}_d with energy dimension $d \equiv \text{dim-}d$ operator
- ▶ $\mathcal{L}(\mathcal{O}_d) = C_{(\mathcal{O}_d)} \mathcal{O}_d \implies C_{(\mathcal{O}_d)} \sim [E]^{4-d}$
- ▶ $\mathcal{O}_{d>4}$ are not renormalizable

- ▶ SM Lagrangian includes all $\mathcal{O}_{d \leq 4}$ invariant under $SU(2)_L \times U(1)_Y$
- ▶ SM cannot be considered as the final theory of everything
- ▶ SM is an effective low-energy theory
- ▶ It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- ▶ It is plausible that at low-energy there are effective non-renormalizable $\mathcal{O}_{d > 4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- ▶ All \mathcal{O}_d must respect $SU(2)_L \times U(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- ▶ $\mathcal{O}_{d>4}$ is suppressed by a coefficient \mathcal{M}^{4-d} , where \mathcal{M} is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}} \mathcal{O}_5 + \frac{g_6}{\mathcal{M}^2} \mathcal{O}_6 + \dots$$

- ▶ Analogy with $\mathcal{L}_{\text{eff}}^{(\text{CC})} \propto G_{\text{F}} (\overline{\nu_{eL}} \gamma^\rho e_L) (\overline{e_L} \gamma_\rho \nu_{eL}) + \dots$

$$\mathcal{O}_6 \rightarrow (\overline{\nu_{eL}} \gamma^\rho e_L) (\overline{e_L} \gamma_\rho \nu_{eL}) + \dots \quad \frac{g_6}{\mathcal{M}^2} \rightarrow \frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

- ▶ \mathcal{M}^{4-d} is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM
- ▶ The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality
- ▶ $\mathcal{O}_5 \implies$ Majorana neutrino masses (Lepton number violation)
- ▶ $\mathcal{O}_6 \implies$ Baryon number violation (proton decay)

- ▶ Only one dim-5 operator:

$$\begin{aligned} \mathcal{O}_5 &= (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.} \\ &= \frac{1}{2} (L_L^T C^\dagger \sigma_2 \vec{\sigma} L_L) \cdot (\Phi^T \sigma_2 \vec{\sigma} \Phi) + \text{H.c.} \end{aligned}$$

$$\mathcal{L}_5 = \frac{g_5}{2\mathcal{M}} (L_L^T C^\dagger \sigma_2 \vec{\sigma} L_L) \cdot (\Phi^T \sigma_2 \vec{\sigma} \Phi) + \text{H.c.}$$

- ▶ Electroweak Symmetry Breaking: $\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\mathcal{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \frac{g_5 v^2}{\mathcal{M}} \nu_L^T C^\dagger \nu_L + \text{H.c.} \implies m = \frac{g_5 v^2}{\mathcal{M}}$$

- ▶ The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM
- ▶ $m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_D^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses
(special case: See-Saw Mechanism)
- ▶ Example: $m_D \sim v \sim 10^2 \text{ GeV}$ and $\mathcal{M} \sim 10^{15} \text{ GeV} \implies m \sim 10^{-2} \text{ eV}$

Mixing of Three Majorana Neutrinos

▶ $\nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$

$$\mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \nu'^T_L C^\dagger M^L \nu'_L + \text{H.c.}$$
$$= \frac{1}{2} \sum_{\alpha, \beta = e, \mu, \tau} \nu'^T_{\alpha L} C^\dagger M^L_{\alpha\beta} \nu'_{\beta L} + \text{H.c.}$$

▶ In general, the matrix M^L is a complex symmetric matrix

$$\begin{aligned} \sum_{\alpha, \beta} \nu'^T_{\alpha L} C^\dagger M^L_{\alpha\beta} \nu'_{\beta L} &= \sum_{\alpha, \beta} \left(\nu'^T_{\alpha L} C^\dagger M^L_{\alpha\beta} \nu'_{\beta L} \right)^T \\ &= - \sum_{\alpha, \beta} \nu'^T_{\beta L} M^L_{\alpha\beta} (C^\dagger)^T \nu'_{\alpha L} = \sum_{\alpha, \beta} \nu'^T_{\beta L} C^\dagger M^L_{\alpha\beta} \nu'_{\alpha L} \\ &= \sum_{\alpha, \beta} \nu'^T_{\alpha L} C^\dagger M^L_{\beta\alpha} \nu'_{\beta L} \end{aligned}$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \iff M^L = M^{L^T}$$

