

Neutrinos: from Particle to Astroparticle Physics

Part III: Phenomenology

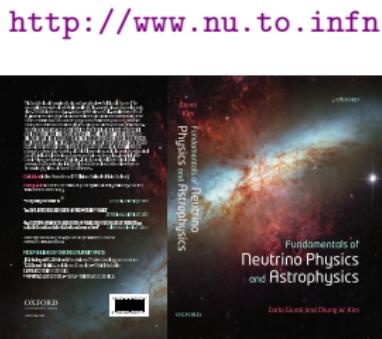
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Neutrino Unbound: <http://www.nu.to.infn.it>

Torino Graduate School in Physics and Astrophysics, January 2016



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and
Astrophysics
Oxford University Press
15 March 2007 – 728 pages

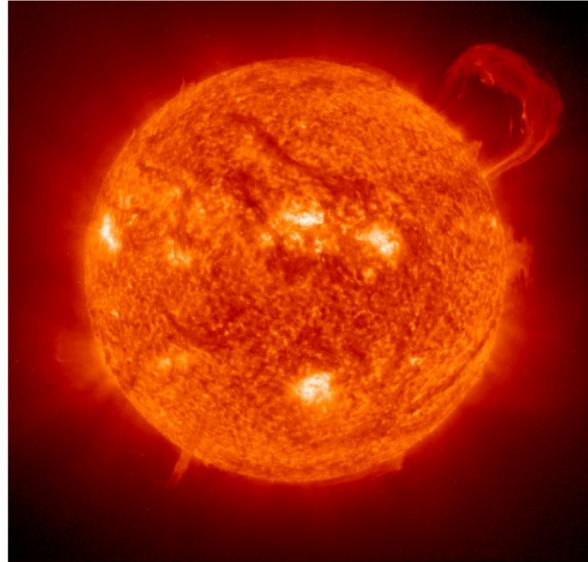
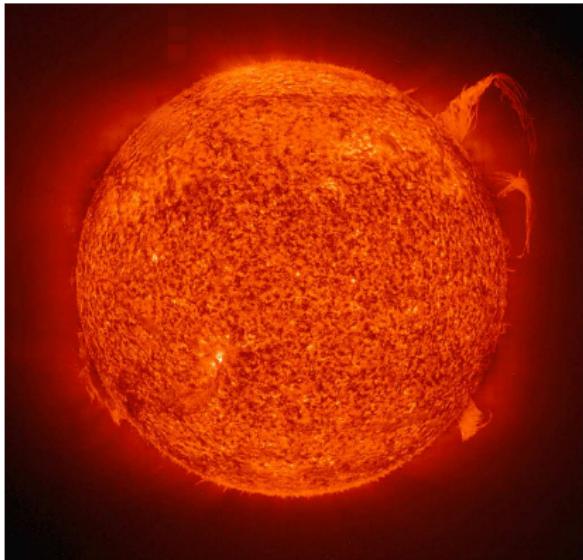
Part III: Phenomenology

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Absolute Scale of Neutrino Masses
- Light Sterile Neutrinos
- Conclusions

Solar Neutrinos and KamLAND

- Solar Neutrinos and KamLAND
 - Standard Solar Model (SSM)
 - Homestake
 - Gallium Experiments
 - Kamiokande
 - Super-Kamiokande
 - SNO: Sudbury Neutrino Observatory
 - KamLAND
 - LMA Solar Neutrino Oscillations
 - BOREXino
- Atmospheric and LBL Oscillation Experiments
- Absolute Scale of Neutrino Masses
- Light Sterile Neutrinos

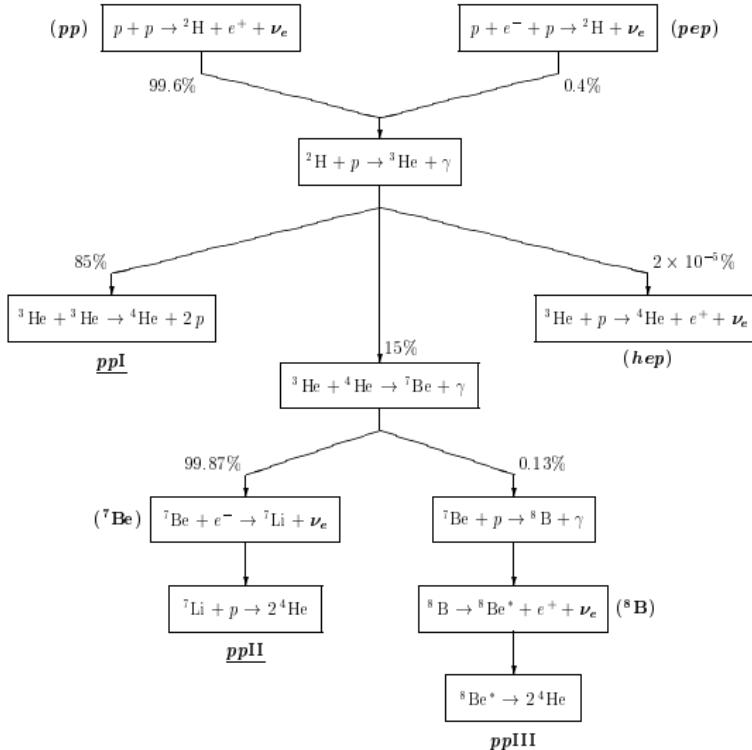
The Sun



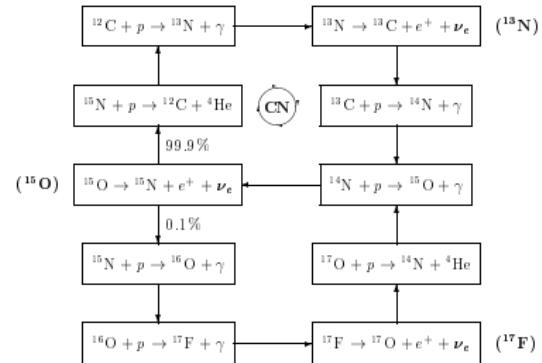
Extreme ultraviolet Imaging Telescope (EIT) 304 Å images of the Sun emission in this spectral line (He II) shows the upper chromosphere at a temperature of about 60,000 K

[The Solar and Heliospheric Observatory (SOHO), <http://sohowww.nascom.nasa.gov/>]

Standard Solar Model (SSM)

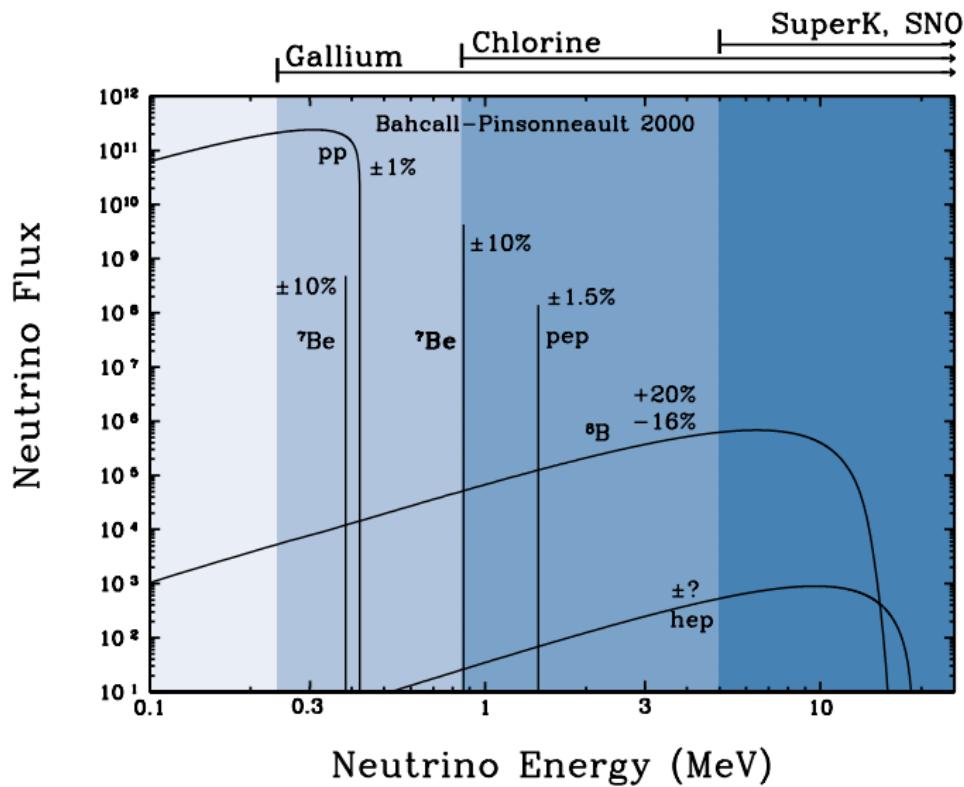


pp chain and CNO cycle

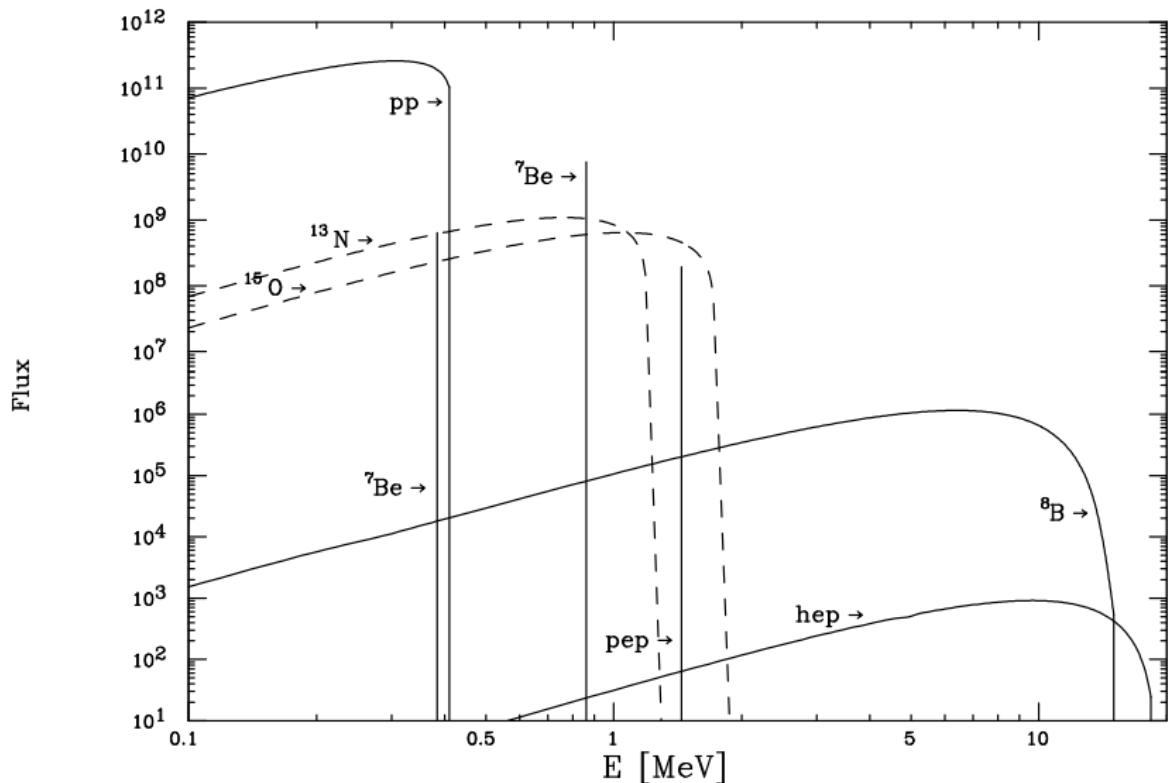


Bahcall SSMs

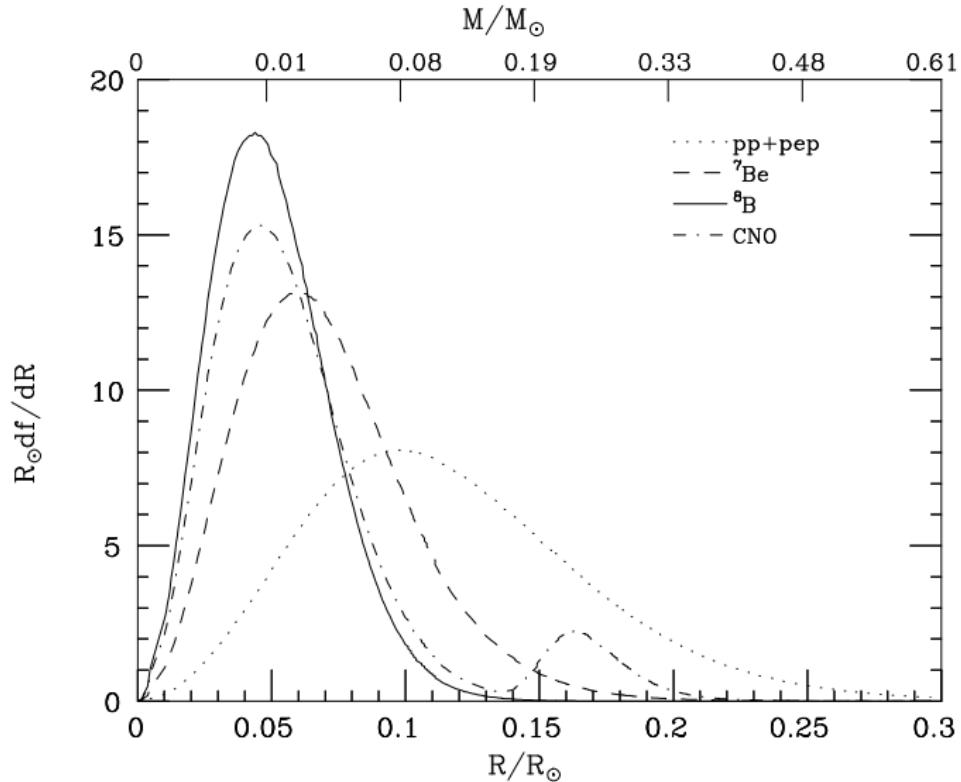
[J.N. Bahcall, <http://www.sns.ias.edu/~jnb/>]



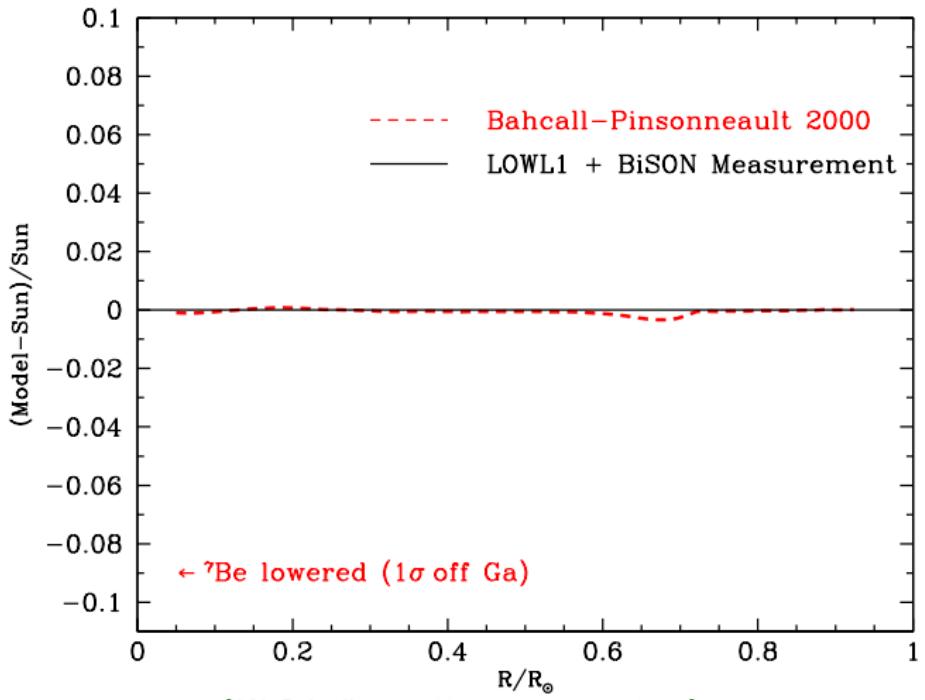
[J.N. Bahcall, <http://www.sns.ias.edu/~jnb>]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between $0.05 R_\odot$ and $0.95 R_\odot$ and is 0.08% for the deep interior region, $r < 0.25 R_\odot$, in which neutrinos are produced

Homestake



[Pontecorvo (1946), Alvarez (1949)]

radiochemical experiment

Homestake Gold Mine (South Dakota)

1478 m deep, 4200 m.w.e. $\Rightarrow \Phi_\mu \simeq 4 \text{ m}^{-2} \text{ day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long (6×10^5 liters)

615 tons of tetrachloroethylene (C_2Cl_4), 2.16×10^{30} atoms of ${}^{37}\text{Cl}$ (133 tons)

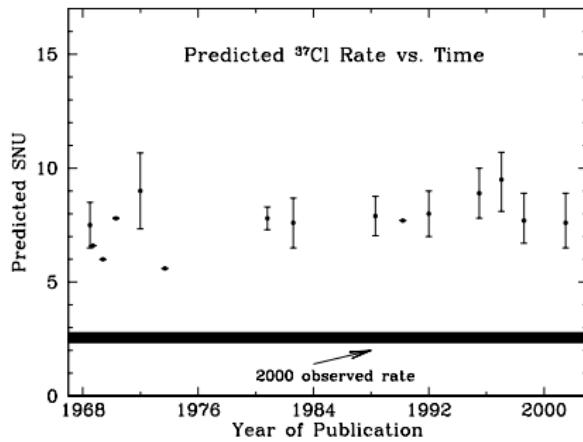
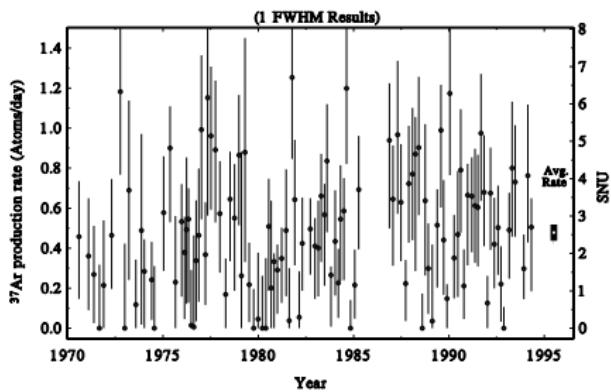
energy threshold: $E_{\text{th}}^{\text{Cl}} = 0.814 \text{ MeV} \Rightarrow {}^8\text{B}, {}^7\text{Be}, \text{ pep, hep, } {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F}$

1970–1994, 108 extractions \Rightarrow

$$\frac{R_{\text{Cl}}^{\text{exp}}}{R_{\text{Cl}}^{\text{SSM}}} = 0.34 \pm 0.03 \quad [\text{APJ 496 (1998) 505}]$$

$$R_{\text{Cl}}^{\text{exp}} = 2.56 \pm 0.23 \text{ SNU}$$

$$1 \text{ SNU} = 10^{-36} \text{ events atom}^{-1} \text{ s}^{-1}$$



Gallium Experiments

SAGE, GALLEX, GNO

radiochemical experiments



threshold: $E_{\text{th}}^{\text{Ga}} = 0.233 \text{ MeV} \implies pp, {}^7\text{Be}, {}^8\text{B}, pep, hep, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F}$

$$\text{SAGE+GALLEX+GNO} \implies \frac{R_{\text{Ga}}^{\text{exp}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.56 \pm 0.03$$

$$R_{\text{Ga}}^{\text{exp}} = 72.4 \pm 4.7 \text{ SNU} \quad R_{\text{Ga}}^{\text{SSM}} = 128^{+9}_{-7} \text{ SNU}$$

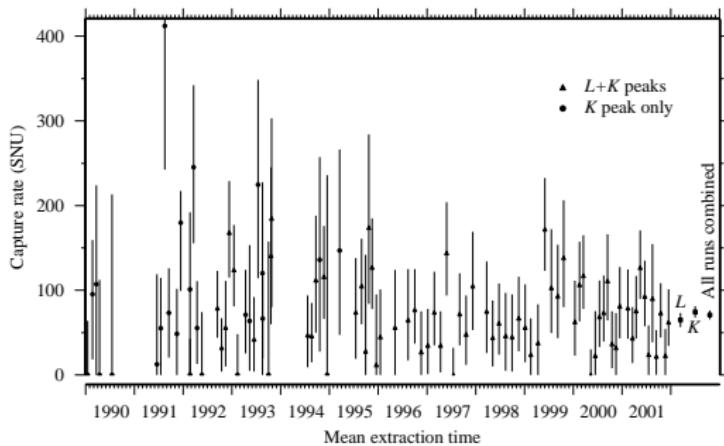
SAGE: Soviet-American Gallium Experiment

Baksan Neutrino Observatory, northern Caucasus

50 tons of metallic ^{71}Ga , 2000 m deep, 4700 m.w.e. $\Rightarrow \Phi_\mu \simeq 2.6 \text{ m}^{-2} \text{ day}^{-1}$

detector test: ^{51}Cr Source: $R = 0.95^{+0.11+0.06}_{-0.10-0.05}$ [PRC 59 (1999) 2246]

$$1990 - 2001 \quad \Rightarrow \quad \frac{R_{\text{Ga}}^{\text{SAGE}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.54 \pm 0.05 \quad [\text{astro-ph/0204245}]$$



GALLEX: GALLium EXperiment

Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

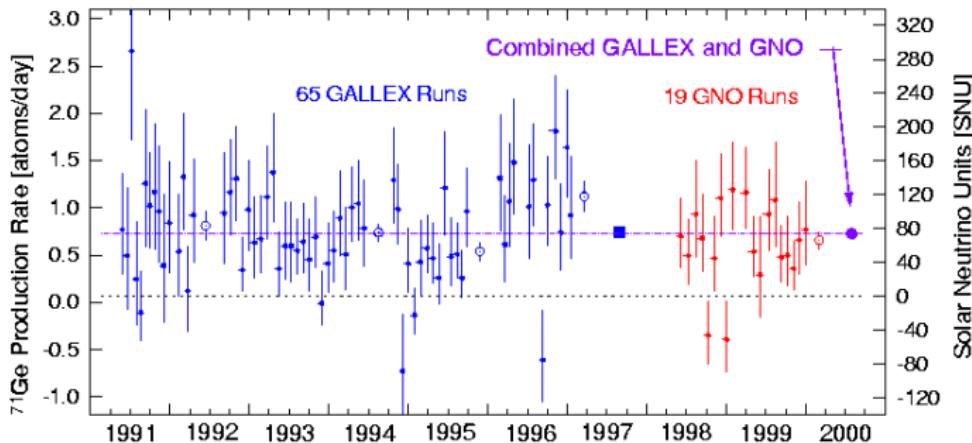
30.3 tons of gallium in 101 tons of gallium chloride ($\text{GaCl}_3\text{-HCl}$) solution

May 1991 – Jan 1997 $\implies \frac{R_{\text{Ga}}^{\text{GALLEX}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.61 \pm 0.06$ [PLB 477 (1999) 127]

GNO: Gallium Neutrino Observatory

continuation of GALLEX: 30.3 tons of gallium

May 1998 – Jan 2000 $\implies \frac{R_{\text{Ga}}^{\text{GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.51 \pm 0.08$ [PLB 490 (2000) 16]



$$\frac{R_{\text{Ga}}^{\text{GALLEX+GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.58 \pm 0.05$$

Kamiokande

water Cherenkov detector

$$\nu + e^- \rightarrow \nu + e^-$$

Sensitive to ν_e , ν_μ , ν_τ , but $\sigma(\nu_e) \simeq 6\sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 6.75 \text{ MeV} \implies {}^8\text{B}$, hep

Jan 1987 – Feb 1995 (2079 days)

$$\frac{R_{\nu e}^{\text{Kam}}}{R_{\nu e}^{\text{SSM}}} = 0.55 \pm 0.08$$

[PRL 77 (1996) 1683]

Super-Kamiokande

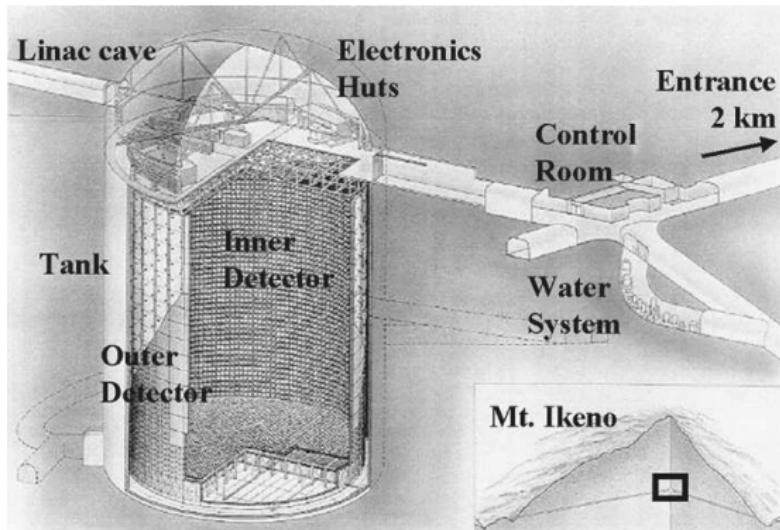
continuation of Kamiokande

50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 4.75 \text{ MeV} \implies {}^8\text{B}, \text{ hep}$

1996 – 2001 (1496 days)

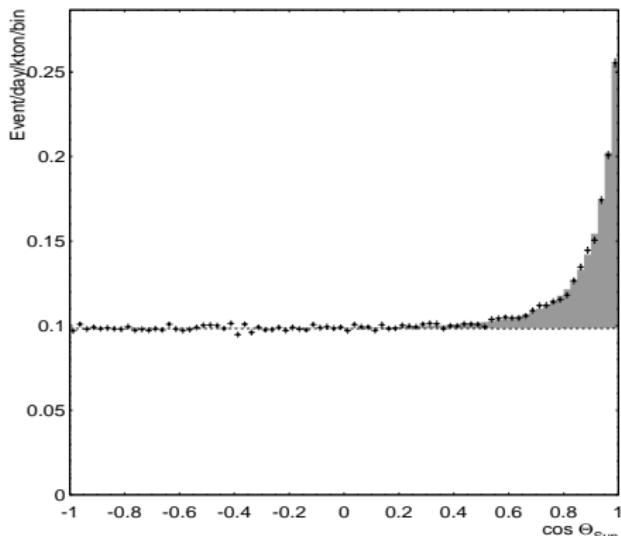
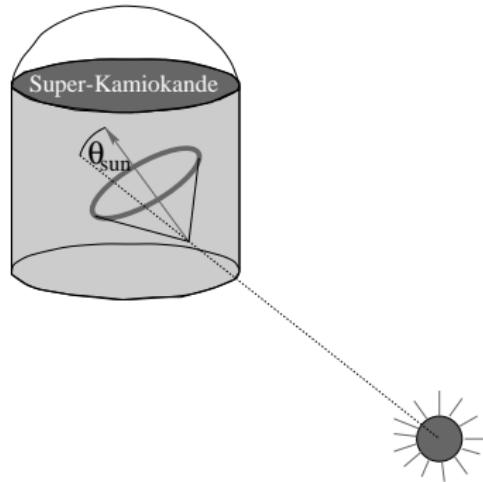
$$\frac{R_{\nu e}^{\text{SK}}}{R_{\nu e}^{\text{SSM}}} = 0.465 \pm 0.015 \quad [\text{SK, PLB 539 (2002) 179}]$$



the Super-Kamiokande underground water Cherenkov detector
located near Higashi-Mozumi, Gifu Prefecture, Japan
access is via a 2 km long truck tunnel

[R. J. Wilkes, SK, hep-ex/0212035]

Super-Kamiokande $\cos \theta_{\text{sun}}$ distribution

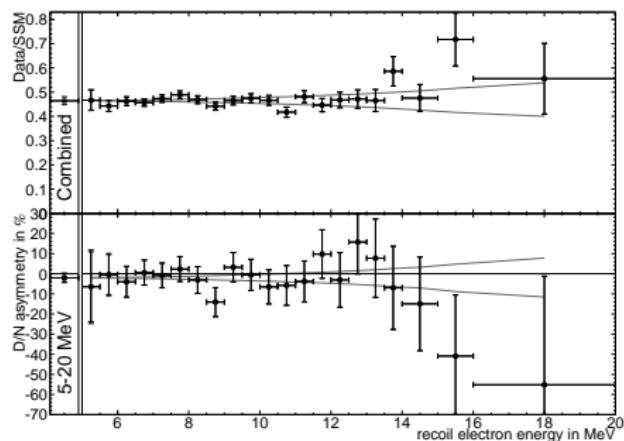


the points represent observed data,
the histogram shows the best-fit signal
(shaded) plus background, the horizontal
dashed line shows the estimated back-
ground

the peak at $\cos \theta_{\text{sun}} = 1$ is due to solar
neutrinos

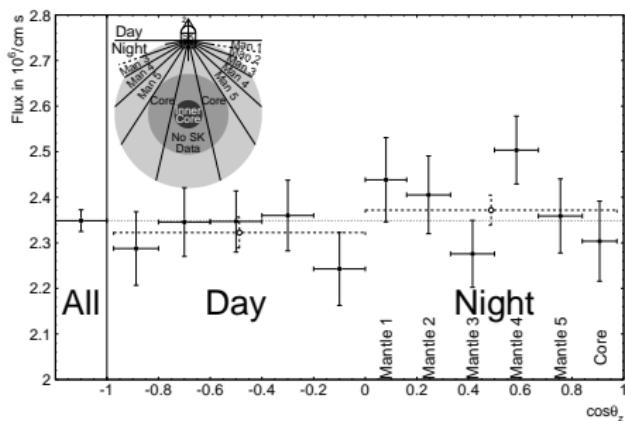
[Smy, hep-ex/0208004]

Super-Kamiokande energy spectrum normalized to BP2000 SSM



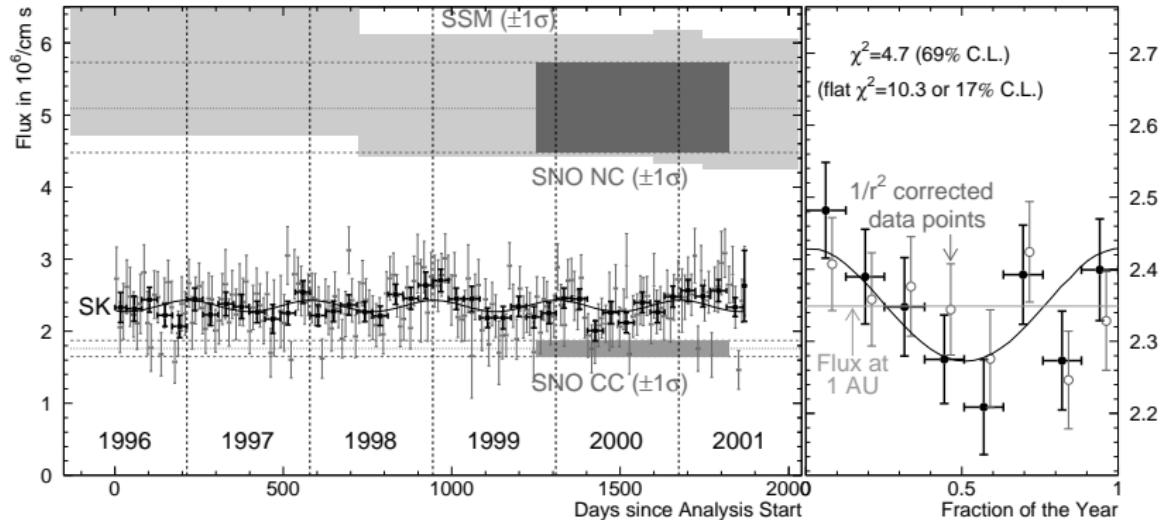
Day-Night asymmetry
as a function of energy

solar zenith angle (θ_z) dependence
of Super-Kamiokande data



[Smy, hep-ex/0208004]

Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days.