▶ $\mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \nu_L^{\prime T} C^\dagger M^L \nu_L' + \text{H.c.}$

▶ $\nu_L' = V_L^\nu \mathbf{n}_L \quad \Rightarrow \quad \mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \nu_L^{\nu T} (V_L^\nu)^T C^\dagger M^L V_L^\nu \nu_L' + \text{H.c.}$

▶ $(V_L^\nu)^T M^L V_L^\nu = M, \quad M_{kj} = m_k \delta_{kj} \quad (k, j = 1, 2, 3)$

▶ Left-handed chiral fields with definite mass: $\mathbf{n}_L = V_L^{\nu \dagger} \nu_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{M}} &= \frac{1}{2} \left(\mathbf{n}_L^T C^\dagger M \mathbf{n}_L - \overline{\mathbf{n}}_L M C \mathbf{n}_L^T \right) \\ &= \frac{1}{2} \sum_{k=1}^3 m_k \left(\nu_{kL}^T C^\dagger \nu_{kL} - \overline{\nu}_{kL} C \nu_{kL}^T \right) \end{aligned}$$

▶ Majorana fields of massive neutrinos: $\nu_k = \nu_{kL} + \nu_{kL}^c$

$\nu_k^c = \nu_k$

▶ $\mathbf{n} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Rightarrow \mathcal{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^3 \overline{\nu}_k (i \not{\partial} - m_k) \nu_k |_{\nu_k = \nu_k^c}$

Mixing Matrix

- ▶ Leptonic Weak Charged Current:

$$j_{W,L}^{\rho\dagger} = 2 \bar{\ell}_L \gamma^\rho U \mathbf{n}_L \quad \text{with} \quad U = V_L^{\ell\dagger} V_L^\nu$$

- ▶ As in the Dirac case, we define the left-handed flavor neutrino fields as

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- ▶ In this way, as in the Dirac case, the Leptonic Weak Charged Current has the SM form

$$j_{W,L}^{\rho\dagger} = 2 \bar{\ell}_L \gamma^\rho \nu_L = 2 \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha L} \gamma^\rho \nu_{\alpha L}$$

- ▶ Important difference with respect to Dirac case:
Two additional CP-violating phases: Majorana phases

- ▶ Majorana Mass Term $\mathcal{L}^M = \frac{1}{2} \sum_{k=1}^3 m_k \nu_{kL}^T C^\dagger \nu_{kL} + \text{H.c.}$ is not invariant under the global U(1) gauge transformations

$$\nu_{kL} \rightarrow e^{i\varphi_k} \nu_{kL} \quad (k = 1, 2, 3)$$

- ▶ For eliminating some of the 6 phases of the unitary mixing matrix we can use only the global phase transformations (3 arbitrary phases)

$$l_\alpha \rightarrow e^{i\varphi_\alpha} l_\alpha \quad (\alpha = e, \mu, \tau)$$

▶ Weak Charged Current: $j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{l_{\alpha L}} \gamma^\rho U_{\alpha k} \nu_{kL}$

▶ Performing the transformation $l_\alpha \rightarrow e^{i\varphi_\alpha} l_\alpha$ we obtain

$$j_{W,L}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{l_{\alpha L}} e^{-i\varphi_\alpha} \gamma^\rho U_{\alpha k} \nu_{kL}$$

$$j_{W,L}^{\rho\dagger} = 2 \underbrace{e^{-i\varphi_e}}_1 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^3 \overline{l_{\alpha L}} \underbrace{e^{-i(\varphi_\alpha - \varphi_e)}}_2 \gamma^\rho U_{\alpha k} \nu_{kL}$$

▶ We can eliminate **3 phases** of the mixing matrix: one overall phase and two phases which can be factorized on the left.

▶ In the Dirac case we could eliminate also two phases which can be factorized on the right.

- ▶ In the Majorana case there are two additional physical Majorana phases which can be factorized on the right of the mixing matrixd:

$$U = U^D D^M \quad D^M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- ▶ U^D is a Dirac mixing matrix, with one Dirac phase
- ▶ Standard parameterization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- ▶ $D^M = \text{diag}(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3})$, but only two Majorana phases are physical

- ▶ All measurable quantities depend only on the differences of the Majorana phases

$$l_\alpha \rightarrow e^{i\varphi} l_\alpha \implies e^{i\lambda_k} \rightarrow e^{i(\lambda_k - \varphi)}$$

$e^{i(\lambda_k - \lambda_j)}$ remains constant

- ▶ Our convention: $\lambda_1 = 0 \implies D^M = \text{diag}(1, e^{i\lambda_2}, e^{i\lambda_3})$

- ▶ CP is conserved if all the elements of each column of the mixing matrix are either real or purely imaginary:

$$\delta_{13} = 0 \text{ or } \pi \quad \text{and} \quad \lambda_k = 0 \text{ or } \pi/2 \text{ or } \pi \text{ or } 3\pi/2$$

Dirac-Majorana Mass Term

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
 - One Generation
 - See-Saw Mechanism
 - Three-Generation Mixing
- Sterile Neutrinos

One Generation

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{L}} + \mathcal{L}_{\text{mass}}^{\text{R}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = -m_{\text{D}} \bar{\nu}_R \nu_L + \text{H.c.} \quad \text{Dirac Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{L}} = \frac{1}{2} m_{\text{L}} \nu_L^T C^\dagger \nu_L + \text{H.c.} \quad \text{Majorana Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{R}} = \frac{1}{2} m_{\text{R}} \nu_R^T C^\dagger \nu_R + \text{H.c.} \quad \text{New Majorana Mass Term!}$$

- ▶ Column matrix of left-handed chiral fields: $N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ C \overline{\nu_R^T} \end{pmatrix}$

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} N_L^T C^\dagger M N_L + \text{H.c.} \quad M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

- ▶ The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- ▶ Diagonalization: $n_L = U^\dagger N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \text{Real } m_k \geq 0$$

- ▶ $\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \sum_{k=1,2} m_k \nu_{kL}^T C^\dagger \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k \overline{\nu_k} \nu_k$

$$\nu_k = \nu_{kL} + \nu_{kL}^c$$

- ▶ Massive neutrinos are Majorana! $\nu_k = \nu_k^c$

Real Mass Matrix

- ▶ CP is conserved if the mass matrix is real: $M = M^*$

- ▶ $M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$ we consider real and positive m_R and m_D and real m_L

- ▶ A real symmetric mass matrix can be diagonalized with $U = \mathcal{O} \rho$

$$\mathcal{O} = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \quad \rho_k^2 = \pm 1$$

- ▶ $\mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix}$ $\tan 2\vartheta = \frac{2m_D}{m_R - m_L}$

$$m'_{2,1} = \frac{1}{2} \left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2} \right]$$

- ▶ m'_1 is negative if $m_L m_R < m_D^2$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m'_1 & 0 \\ 0 & \rho_2^2 m'_2 \end{pmatrix} \implies \boxed{m_k = \rho_k^2 m'_k}$$

- ▶ m'_2 is always positive:

$$m_2 = m'_2 = \frac{1}{2} \left[m_L + m_R + \sqrt{(m_L - m_R)^2 + 4 m_D^2} \right]$$

- ▶ If $m_L m_R \geq m_D^2$, then $m'_1 \geq 0$ and $\rho_1^2 = 1$

$$m_1 = \frac{1}{2} \left[m_L + m_R - \sqrt{(m_L - m_R)^2 + 4 m_D^2} \right]$$

$$\rho_1 = 1 \text{ and } \rho_2 = 1 \implies U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

- ▶ If $m_L m_R < m_D^2$, then $m'_1 < 0$ and $\rho_1^2 = -1$

$$m_1 = \frac{1}{2} \left[\sqrt{(m_L - m_R)^2 + 4 m_D^2} - (m_L + m_R) \right]$$

$$\rho_1 = i \text{ and } \rho_2 = 1 \implies U = \begin{pmatrix} i \cos \vartheta & \sin \vartheta \\ -i \sin \vartheta & \cos \vartheta \end{pmatrix}$$

- ▶ If Δm^2 is small, there are oscillations between active ν_a generated by ν_L and sterile ν_s generated by ν_R^c :

$$P_{\nu_a \rightarrow \nu_s}(L, E) = \sin^2 2\vartheta \sin^2 \left(\frac{\Delta m^2 L}{4 E} \right)$$

$$\Delta m^2 = m_2^2 - m_1^2 = (m_L + m_R) \sqrt{(m_L - m_R)^2 + 4 m_D^2}$$

- ▶ It can be shown that the CP parity of ν_k is $\xi_k^{\text{CP}} = i \rho_k^2$:

$$U_{\text{CP}} \nu_k(x) U_{\text{CP}}^{-1} = i \rho_k^2 \gamma^0 \nu_k(x_{\text{P}})$$