The black data points are measured every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points by subtracting the expected 7% variation.

[Smy, hep-ex/0208004]

SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of D₂O, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.



$$\left. \begin{array}{l} \text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2 \text{ MeV} \\ \text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2 \text{ MeV} \\ \text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0 \text{ MeV} \end{array} \right\} \Rightarrow {}^8\text{B, hep}$$

D₂O phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{SSM}}^{\text{CC}}} = 0.35 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{SSM}}^{\text{CC}}} = 1.01 \pm 0.13$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{SSM}}^{\text{ES}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

NaCl phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{SSM}}^{\text{CC}}} = 0.31 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{SSM}}^{\text{CC}}} = 1.03 \pm 0.09$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{SSM}}^{\text{ES}}} = 0.44 \pm 0.06$$

[PRL 92 (2004) 181301]

$$\phi_{\nu_e}^{\text{SNO}} = 1.76 \pm 0.11 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

$$\phi_{\nu_\mu, \nu_\tau}^{\text{SNO}} = 5.41 \pm 0.66 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

SNO solved
solar neutrino problem



Neutrino Physics
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

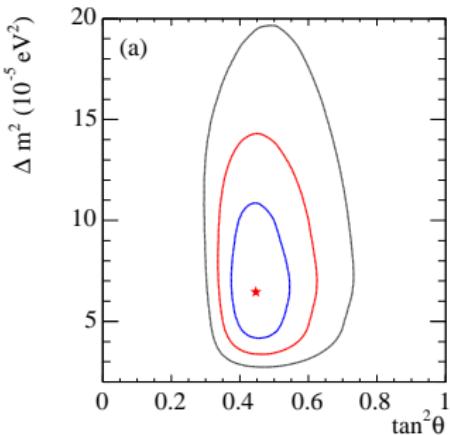
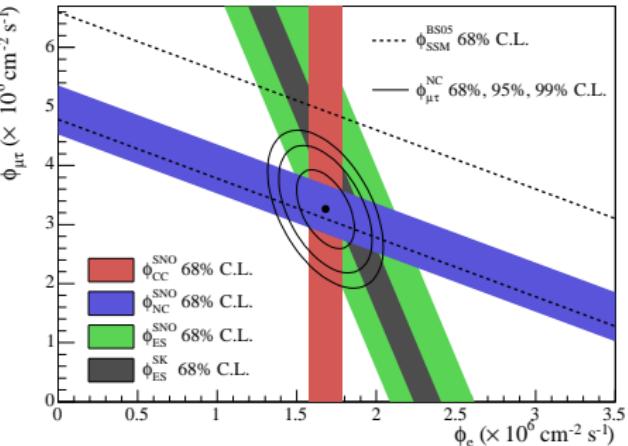
$\nu_e \rightarrow \nu_\mu, \nu_\tau$ oscillations



Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \vartheta \simeq 0.45$$



[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

6.7% of flux from one reactor at 88 km

average distance from reactors: 180 km 79% of flux from 26 reactors at 138–214 km
14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \rightarrow e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8 \text{ MeV}$

data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.):

$$N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$$

expected number of background events:

$$N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$$

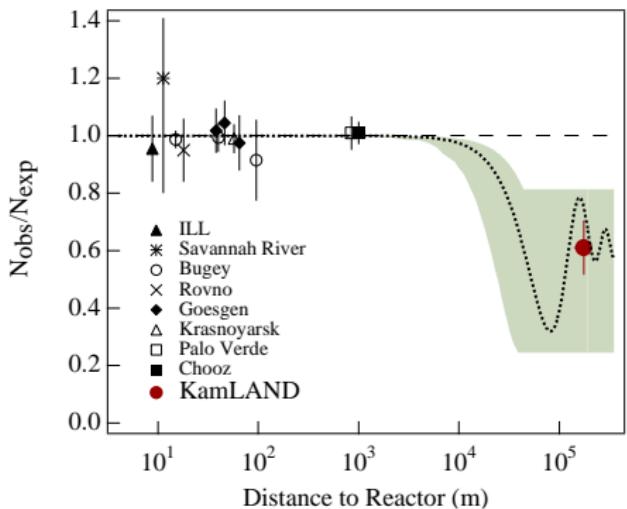
observed number of neutrino events:

$$N_{\text{observed}}^{\text{KamLAND}} = 54$$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

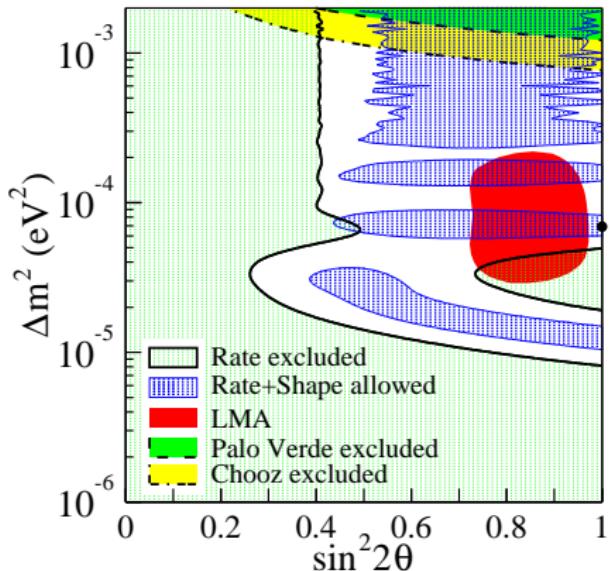
99.95% C.L. evidence
of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



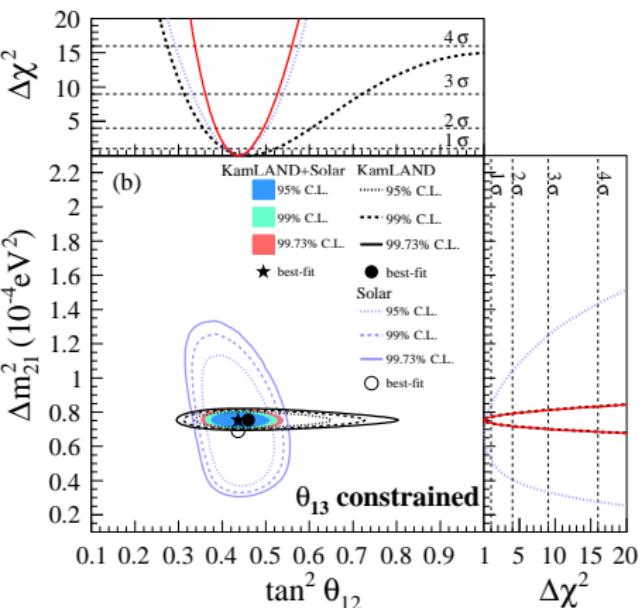
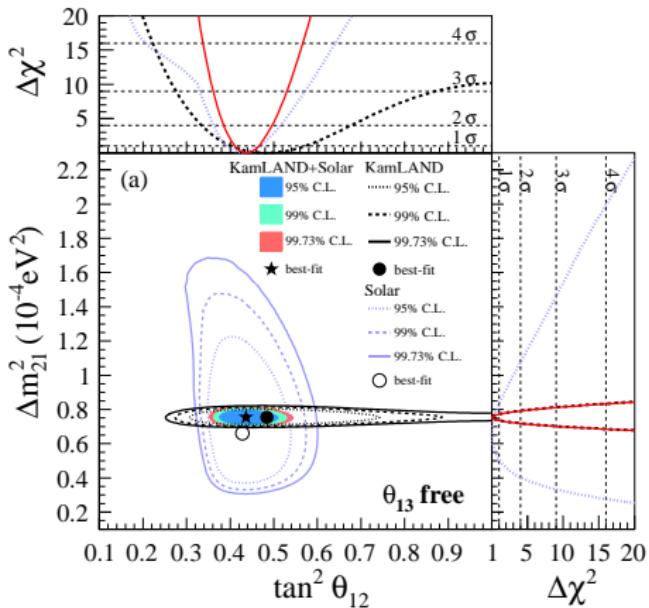
Shade: 95% C.L. LMA

Curve: $\left\{ \begin{array}{l} \Delta m^2 = 5.5 \times 10^{-5} \text{ eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{array} \right.$



95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]



$$\Delta m_{21}^2 = 7.53^{+0.19}_{-0.18} \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \vartheta_{12} = 0.437^{+0.029}_{-0.026}$$

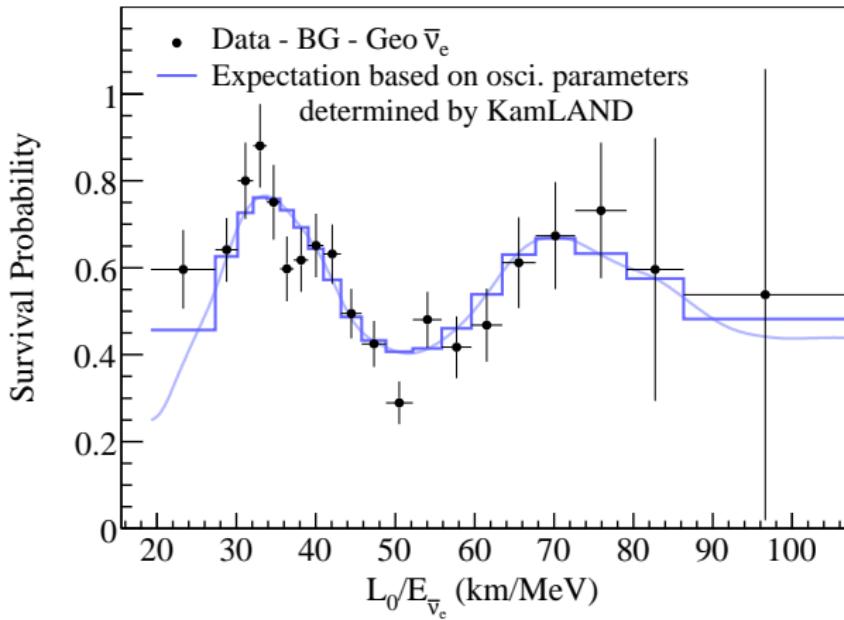
$$\sin^2 \vartheta_{13} = 0.023 \pm 0.015$$

$$\Delta m_{21}^2 = 7.53 \pm 0.18 \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \vartheta_{12} = 0.436^{+0.029}_{-0.025}$$

$$\sin^2 \vartheta_{13} = 0.023 \pm 0.002$$

[KamLAND, PRD 88 (2013) 033001]



[KamLAND, PRL 100 (2008) 221803]

LMA Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5} \text{ eV}^2$ $\tan^2 \vartheta \simeq 0.4$

$$\overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} = \frac{1}{2} + \left(\frac{1}{2} - P_c \right) \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)} \quad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left| \frac{d \ln A}{dx} \right|_R} \quad F = 1 - \tan^2 \vartheta$$

$$A_{CC} \simeq 2\sqrt{2}EG_F N_e^c \exp\left(-\frac{x}{x_0}\right) \implies \left| \frac{d \ln A}{dx} \right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15} \text{ eV}$$

$$\tan^2 \vartheta \simeq 0.4 \implies \sin^2 2\vartheta \simeq 0.82, \cos 2\vartheta \simeq 0.43 \quad \gamma \simeq 2 \times 10^4 \left(\frac{E}{\text{MeV}} \right)^{-1}$$

$$\gamma \gg 1 \implies P_c \ll 1 \implies \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun,LMA}} \simeq \frac{1}{2} + \frac{1}{2} \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$\cos 2\vartheta_M^0 = \frac{\Delta m^2 \cos 2\vartheta - A_{CC}^0}{\sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC}^0)^2 + (\Delta m^2 \sin 2\vartheta)^2}}$$

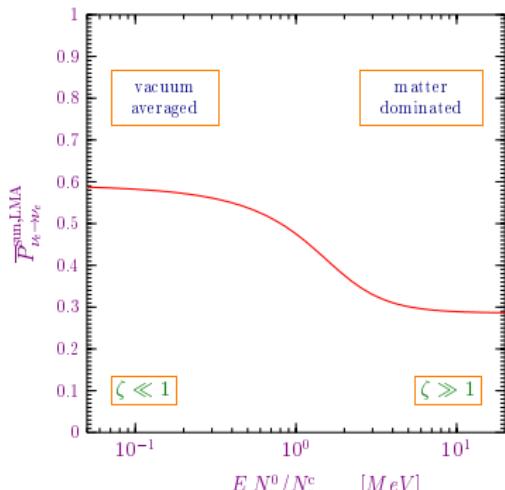
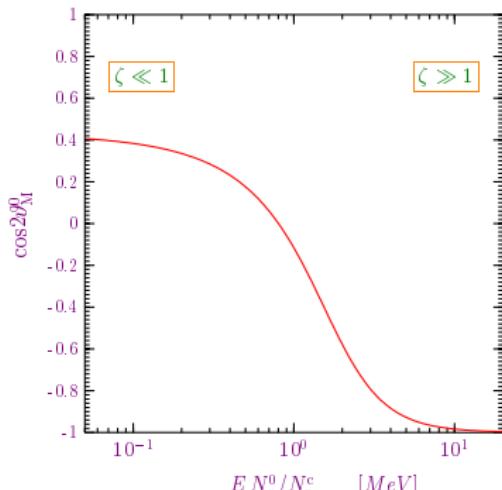
critical parameter [Bahcall, Peña-Garay, JHEP 0311 (2003) 004]

$$\zeta = \frac{A_{CC}^0}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2}EG_F N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\text{MeV}} \right) \left(\frac{N_e^0}{N_e^c} \right)$$

$$\zeta \ll 1 \implies \vartheta_M^0 \simeq \vartheta \implies \bar{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} \simeq 1 - \frac{1}{2} \sin^2 2\vartheta$$

$$\zeta \gg 1 \implies \vartheta_M^0 \simeq \pi/2 \implies \bar{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} \simeq \sin^2 \vartheta$$

vacuum averaged
survival probability
matter dominated
survival probability

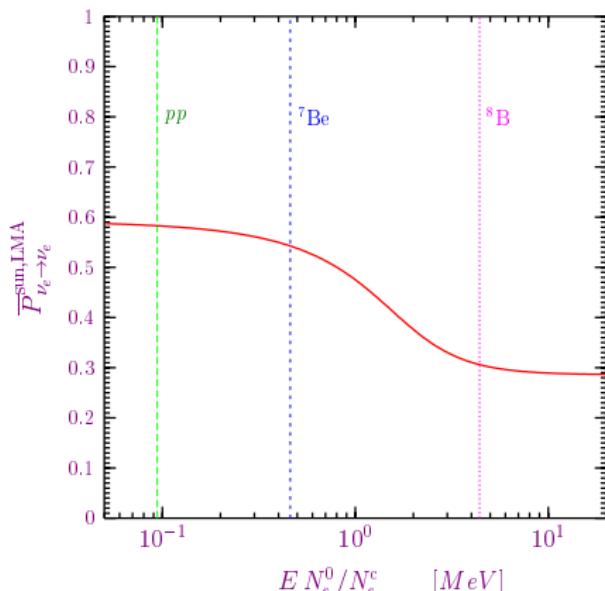


$$\zeta = \frac{A_{CC}^0}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2}EG_F N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\text{MeV}} \right) \left(\frac{N_e^0}{N_e^c} \right)$$

$$\langle E \rangle_{pp} \simeq 0.27 \text{ MeV}, \langle r_0 \rangle_{pp} \simeq 0.1 R_\odot \implies \langle E N_e^0 / N_e^c \rangle_{pp} \simeq 0.094 \text{ MeV}$$

$$E_{^7\text{Be}} \simeq 0.86 \text{ MeV}, \langle r_0 \rangle_{^7\text{Be}} \simeq 0.06 R_\odot \implies \langle E N_e^0 / N_e^c \rangle_{^7\text{Be}} \simeq 0.46 \text{ MeV}$$

$$\langle E \rangle_{^8\text{B}} \simeq 6.7 \text{ MeV}, \langle r_0 \rangle_{^8\text{B}} \simeq 0.04 R_\odot \implies \langle E N_e^0 / N_e^c \rangle_{^8\text{B}} \simeq 4.4 \text{ MeV}$$

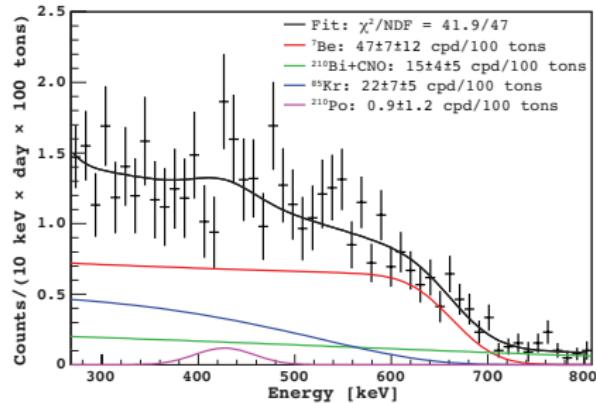
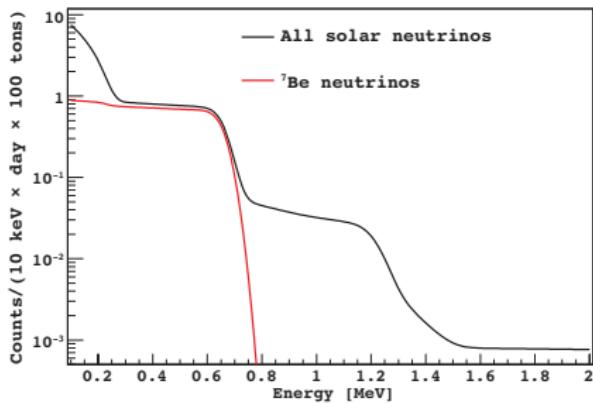


BOREXino

[BOREXino, PLB 658 (2008) 101]

Real-time measurement of ${}^7\text{Be}$ solar neutrinos (0.862 MeV)

$$\nu + e \rightarrow \nu + e \quad E = 0.862 \text{ MeV} \quad \Rightarrow \quad \sigma_{\nu_e} \simeq 5.5 \sigma_{\nu_\mu, \nu_\tau}$$

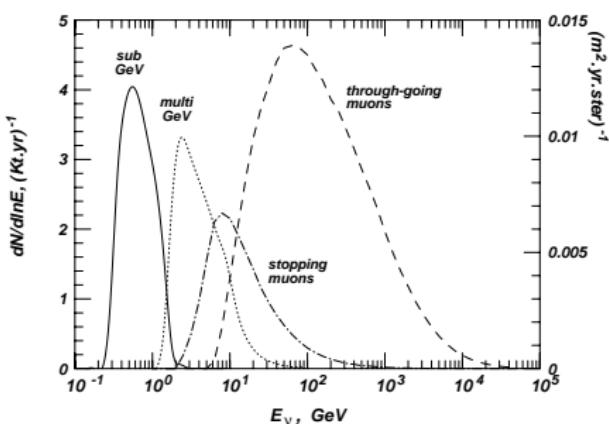
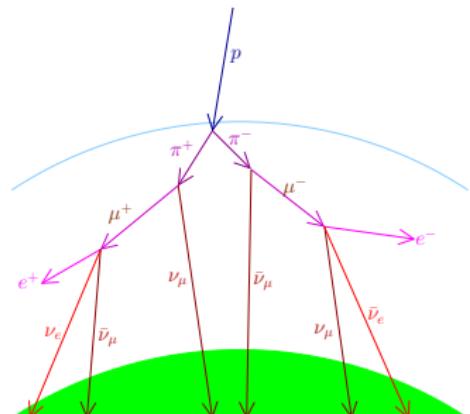


$$n_{\text{the}}^{\text{no-osc}} = 75 \pm 4 \text{ day}^{-1} (100 \text{ tons})^{-1} \quad n_{\text{exp}} = 47 \pm 7 \pm 12 \text{ day}^{-1} (100 \text{ tons})^{-1}$$
$$n_{\text{the}}^{\text{osc}} = 49 \pm 4 \text{ day}^{-1} (100 \text{ tons})^{-1}$$

Atmospheric and LBL Oscillation Experiments

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
 - Atmospheric Neutrinos
 - Super-Kamiokande Up-Down Asymmetry
 - Fit of Super-Kamiokande Atmospheric Data
 - Kamiokande, Soudan-2, MACRO and MINOS
 - K2K
 - MINOS
- Absolute Scale of Neutrino Masses
- Light Sterile Neutrinos
- Conclusions

Atmospheric Neutrinos



$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1 \text{ GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

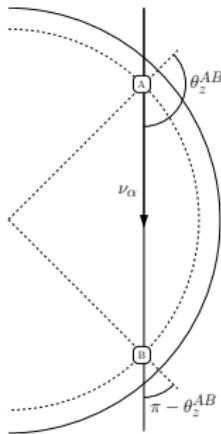
$$R_{\text{sub-GeV}}^K = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R_{\text{multi-GeV}}^K = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1 \text{ GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi_{\nu_\alpha}^{(A)}(\theta_z^{AB}) = \phi_{\nu_\alpha}^{(B)}(\pi - \theta_z^{AB}) \quad \phi_{\nu_\alpha}^{(A)}(\theta_z^{AB}) = \phi_{\nu_\alpha}^{(B)}(\theta_z^{AB})$$

\downarrow

$$\phi_{\nu_\alpha}^{(A)}(\theta_z) = \phi_{\nu_\alpha}^{(A)}(\pi - \theta_z)$$

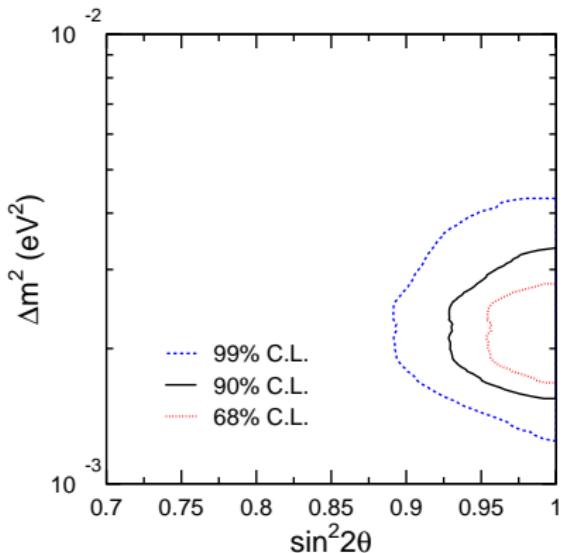
(December 1998)

$$A_{\nu_\mu}^{\text{up-down}}(\text{SK}) = \left(\frac{N_{\nu_\mu}^{\text{up}} - N_{\nu_\mu}^{\text{down}}}{N_{\nu_\mu}^{\text{up}} + N_{\nu_\mu}^{\text{down}}} \right) = -0.296 \pm 0.048 \pm 0.01$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



Best Fit: $\left\{ \begin{array}{l} \nu_\mu \rightarrow \nu_\tau \\ \Delta m^2 = 2.1 \times 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta = 1.0 \end{array} \right.$
1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5 \text{ GeV} \implies \sim 20 \text{ events/yr}$
- τ -Decay \implies Many Final States

ν_τ -Enriched Sample

$$N_{\nu_\tau}^{\text{the}} = 78 \pm 26 \text{ @ } \Delta m^2 = 2.4 \times 10^{-3} \text{ eV}^2$$

$$N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}$$

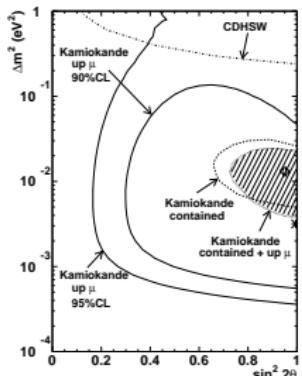
$$N_{\nu_\tau} > 0 \text{ @ } 2.4\sigma$$

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

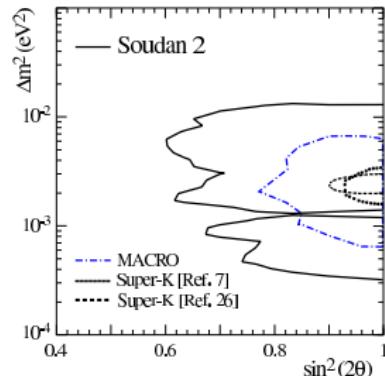
Check: OPERA ($\nu_\mu \rightarrow \nu_\tau$)
CERN to Gran Sasso (CNGS)
 $L \simeq 732 \text{ km}$ $\langle E \rangle \simeq 18 \text{ GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

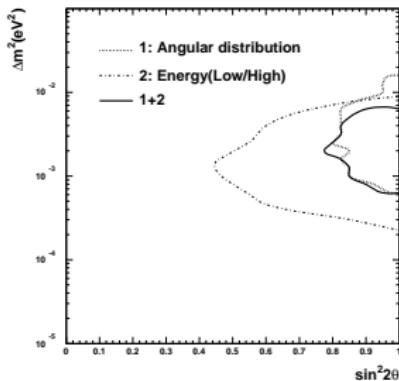
Kamiokande, Soudan-2, MACRO and MINOS



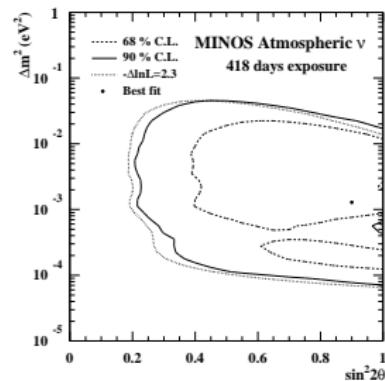
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]

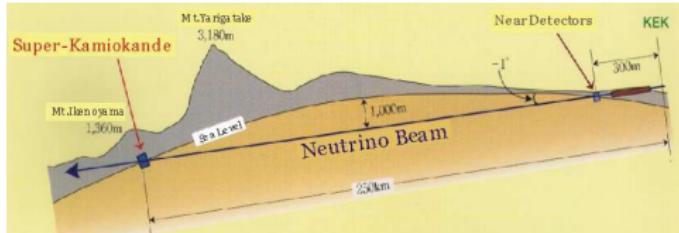


[MACRO, hep-ex/0304037]



[MINOS, hep-ex/0512036]

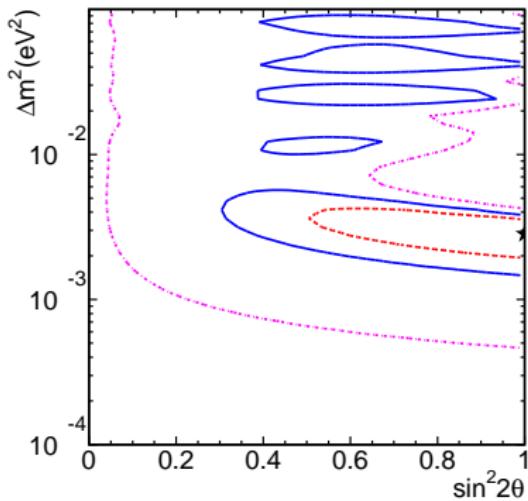
confirmation of atmospheric allowed region (June 2002)



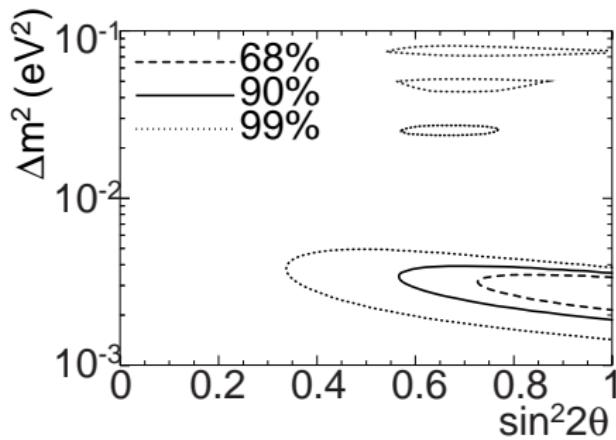
KEK to Kamioka
(Super-Kamiokande)

250 km

$\nu_\mu \rightarrow \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]

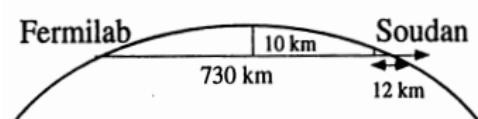


[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

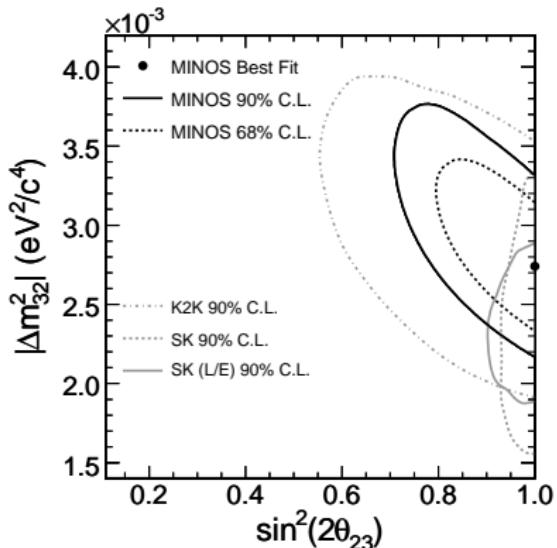
MINOS

May 2005 – Feb 2006

<http://www-numi.fnal.gov/>



Near Detector: 1 km

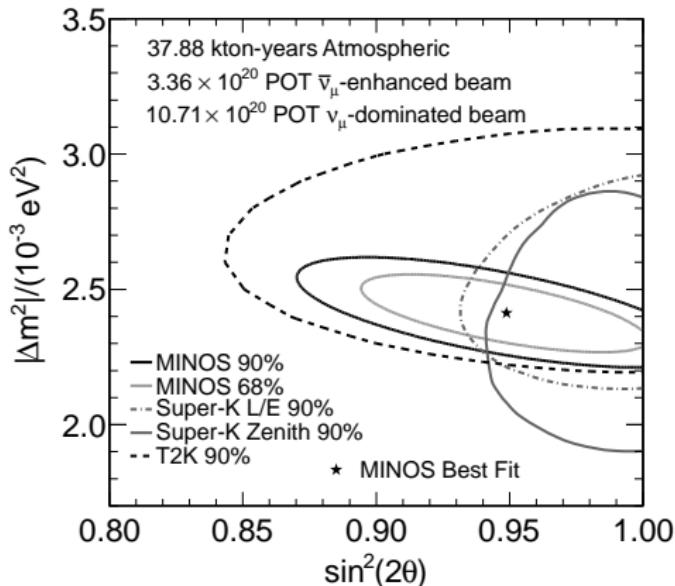


$$\nu_\mu \rightarrow \nu_\mu$$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\vartheta > 0.87 @ 68\% CL$$

[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]



$$|\Delta m_{31}^2| = (2.41^{+0.09}_{-0.10}) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\vartheta_{23} = 0.950^{+0.035}_{-0.036}$$

[MINOS, PRL 110 (2013) 251801]

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$\vartheta_{23} \simeq \vartheta_{\text{ATM}}$ $\vartheta_{12} \simeq \vartheta_{\text{SOL}}$ $\beta\beta_{0\nu}$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\Delta m_{21}^2 = (7.65^{+0.23}_{-0.20}) \times 10^{-5} \text{ eV}^2 \quad |\Delta m_{31}^2| = (2.40^{+0.12}_{-0.11}) \times 10^{-3} \text{ eV}^2$$

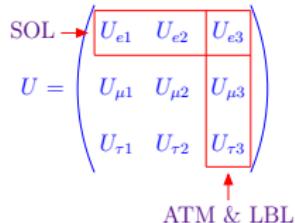
$$\sin^2 \vartheta_{12} = 0.304^{+0.022}_{-0.016} \quad \sin^2 \vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

$$\sin^2 \vartheta_{13} < 0.035 \quad (90\% \text{ C.L.})$$

[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]

Small ϑ_{13}

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2|$$



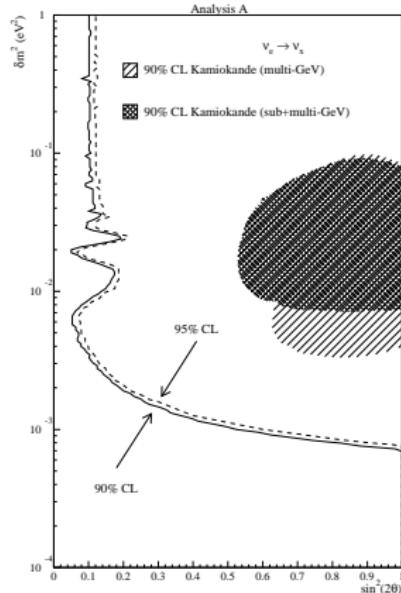
CHOOZ: $\left\{ \begin{array}{l} \Delta m_{\text{CHOOZ}}^2 = \Delta m_{31}^2 = \Delta m_{\text{ATM}}^2 \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{array} \right.$

↓

$|U_{e3}|^2 \lesssim 5 \times 10^{-2}$

[Bilenky, Giunti, PLB 444 (1998) 379]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS
ARE PRACTICALLY DECOUPLED!



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

$$|U_{e1}|^2 \simeq \cos^2 \vartheta_{\text{SOL}} \quad |U_{e2}|^2 \simeq \sin^2 \vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2 \vartheta_{\text{ATM}} \quad |U_{\tau 3}|^2 \simeq \cos^2 \vartheta_{\text{ATM}}$$

Effective ATM and LBL Oscillation Probabilities

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta} &= \left| \sum_{k=1}^3 U_{\alpha k}^* U_{\beta k} e^{-im_k^2 L/2E} \right|^2 * \left| e^{im_1^2 L/2E} \right|^2 \\ &= \left| \sum_{k=1}^3 U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E}\right) \right|^2 \end{aligned}$$

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1$$

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E}\right) \right|^2$$

$$U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^* U_{\beta 3}$$

$$\begin{aligned}
P_{\nu_\alpha \rightarrow \nu_\beta} &= \left| \delta_{\alpha\beta} - U_{\alpha 3}^* U_{\beta 3} \left[1 - \exp\left(-i \frac{\Delta m_{31}^2 L}{2E}\right) \right] \right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \left(2 - 2 \cos \frac{\Delta m_{31}^2 L}{2E} \right) \\
&\quad - 2\delta_{\alpha\beta} |U_{\alpha 3}|^2 \left(1 - \cos \frac{\Delta m_{31}^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2 (\delta_{\alpha\beta} - |U_{\beta 3}|^2) \left(1 - \cos \frac{\Delta m_{31}^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 (\delta_{\alpha\beta} - |U_{\beta 3}|^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E}
\end{aligned}$$

$$\alpha \neq \beta \implies P_{\nu_\alpha \rightarrow \nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

$$\alpha = \beta \implies P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \quad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & \boxed{U_{e3}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & \boxed{U_{\tau 3}} \end{pmatrix}$$

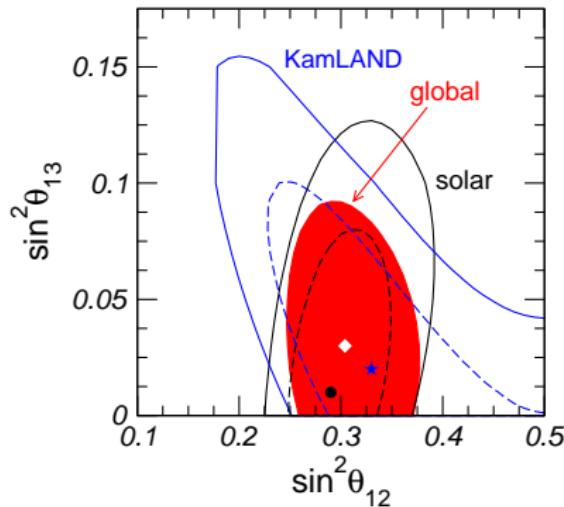
$\sin^2 2\vartheta_{ee} \ll 1$
 \Downarrow
 $|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$

↑
LBL

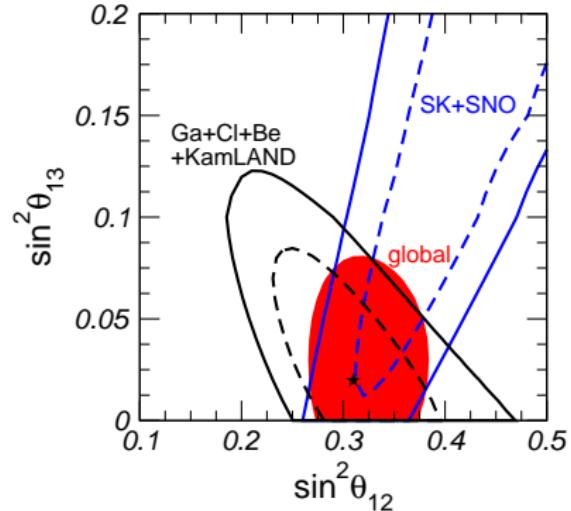
2008 Hint of $\sin^2 \vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$$\sin^2 \vartheta_{13} = 0.016 \pm 0.010 \quad [\text{Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801}]$$



[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]



[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

$$P_{\nu_e \rightarrow \nu_e}^{(-)} \simeq \begin{cases} (1 - \sin^2 \vartheta_{13})^2 (1 - 0.5 \sin^2 \vartheta_{12}) & \text{SOL low-energy \& KamLAND} \\ (1 - \sin^2 \vartheta_{13})^2 \sin^2 \vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

Measurements of ϑ_{13}

$0.03(0.04) < \sin^2 2\vartheta_{13} < 0.28(0.34)$ T2K, arXiv:1106.2822 (90% CL)

$\sin^2 2\vartheta_{13} = 0.041^{+0.047}_{-0.031} (0.079^{+0.071}_{-0.053})$ MINOS, arXiv:1108.0015

$\sin^2 \vartheta_{13} = 0.022 \pm 0.013$ Double Chooz, arXiv:1112.6353

$\sin^2 \vartheta_{13} = 0.024 \pm 0.004$ Daya Bay, arXiv:1203.1669

$\sin^2 \vartheta_{13} = 0.029 \pm 0.006$ RENO, arXiv:1204.0626

$\sin^2 \vartheta_{13} > 0 \implies$ CP violation, matter effects, mass ordering

Experimental Evidences of Neutrino Oscillations

Solar $\nu_e \rightarrow \nu_\mu, \nu_\tau$	$\left(\begin{array}{l} \text{SNO, BOREXino} \\ \text{Super-Kamiokande} \\ \text{GALLEX/GNO, SAGE} \\ \text{Homestake, Kamiokande} \\ \text{(KamLAND)} \end{array} \right)$	$\left\{ \begin{array}{l} \Delta m_S^2 = \Delta m_{21}^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2 \\ \sin^2 \vartheta_S = \sin^2 \vartheta_{12} \simeq 0.30 \end{array} \right.$
--	--	--

VLBL Reactor
 $\bar{\nu}_e$ disappearance

Atmospheric

$$\nu_\mu \rightarrow \nu_\tau$$

LBL Accelerator
 ν_μ disappearance

LBL Accelerator

$$\nu_\mu \rightarrow \nu_\tau$$

$$\left(\begin{array}{l} \text{Super-Kamiokande} \\ \text{Kamiokande, IMB} \\ \text{MACRO, Sudan-2} \\ \left(\begin{array}{l} \text{K2K, MINOS} \\ \text{T2K, NO}\nu\text{A} \end{array} \right) \\ \text{(Opera)} \end{array} \right)$$

$$\left\{ \begin{array}{l} \Delta m_A^2 = |\Delta m_{31}^2| \simeq 2.4 \times 10^{-3} \text{ eV}^2 \\ \sin^2 \vartheta_A = \sin^2 \vartheta_{23} \simeq 0.50 \end{array} \right.$$

LBL Accelerator

$$\nu_\mu \rightarrow \nu_e$$

$$(\text{T2K, MINOS, NO}\nu\text{A})$$

LBL Reactor

$\bar{\nu}_e$ disappearance

$$\left(\begin{array}{l} \text{Daya Bay, RENO} \\ \text{Double Chooz} \end{array} \right)$$

$$\left\{ \begin{array}{l} \Delta m_A^2 = |\Delta m_{31}^2| \\ \sin^2 \vartheta_{13} \simeq 0.023 \end{array} \right.$$

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$c_{ab} \equiv \cos \vartheta_{ab} \quad s_{ab} \equiv \sin \vartheta_{ab} \quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$$

OSCILLATION
PARAMETERS

{ 3 Mixing Angles: ϑ_{12} , ϑ_{23} , ϑ_{13}
1 CPV Dirac Phase: δ_{13}
2 independent $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$: Δm_{21}^2 , Δm_{31}^2

2 CPV Majorana Phases: λ_{21} , $\lambda_{31} \iff |\Delta L| = 2$ processes

Recent Experimental Results

- ▶ OPERA observed a fifth ν_τ candidate event:
 5σ evidence of long-baseline $\nu_\mu \rightarrow \nu_\tau$ transitions!
arXiv:1507.01417
- ▶ NO ν A observed first long-baseline neutrino events:
 ν_μ disappearance (33 ν_μ events vs 201 without oscillations)
and
 ν_e appearance (6 ν_e events with 1 background).
7 August 2015 Press Release

Recent Global Fits

- ▶ Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo
Status of three-neutrino oscillation parameters, circa 2013
Phys.Rev. D89 (2014) 093018, arXiv:1312.2878
- ▶ Forero, Tortola, Valle
Neutrino oscillations refitted
Phys.Rev. D90 (2014) 093006, arXiv:1405.7540
- ▶ Gonzalez-Garcia, Maltoni, Schwetz
Updated fit to three neutrino mixing: status of leptonic CP violation
JHEP 1411 (2014) 052, arXiv:1409.5439
- ▶ Bergstrom, Gonzalez-Garcia, Maltoni, Schwetz
Bayesian global analysis of neutrino oscillation data
arXiv:1507.04366



www.nu-fit.org

$$\Delta m_S^2 = \Delta m_{21}^2 \simeq 7.5 \pm 0.3 \times 10^{-5} \text{ eV}^2 \quad \text{uncertainty} \simeq 3\%$$

$$\Delta m_A^2 = |\Delta m_{31}^2| \simeq |\Delta m_{32}^2| \simeq 2.4 \pm 0.1 \times 10^{-3} \text{ eV}^2 \quad \text{uncertainty} \simeq 4\%$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$\vartheta_{23} = \vartheta_A$$

Daya Bay, RENO

$$\vartheta_{12} = \vartheta_S$$

$$\beta\beta_{0\nu}$$

$$\sin^2 \vartheta_{23} \simeq 0.4 - 0.6$$

Double Chooz

$$\sin^2 \vartheta_{12} \simeq 0.30 \pm 0.01$$

$$P_{\text{osc}} \propto \sin^2 2\vartheta_{23}$$

T2K, MINOS

maximal and flat

$$\sin^2 \vartheta_{13} \simeq 0.023 \pm 0.002$$

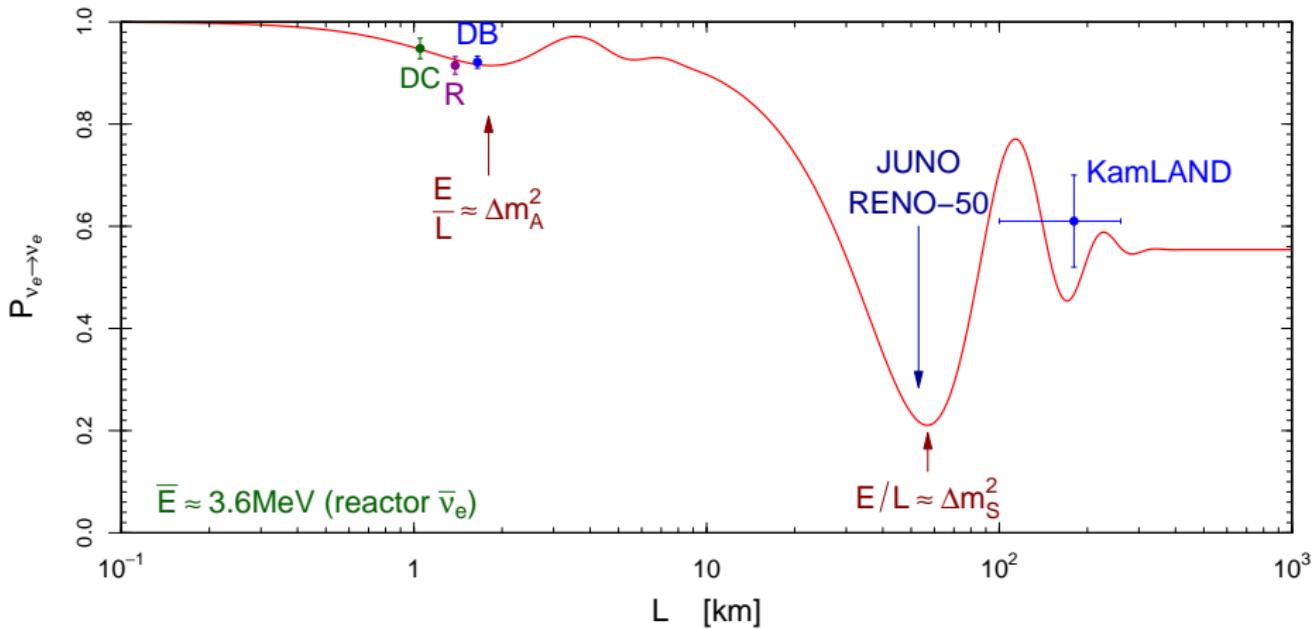
at $\vartheta_{23} = 45^\circ$

$$\delta_{13} \approx 3\pi/2?$$

$$\frac{\delta \sin^2 \vartheta_{23}}{\sin^2 \vartheta_{23}} \approx 40\%$$

$$\frac{\delta \sin^2 \vartheta_{13}}{\sin^2 \vartheta_{13}} \approx 10\%$$

$$\frac{\delta \sin^2 \vartheta_{12}}{\sin^2 \vartheta_{12}} \approx 5\%$$



Effective VLBL ν_e Survival Probability

$$P_{\nu_e \rightarrow \nu_e} = \left| \sum_{k=1}^3 |U_{ek}|^2 e^{-im_k^2 L/2E} \right|^2$$

$$|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2 \implies |U_{e1}|^2 \simeq \cos^2 \vartheta_{12}, |U_{e2}|^2 \simeq \sin^2 \vartheta_{12}$$

$$\begin{aligned} P_{\nu_e \rightarrow \nu_e} &\simeq \left| \sum_{k=1}^2 |U_{ek}|^2 e^{-im_k^2 L/2E} \right|^2 \\ &\simeq \left| \cos^2 \vartheta_{12} e^{-im_1^2 L/2E} + \sin^2 \vartheta_{12} e^{-im_2^2 L/2E} \right|^2 \\ &= \cos^4 \vartheta_{12} + \sin^4 \vartheta_{12} + 2 \cos^2 \vartheta_{12} \cos^2 \vartheta_{12} \cos \left(\frac{\Delta m_{21}^2 L}{2E} \right) \\ &= 1 - \sin^2 2\vartheta_{12} \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \end{aligned}$$

Effective ATM and LBL Oscillation Amplitudes

- ν_e disappearance:

Daya Bay, RENO, Double Chooz

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 (1 - |U_{e3}|^2) = 4s_{13}^2 c_{13}^2 = \sin^2 2\vartheta_{13} \simeq 0.09$$

- ν_μ disappearance:

K2K, MINOS, T2K, NO ν A

$$\begin{aligned}\sin^2 2\vartheta_{\mu\mu} &= 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) = 4c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \\ &\simeq 4s_{23}^2 (1 - s_{23}^2) = \sin^2 2\vartheta_{23} \simeq 1\end{aligned}$$

- $\nu_\mu \rightarrow \nu_e$:

T2K, MINOS, NO ν A

$$\begin{aligned}\sin^2 2\vartheta_{\mu e} &= 4|U_{e3}|^2 |U_{\mu 3}|^2 = 4s_{13}^2 c_{13}^2 s_{23}^2 = \sin^2 2\vartheta_{13} \sin^2 \vartheta_{23} \\ &\simeq \frac{1}{2} \sin^2 2\vartheta_{13} \simeq 0.045\end{aligned}$$

- $\nu_\mu \rightarrow \nu_\tau$:

OPERA

$$\sin^2 2\vartheta_{\mu\tau} = 4|U_{\mu 3}|^2 |U_{\tau 3}|^2 = 4c_{13}^4 s_{23} c_{23} = c_{13}^4 \sin^2 2\vartheta_{23} \simeq \sin^2 2\vartheta_{23} \simeq 1$$

CP Violation?

- ▶ In this approximation there is no observable CP-violation effect!
- ▶ CP-violation can be observed only with sensitivity to Δm_{21}^2 : in vacuum

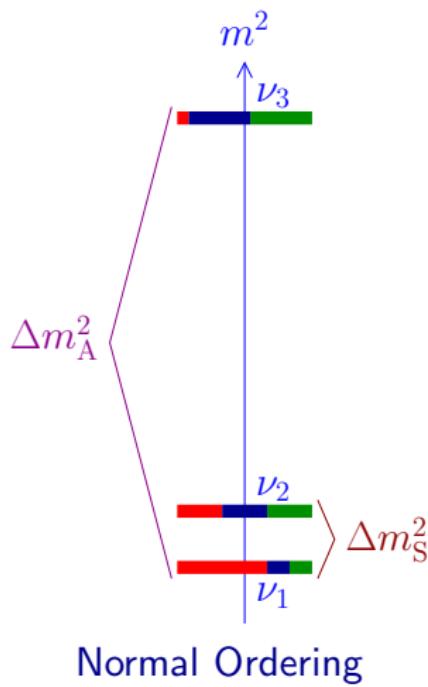
$$\begin{aligned} A_{\alpha\beta}^{\text{CP}} &= P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} \\ &= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) \end{aligned}$$

$$J_{\alpha\beta} = \text{Im}(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin \delta_{13}$$

- ▶ Necessary conditions for observation of CP violation:
 - ▶ Sensitivity to all mixing angles, including small ϑ_{13}
 - ▶ Sensitivity to oscillations due to Δm_{21}^2 and Δm_{31}^2

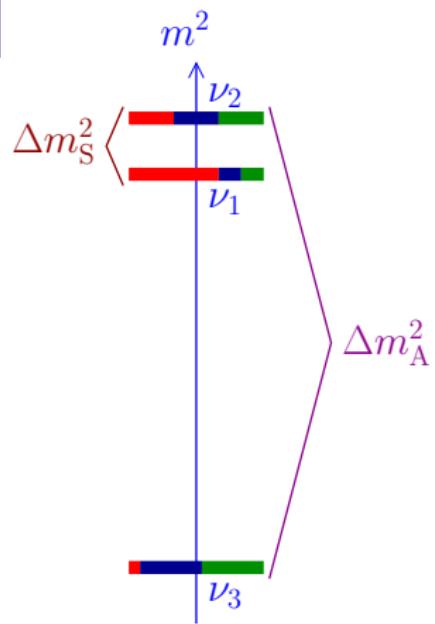
Mass Ordering



Normal Ordering

$$\Delta m_{31}^2 > \Delta m_{32}^2 > 0$$

absolute scale is not determined by neutrino oscillation data



Inverted Ordering

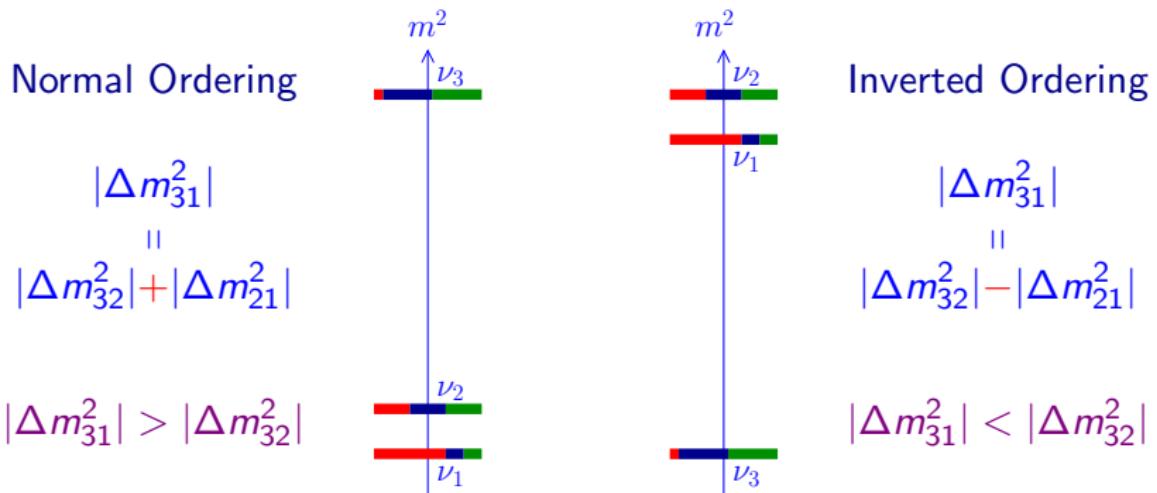
$$\Delta m_{32}^2 < \Delta m_{31}^2 < 0$$

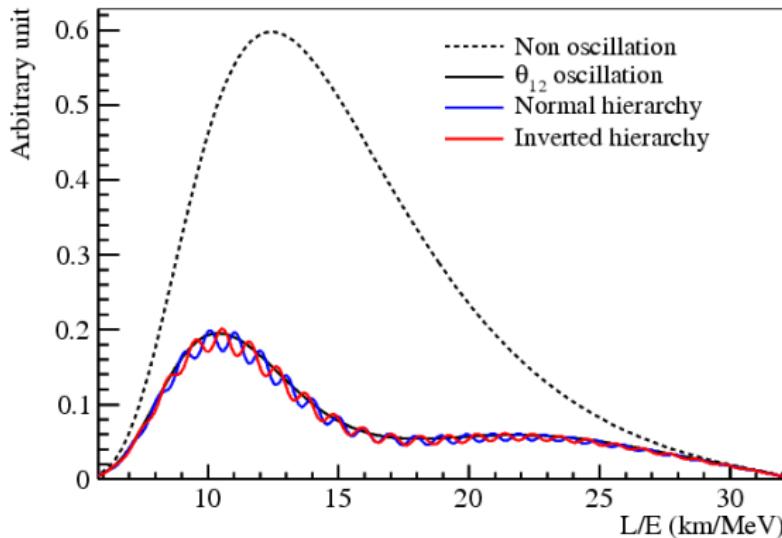
Determination of Mass Ordering

1. Matter Effects: Atmospheric (PINGU, ORCA), Long-Baseline, Supernova Experiments

- $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \frac{\Delta m_{13}^2 \cos 2\vartheta_{13}}{2E} \Leftrightarrow \Delta m_{13}^2 > 0$ NO
- $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\frac{\Delta m_{13}^2 \cos 2\vartheta_{13}}{2E} \Leftrightarrow \Delta m_{13}^2 < 0$ IO

2. Phase Difference: Reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ (JUNO, RENO-50)





Neutrino Physics with JUNO, arXiv:1507.05613

$$\begin{aligned}
 P_{\substack{(-) \\ \nu_e \rightarrow \nu_e}} &= 1 - \cos^4 \vartheta_{13} \sin^2 2\vartheta_{12} \sin^2 (\Delta m_{21}^2 L / 4E) \\
 &\quad - \cos^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 (\Delta m_{31}^2 L / 4E) \\
 &\quad - \sin^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 (\Delta m_{32}^2 L / 4E)
 \end{aligned}$$

[Petcov, Piai, PLB 533 (2002) 94; Choubey, Petcov, Piai, PRD 68 (2003) 113006; Learned, Dye, Pakvasa, Svoboda, PRD 78 (2008) 071302; Zhan, Wang, Cao, Wen, PRD 78 (2008) 111103, PRD 79 (2009) 073007]

LBL Oscillation Probabilities

$$\Delta = \frac{\Delta m_{31}^2 L}{4E} \quad \alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \quad A = \frac{2EV}{\Delta m_{31}^2} \quad V = \sqrt{2}G_F N_e$$

$$\sin \theta_{13} \ll 1 \quad \alpha \ll 1$$

$$P_{\nu_e \rightarrow \nu_e}^{\text{LBL}} \simeq 1 - \sin^2 2\vartheta_{13} \sin^2 \Delta - \alpha^2 \Delta^2 \sin^2 2\vartheta_{12}$$

$$\begin{aligned} P_{\nu_\mu \rightarrow \nu_e}^{\text{LBL}} &\simeq \sin^2 2\vartheta_{13} \sin^2 \vartheta_{23} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ &+ \alpha \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \delta_{13}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ &+ \alpha^2 \sin^2 2\vartheta_{12} \cos^2 \vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

$$\text{NO: } \Delta m_{31}^2 > 0 \quad \text{IO: } \Delta m_{31}^2 < 0$$

for antineutrinos: $\delta_{13} \rightarrow -\delta_{13}$ and $A \rightarrow -A$

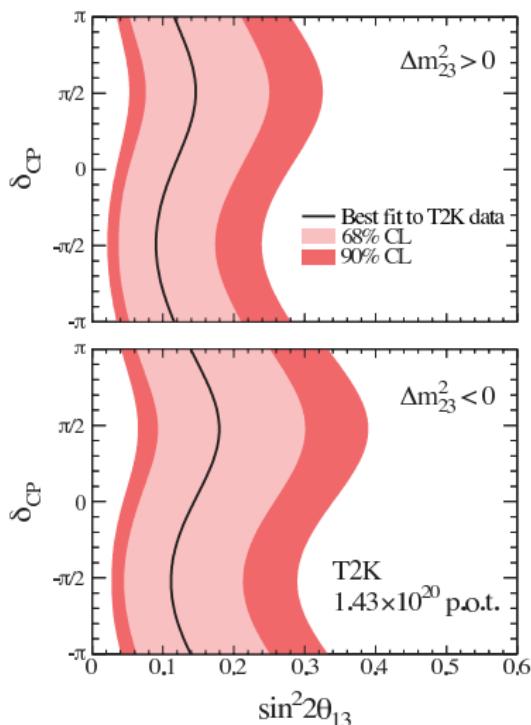
[Mezzetto, Schwetz, JPG 37 (2010) 103001]

T2K

[PRL 107 (2011) 041801, arXiv:1106.2822]

ND at 280 m FD at 295 km

2.5° off-axis \Rightarrow NBB with $\langle E \rangle \simeq 0.6 \text{ GeV} \simeq |\Delta m_{31}^2|L/2\pi$



$$\nu_\mu \rightarrow \nu_e$$

6 ν_e events in FD

background: 1.5 ± 0.3

2.5σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.11^{+0.17}_{-0.08} & (\text{NO}) \\ 0.14^{+0.20}_{-0.10} & (\text{IO}) \end{cases}$$

90% C.L. $\delta_{13} = 0$

Assumptions

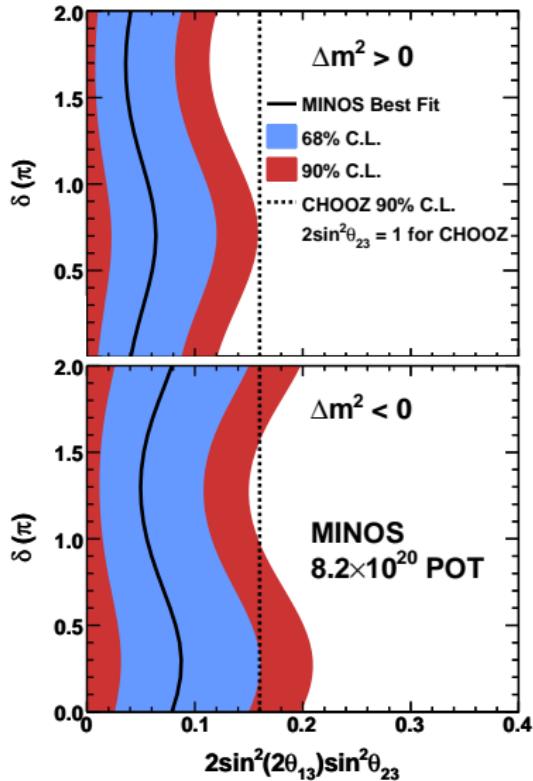
$$\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}, \sin^2 2\vartheta_{12} = 0.87$$

$$|\Delta m_{31}^2| = 2.4 \times 10^{-3} \text{ eV}, \sin^2 2\vartheta_{23} = 1$$

MINOS

[PRL 107 (2011) 181802, arXiv:1108.0015]

ND at 1.04 km



FD at 735 km

$\langle E \rangle \simeq 3 \text{ GeV}$

$$\nu_\mu \rightarrow \nu_e$$

62 ν_e events in FD

background: 49.6 ± 7.5

1.6 σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.041^{+0.047}_{-0.031} & (\text{NO}) \\ 0.079^{+0.071}_{-0.053} & (\text{IO}) \end{cases}$$

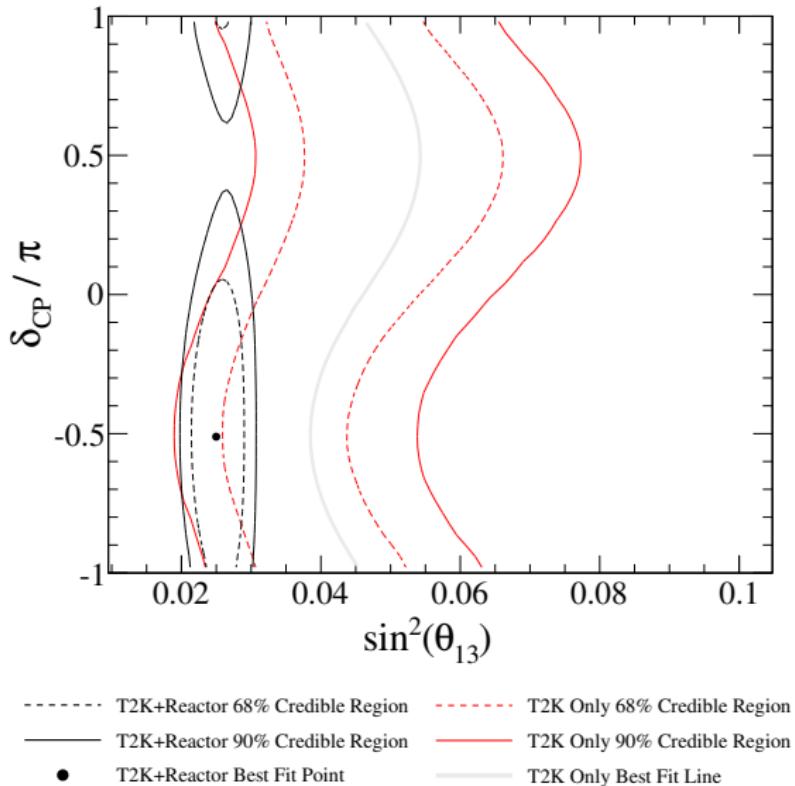
68% C.L. $\delta_{13} = 0$

Assumptions

$$\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}, \sin^2 2\vartheta_{12} = 0.87$$

$$|\Delta m_{31}^2| = 2.3 \times 10^{-3} \text{ eV}, \sin^2 2\vartheta_{23} = 1$$

Large CP Violation?