- ▶ Special cases:

- ▶ $m_L = m_R \implies$ Maximal Mixing
- ▶ $m_L = m_R = 0 \implies$ Dirac Limit
- ▶ $|m_L|, m_R \ll m_D \implies$ Pseudo-Dirac Neutrinos
- ▶ $m_L = 0 \quad m_D \ll m_R \implies$ See-Saw Mechanism

Maximal Mixing

$$m_L = m_R$$

$$\tan 2\vartheta = \frac{2m_D}{m_R - m_L} \implies \vartheta = \pi/4$$

$$m'_{2,1} = m_L \pm m_D$$

$$\left\{ \begin{array}{ll} \rho_1^2 = +1, & m_1 = m_L - m_D \quad \text{if } m_L \geq m_D \\ \rho_1^2 = -1, & m_1 = m_D - m_L \quad \text{if } m_L < m_D \end{array} \right.$$
$$m_2 = m_L + m_D$$

$$\underline{m_L < m_D}$$

$$\left\{ \begin{array}{l} \nu_{1L} = \frac{-i}{\sqrt{2}} (\nu_L - \nu_R^c) \\ \nu_{2L} = \frac{1}{\sqrt{2}} (\nu_L + \nu_R^c) \end{array} \right.$$

$$\left\{ \begin{array}{l} \nu_1 = \nu_{1L} + \nu_{1L}^c = \frac{-i}{\sqrt{2}} [(\nu_L + \nu_R) - (\nu_L^c + \nu_R^c)] \\ \nu_2 = \nu_{2L} + \nu_{2L}^c = \frac{1}{\sqrt{2}} [(\nu_L + \nu_R) + (\nu_L^c + \nu_R^c)] \end{array} \right.$$

Dirac Limit

$$m_L = m_R = 0$$

$$\blacktriangleright m'_{2,1} = \pm m_D \implies \begin{cases} \rho_1^2 = -1, & m_1 = m_D \\ \rho_2^2 = +1, & m_2 = m_D \end{cases}$$

- ▶ The two Majorana fields ν_1 and ν_2 can be combined to give one Dirac field:

$$\nu = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) = \nu_L + \nu_R$$

- ▶ A Dirac field ν can always be split in two Majorana fields:

$$\begin{aligned} \nu &= \frac{1}{2} [(\nu - \nu^c) + (\nu + \nu^c)] \\ &= \frac{i}{\sqrt{2}} \left(-i \frac{\nu - \nu^c}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left(\frac{\nu + \nu^c}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) \end{aligned}$$

- ▶ A Dirac field is equivalent to two Majorana fields with the same mass and opposite CP parities

Pseudo-Dirac Neutrinos

$$|m_L|, m_R \ll m_D$$

$$\blacktriangleright m'_{2,1} \simeq \frac{m_L + m_R}{2} \pm m_D$$

$$\blacktriangleright m'_1 < 0 \implies \rho_1^2 = -1 \implies m_{2,1} \simeq m_D \pm \frac{m_L + m_R}{2}$$

- ▶ The two massive Majorana neutrinos have opposite CP parities and are almost degenerate in mass
- ▶ The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_D (m_L + m_R)$$

- ▶ The oscillations occur with practically maximal mixing:

$$\tan 2\vartheta = \frac{2m_D}{m_R - m_L} \gg 1 \implies \vartheta \simeq \pi/4$$

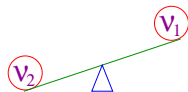
See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

$$m_L = 0 \quad m_D \ll m_R$$

- ▶ $\mathcal{L}_{\text{mass}}^L$ is forbidden by SM symmetries $\implies m_L = 0$
- ▶ $m_D \lesssim v \sim 100 \text{ GeV}$ is generated by SM Higgs Mechanism (protected by SM symmetries)
- ▶ m_R is not protected by SM symmetries $\implies m_R \sim \mathcal{M}_{\text{GUT}} \gg v$

$$\left. \begin{array}{l} m'_1 \simeq -\frac{m_D^2}{m_R} \\ m'_2 \simeq m_R \end{array} \right\} \implies \left\{ \begin{array}{l} \rho_1^2 = -1, \quad m_1 \simeq \frac{m_D^2}{m_R} \\ \rho_2^2 = +1, \quad m_2 \simeq m_R \end{array} \right.$$



- ▶ Natural explanation of smallness of neutrino masses
- ▶ Mixing angle is very small: $\tan 2\vartheta = 2 \frac{m_D}{m_R} \ll 1$
- ▶ ν_1 is composed mainly of active ν_L : $\nu_{1L} \simeq -i \nu_L$
- ▶ ν_2 is composed mainly of sterile ν_R : $\nu_{2L} \simeq \nu_R^c$

Connection with Effective Lagrangian Approach

- ▶ Dirac–Majorana neutrino mass term with $m_L = 0$:

$$\mathcal{L}^{\text{D+M}} = -m_D (\overline{\nu_R} \nu_L + \overline{\nu_L} \nu_R) + \frac{1}{2} m_R \left(\nu_R^T C^\dagger \nu_R + \nu_R^\dagger C \nu_R^* \right)$$

- ▶ Above the electroweak symmetry-breaking scale:

$$\mathcal{L}^{\text{D+M}} = -y^\nu \left(\overline{\nu_R} \tilde{\Phi}^\dagger L_L + \overline{L_L} \tilde{\Phi} \nu_R \right) + \frac{1}{2} m_R \left(\nu_R^T C^\dagger \nu_R + \nu_R^\dagger C \nu_R^* \right)$$

- ▶ If $m_R \gg v \implies \nu_R$ is static \implies kinetic term in equation of motion can be neglected:

$$0 \simeq \frac{\partial \mathcal{L}^{\text{D+M}}}{\partial \nu_R} = m_R \nu_R^T C^\dagger - y^\nu \overline{L_L} \tilde{\Phi}$$

$$\nu_R \simeq -\frac{y^\nu}{m_R} \tilde{\Phi}^T C \overline{L_L}^T$$

$$\mathcal{L}^{\text{D+M}} \rightarrow \mathcal{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \frac{(y^\nu)^2}{m_R} (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.}$$

$$\mathcal{L}_5 = \frac{g}{\mathcal{M}} (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.}$$

$$\mathcal{L}_5^{\text{D+M}} \simeq -\frac{1}{2} \frac{(y^\nu)^2}{m_R} (L_L^T \sigma_2 \Phi) C^\dagger (\Phi^T \sigma_2 L_L) + \text{H.c.}$$

$$g = -\frac{(y^\nu)^2}{2} \quad \mathcal{M} = m_R$$

- ▶ See-saw mechanism is a particular case of the effective Lagrangian approach.
- ▶ See-saw mechanism is obtained when dimension-five operator is generated only by the presence of ν_R with $m_R \sim \mathcal{M}$.
- ▶ In general, other terms can contribute to \mathcal{L}_5 .

Generalized Seesaw

- ▶ General effective Dirac-Majorana mass matrix:

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

- ▶ m_L generated by dim-5 operator:

$$m_L \ll m_D \ll m_R$$

- ▶ Eigenvalues:

$$\begin{vmatrix} m_L - \mu & m_D \\ m_D & m_R - \mu \end{vmatrix} = 0$$

$$\mu^2 - (\cancel{m_L} + m_R)\mu + m_L m_R - m_D^2 = 0$$

$$\mu = \frac{1}{2} \left[m_R \pm \sqrt{m_R^2 - 4(m_L m_R - m_D^2)} \right]$$

$$\begin{aligned}
\mu &= \frac{1}{2} \left[m_R \pm \sqrt{m_R^2 - 4(m_L m_R - m_D^2)} \right] \\
&= \frac{1}{2} \left[m_R \pm m_R \left(1 - 4 \frac{m_L m_R - m_D^2}{m_R^2} \right)^{1/2} \right] \\
&\simeq \frac{1}{2} \left[m_R \pm m_R \left(1 - 2 \frac{m_L m_R - m_D^2}{m_R^2} \right) \right]
\end{aligned}$$