T2K, Phys.Rev. D91 (2015) 072010, arXiv:1502.01550

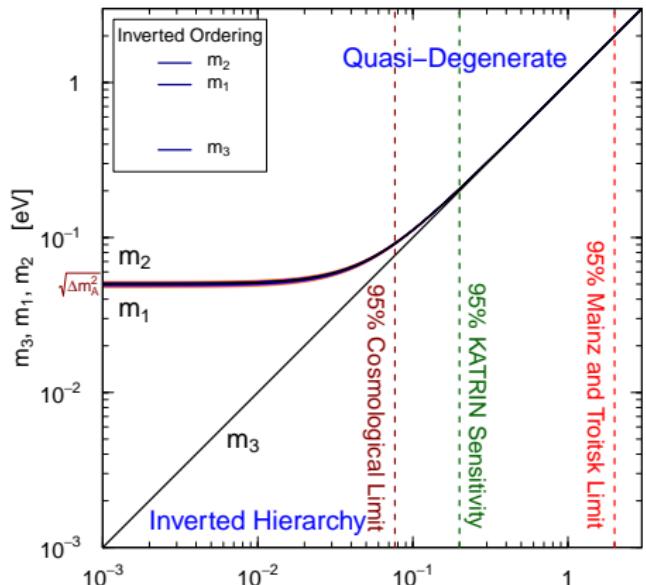
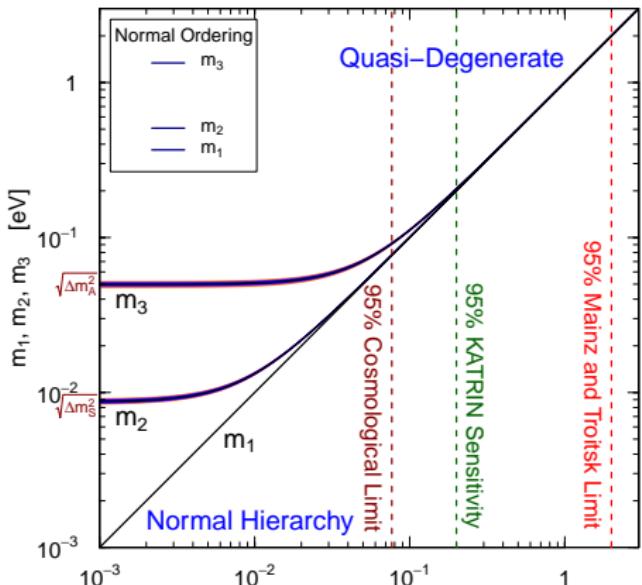
Open Problems

- ▶ $\vartheta_{23} \leqslant 45^\circ$?
 - ▶ T2K (Japan), NO ν A (USA), PINGU (Antarctica), ORCA (EU), INO (India), ...
- ▶ Mass Ordering ?
 - ▶ NO ν A (USA), JUNO (China), RENO-50 (Korea), PINGU (Antarctica), ORCA (EU), INO (India), ...
- ▶ CP violation ? $\delta_{13} \approx 3\pi/2$?
 - ▶ T2K (Japan), NO ν A (USA), DUNE (USA), HyperK (Japan), ...
- ▶ Absolute Mass Scale ?
 - ▶ β Decay, Neutrinoless Double- β Decay, Cosmology, ...
- ▶ Dirac or Majorana ?
 - ▶ Neutrinoless Double- β Decay, ...
- ▶ Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Absolute Scale of Neutrino Masses

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Absolute Scale of Neutrino Masses
 - Tritium Beta-Decay
 - Neutrinoless Double-Beta Decay
- Light Sterile Neutrinos
- Conclusions

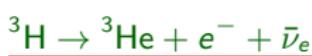
Mass Hierarchy or Degeneracy?



Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}$ eV

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

Tritium Beta-Decay

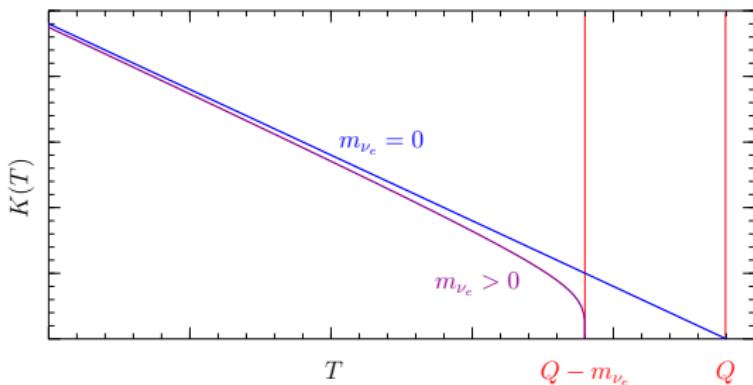


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE (Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58 \text{ keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE}} = \left[(Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2} \right]^{1/2}$$



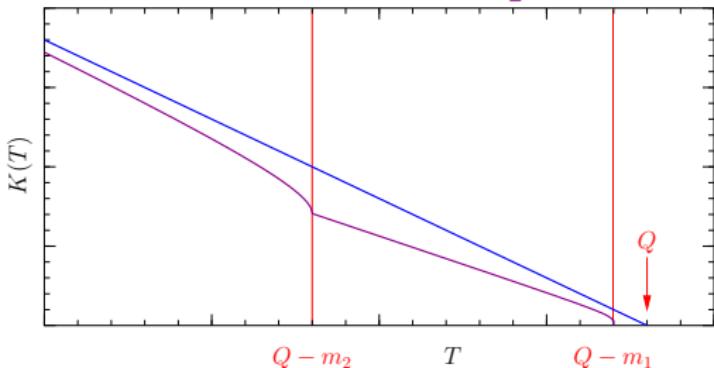
$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

future: KATRIN
www.katrin.kit.edu
 start data taking 2016?
 sensitivity: $m_{\nu_e} \simeq 0.2 \text{ eV}$

Neutrino Mixing $\implies K(T) = \left[(Q - T) \sum_k |U_{ek}|^2 \sqrt{(Q - T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case:
 $2N - 1$ parameters
 $\left(\sum_k |U_{ek}|^2 = 1 \right)$

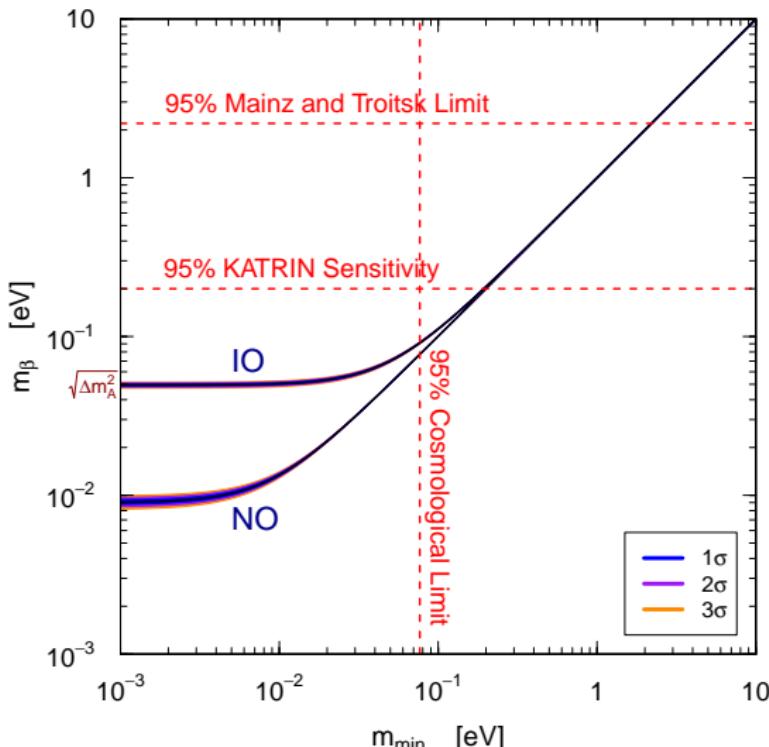
if experiment is not sensitive to masses ($m_k \ll Q - T$)

effective mass:
$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$\begin{aligned} K^2 &= (Q - T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q - T)^2}} \simeq (Q - T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q - T)^2} \right] \\ &= (Q - T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q - T)^2} \right] \simeq (Q - T) \sqrt{(Q - T)^2 - m_\beta^2} \end{aligned}$$

Predictions of 3ν -Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2$$



► Quasi-Degenerate:

$$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

► Inverted Hierarchy:

$$m_\beta^2 \simeq (1 - s_{13}^2) \Delta m_A^2 \simeq \Delta m_A^2$$

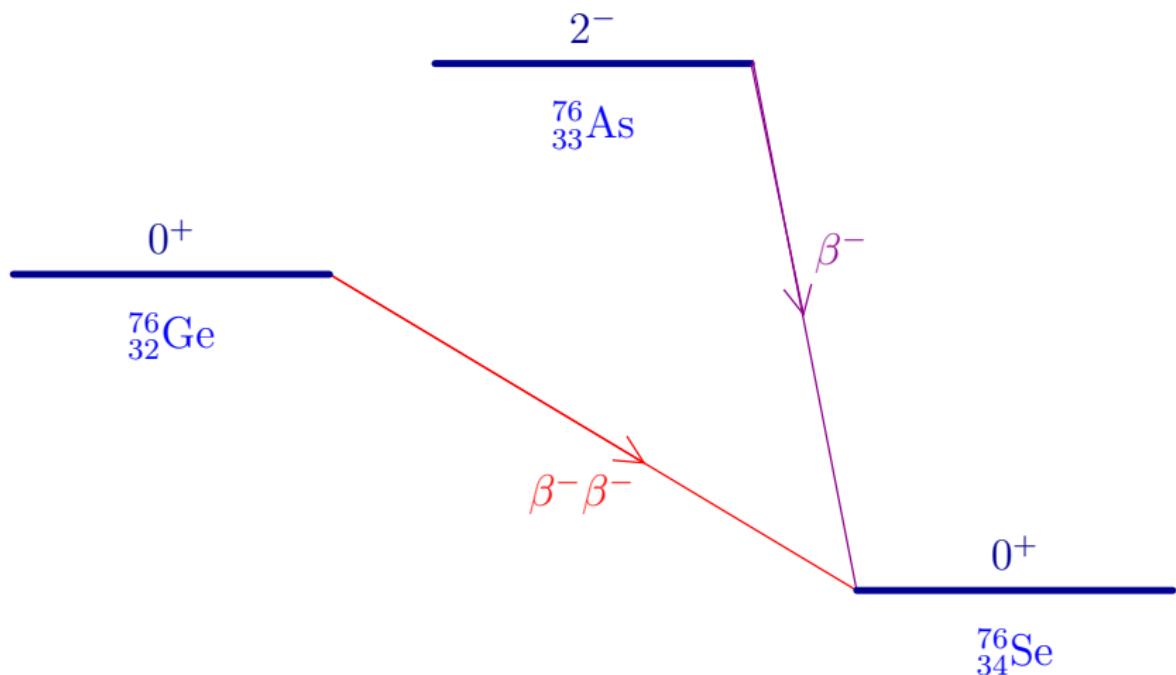
► Normal Hierarchy:

$$\begin{aligned} m_\beta^2 &\simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2 \\ &\simeq 2 \times 10^{-5} + 6 \times 10^{-5} \text{ eV}^2 \end{aligned}$$

► If $m_\beta \lesssim 4 \times 10^{-2}$ eV
↓

Normal Spectrum

Neutrinoless Double-Beta Decay



Effective Majorana Neutrino Mass:

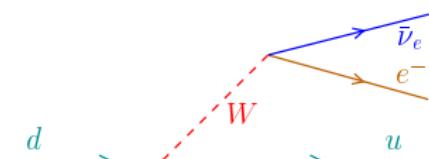
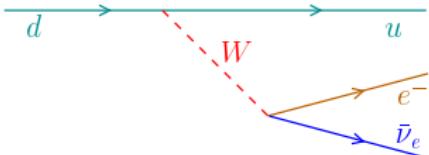
$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

Two-Neutrino Double- β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



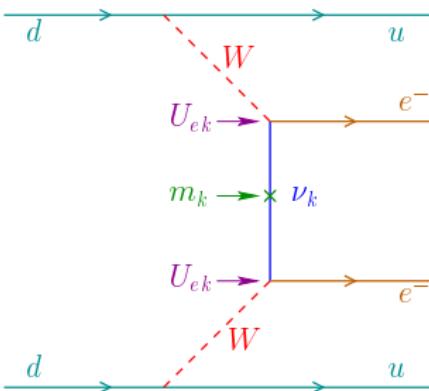
Neutrinoless Double- β Decay: $\Delta L = 2$

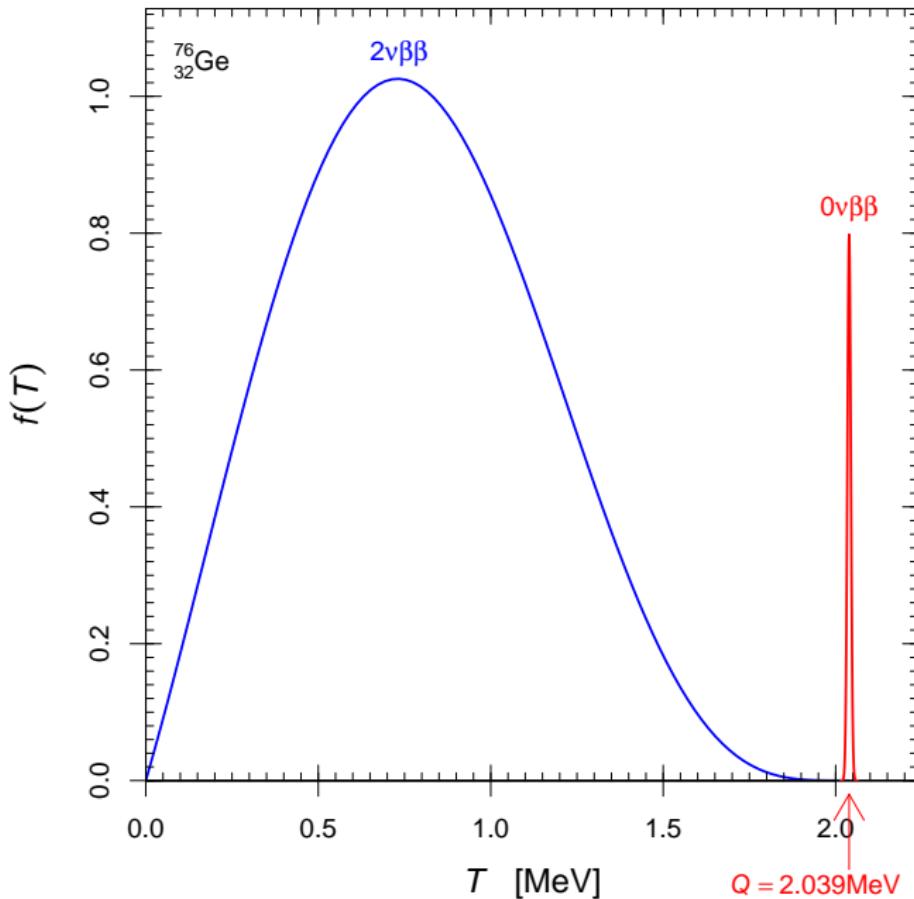
$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |\mathcal{M}_{0\nu}|^2 |m_{\beta\beta}|^2$$

effective
Majorana
mass

$$|m_{\beta\beta}| = \left| \sum_k U_{ek}^2 m_k \right|$$



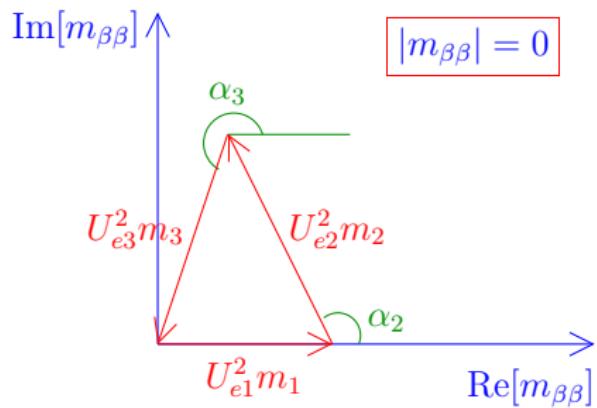
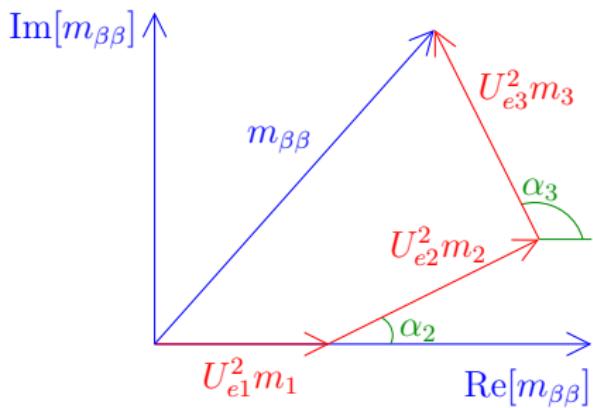


Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k \quad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

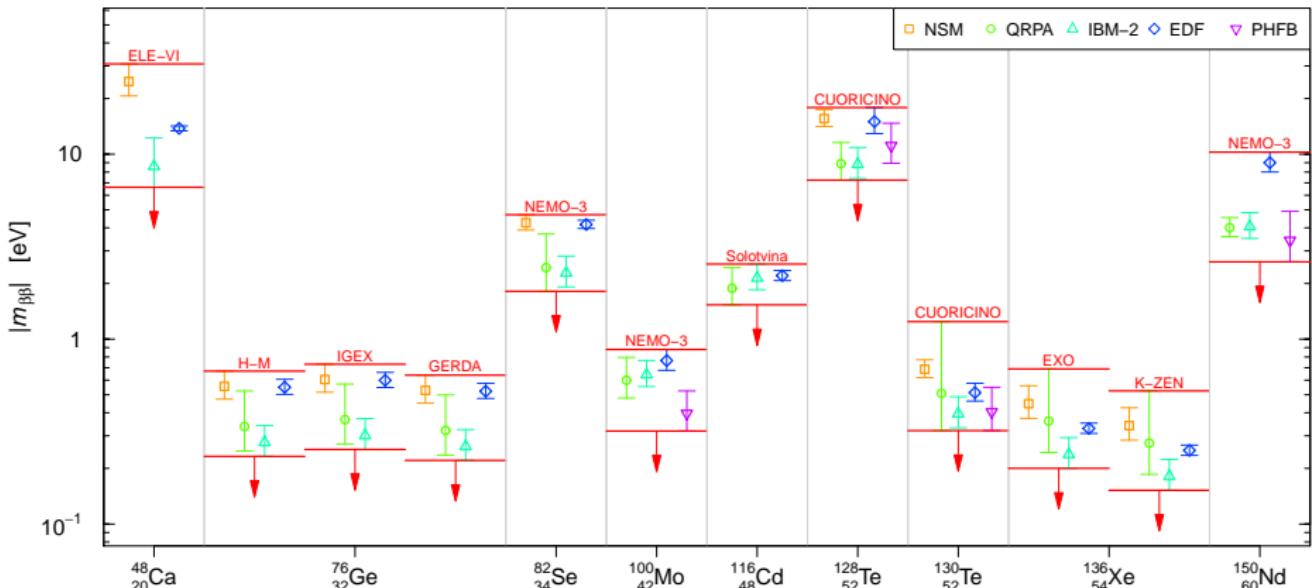
$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$

$$\alpha_2 = 2\lambda_2 \quad \alpha_3 = 2(\lambda_3 - \delta_{13})$$



90% C.L. Experimental Bounds

$\beta\beta^-$ decay	experiment	$T_{1/2}^{0\nu}$ [y]	$m_{\beta\beta}$ [eV]
${}_{20}^{48}\text{Ca} \rightarrow {}_{22}^{48}\text{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
${}_{32}^{76}\text{Ge} \rightarrow {}_{34}^{76}\text{Se}$	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	GERDA	$> 2.1 \times 10^{25}$	$< 0.22 - 0.64$
${}_{34}^{82}\text{Se} \rightarrow {}_{36}^{82}\text{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}_{42}^{100}\text{Mo} \rightarrow {}_{44}^{100}\text{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}_{48}^{116}\text{Cd} \rightarrow {}_{50}^{116}\text{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}_{52}^{128}\text{Te} \rightarrow {}_{54}^{128}\text{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}_{52}^{130}\text{Te} \rightarrow {}_{54}^{130}\text{Xe}$	CUORICINO	$> 2.8 \times 10^{24}$	$< 0.32 - 1.2$
${}_{54}^{136}\text{Xe} \rightarrow {}_{56}^{136}\text{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.2 - 0.69$
	KamLAND-Zen	$> 1.9 \times 10^{25}$	$< 0.15 - 0.52$
${}_{60}^{150}\text{Nd} \rightarrow {}_{62}^{150}\text{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



[Bilenky, Giunti, IJMPA 30 (2015) 0001]

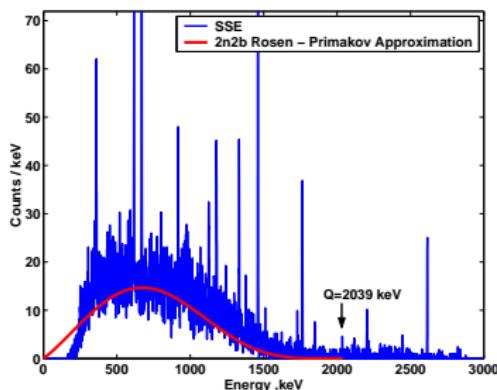
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409]

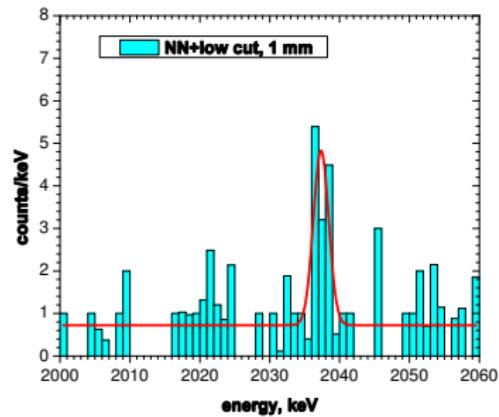
$$T_{1/2}^{0\nu} = (2.23^{+0.44}_{-0.31}) \times 10^{25} \text{ y}$$

6.5 σ evidence

[MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



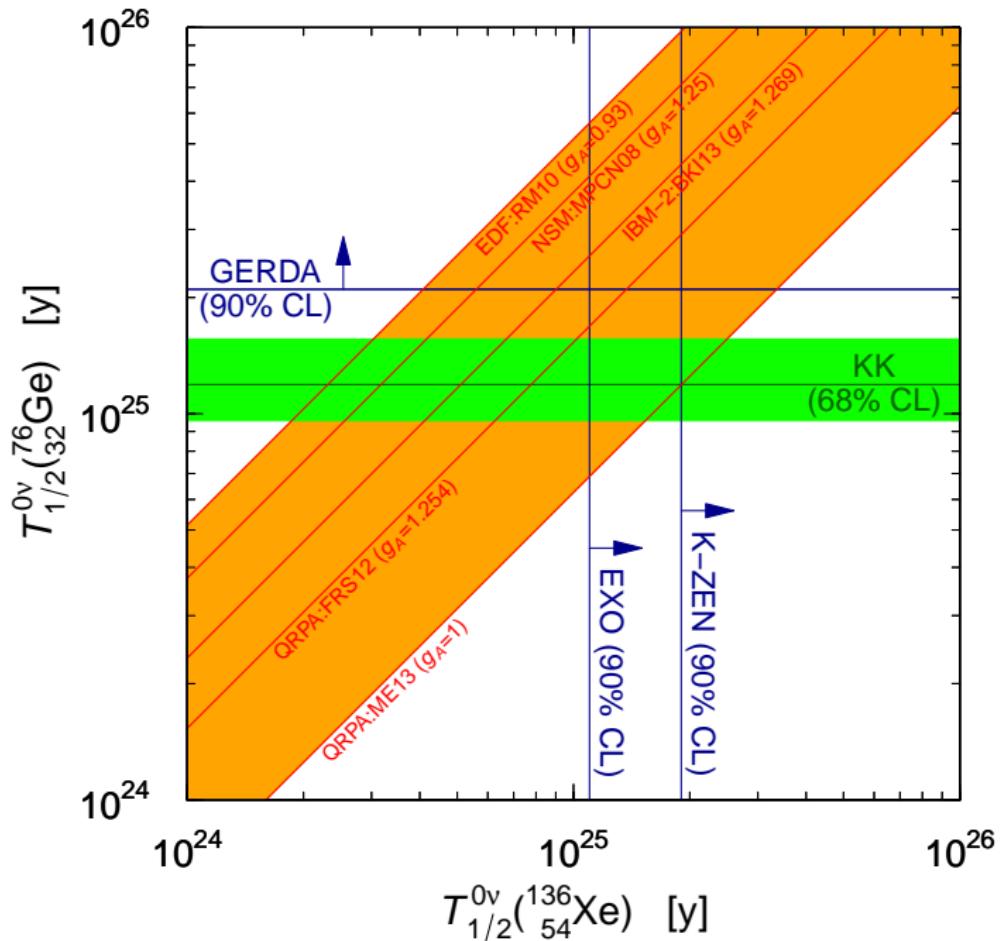
[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

$$|m_{\beta\beta}| = 0.32 \pm 0.03 \text{ eV}$$

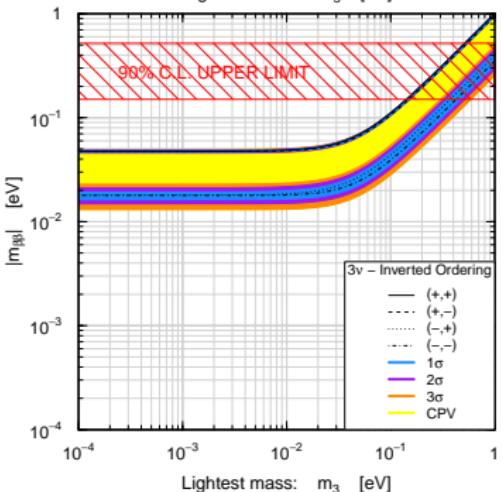
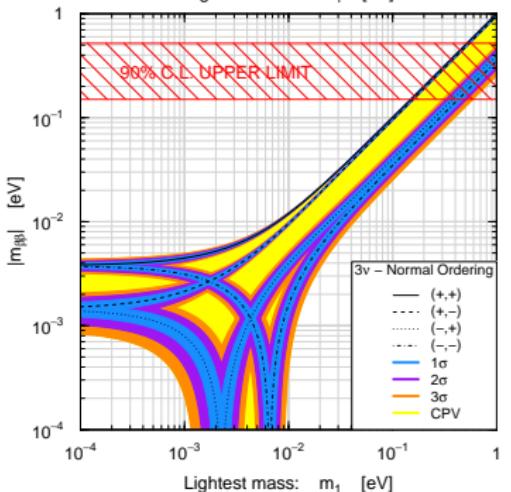
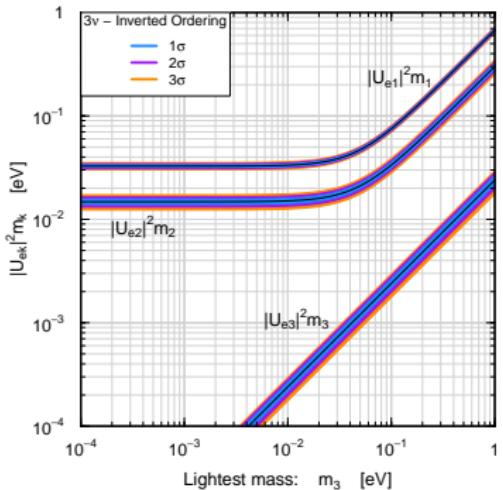
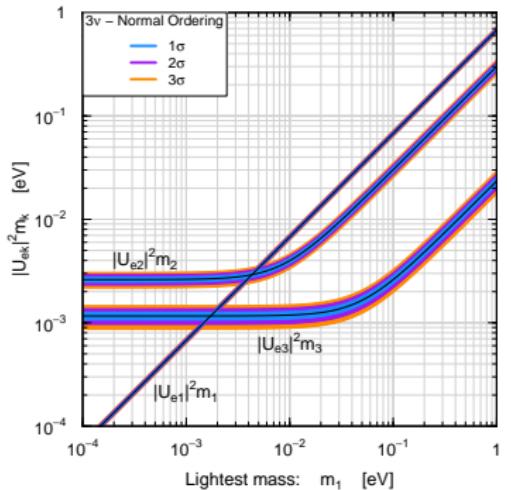
[MPLA 21 (2006) 1547]