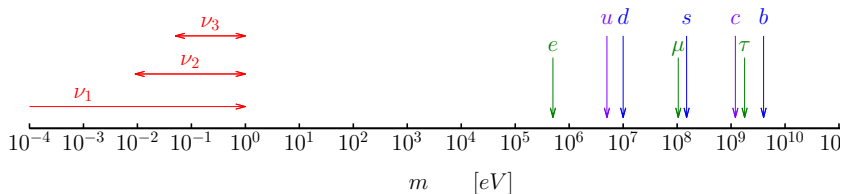
$$+ \rightarrow m_{\text{heavy}} \simeq m_R$$

$$- \rightarrow m_{\text{light}} \simeq m_L - \frac{m_D^2}{m_R}$$

$$\text{Type I seesaw: } m_L \ll \frac{m_D^2}{m_R} \implies m_{\text{light}} \simeq \frac{m_D^2}{m_R}$$

$$\text{Type II seesaw: } m_L \gg \frac{m_D^2}{m_R} \implies m_{\text{light}} \simeq m_L$$

Majorana Neutrino Mass?



known natural explanation of smallness of ν masses

New High Energy Scale $\mathcal{M} \Rightarrow \left\{ \begin{array}{l} \text{See-Saw Mechanism (if } \nu_R \text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{array} \right.$

both imply $\left\{ \begin{array}{l} \text{Majorana } \nu \text{ masses} \iff |\Delta L| = 2 \iff \beta\beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \frac{\mathcal{M}_{EW}^2}{\mathcal{M}} \end{array} \right.$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Right-Handed Neutrino Mass Term

Majorana mass term for ν_R respects the $SU(2)_L \times U(1)_Y$ Standard Model Symmetry!

$$\mathcal{L}_R^M = -\frac{1}{2} m (\overline{\nu_R^c} \nu_R + \overline{\nu_R} \nu_R^c)$$

Majorana mass term for ν_R breaks Lepton number conservation!

Three possibilities:

- ▶ Lepton number can be explicitly broken
- ▶ Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- ▶ Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\begin{aligned} \mathcal{L}_\Phi &= -y_d (\overline{L}_L \Phi \nu_R + \overline{\nu}_R \Phi^\dagger L_L) \xrightarrow{\langle \Phi \rangle \neq 0} -m_D (\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L) \\ \mathcal{L}_\eta &= -y_s (\eta \overline{\nu}_R^c \nu_R + \eta^\dagger \overline{\nu}_R \nu_R^c) \xrightarrow{\langle \eta \rangle \neq 0} -\frac{1}{2} m_R (\overline{\nu}_R^c \nu_R + \overline{\nu}_R \nu_R^c) \end{aligned}$$

$$\eta = 2^{-1/2} (\langle \eta \rangle + \rho + i\chi) \quad \mathcal{L}_{\text{mass}} = -\frac{1}{2} (\overline{\nu}_L^c \ \overline{\nu}_R) \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$m_R \text{ (scale of } L \text{ violation)} \gg m_D \text{ (EW scale)} \implies \text{See-Saw: } m_1 \simeq \frac{m_D^2}{m_R}$$

$\rho =$ massive scalar, $\chi =$ Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{iy_s}{\sqrt{2}} \chi \left[\overline{\nu}_2 \gamma^5 \nu_2 - \frac{m_D}{m_R} [\overline{\nu}_2 \gamma^5 \nu_1 + \overline{\nu}_1 \gamma^5 \nu_2] + \left(\frac{m_D}{m_R} \right)^2 \overline{\nu}_1 \gamma^5 \nu_1 \right]$$

Three-Generation Mixing

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{L}} + \mathcal{L}_{\text{mass}}^{\text{R}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = - \sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}} M_{s\alpha}^{\text{D}} \nu'_{\alpha L} + \text{H.c.}$$

$$\mathcal{L}_{\text{mass}}^{\text{L}} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'_{\alpha L} C^\dagger M_{\alpha\beta}^{\text{L}} \nu'_{\beta L} + \text{H.c.}$$

$$\mathcal{L}_{\text{mass}}^{\text{R}} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'_{sR} C^\dagger M_{ss'}^{\text{R}} \nu'_{s'R} + \text{H.c.}$$

$$\mathbf{N}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'^{\text{C}}_R \equiv \begin{pmatrix} \nu'^{\text{C}}_{1R} \\ \vdots \\ \nu'^{\text{C}}_{N_S R} \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \mathbf{N}'_L{}^T C^\dagger M^{\text{D+M}} \mathbf{N}'_L + \text{H.c.} \quad M^{\text{D+M}} = \begin{pmatrix} M^{\text{L}} & M^{\text{D}T} \\ M^{\text{D}} & M^{\text{R}} \end{pmatrix}$$