if confirmed, very exciting: Majorana ν and large mass scale

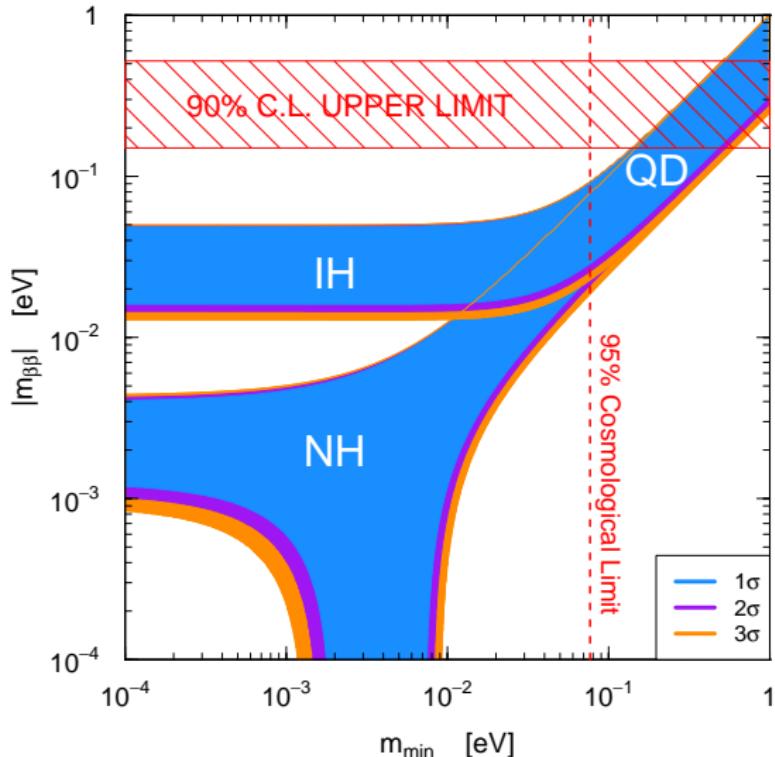


Predictions of 3ν -Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$



$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$



► Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2}$$

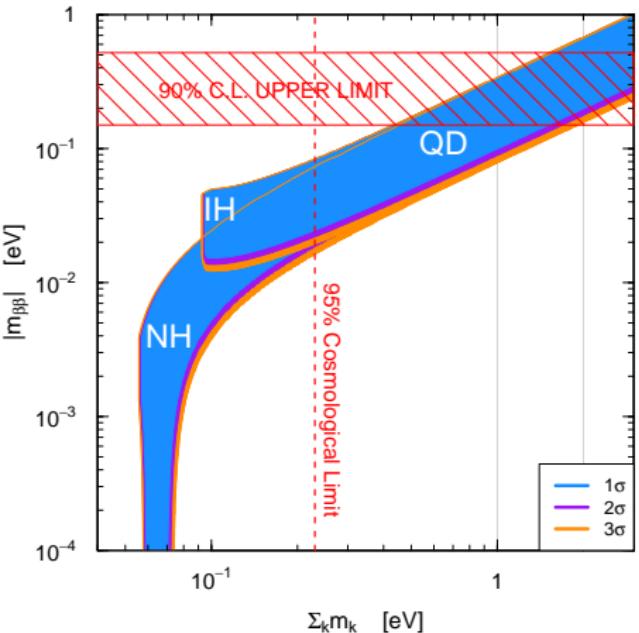
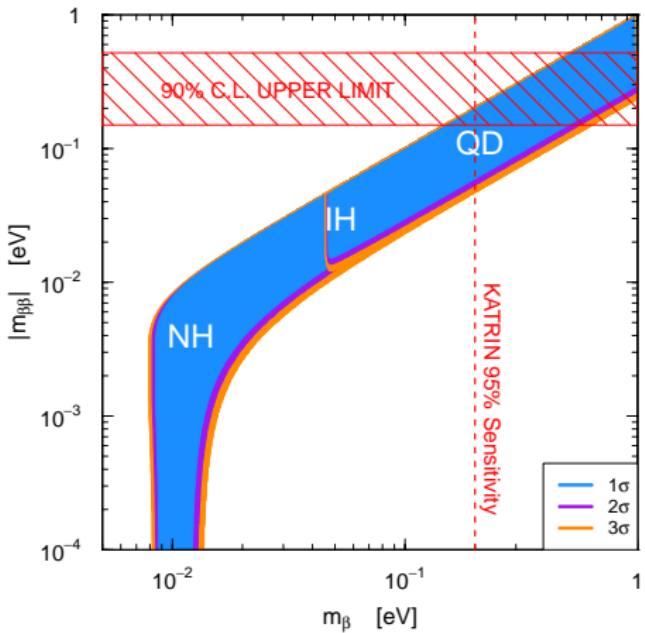
► Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m_A^2 (1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2)}$$

► Normal Hierarchy:

$$\begin{aligned} |m_{\beta\beta}| &\simeq |s_{12}^2 \sqrt{\Delta m_S^2} + e^{i\alpha} s_{13}^2 \sqrt{\Delta m_A^2}| \\ &\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3} \text{ eV} \end{aligned}$$

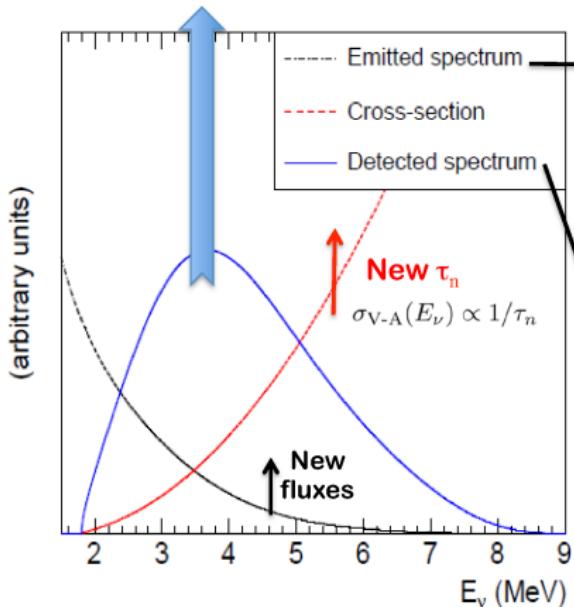
$$|m_{\beta\beta}| \lesssim 10^{-2} \text{ eV} \implies \text{Normal Spectrum}$$



Light Sterile Neutrinos

New Reactor $\bar{\nu}_e$ Fluxes

Increased prediction of detected flux by 6.5%



i)

Neutrino Emission:

- Improved reactor neutrino spectra → +3.5%
- Accounting for long-lived isotopes in reactors → +1%

ii)

Neutrino Detection:

- Reevaluation of σ_{IBD} → +1.5% (evolution of the neutron life time)
- Reanalysis of all SBL experiments

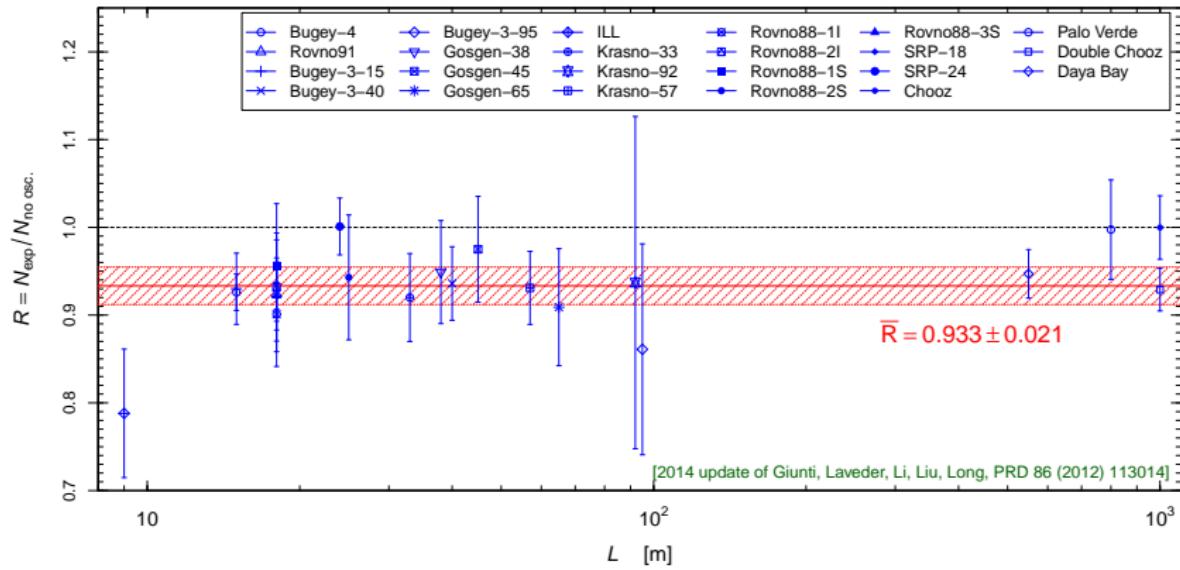
[T. Lasserre, TAUP 2013]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006; update in White Paper, arXiv:1204.5379]

New reactor $\bar{\nu}_e$ fluxes

[Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]



$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

$$L \sim 10 - 100 \text{ m}$$

$$E \sim 4 \text{ MeV}$$

$$\text{Nominal} \approx 3.1\sigma \text{ deficit}$$

$$\Delta m^2 \gtrsim 0.5 \text{ eV}^2$$

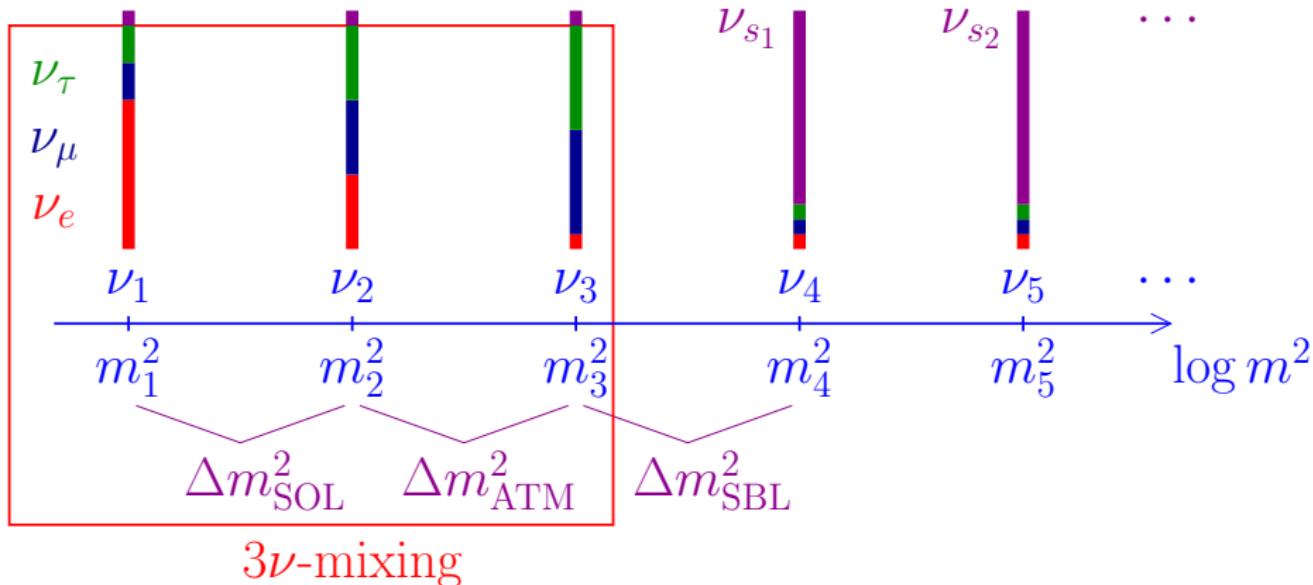
$$(\gg \Delta m_A^2 \gg \Delta m_S^2)$$

[see also: Sinev, arXiv:1103.2452; Ciuffoli, Evslin, Li, JHEP 12 (2012) 110; Zhang, Qian, Vogel, PRD 87 (2013) 073018; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Ivanov et al, PRC 88 (2013) 055501]

Problem: unknown $\bar{\nu}_e$ flux uncertainties?

[Hayes, Friar, Garvey, Jonkmans, PRL 112 (2014) 202501; Dwyer, Langford, PRL 114 (2015) 012502]

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Terminology: a eV-scale sterile neutrino
means: a eV-scale massive neutrino which is mainly sterile

Sterile Neutrinos from Physics Beyond the SM

- ▶ Neutrinos are special in the Standard Model: the only **neutral fermions**
- ▶ In extensions of SM neutrinos can mix with non-SM fermions
- ▶ SM doublets: $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$ $\tilde{\Phi} = i\sigma_2 \Phi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \frac{1}{\sqrt{2}} \begin{pmatrix} \nu \\ 0 \end{pmatrix}$
- ▶ SM singlet: $\overline{L}_L \tilde{\Phi} = (\overline{\nu}_L \quad \overline{\ell}_L) \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \overline{\nu}_L \phi^0 + \overline{\ell}_L \phi^- \xrightarrow[\text{Breaking}]{\text{Symmetry}} \frac{\nu}{\sqrt{2}} \overline{\nu}_L$
- ▶ SM singlet $\overline{L}_L \tilde{\Phi}$ can couple to new singlet (**sterile**) fermion field ν_R (right-handed neutrino) related to physics beyond the SM
- ▶ $\mathcal{L}^D \sim \overline{L}_L \tilde{\Phi} \nu_R \xrightarrow[\text{Breaking}]{\text{Symmetry}} \frac{\nu}{\sqrt{2}} \overline{\nu}_L \nu_R$ **Dirac mass term**
- ▶ Surprise: Majorana mass term $\mathcal{L}^M \sim \overline{\nu}_R^c \nu_R$ allowed by SM symmetries
- ▶ In general: Dirac mass term $\sim \overline{L}_L \tilde{\Phi} \nu_R$ + Majorana mass term $\sim \overline{\nu}_R^c \nu_R$
- ▶ Diagonalization of mass matrix $\sim \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \implies$ 2 massive Majorana neutrinos
- ▶ If mass splitting is small we have active-sterile $\nu_L \rightarrow \nu_R^c$ oscillations

- ▶ 3 left-handed + N_s right-handed fields $\implies (3 + N_s) \times (3 + N_s)$ mass matrix
- ▶ Diagonalization $\implies 3 + N_s$ massive Majorana neutrinos
- ▶ Light anti- ν_R are light sterile neutrinos

$$\nu_R^c \rightarrow \nu_{sL} \quad (\text{left-handed})$$

- ▶ Sterile means no standard model interactions
[Pontecorvo, Sov. Phys. JETP 26 (1968) 984]
- ▶ Active neutrinos (ν_e, ν_μ, ν_τ) can oscillate into light sterile neutrinos (ν_s)
- ▶ Observables:
 - ▶ Disappearance of active neutrinos (neutral current deficit)
 - ▶ Indirect evidence through combined fit of data (current indication)
- ▶ Short-baseline anomalies + 3ν -mixing:

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2| \ll |\Delta m_{41}^2| \leq \dots$$

ν_1	ν_2	ν_3	ν_4	\dots
ν_e	ν_μ	ν_τ	ν_{s1}	\dots

- ▶ Here I consider sterile neutrinos with mass scale $\sim 1\text{ eV}$ in light of short-baseline Reactor Anomaly, Gallium Anomaly, LSND.
- ▶ Other possibilities (not incompatible):
 - ▶ Very light sterile neutrinos with mass scale $\ll 1\text{ eV}$: important for solar neutrino phenomenology

[de Holanda, Smirnov, PRD 69 (2004) 113002; PRD 83 (2011) 113011]

[Das, Pulido, Picariello, PRD 79 (2009) 073010]

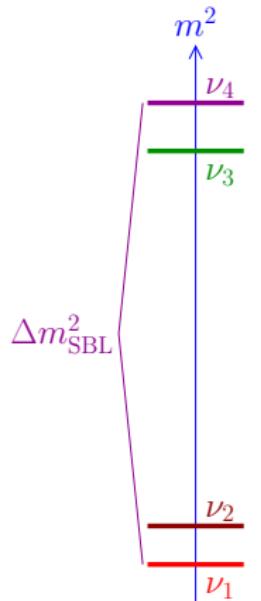
Recent Daya Bay constraints for $10^{-3} \lesssim \Delta m^2 \lesssim 10^{-1}\text{ eV}^2$ [PRL 113 (2014) 141802]

- ▶ Heavy sterile neutrinos with mass scale $\gg 1\text{ eV}$: could be Warm Dark Matter

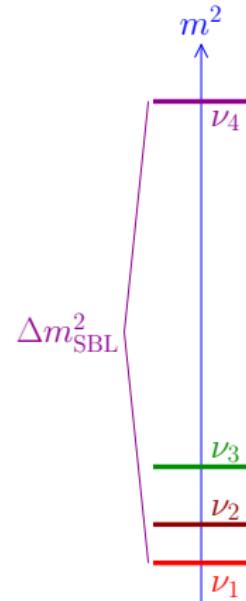
[Asaka, Blanchet, Shaposhnikov, PLB 631 (2005) 151; Asaka, Shaposhnikov, PLB 620 (2005) 17; Asaka, Shaposhnikov, Kusenko, PLB 638 (2006) 401; Asaka, Laine, Shaposhnikov, JHEP 0606 (2006) 053, JHEP 0701 (2007) 091]

[Reviews: Kusenko, Phys. Rept. 481 (2009) 1; Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191; Boyarsky, Iakubovskyi, Ruchayskiy, Phys. Dark Univ. 1 (2012) 136; Drewes, IJMPE, 22 (2013) 1330019]

Four-Neutrino Schemes: 2+2 and 3+1

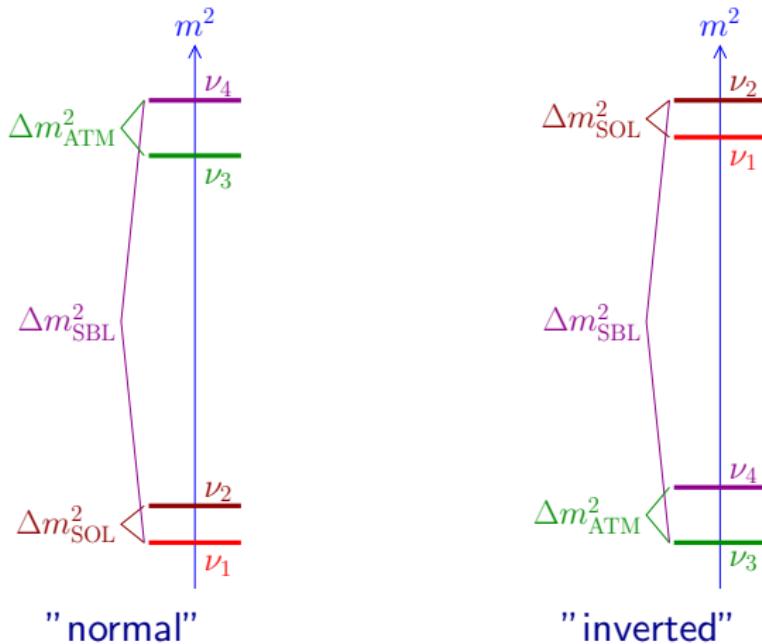


"2+2"

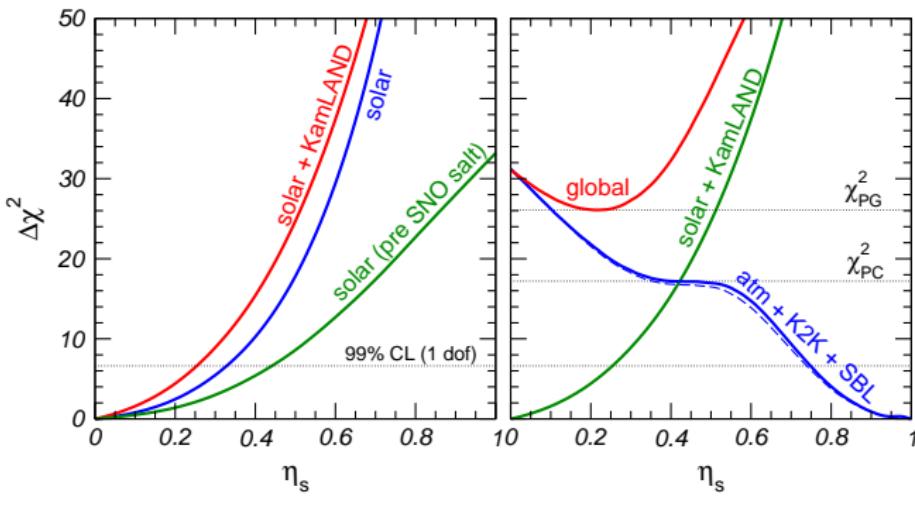


"3+1"

2+2 Four-Neutrino Schemes



2+2 Schemes are strongly disfavored by solar and atmospheric data

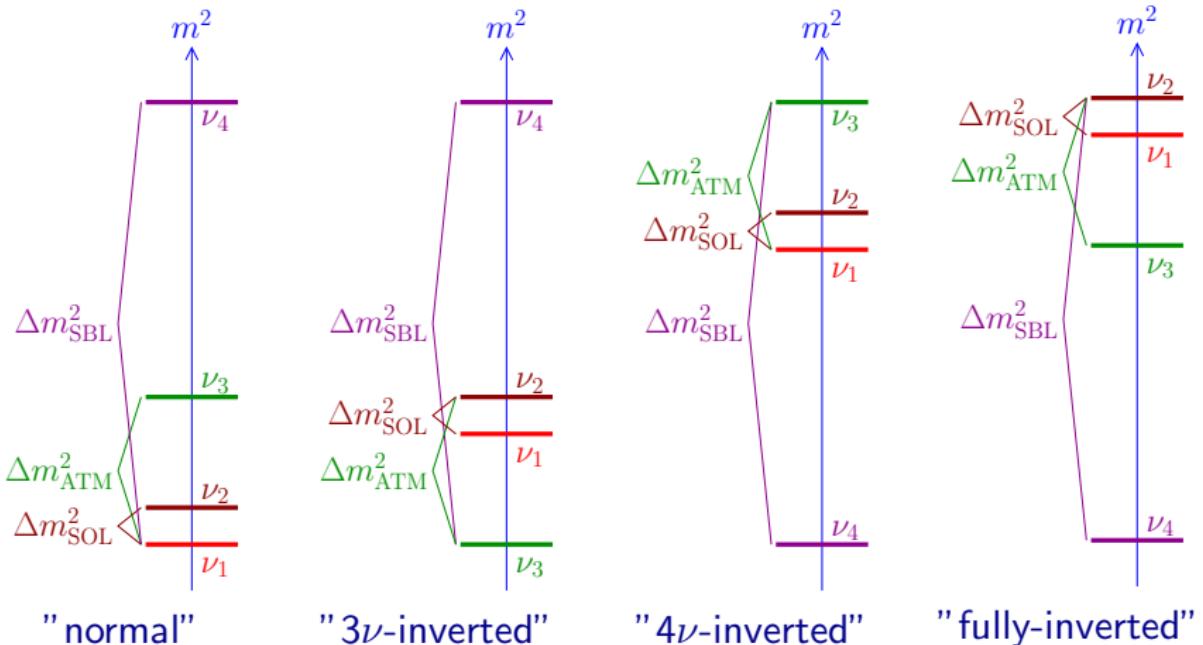


$$\begin{aligned}\eta_s &= |U_{s1}|^2 + |U_{s2}|^2 \\ 1 - \eta_s &= |U_{s3}|^2 + |U_{s4}|^2\end{aligned}$$

$$99\% \text{ CL: } \begin{cases} \eta_s < 0.25 & (\text{solar + KamLAND}) \\ \eta_s > 0.75 & (\text{atmospheric + K2K}) \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122, arXiv:hep-ph/0405172]

3+1 Four-Neutrino Schemes



Perturbation of 3- ν Mixing

$$|U_{e4}|^2 \ll 1$$

$$|U_{\mu 4}|^2 \ll 1$$

$$|U_{\tau 4}|^2 \ll 1$$

$$|U_{s4}|^2 \simeq 1$$

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$\begin{aligned}
 P_{\nu_\alpha \rightarrow \nu_\beta} &= \left| \sum_{k=1}^4 U_{\alpha k}^* U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2 \\
 &= \left| \sum_{k=1}^4 U_{\alpha k}^* U_{\beta k} e^{-i(E_k - E_1)t} \right|^2 \rightarrow \left| \sum_{k=1}^4 U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E}\right) \right|^2
 \end{aligned}$$

$$E_k \simeq E + \frac{m_k^2}{2E} \quad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \quad \frac{\Delta m_{31}^2 L}{2E} \ll 1 \quad \Delta m_{41}^2 \rightarrow \Delta m^2$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{SBL}} \simeq \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m^2 L}{2E}\right) \right|^2$$

$$U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^* U_{\beta 4}$$

$$\begin{aligned}
P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{SBL}} &\simeq \left| \delta_{\alpha\beta} - U_{\alpha 4}^* U_{\beta 4} \left[1 - \exp\left(-i \frac{\Delta m^2 L}{2E}\right) \right] \right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 4}|^2 |U_{\beta 4}|^2 \left(2 - 2 \cos \frac{\Delta m^2 L}{2E} \right) \\
&\quad - 2\delta_{\alpha\beta} |U_{\alpha 4}|^2 \left(1 - \cos \frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 4}|^2 (\delta_{\alpha\beta} - |U_{\beta 4}|^2) \left(1 - \cos \frac{\Delta m^2 L}{2E} \right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 (\delta_{\alpha\beta} - |U_{\beta 4}|^2) \sin^2 \frac{\Delta m^2 L}{4E}
\end{aligned}$$

$$\begin{aligned}
\alpha \neq \beta \quad \Rightarrow \quad P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{SBL}} &\simeq 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \\
\alpha = \beta \quad \Rightarrow \quad P_{\nu_\alpha \rightarrow \nu_\alpha}^{\text{SBL}} &\simeq 1 - 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)
\end{aligned}$$

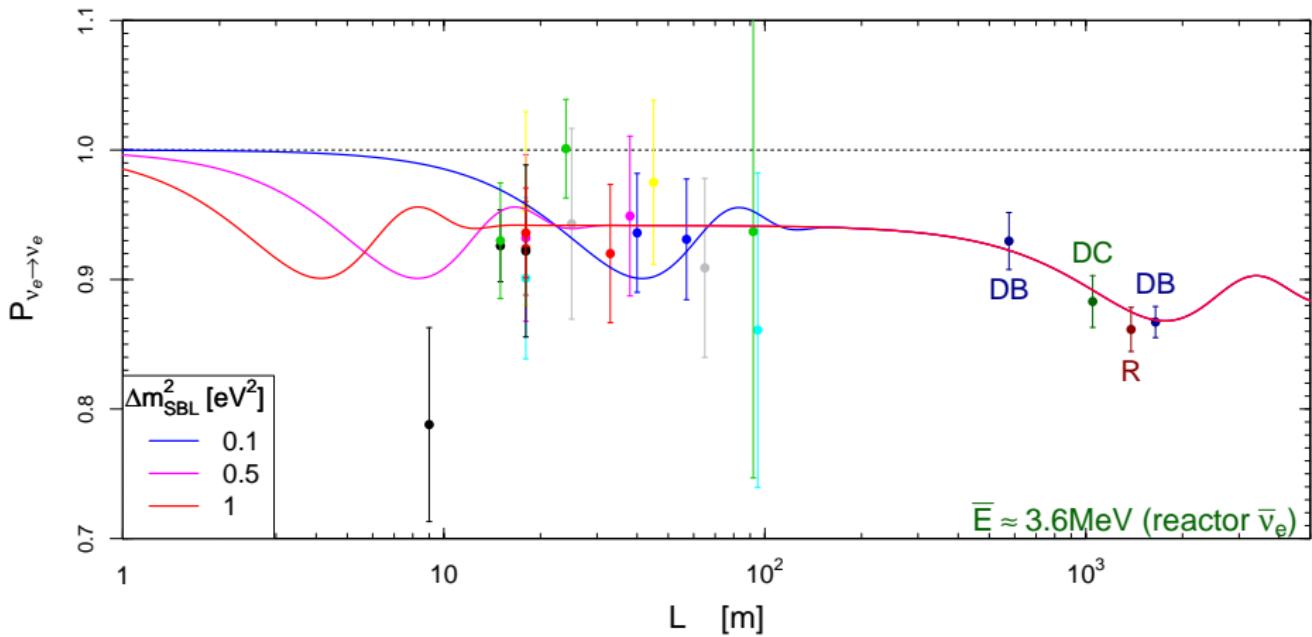
Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\beta}}^{\text{SBL}} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \quad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\alpha}}^{\text{SBL}} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \quad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2)$$

Perturbation of 3ν Mixing: $|U_{e4}|^2 \ll 1$, $|U_{\mu 4}|^2 \ll 1$, $|U_{\tau 4}|^2 \ll 1$, $|U_{s4}|^2 \simeq 1$

- ▶ 6 mixing angles
 - ▶ 3 Dirac CP phases
 - ▶ 3 Majorana CP phases
 - ▶ But CP violation is not observable in current SBL experiments!
 - ▶ Observable in LBL accelerator exp. sensitive to Δm_{ATM}^2 [de Gouvea, Kelly, Kobach, PRD 91 (2015) 053005; Klop, Palazzo, PRD 91 (2015) 073017; Berryman, de Gouvea, Kelly, Kobach, PRD 92 (2015) 073012, Palazzo, arXiv:1509.03148] and solar exp. sensitive to Δm_{SOL}^2 [Long, Li, Giunti, PRD 87, 113004 (2013) 113004]
- $U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$
- ↑
SBL

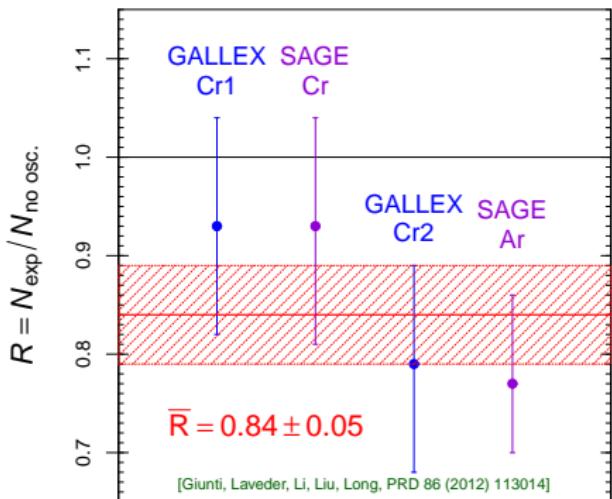


Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \rightarrow {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \rightarrow {}^{37}\text{Cl} + \nu_e$



$\bar{\nu}_e \rightarrow \bar{\nu}_e$ $E \sim 0.7 \text{ MeV}$

$\langle L \rangle_{\text{GALLEX}} = 1.9 \text{ m}$

$\langle L \rangle_{\text{SAGE}} = 0.6 \text{ m}$

Nominal $\approx 2.9\sigma$ anomaly

$\Delta m^2 \gtrsim 1 \text{ eV}^2$ ($\gg \Delta m_A^2 \gg \Delta m_S^2$)

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807]

[Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344;
MPLA 22 (2007) 2499; PRD 78 (2008) 073009;
PRC 83 (2011) 065504]

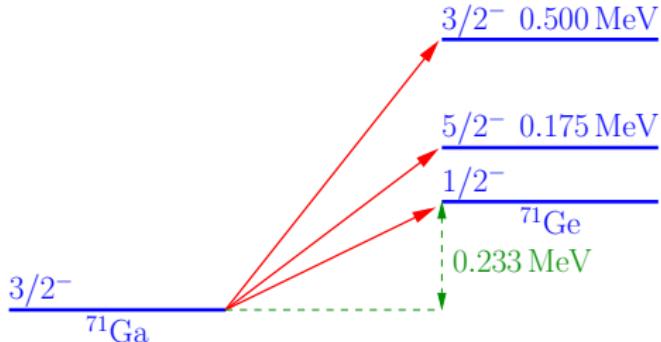
[Mention et al, PRD 83 (2011) 073006]

- ${}^3\text{He} + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + {}^3\text{H}$ cross section measurement [Frekers et al., PLB 706 (2011) 134]
- $E_{\text{th}}(\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-) = 233.5 \pm 1.2 \text{ keV}$ [Frekers et al., PLB 722 (2013) 233]

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\text{G.S.}}$ from $T_{1/2}({}^{71}\text{Ge}) = 11.43 \pm 0.03$ days [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\text{G.S.}}({}^{51}\text{Cr}) = 55.3 \times 10^{-46} \text{ cm}^2 (1 \pm 0.004)_{3\sigma}$$

$$\sigma({}^{51}\text{Cr}) = \sigma_{\text{G.S.}}({}^{51}\text{Cr}) \left(1 + 0.669 \frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}} + 0.220 \frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}} \right)$$

- Contribution of Excited States only 5%!