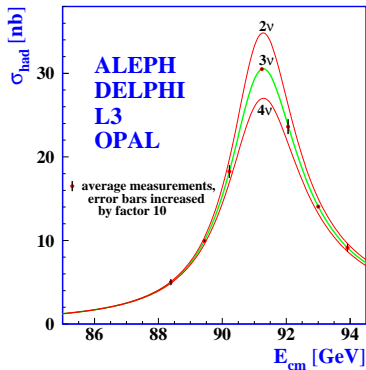
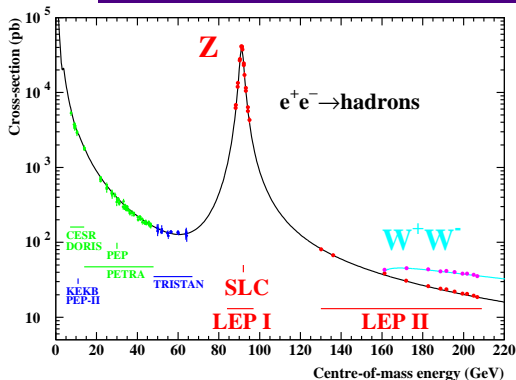
- ▶ Diagonalization of the **Dirac-Majorana Mass Term** \implies massive Majorana neutrinos
- ▶ **See-Saw Mechanism** \implies right-handed neutrinos have large Majorana masses and are decoupled from the low-energy phenomenology.
- ▶ If all right-handed neutrinos have large Majorana masses, at low energy we have an effective mixing of three Majorana neutrinos.
- ▶ It is possible that not all right-handed neutrinos have large Majorana masses: some right-handed neutrinos may correspond to low-energy Majorana particles which belong to new physics beyond the Standard Model (as a light neutralino in supersymmetric models).
- ▶ Light anti- ν_R are called **sterile neutrinos**

$$\nu_R^c \rightarrow \nu_{sL} \quad (\text{left-handed})$$

Sterile Neutrinos

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Sterile Neutrinos
 - Number of Flavor and Massive Neutrinos?
 - Sterile Neutrinos

Number of Flavor and Massive Neutrinos?



[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z \rightarrow \ell \bar{\ell}} + \sum_{q \neq t} \Gamma_{Z \rightarrow q \bar{q}} + \Gamma_{\text{inv}}$$

$$\Gamma_{\text{inv}} = N_\nu \Gamma_{Z \rightarrow \nu \bar{\nu}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

$$e^+e^- \rightarrow Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \implies \nu_e \nu_\mu \nu_\tau$$

3 light active flavor neutrinos

mixing $\implies \nu_{\alpha L} = \sum_{k=1}^N U_{\alpha k} \nu_{kL} \quad \alpha = e, \mu, \tau$ $N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	\dots
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	\dots
	ACTIVE			STERILE		

$$\nu_{\alpha L} = \sum_{k=1}^N U_{\alpha k} \nu_{kL} \quad \alpha = e, \mu, \tau, s_1, s_2, \dots$$

Sterile Neutrinos

- ▶ Sterile means no standard model interactions
- ▶ Obviously no electromagnetic interactions as normal active neutrinos
- ▶ Thus sterile means no standard weak interactions
- ▶ But sterile neutrinos are not absolutely sterile:
 - ▶ Gravitational Interactions
 - ▶ New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos
- ▶ Active neutrinos (ν_e, ν_μ, ν_τ) can oscillate into sterile neutrinos (ν_s)
- ▶ Observables:
 - ▶ Disappearance of active neutrinos
 - ▶ Indirect evidence through combined fit of data
- ▶ Powerful window on new physics beyond the Standard Model

No GIM with Sterile Neutrinos

[Lee, Shrock, PRD 16 (1977) 1444; Schechter, Valle PRD 22 (1980) 2227]

- ▶ Neutrino Neutral-Current Weak Interaction Lagrangian:

$$\mathcal{L}_1^{(\text{NC})} = -\frac{g}{2 \cos \vartheta_W} Z_\rho \overline{\nu'_L} \gamma^\rho \nu'_L$$

- ▶ The transformation to active flavor neutrino fields is independent of the existence of sterile neutrinos: $\nu'_L = V_L^\ell \nu_L$

$$\mathcal{L}_1^{(\text{NC})} = -\frac{g}{2 \cos \vartheta_W} Z_\rho \overline{\nu_L} \gamma^\rho \nu_L = -\frac{g}{2 \cos \vartheta_W} Z_\rho \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \gamma^\rho \nu_{\alpha L}$$

- ▶ Mixing with sterile neutrinos: $\nu_{\alpha L} = \sum_{k=1}^{3+N_s} U_{\alpha k} \nu_{kL}$

- ▶ No GIM: $\mathcal{L}_1^{(\text{NC})} = -\frac{g}{2 \cos \vartheta_W} Z_\rho \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \overline{\nu_{jL}} \gamma^\rho \nu_{kL} \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k}$

- ▶ $\sum_{\alpha=e,\mu,\tau,s_1,\dots} U_{\alpha j}^* U_{\alpha k} = \delta_{jk}$ but $\sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \neq \delta_{jk}$

Effect on Invisible Width of Z Boson?