		$\frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}}$	$\frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}}$
Krofcheck et al. PRL 55 (1985) 1051	$^{71}\text{Ga}(p, n)^{71}\text{Ge}$	< 0.056	0.126 ± 0.023
Haxton PLB 431 (1998) 110	Shell Model	0.19 ± 0.18	
Frekers et al. PLB 706 (2011) 134	$^{71}\text{Ga}({}^3\text{He}, {}^3\text{H})^{71}\text{Ge}$	0.039 ± 0.030	0.202 ± 0.016

- ▶ Haxton: [Haxton, PLB 431 (1998) 110]

“a sophisticated shell model calculation is performed ... for the transition to the first excited state in ^{71}Ge . The calculation predicts destructive interference between the (p, n) spin and spin-tensor matrix elements”
- ▶ Does Haxton argument apply also to $({}^3\text{He}, {}^3\text{H})$ measurements?
- ▶ 2.7σ discrepancy of $\text{BGT}_{500}/\text{BGT}_{\text{G.S.}}$ measurements!
- ▶ Anyhow, new $^{71}\text{Ga}({}^3\text{He}, {}^3\text{H})^{71}\text{Ge}$ data support Gallium Anomaly!

Solar bound on $|U_{e4}|^2$

[Giunti, Li, PRD 80 (2009) 113007; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301]

$$P_{\nu_e \rightarrow \nu_e}^{\text{SOL}} \simeq \left(1 - \sum_{k \geq 3} |U_{ek}|^2 \right)^2 P_{\nu_e \rightarrow \nu_e}^{\text{SOL}, 2\nu} + \sum_{k \geq 3} |U_{ek}|^4$$

$$P_{\nu_e \rightarrow \nu_s}^{\text{SOL}} \simeq \left(1 - \sum_{k \geq 3} |U_{ek}|^2 \right) \left(1 - \sum_{k \geq 3} |U_{sk}|^2 \right) P_{\nu_e \rightarrow \nu_s}^{\text{SOL}, 2\nu} + \sum_{k \geq 3} |U_{ek}|^2 |U_{sk}|^2$$

3+1 with simplifying assumptions: $U_{\mu 4} = U_{\tau 4} = 0$, no CP violation

$$U_{e1} = c_{12}c_{13}c_{14} \quad U_{e2} = s_{12}c_{13}c_{14} \quad U_{e3} = s_{13}c_{14} \quad U_{e4} = s_{14}$$

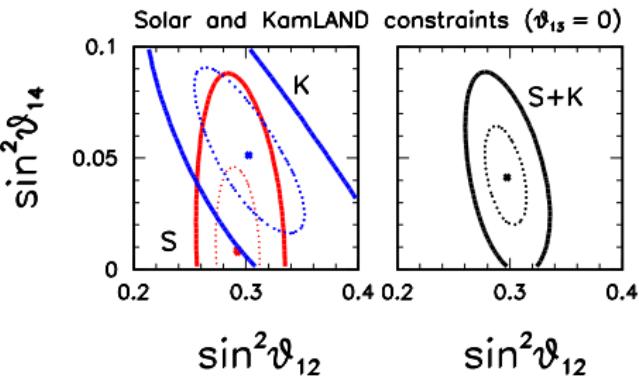
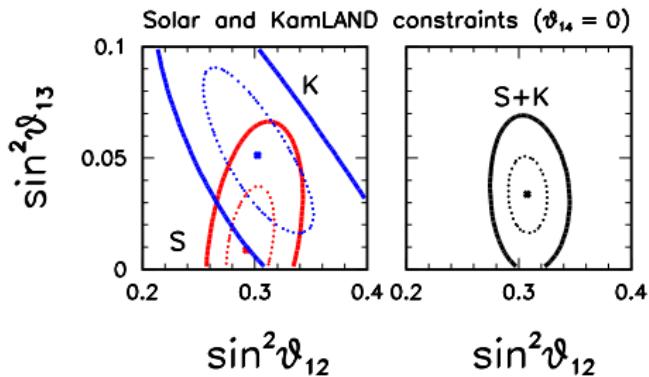
$$U_{s1} = -c_{12}c_{13}s_{14} \quad U_{s2} = -s_{12}c_{13}s_{14} \quad U_{s3} = -s_{13}s_{14} \quad U_{s4} = c_{14}$$

$$P_{\nu_e \rightarrow \nu_e}^{\text{SOL}} \simeq c_{13}^4 c_{14}^4 P_{\nu_e \rightarrow \nu_e}^{\text{SOL}, 2\nu} + s_{13}^4 c_{14}^4 + s_{14}^4$$

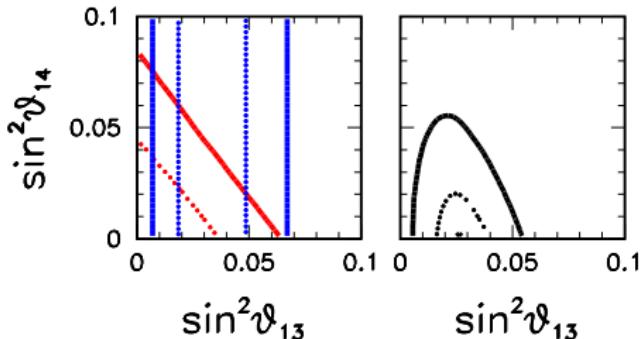
$$P_{\nu_e \rightarrow \nu_s}^{\text{SOL}} \simeq c_{14}^2 s_{14}^2 \left(c_{13}^4 P_{\nu_e \rightarrow \nu_s}^{\text{SOL}, 2\nu} + s_{13}^4 + 1 \right)$$

$$V = c_{13}^2 c_{14}^2 V_{\text{CC}} - c_{13}^2 s_{14}^2 V_{\text{NC}}$$

$$= (|U_{e1}|^2 + |U_{e2}|^2) V_{\text{CC}} - (|U_{s1}|^2 + |U_{s2}|^2) V_{\text{NC}}$$



[Palazzo, PRD 83 (2011) 113013]



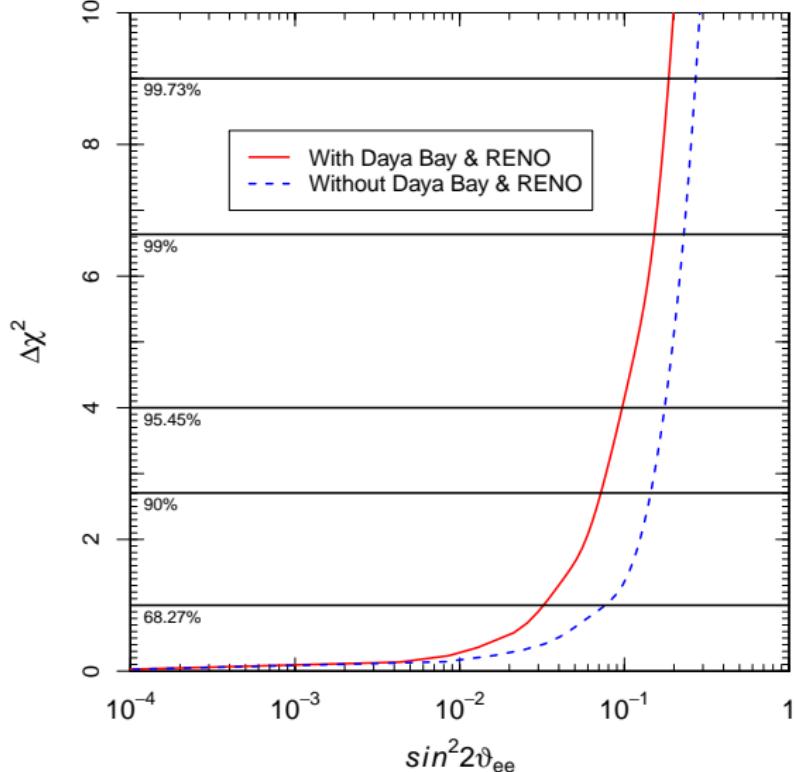
[Palazzo, PRD 85 (2012) 077301]

Daya Bay and RENO

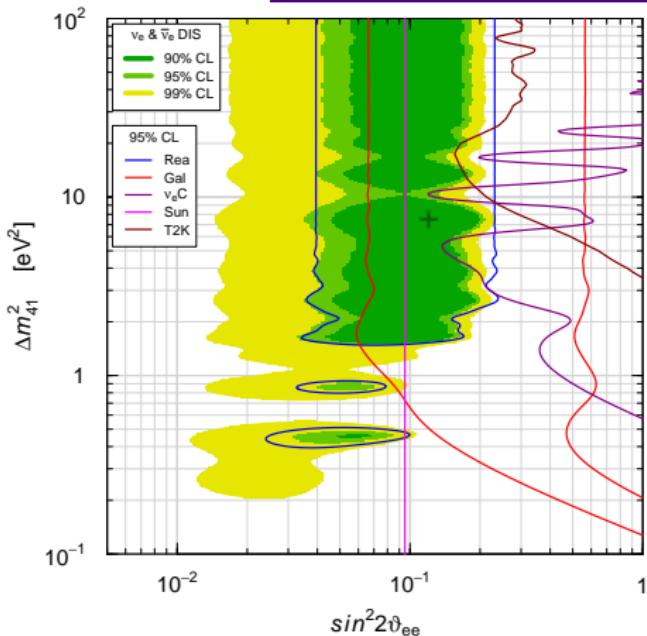
$$\sin^2 \vartheta_{13} = 0.025 \pm 0.004$$

$$|U_{e4}|^2 = \sin^2 \vartheta_{14} \lesssim 0.02 (1\sigma)$$

Fit of solar and KamLAND data with
 Daya Bay and RENO constraint $\sin^2 \vartheta_{13} = 0.025 \pm 0.004$
 and free $|U_{\mu 4}|$ and $|U_{\tau 4}|$ (neglecting small CP violation effects)



Global ν_e and $\bar{\nu}_e$ Disappearance



KARMEN + LSND $\nu_e + {}^{12}\text{C} \rightarrow {}^{12}\text{N}_{\text{g.s.}} + e^-$

[Conrad, Shaevitz, PRD 85 (2012) 013017]

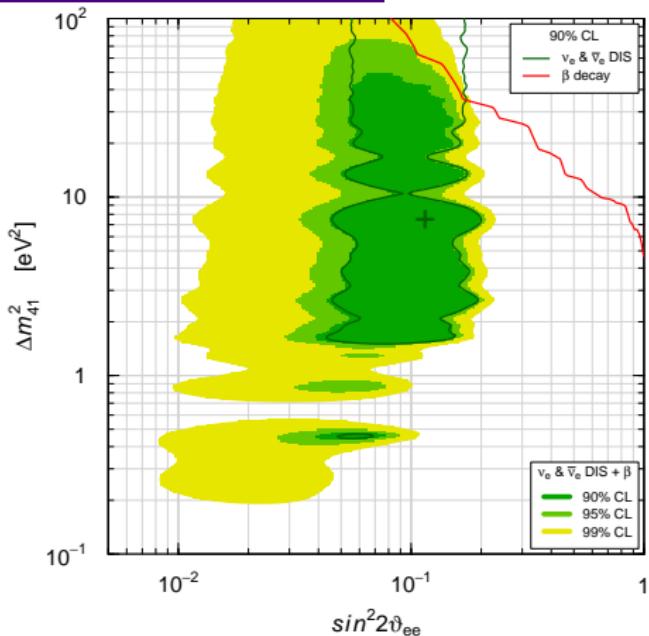
[Giunti, Laveder, PLB 706 (2011) 200]

solar ν_e + KamLAND $\bar{\nu}_e + \vartheta_{13}$

[Giunti, Li, PRD 80 (2009) 113007]

[Palazzo, PRD 83 (2011) 113013; PRD 85 (2012) 077301]

[Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]



T2K Near Detector ν_e disappearance

[T2K, PRD 91 (2015) 051102]

Mainz + Troitsk Tritium β decay

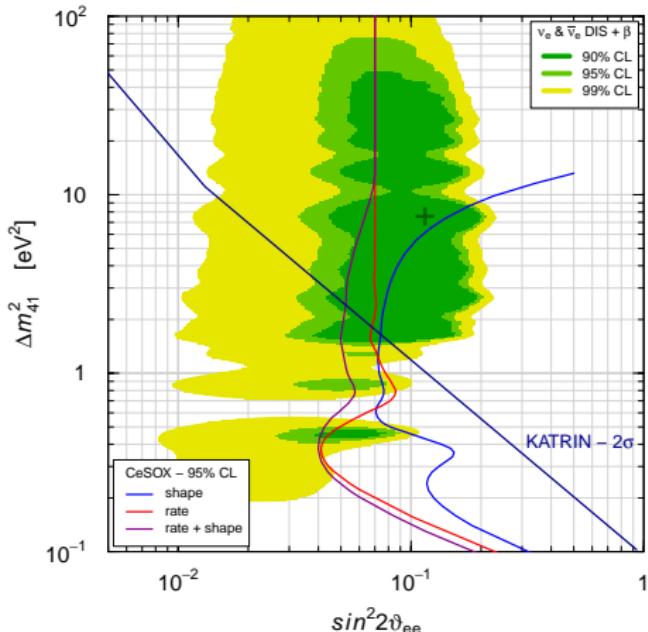
[Mainz, EPJC 73 (2013) 2323]

[Troitsk, JETPL 97 (2013) 67; JPG 41 (2014) 015001]

No Osc. excluded at 2.9σ

$$\Delta\chi^2/\text{NDF} = 11.2/2$$

Near-Future Experiments



CeSOX (BOREXINO, Italy)

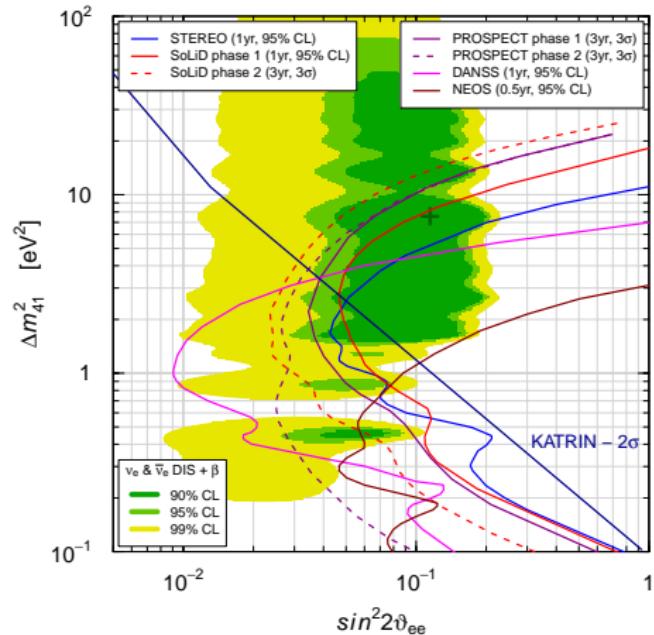
^{144}Ce – 100 kCi [Vivier@TAUP2015]

rate: 1% normalization uncertainty

8.5 m from detector center

KATRIN (Germany)

Tritium β^- decay [Mertens@TAUP2015]



STEREO (France) $L \simeq 8\text{-}12\text{m}$ [Sanchez@EPSHEP2015]

SoLid (Belgium) $L \simeq 5\text{-}8\text{m}$ [Yermia@TAUP2015]

PROSPECT (USA) $L \simeq 7\text{-}12\text{m}$ [Heeger@TAUP2015]

DANSS (Russia) $L \simeq 10\text{-}12\text{m}$ [arXiv:1412.0817]

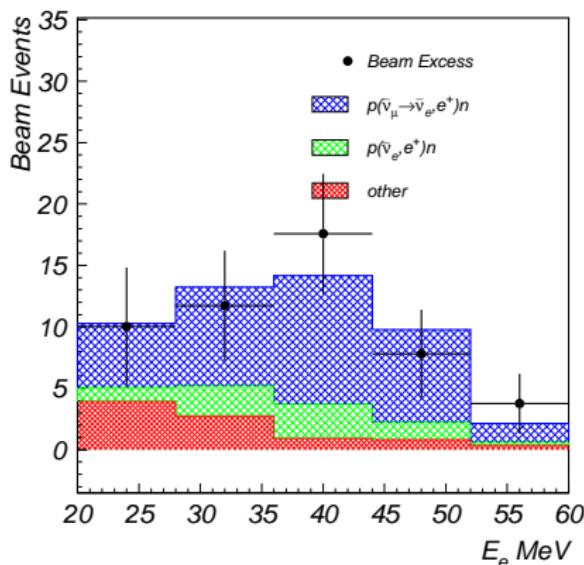
NEOS (Korea) $L \simeq 25\text{m}$ [Oh@WIN2015]

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e \quad L \simeq 30 \text{ m}$$

$$20 \text{ MeV} \leq E \leq 60 \text{ MeV}$$



Nominal $\approx 3.8\sigma$ excess

$$\Delta m^2 \gtrsim 0.2 \text{ eV}^2 \quad (\gg \Delta m_A^2 \gg \Delta m_\zeta^2)$$

- ▶ Well known source of $\bar{\nu}_\mu$:
 μ^+ at rest $\rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- ▶ $\bar{\nu}_\mu \xrightarrow[L \simeq 30 \text{ m}]{} \bar{\nu}_e$
- ▶ Well known detection process of $\bar{\nu}_e$:
 $\bar{\nu}_e + p \rightarrow n + e^+$
- ▶ But signal not seen by KARMEN with same method at $L \simeq 18 \text{ m}$

[PRD 65 (2002) 112001]

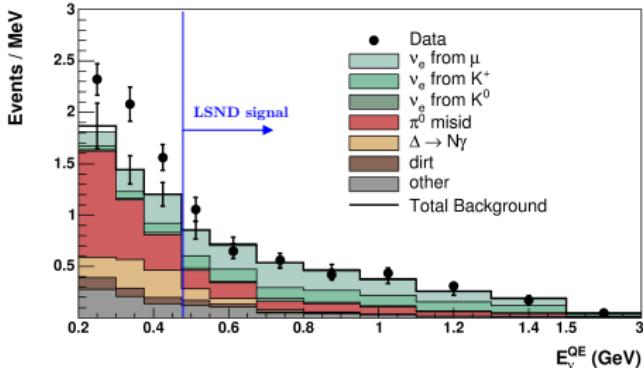
MiniBooNE

$L \simeq 541 \text{ m}$

$200 \text{ MeV} \leq E \lesssim 3 \text{ GeV}$

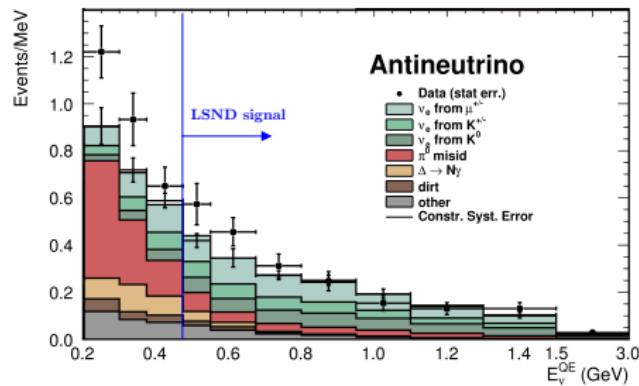
$$\nu_\mu \rightarrow \nu_e$$

[PRL 102 (2009) 101802]



$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

[PRL 110 (2013) 161801]



- ▶ Purpose: check LSND signal.
- ▶ LSND signal: $E > 475 \text{ MeV}$.
- ▶ Different L and E .
- ▶ Agreement with LSND signal?
- ▶ Similar L/E (oscillations).
- ▶ CP violation?
- ▶ No money, no Near Detector.
- ▶ Low-energy anomaly!

3+1: Appearance vs Disappearance

- ▶ Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 (1 - |U_{e4}|^2) \simeq 4|U_{e4}|^2$$

- ▶ Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2 (1 - |U_{\mu 4}|^2) \simeq 4|U_{\mu 4}|^2$$

- ▶ Amplitude of $\nu_\mu \rightarrow \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

- ▶ Upper bounds on ν_e and ν_μ disappearance \Rightarrow strong limit on $\nu_\mu \rightarrow \nu_e$

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, Giunti, Grimus, EPJC 1 (1998) 247]

- ▶ Similar constraint in 3+2, 3+3, ..., 3+N_s!

[Giunti, Zavaini, MPLA 31 (2015) 1650003]

Appearance vs Disappearance in $N = 3 + N_s$ Mixing

[Giunti, Zavanin, MPLA 31 (2015) 1650003]

$$\frac{\Delta m_{21}^2 L}{4E} \ll \frac{\Delta m_{31}^2 L}{4E} \ll 1$$

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\beta}}^{\text{SBL}} \simeq \delta_{\alpha\beta} - 4 \sum_{k=4}^N |U_{\alpha k}|^2 (\delta_{\alpha\beta} - |U_{\beta k}|^2) \sin^2 \Delta_{k1} \\ + 8 \sum_{k=4}^N \sum_{j=k+1}^N |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk}^{(+)}) \eta_{\alpha\beta jk}$$

$$\Delta_{jk} = \frac{\Delta m_{jk}^2 L}{4E} \quad \eta_{\alpha\beta jk} = \arg[U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*]$$

Survival Probabilities

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\alpha}}^{\text{SBL}} \simeq 1 - 4 \sum_{k=4}^N |U_{\alpha k}|^2 (1 - |U_{\alpha k}|^2) \sin^2 \Delta_{k1} \\ + 8 \sum_{k=4}^N \sum_{j=k+1}^N |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin \Delta_{j1} \sin \Delta_{k1} \cos \Delta_{jk}$$

Effective amplitude of $\nu_\alpha^{(-)}$ disappearance due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta_{\alpha\alpha}^{(k)} = 4|U_{\alpha k}|^2 (1 - |U_{\alpha k}|^2) \simeq 4|U_{\alpha k}|^2$$

$$|U_{\alpha k}|^2 \ll 1 \quad (\alpha = e, \mu, \tau; \quad k = 4, \dots, N)$$

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\alpha}}^{\text{SBL}} \simeq 1 - \sum_{k=4}^N \sin^2 2\vartheta_{\alpha\alpha}^{(k)} \sin^2 \Delta_{k1}$$

Appearance Probabilities ($\alpha \neq \beta$)

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\beta}}^{\text{SBL}} \simeq 4 \sum_{k=4}^N |U_{\alpha k}|^2 |U_{\beta k}|^2 \sin^2 \Delta_{k1} + 8 \sum_{k=4}^N \sum_{j=k+1}^N |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk}^{(+)} - \eta_{\alpha \beta jk})$$

Effective amplitude of $\overset{(-)}{\nu_\alpha} \rightarrow \overset{(-)}{\nu_\beta}$ transitions due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta_{\alpha\beta}^{(k)} = 4|U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\beta}}^{\text{SBL}} \simeq \sum_{k=4}^N \sin^2 2\vartheta_{\alpha\beta}^{(k)} \sin^2 \Delta_{k1} + 2 \sum_{k=4}^N \sum_{j=k+1}^N \sin 2\vartheta_{\alpha\beta}^{(k)} \sin 2\vartheta_{\alpha\beta}^{(j)} \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk}^{(+)} - \eta_{\alpha \beta jk})$$

$$\sin^2 2\vartheta_{\alpha\alpha}^{(k)} = 4|U_{\alpha k}|^2 (1 - |U_{\alpha k}|^2) \simeq 4|U_{\alpha k}|^2$$

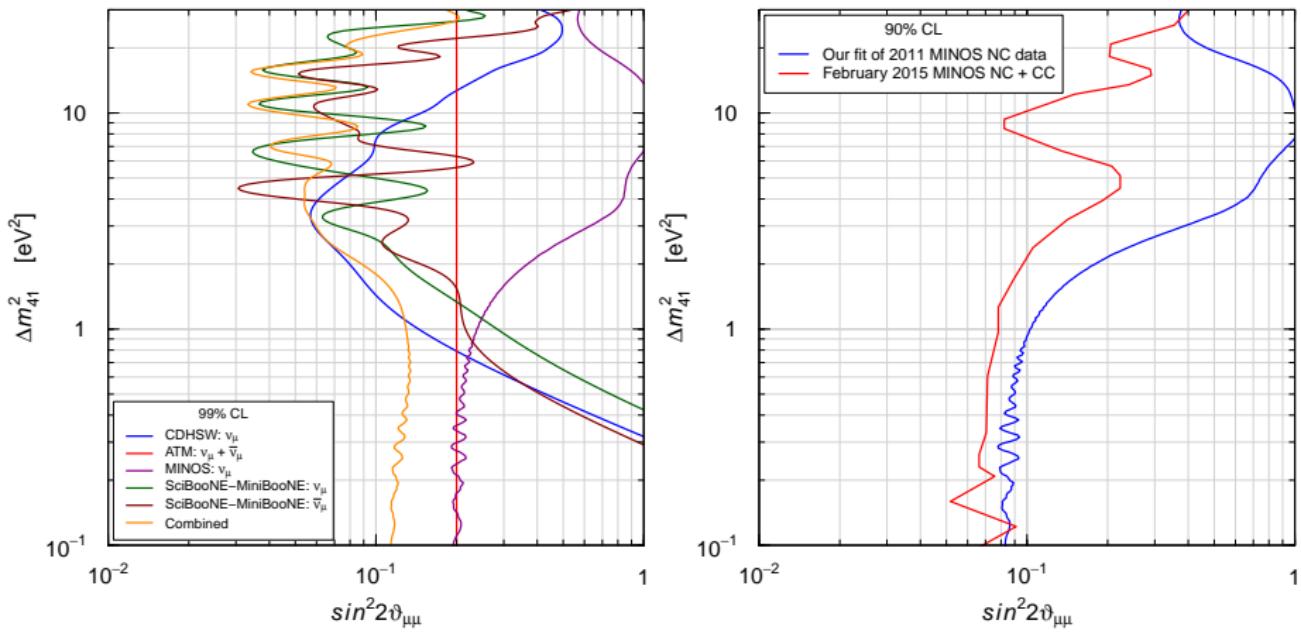
$$\sin^2 2\vartheta_{\alpha\beta}^{(k)} = 4|U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$\boxed{\sin^2 2\vartheta_{\alpha\beta}^{(k)} \simeq \frac{1}{4} \sin^2 2\vartheta_{\alpha\alpha}^{(k)} \sin^2 2\vartheta_{\beta\beta}^{(k)}}$$

$$\left. \begin{array}{l} \sin^2 2\vartheta_{ee}^{(k)} \ll 1 \\ \sin^2 2\vartheta_{\mu\mu}^{(k)} \ll 1 \end{array} \right\} \Rightarrow \sin^2 2\vartheta_{e\mu}^{(k)} \quad \text{is quadratically suppressed}$$

on the other hand, observation of $\overset{(-)}{\nu_\alpha} \rightarrow \overset{(-)}{\nu_\beta}$ transitions due to Δm_{k1}^2 imply
that the corresponding $\overset{(-)}{\nu_\alpha}$ and $\overset{(-)}{\nu_\beta}$ disappearances must be observed

ν_μ and $\bar{\nu}_\mu$ Disappearance

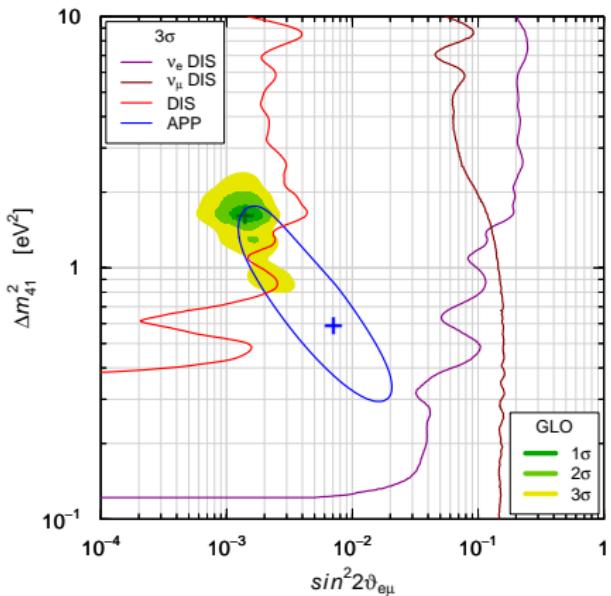


MINOS: $L_{\text{decay}} \simeq 0.675 \text{ km}$ $L_{\text{ND}} \simeq 1.04 \text{ km}$ $L_{\text{FD}} \simeq 735 \text{ km}$

$$E \approx 4 \text{ GeV} \implies \frac{L_{\text{osc}}}{L_{\text{ND}}} \approx \frac{10}{\Delta m_{41}^2 [\text{eV}^2]}$$

Global 3+1 Fit

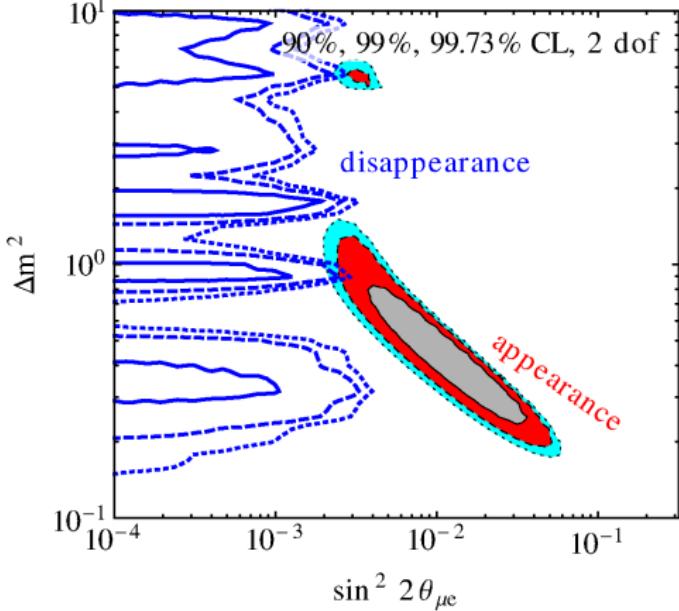
Our Fit



GoF = 5%

PGoF = 0.1%

Kopp, Machado, Maltoni, Schwetz



GoF = 19%

PGoF = 0.01%

[Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

There is no globally allowed region
in this paper!