- ▶ Amplitude of $Z \rightarrow \nu_j \bar{\nu}_k$ decay:

$$\begin{aligned} A(Z \rightarrow \nu_j \bar{\nu}_k) &= \langle \nu_j \bar{\nu}_k | - \int d^4x \mathcal{L}_1^{(\text{NC})}(x) | Z \rangle \\ &= \frac{g}{2 \cos \vartheta_W} \langle \nu_j \bar{\nu}_k | \int d^4x \bar{\nu}_{jL}(x) \gamma^\rho \nu_{kL}(x) Z_\rho(x) | Z \rangle \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \end{aligned}$$

- ▶ If $m_k \ll m_Z/2$ for all k 's, the neutrino masses are negligible in all the matrix elements and we can approximate

$$\frac{g}{2 \cos \vartheta_W} \langle \nu_j \bar{\nu}_k | \int d^4x \bar{\nu}_{jL}(x) \gamma^\rho \nu_{kL}(x) Z_\rho(x) | Z \rangle \simeq A_{\text{SM}}(Z \rightarrow \nu_\ell \bar{\nu}_\ell)$$

- ▶ $A_{\text{SM}}(Z \rightarrow \nu_\ell \bar{\nu}_\ell)$ is the Standard Model amplitude of Z decay into a massless neutrino-antineutrino pair of any flavor $\ell = e, \mu, \tau$

- ▶ $A(Z \rightarrow \nu_j \bar{\nu}_k) \simeq A_{\text{SM}}(Z \rightarrow \nu_\ell \bar{\nu}_\ell) \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k}$

- ▶ $P(Z \rightarrow \nu \bar{\nu}) = \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} |A(Z \rightarrow \nu_j \bar{\nu}_k)|^2$

▶ $P(Z \rightarrow \nu\bar{\nu}) \simeq P_{\text{SM}}(Z \rightarrow \nu_\ell\bar{\nu}_\ell) \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \right|^2$

▶ Effective number of neutrinos in Z decay:

$$N_\nu^{(Z)} = \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \right|^2$$

▶ Using the unitarity relation $\sum_{k=1}^{3+N_s} U_{\alpha k} U_{\beta k}^* = \delta_{\alpha\beta}$ we obtain

$$\begin{aligned} N_\nu^{(Z)} &= \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \sum_{\beta=e,\mu,\tau} U_{\beta j} U_{\beta k}^* \\ &= \sum_{\alpha=e,\mu,\tau} \sum_{\beta=e,\mu,\tau} \underbrace{\sum_{j=1}^{3+N_s} U_{\alpha j}^* U_{\beta j}}_{\delta_{\alpha\beta}} \underbrace{\sum_{k=1}^{3+N_s} U_{\alpha k} U_{\beta k}^*}_{\delta_{\alpha\beta}} = \sum_{\alpha=e,\mu,\tau} 1 = 3 \end{aligned}$$

▶ $N_\nu^{(Z)} = 3$ independently of the number of light sterile neutrinos!

Effect of Heavy Sterile Neutrinos

[Jarlskog, PLB 241 (1990) 579; Bilenky, Grimus, Neufeld, PLB 252 (1990) 119]

$$\blacktriangleright N_{\nu}^{(Z)} = \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \right|^2 R_{jk} \quad \text{with}$$

$$R_{jk} = \left(1 - \frac{m_j^2 + m_k^2}{2m_Z^2} - \frac{(m_j^2 - m_k^2)^2}{2m_Z^4} \right) \frac{\lambda(m_Z^2, m_j^2, m_k^2)}{m_Z^2} \theta(m_Z - m_j - m_k)$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

$$\blacktriangleright R_{jk} \leq 1 \quad \Rightarrow \quad \boxed{N_{\nu}^{(Z)} \leq 3}$$