Goodness of Fit

- ▶ Assumption or approximation: Gaussian uncertainties and linear model
- ▶ χ_{\min}^2 has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_D - N_P$$

N_D = Number of Data N_P = Number of Fitted Parameters

- ▶ $\langle \chi_{\min}^2 \rangle = \text{NDF}$ $\text{Var}(\chi_{\min}^2) = 2\text{NDF}$

- ▶ $\text{GoF} = \int_{\chi_{\min}^2}^{\infty} p_{\chi^2}(z, \text{NDF}) dz$ $p_{\chi^2}(z, n) = \frac{z^{n/2-1} e^{-z/2}}{2^{n/2} \Gamma(n/2)}$

Parameter Goodness of Fit

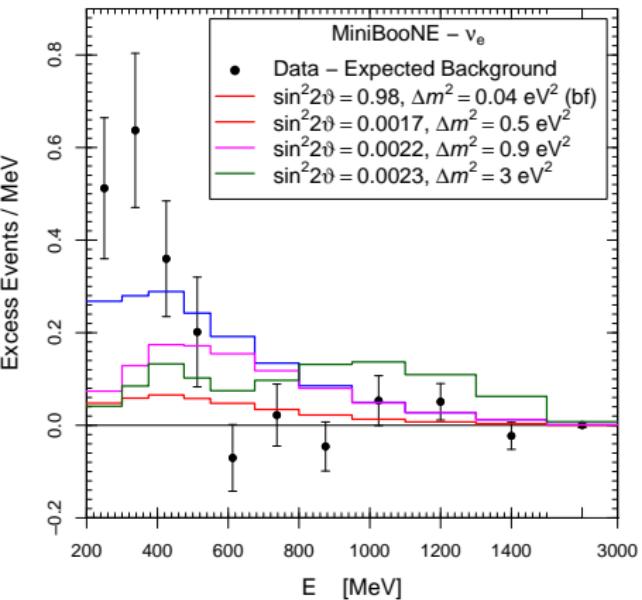
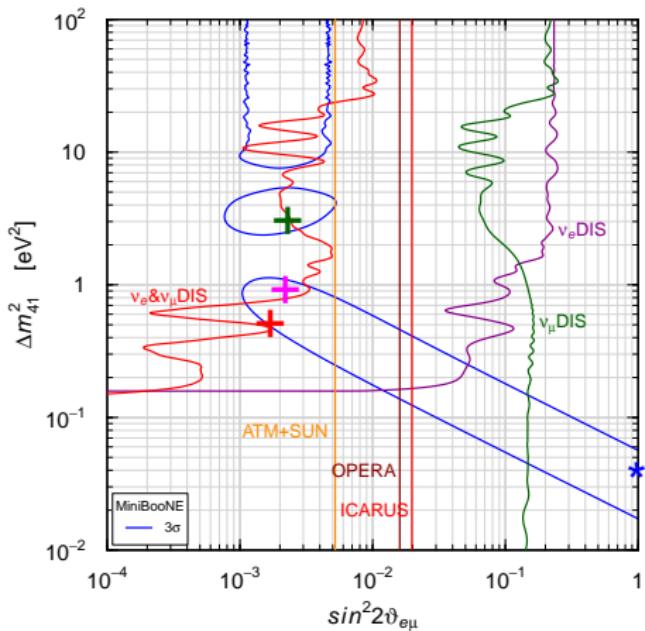
Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

- ▶ Measure compatibility of two (or more) sets of data points A and B under fitting model
- ▶ $\chi_{\text{PGoF}}^2 = (\chi_{\min}^2)_{A+B} - [(\chi_{\min}^2)_A + (\chi_{\min}^2)_B]$
- ▶ χ_{PGoF}^2 has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_P^A + N_P^B - N_P^{A+B}$$

- ▶ $\text{PGoF} = \int_{\chi_{\text{PGoF}}^2}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}}) dz$

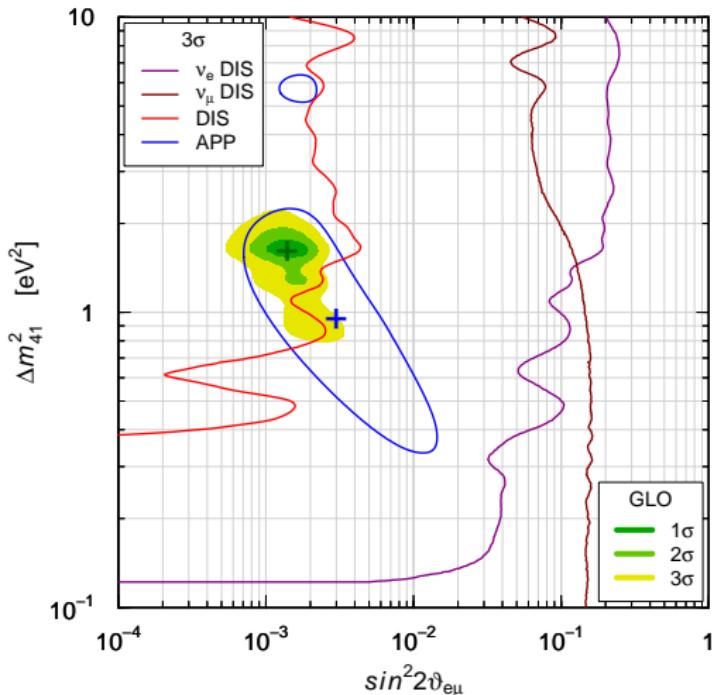
MiniBooNE Low-Energy Excess?



- ▶ No fit of low-energy excess for realistic $\sin^2 2\theta_{e\mu} \lesssim 3 \times 10^{-3}$
- ▶ Neutrino energy reconstruction problem? [Martini, Ericson, Chanfray, PRD 87 (2013) 013009]
- ▶ MB low-energy excess is the main cause of bad APP-DIS PGOf = 0.1%
- ▶ Pragmatic Approach: discard the Low-Energy Excess because it is very likely not due to oscillations

Pragmatic Global 3+1 Fit

[PRD 88 (2013) 073008; arXiv:1507.08204]



MiniBooNE $E > 475 \text{ MeV}$
GoF = 26% PGoF = 7%

- ▶ APP $\nu_\mu \rightarrow \nu_e$ & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$:
LSND (ν_s), MiniBooNE (?),
OPERA ($\cancel{\nu_s}$), ICARUS ($\cancel{\nu_s}$),
KARMEN ($\cancel{\nu_s}$),
NOMAD ($\cancel{\nu_s}$), BNL-E776 ($\cancel{\nu_s}$)
- ▶ DIS ν_e & $\bar{\nu}_e$: Reactors (ν_s),
Gallium (ν_s), $\nu_e C$ ($\cancel{\nu_s}$),
Solar ($\cancel{\nu_s}$)
- ▶ DIS ν_μ & $\bar{\nu}_\mu$: CDHSW ($\cancel{\nu_s}$),
MINOS ($\cancel{\nu_s}$),
Atmospheric ($\cancel{\nu_s}$),
MiniBooNE/SciBooNE ($\cancel{\nu_s}$)

No Osc. nominally disfavored
at $\approx 6.3\sigma$
 $\Delta\chi^2/\text{NDF} = 47.7/3$

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\Delta_{kj} = \Delta m_{kj}^2 L / 4E$$

$$\eta = \arg[U_{e4}^* U_{\mu 4} U_{e5} U_{\mu 5}^*]$$

$$P_{\substack{(-) \\ \nu_\mu \rightarrow \nu_e}}^{\text{SBL}} = 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2 \Delta_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2 \Delta_{51} \\ + 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin \Delta_{41} \sin \Delta_{51} \cos(\Delta_{54} - \eta)$$

$$P_{\substack{(-) \\ \nu_\alpha \rightarrow \nu_\alpha}}^{\text{SBL}} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2 \Delta_{41} + |U_{\alpha 5}|^2 \sin^2 \Delta_{51}) \\ - 4|U_{\alpha 4}|^2|U_{\alpha 5}|^2 \sin^2 \Delta_{54}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Archidiacono et al, PRD 86 (2012) 065028; Jacques, Krauss, Lunardini, PRD 87 (2013) 083515; Conrad et al, AHEP 2013 (2013) 163897; Archidiacono et al, PRD 87 (2013) 125034; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008; Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

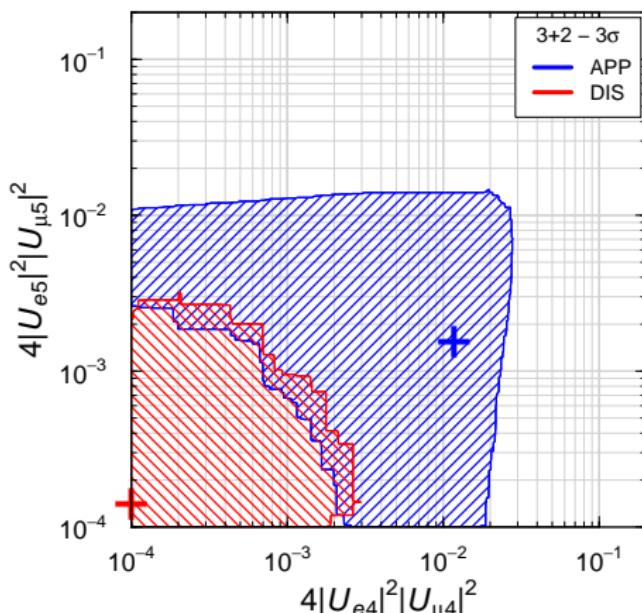
- ▶ Good: CP violation
- ▶ Bad: Two massive sterile neutrinos at the eV scale!

4 more parameters: $\underbrace{\Delta m_{41}^2, |U_{e4}|^2, |U_{\mu 4}|^2, \Delta m_{51}^2, |U_{e5}|^2, |U_{\mu 5}|^2, \eta}_{3+1}$

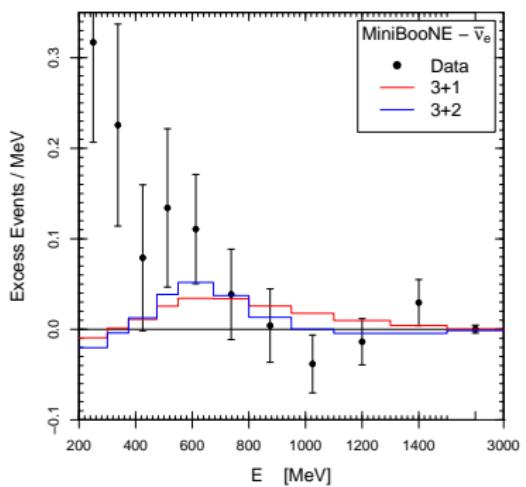
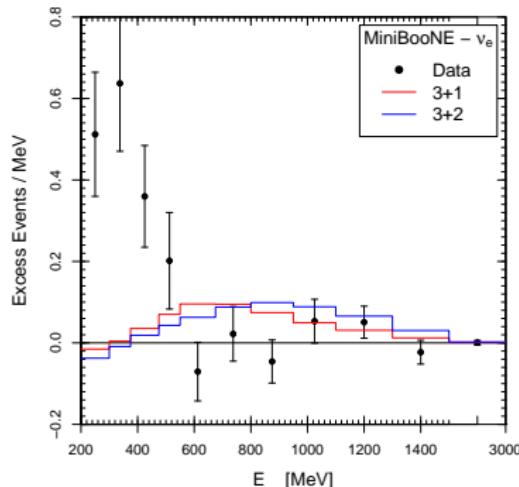
Global Fits	Our Fit		KMMS	
	3+1	3+2	3+1	3+2
GoF	5%	7%	19%	23%
PGoF	0.1%	0.04%	0.01%	0.003%

- Our Fit: Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001
- KMMS: Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050

APP-DIS 3+2 Tension:

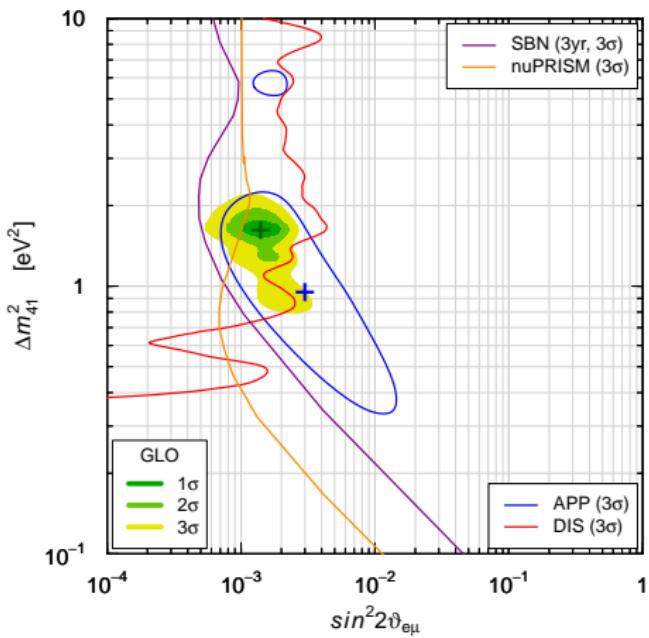


3+2 cannot fit MiniBooNE Low-Energy Excess



- ▶ Note difference between 3+2 ν_e and $\bar{\nu}_e$ histograms due to CP violation
- ▶ 3+2 can fit slightly better the small $\bar{\nu}_e$ excess at about 600 MeV
- ▶ 3+2 fit of low-energy excess as bad as 3+1
- ▶ Claims that 3+2 can fit low-energy excess do not take into account constraints from other data
- ▶ Conclusion: 3+2 is not needed

Future Experiments



SBN (FNAL, USA)

[arXiv:1503.01520]

3 Liquid Argon TPCs

LAr1-ND $L \simeq 100$ m

MicroBooNE $L \simeq 470$ m

ICARUS T600 $L \simeq 600$ m

nuPRISM (J-PARC, Japan)

[Wilking@NNN2015]

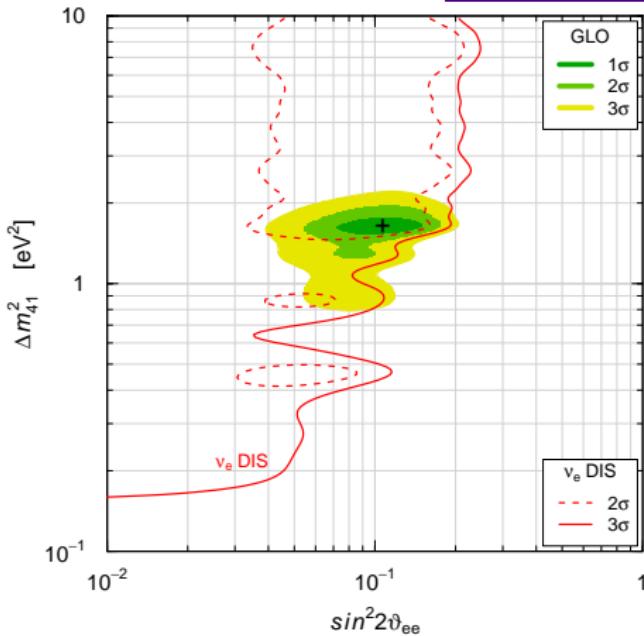
$L \simeq 1$ km

50 m tall water Cherenkov detector

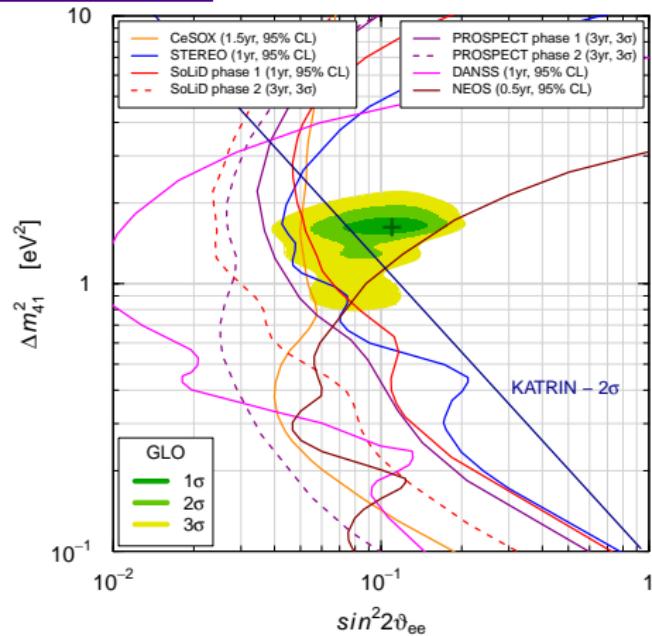
$1^\circ - 4^\circ$ off-axis

can be improved with T2K ND

ν_e Disappearance

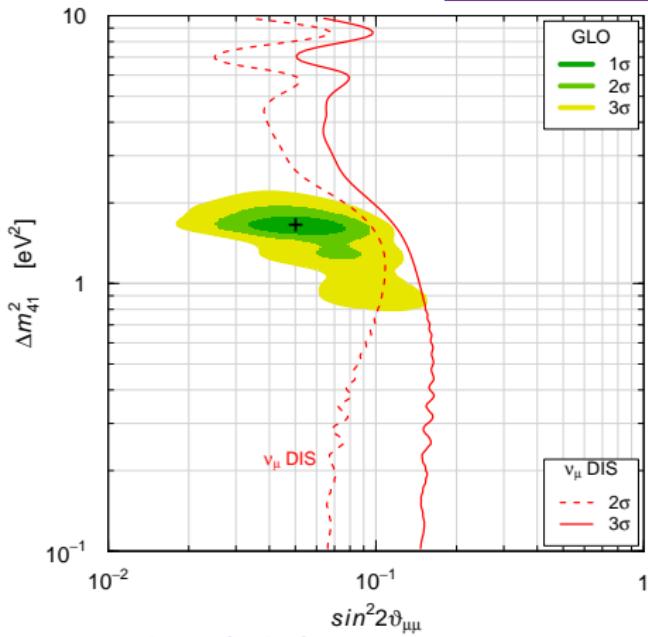


CeSOX (BOREXINO, Italy)
 $^{144}\text{Ce} - 100 \text{ kCi}$ [Vivier@TAUP2015]
 rate: 1% normalization uncertainty
 8.5 m from detector center
KATRIN (Germany)
 Tritium β decay [Mertens@TAUP2015]



STEREO (France) $L \simeq 8\text{-}12\text{m}$ [Sanchez@EPSHEP2015]
SoLid (Belgium) $L \simeq 5\text{-}8\text{m}$ [Yermia@TAUP2015]
PROSPECT (USA) $L \simeq 7\text{-}12\text{m}$ [Heeger@TAUP2015]
DANSS (Russia) $L \simeq 10\text{-}12\text{m}$ [arXiv:1412.0817]
NEOS (Korea) $L \simeq 25\text{m}$ [Oh@WIN2015]

ν_μ Disappearance

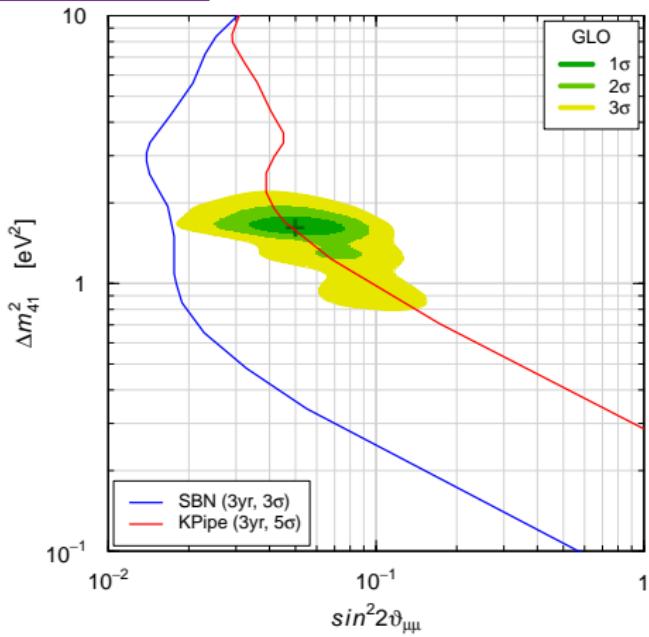


SBN (USA) [arXiv:1503.01520]

LAr1-ND $L \simeq 100$ m

MicroBooNE $L \simeq 470$ m

ICARUS T600 $L \simeq 600$ m



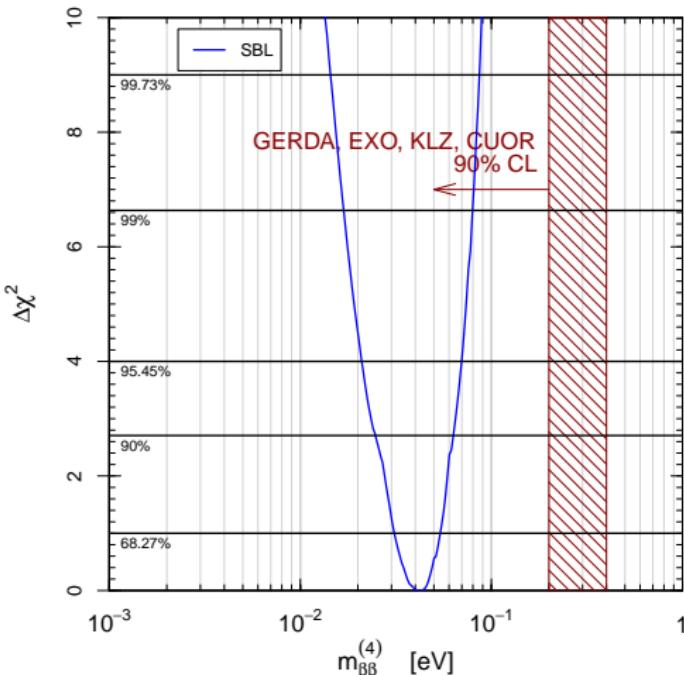
KPipe (Japan) [arXiv:1510.06994]

$L \simeq 30-150$ m

120 m long detector!

Neutrinoless Double- β Decay

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3 + |U_{e4}|^2 e^{i\alpha_{41}} m_4$$



$$m_{\beta\beta}^{(k)} = |U_{ek}|^2 m_k$$

$$\begin{aligned} m_1 &\ll m_4 \\ \downarrow \\ m_{\beta\beta}^{(4)} &\simeq |U_{e4}|^2 \sqrt{\Delta m_{41}^2} \end{aligned}$$

surprise:
possible cancellation
with $m_{\beta\beta}^{(3\nu)}$

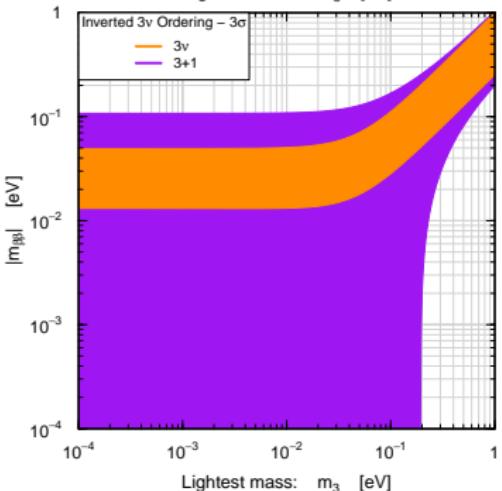
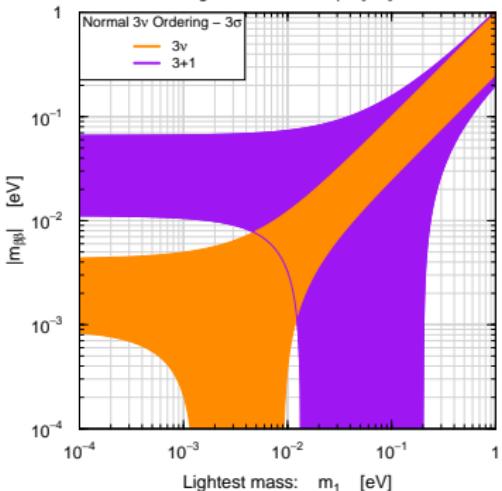
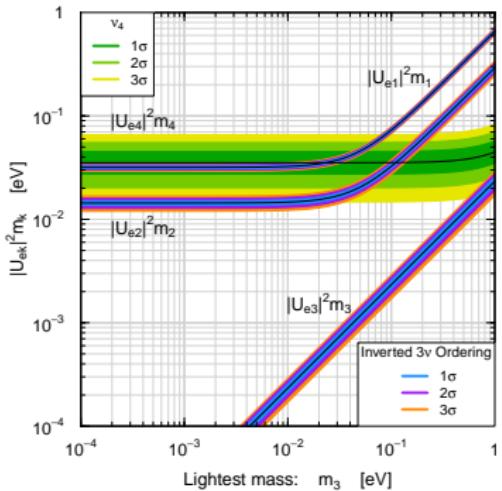
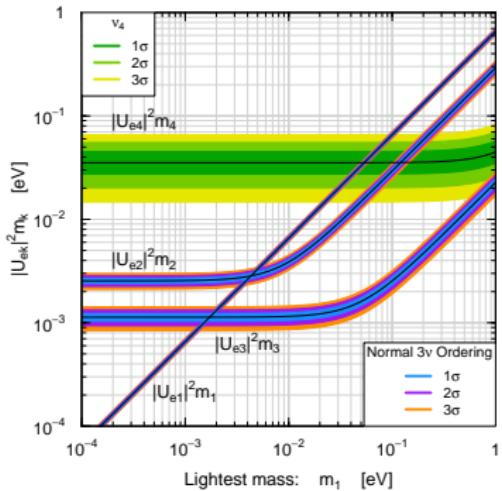
[Barry et al, JHEP 07 (2011) 091]

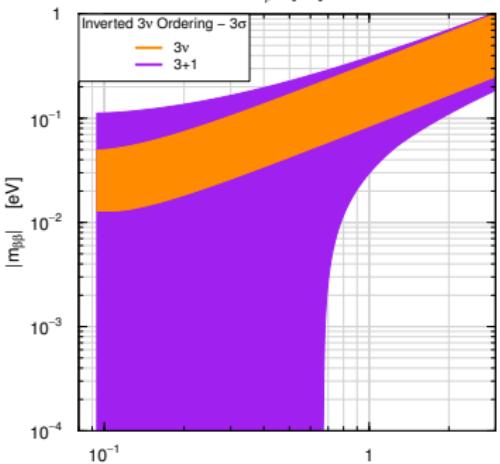
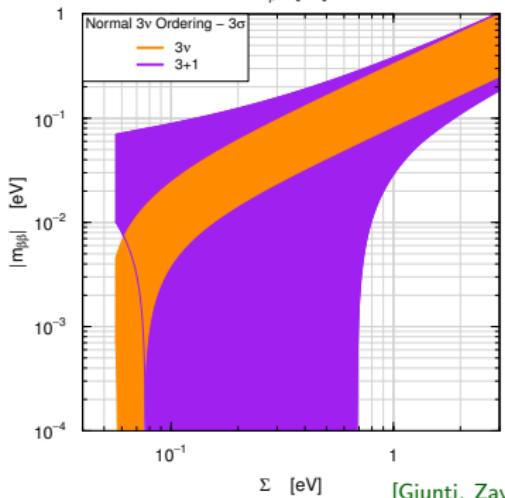
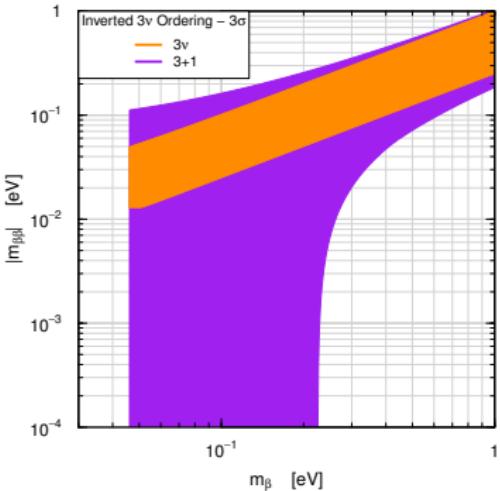
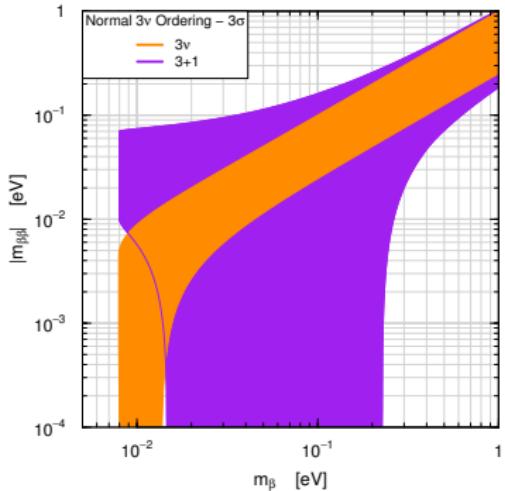
[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, JPG 39 (2012) 124008]

[Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

[Giunti, Zavarni, JHEP 07 (2015) 171]





[Giunti, Zavanin, JHEP 07 (2015) 171]

Cosmology

- neutrinos in equilibrium in early Universe through weak interactions:

$$\begin{array}{lll} \nu\bar{\nu} \leftrightarrows e^+e^- & {}^{(-)}_{\nu}e \leftrightarrows {}^{(-)}_{\nu}e & {}^{(-)}_{\nu}N \leftrightarrows {}^{(-)}_{\nu}N \\ \nu_e n \leftrightarrows pe^- & \bar{\nu}_e p \leftrightarrows ne^+ & n \leftrightarrows pe^-\bar{\nu}_e \end{array}$$

- weak interactions freeze out \implies active $(\nu_e, \nu_\mu, \nu_\tau)$ neutrino decoupling

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H$$

$$T_{\nu\text{-dec}} \sim 1 \text{ MeV} \quad t_{\nu\text{-dec}} \sim 1 \text{ s}$$

- relic neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945 \text{ K} \implies k T_\nu \simeq 1.676 \times 10^{-4} \text{ eV}$
 $(T_\gamma = 2.725 \pm 0.001 \text{ K})$

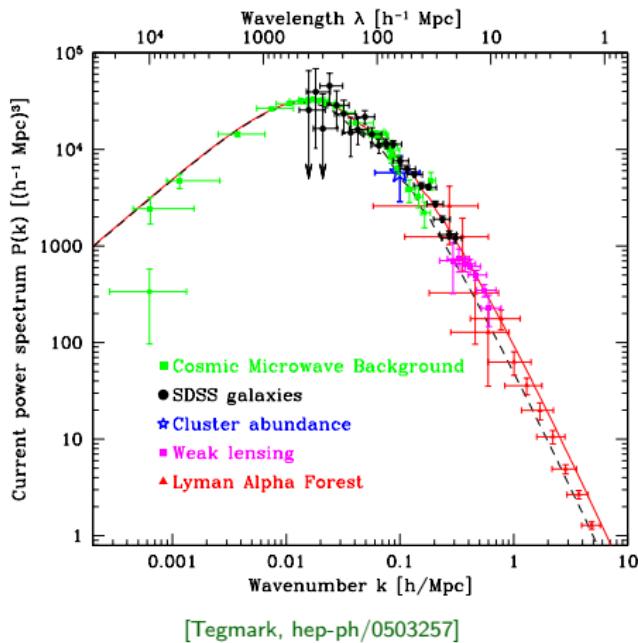
- number density: $n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T_f^3 \implies n_{\nu_k, \bar{\nu}_k} \simeq 0.1827 T_\nu^3 \simeq 112 \text{ cm}^{-3}$

- density contribution: $\Omega_k = \frac{n_{\nu_k, \bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2} \frac{m_k}{94.1 \text{ eV}} \implies$
 $\left(\rho_c = \frac{3H^2}{8\pi G_N} \right)$

$$\boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.1 \text{ eV}}}$$

wsik, McClelland, PRL 29 (1972) 669]

Power Spectrum of Density Fluctuations



Solid Curve: flat Λ CDM model

$$(\Omega_M^0 = 0.28, h = 0.72, \Omega_B^0/\Omega_M^0 = 0.16)$$

Dashed Curve: $\sum_{k=1}^3 m_k = 1 \text{ eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} P(\vec{k})$$

small scale suppression

$$\begin{aligned} \frac{\Delta P(k)}{P(k)} &\approx -8 \frac{\Omega_\nu}{\Omega_m} \\ &\approx -0.8 \left(\frac{\sum_k m_k}{1 \text{ eV}} \right) \left(\frac{0.1}{\Omega_m h^2} \right) \end{aligned}$$

for

$$k \gtrsim k_{nr} \approx 0.026 \sqrt{\frac{m_\nu}{1 \text{ eV}}} \sqrt{\Omega_m} h \text{ Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia \implies Flat Λ CDM

$$T_0 = 13.7 \pm 0.2 \text{ Gyr} \quad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \quad \Omega_b = 0.044 \pm 0.004 \quad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad (\text{95\% conf.}) \quad \implies \quad \sum_{k=1}^3 m_k < 0.71 \text{ eV}$$

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12 \text{ Gyr} \quad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad (\text{95\% C.L.})$$

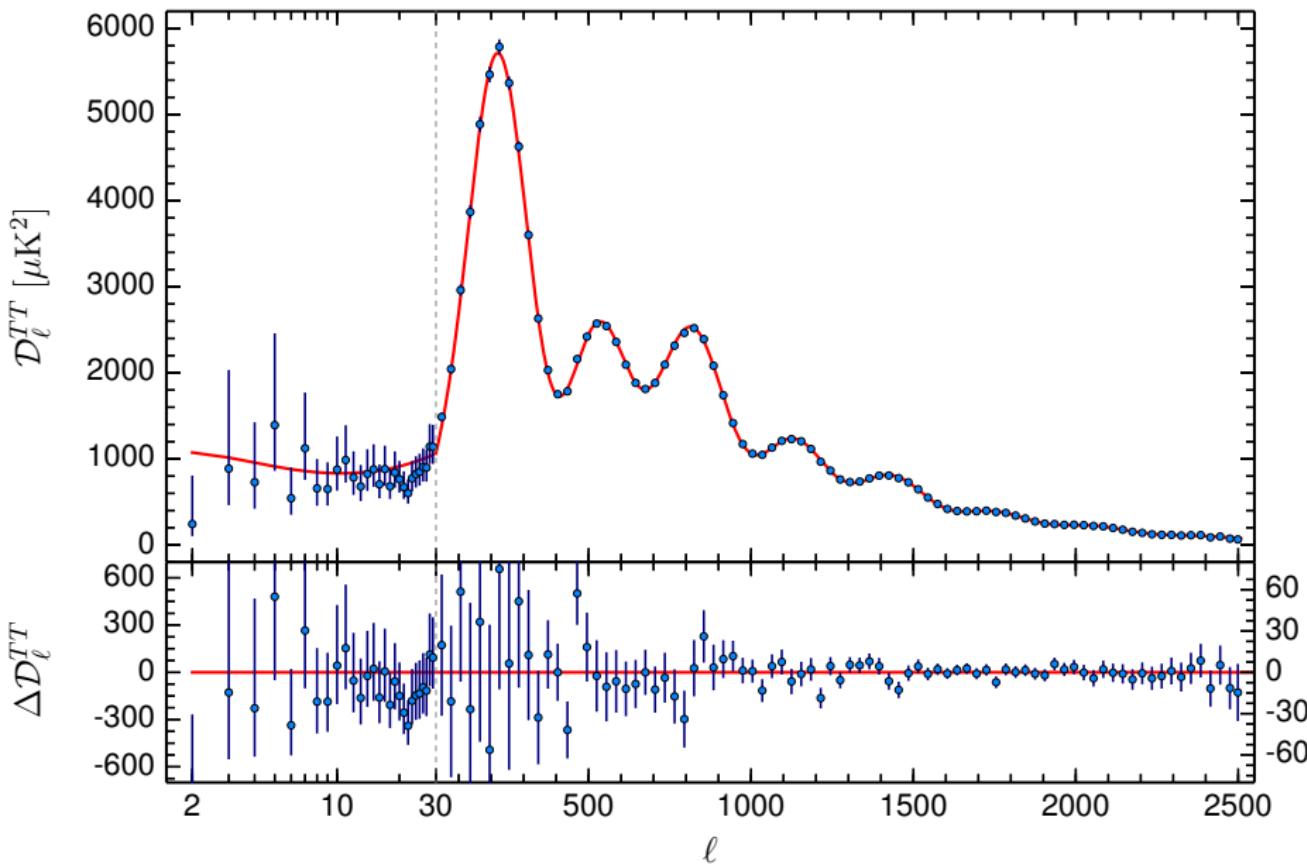
$$\Omega_b = 0.0456 \pm 0.0015 \quad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^3 m_k < 0.67 \text{ eV} \quad (\text{95\% C.L.}) \quad N_{\text{eff}} = 4.4 \pm 1.5$$

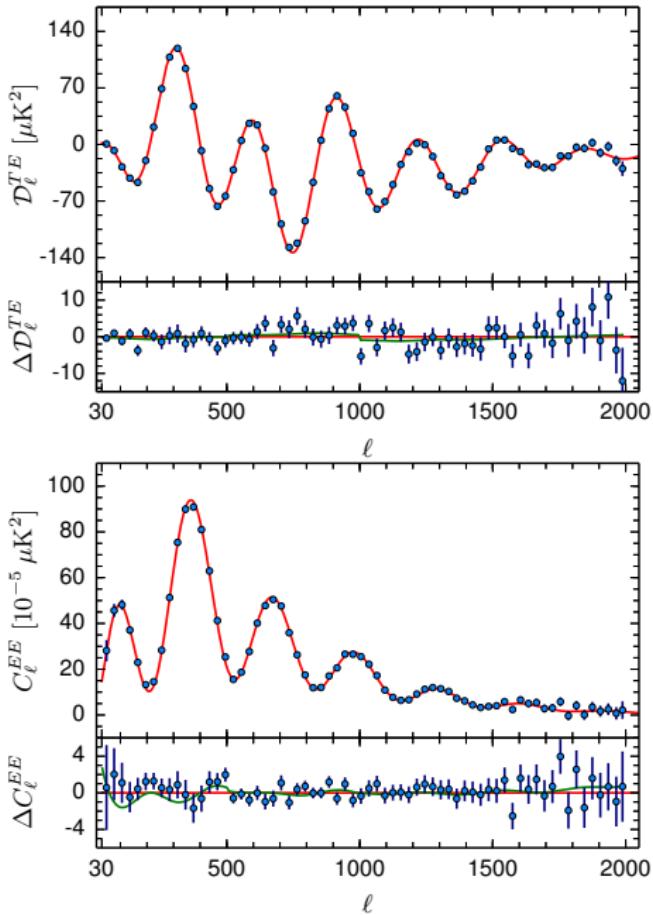
Flat Λ CDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Ly α	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ .

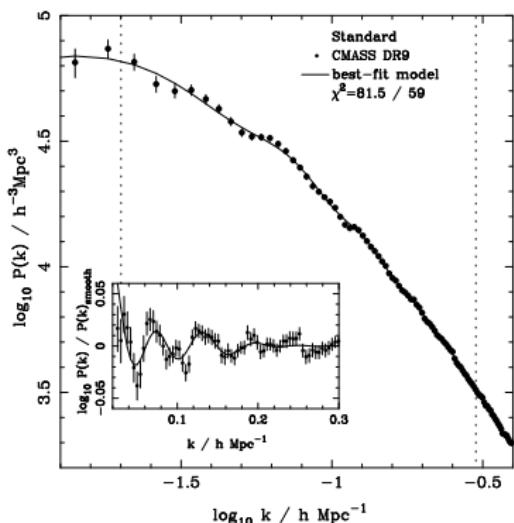


Planck Polarization Data



Planck Terminology

- TT denotes the Planck TT data (low- ℓ for $\ell < 30$ and high- ℓ for $\ell \geq 30$).
- lowP denotes the Planck polarization data at multipoles $\ell < 30$ (low- ℓ).
- TE denotes the Planck TE data at $\ell \geq 30$.
- EE denotes the Planck EE data at $\ell \geq 30$.
- Lensing denotes the Planck weak lensing data.
- BAO denotes the Baryon Acoustic Oscillation data.

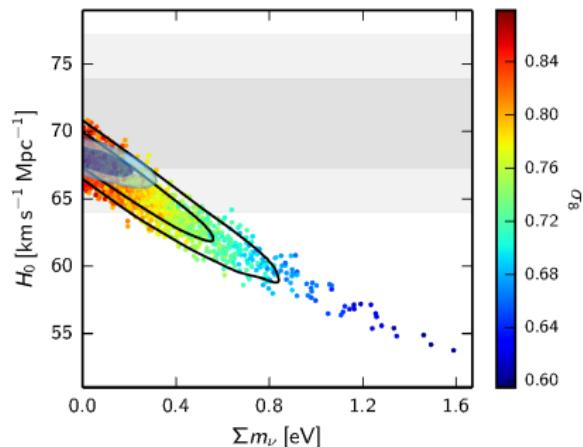
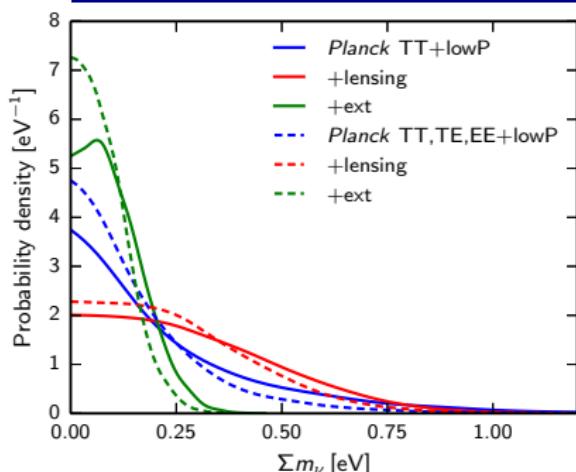


Baryon Oscillation Spectroscopic Survey
(BOSS)
part of the Sloan Digital Sky Survey III
(SDSS-III)
Data Release 9 (DR9) CMASS sample
[arXiv:1203.6594]

Limits on the Sum of Standard Light Neutrino Masses

[Planck, arXiv:1502.01589]

Cosmological data set	Σ (at 95% C.L.)
Plank TT + lowP	< 0.72 eV
Plank TT + lowP + BAO	< 0.21 eV
Plank TT,TE,EE + lowP	< 0.49 eV
Plank TT,TE,EE + lowP + BAO	< 0.17 eV
Plank TT + lowP + lensing	< 0.68 eV
Plank TT,TE,EE + lowP + lensing	< 0.59 eV
Plank TT + lowP + lensing + BAO + H_0	< 0.23 eV



Sterile Neutrinos in Cosmology

- sterile neutrinos can be produced by $\nu_{e,\mu,\tau} \rightarrow \nu_s$ oscillations before active neutrino decoupling ($t_{\nu\text{-dec}} \sim 1\text{ s}$)
- energy density of radiation before matter-radiation equality:

$$\rho_R = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_\gamma \quad (t < t_{\text{eq}} \sim 6 \times 10^4 \text{ y})$$
$$N_{\text{eff}}^{\text{SM}} = 3.046 \quad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$$

- sterile neutrino contribution:

$$\rho_s = (T_s/T_\nu)^4 \rho_\nu \implies \Delta N_{\text{eff}} = (T_s/T_\nu)^4$$

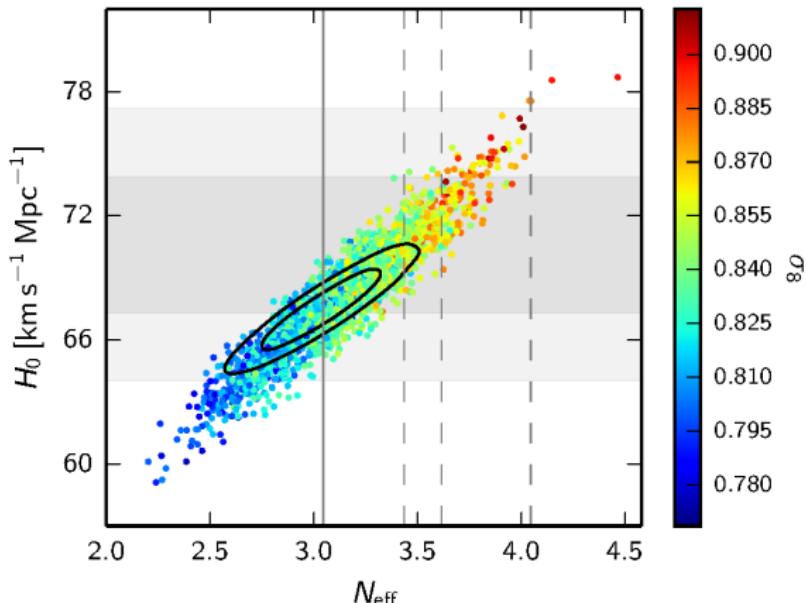
- sterile neutrino $\nu_s \simeq \nu_4$ with mass $m_s = m_4 \simeq \sqrt{\Delta m_{41}^2} \sim 1\text{ eV}$ becomes non-relativistic at $T_\nu \sim m_s/3$, that is at $t_{\nu_s\text{-nr}} \sim 2.0 \times 10^5 \text{ y}$, before recombination at $t_{\text{rec}} \sim 3.8 \times 10^5 \text{ y}$
- current energy density of sterile neutrinos:

$$\Omega_s = \frac{n_s m_s}{\rho_c} \simeq \frac{1}{h^2} \frac{(T_s/T_\nu)^3 m_s}{94.1\text{ eV}} = \frac{1}{h^2} \frac{\Delta N_{\text{eff}}^{3/4} m_s}{94.1\text{ eV}} = \frac{1}{h^2} \frac{m_s^{\text{eff}}}{94.1\text{ eV}}$$
$$m_s^{\text{eff}} = \Delta N_{\text{eff}}^{3/4} m_s = (T_s/T_\nu)^3 m_s$$

Limits on Dark Radiation

[Planck, arXiv:1502.01589]

Cosmological data set	N_{eff}
Plank TT + lowP	3.13 ± 0.32
Plank TT + lowP + BAO	3.15 ± 0.23
Plank TT,TE,EE + lowP	2.99 ± 0.20
Plank TT,TE,EE + lowP + BAO	3.04 ± 0.18

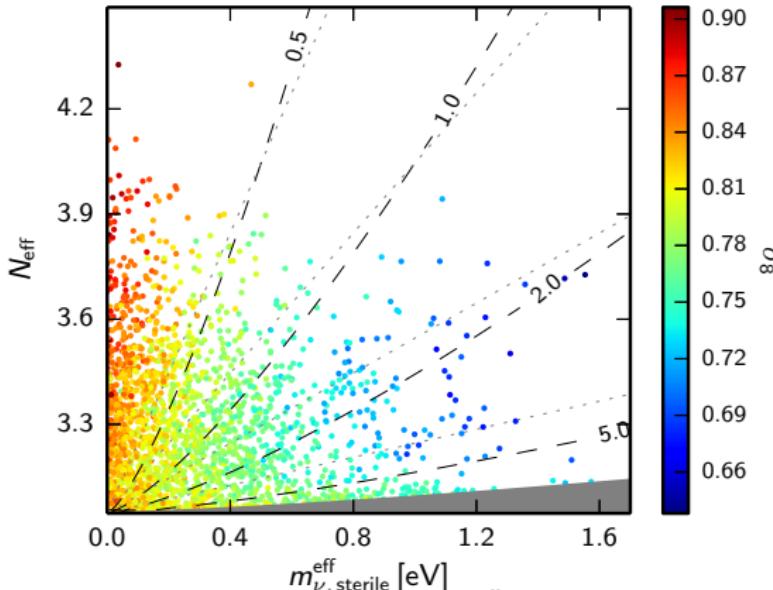


Limits on Massive Sterile Neutrinos

$$N_{\text{eff}} < 3.7$$

$$m_s^{\text{eff}} < 0.52$$

(95%; Plank TT + lowP + lensing + BAO)



Samples from Plank TT + lowP in the $N_{\text{eff}} - m_s^{\text{eff}}$ plane, colour-coded by σ_8 , in models with one massive sterile neutrino family, with effective mass m_s^{eff} , and the three active neutrinos as in the base Λ CDM model. The physical mass of the sterile neutrino in the thermal scenario, m_s^{thermal} , is constant along the grey dashed lines, with the indicated mass in eV; the grey region shows the region excluded by our prior $m_s^{\text{thermal}} < 10$ eV, which excludes most of the area where the neutrinos behave nearly like dark matter. The physical mass in the Dodelson-Widrow scenario, m_s^{DW} , is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

[arXiv:1502.01589]

$$\blacktriangleright m_s^{\text{eff}} \equiv 94.1 \Omega_s h^2 \text{ eV}$$

\blacktriangleright Thermally distributed:

$$f_s(E) = \frac{1}{e^{E/T_s} + 1}$$

$$\begin{aligned} m_s^{\text{eff}} &= \left(\frac{T_s}{T_\nu} \right)^3 m_4 \\ &= (\Delta N_{\text{eff}})^{3/4} m_4 \end{aligned}$$

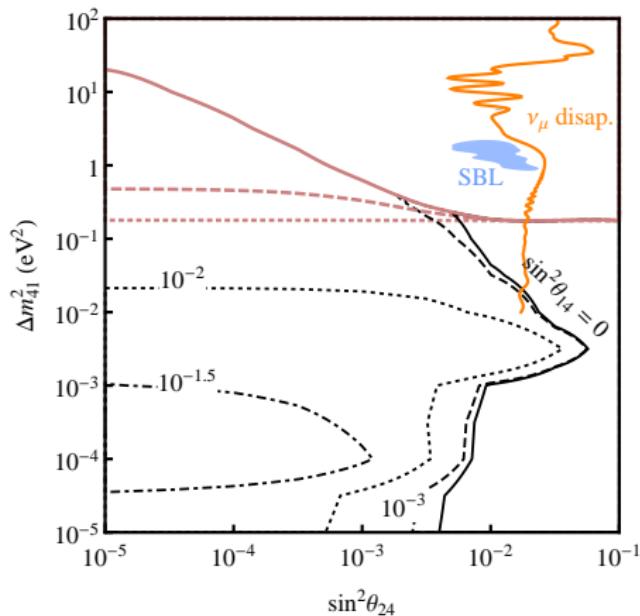
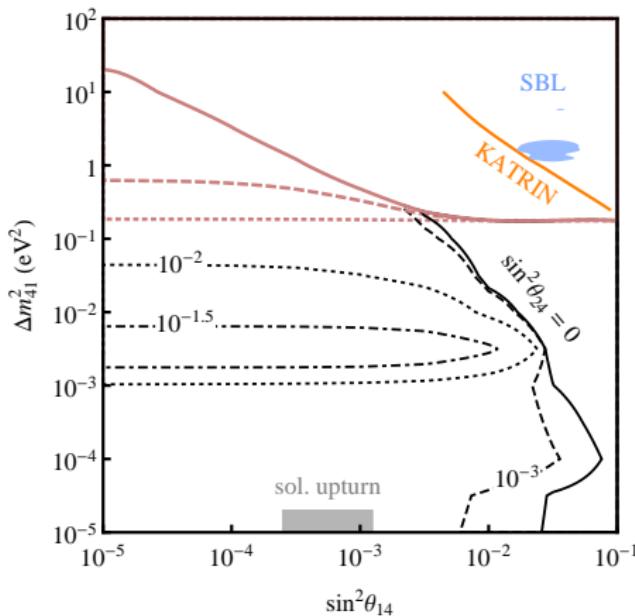
\blacktriangleright Dodelson-Widrow:

$$f_s(E) = \frac{\chi}{e^{E/T_\nu} + 1}$$

$$m_s^{\text{eff}} = \chi_s m_4$$

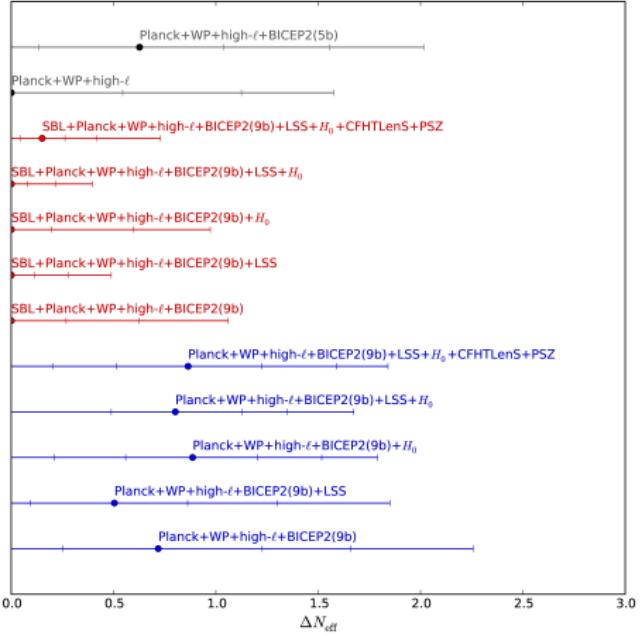
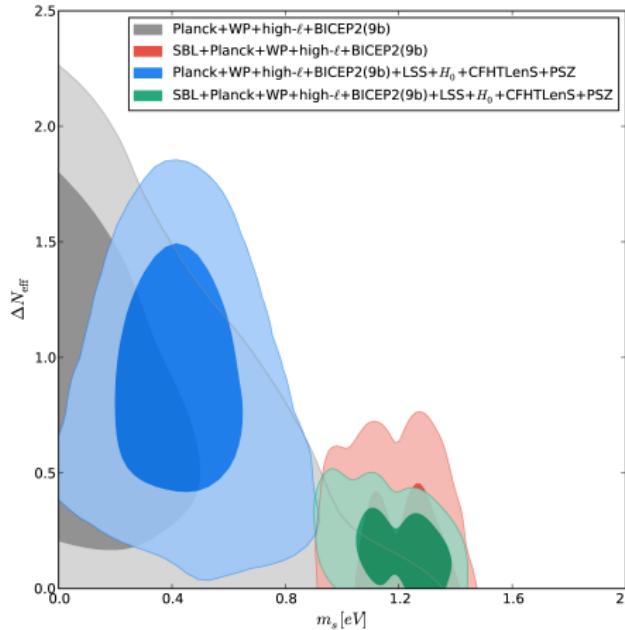
Standard Cosmological Scenario Mixing Bounds

[Mirizzi, Mangano, Saviano, Borriello, Giunti, Miele, Pisanti, arXiv:1303.5368]



Non-standard mechanism for partial thermalization of ν_s is needed
Large primordial neutrino asymmetry?

[Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009;
Saviano, Mirizzi, Pisanti, Serpico, Mangano, Miele, PRD 87 (2013) 073006]



[Archidiacono, Fornengo, Gariazzo, Giunti, Hannestad, Laveder, arXiv:1404.1794]

See also: { [Gariazzo, Giunti, Laveder, JCAP 1504 (2015) 023]
 [Bergstrom, Gonzalez-Garcia, Niro, Salvado, JHEP 1410 (2014) 104]

Without oscillation data: { [Giusarma, Di Valentino, Lattanzi, Melchiorri, Mena, arXiv:1403.4852]
 [Zhang, Li, Zhang, arXiv:1403.7028]
 [Dvorkin, Wyman, Rudd, Hu, arXiv:1403.8049]
 [Zhang, Li, Zhang, arXiv:1404.3598]

Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1 \text{ eV}$

Sterile neutrinos are thermalized ($\Delta N_{\text{eff}} = 1$) by active-sterile oscillations before neutrino decoupling

[Dolgov, Villante, NPB 679 (2004) 261]

Proposed mechanisms to avoid the tension:

- ▶ Large lepton asymmetry [Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009; Saviano et al., PRD 87 (2013) 073006; Hannestad, Hansen, Tram, JCAP 1304 (2013) 032]
- ▶ Enhanced background potential due to interactions in the sterile sector
[Hannestad, Hansen, Tram, PRL 112 (2014) 031802; Dasgupta, Kopp, PRL 112 (2014) 031803; Bringmann, Hasenkamp, Kersten, JCAP 1407 (2014) 042; Ko, Tang, PLB 739 (2014) 62; Archidiacono, Hannestad, Hansen, Tram, PRD 91 (2015) 065021; Mirizzi, Mangano, Pisanti, Saviano, PRD 90 (2014) 113009, PRD 91 (2015) 025019; Tang, arXiv:1501.00059]
- ▶ A larger cosmic expansion rate at the time of sterile neutrino production
[Rehagen, Gelmini JCAP 1406 (2014) 044]
- ▶ MeV dark matter annihilation [Ho, Scherrer, PRD 87 (2013) 065016]
- ▶ Invisible decay [Gariazzo, Giunti, Laveder, arXiv:1404.6160]
- ▶ Free primordial power spectrum of scalar fluctuations (Inflationary Freedom) [Gariazzo, Giunti, Laveder, JCAP 1504 (2015) 023]

Conclusions

$\nu_e \rightarrow \nu_\mu, \nu_\tau$ with $\Delta m_{\text{SOL}}^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2$ [SOL, KamLAND]

$\nu_\mu \rightarrow \nu_\tau$ with $\Delta m_{\text{ATM}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$ [ATM, K2K, MINOS]

$\sin^2 \vartheta_{12} \simeq 0.3$ $\sin^2 \vartheta_{23} \simeq 0.5$ $\sin^2 \vartheta_{13} \simeq 0.02$ [Daya Bay]

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\implies m_\nu \lesssim 1 \text{ eV}$

To Do

Theory: Why lepton mixing \neq quark mixing?

(Due to Majorana nature of ν 's?)

Why $0 < \sin^2 \vartheta_{13} \ll \sin^2 \vartheta_{12} < \sin^2 \vartheta_{23} \simeq 0.5$?

Exp.&Pheno.: Measure mass ordering and CP violation.

Find absolute mass scale and Majorana or Dirac.

Find if sterile neutrinos exist.

Conclusions on Light Sterile Neutrinos

- ▶ Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance:
 - ▶ Experimental data agree on Reactor $\bar{\nu}_e$ and Gallium ν_e disappearance.
 - ▶ Problem: total rates may have unknown systematic uncertainties.
 - ▶ Many promising projects to test unambiguously short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources.
 - ▶ Independent tests through effect of m_4 in β -decay and $\beta\beta_{0\nu}$ -decay.
- ▶ Short-Baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ LSND Signal:
 - ▶ Not seen by other SBL $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ experiments.
 - ▶ MiniBooNE experiment has been inconclusive.
 - ▶ Experiments with near detector are needed to check LSND signal!
 - ▶ Promising Fermilab program aimed at a conclusive solution of the mystery: a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-T600), all Liquid Argon Time Projection Chambers.
- ▶ Pragmatic 3+1 Fit is fine: moderate APP-DIS tension.
- ▶ 3+2 is not needed: same APP-DIS tension and no exp. CP violation.
- ▶ Cosmology:
 - ▶ Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1 \text{ eV}$.
 - ▶ Cosmological and oscillation data may be reconciled by a non-standard cosmological mechanism.