Sterile Neutrinos

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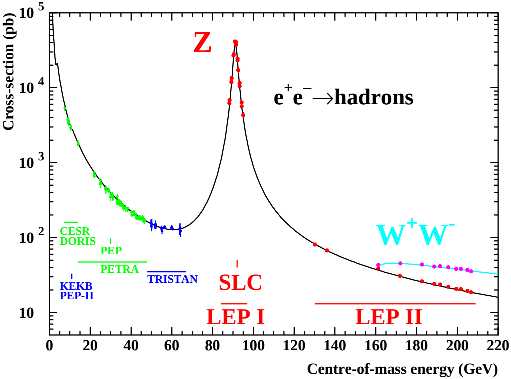
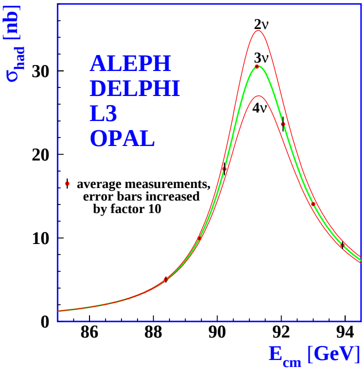
52nd Winter School of Theoretical Physics

Ladek Zdroj, Poland

14-21 February 2016

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

$$e^+ e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \; \nu_\mu \; \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots}$$

Sterile Neutrinos

[Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

- Sterile means no standard model interactions.
- Obviously no electromagnetic interactions as normal active neutrinos.
- Thus sterile means no standard weak interactions.
- But sterile neutrinos are not absolutely sterile:
  - Gravitational interactions (cosmology).
  - New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos.
- Observables in terrestrial experiments:
  - Disappearance of active neutrinos ν_e, ν_μ, ν_τ into sterile neutrinos ν_s due to active-sterile oscillations (neutral current deficit).
  - Oscillations (disappearance and transitions) of active neutrinos due to the new masses.
  - Kinematical effects of the new masses (e.g. β decay).
  - Contribution of the new masses to some process (e.g. neutrinoless double-β decay).

Extended Lepton Sector

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
$L_L = \begin{pmatrix} \nu_{\alpha L} \\ \ell_{\alpha L} = \alpha_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
$\ell_{\alpha R} = \alpha_R$	0	0	-2	-1
$\nu_{s_a R}$	0	0	0	0

$\alpha = e, \mu, \tau$ $\qquad$ $a = 1, \ldots, N_s$

- The right-handed sterile fields $\nu_{s_a R}$ belong to new physics beyond the Standard Model.

- Sterile neutrinos allow us to probe the new physics beyond the Standard Model.

General Dirac-Majorana Mass Lagrangian

$$\mathscr{L}_{\text{mass}} = \mathscr{L}^{\text{D}}_{\text{mass}} + \mathscr{L}^{L}_{\text{mass}} + \mathscr{L}^{R}_{\text{mass}}$$

$$\mathscr{L}^{\text{D}}_{\text{mass}} = - \sum_{\alpha=e,\mu,\tau} \sum_{a=1}^{N_S} \overline{\nu_{\alpha L}} M^{\text{D}}_{\alpha a} \nu_{s_a R} + \text{H.c.}$$

$$\mathscr{L}^{L}_{\text{mass}} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu^{T}_{\alpha L} \mathcal{C}^{\dagger} M^{L}_{\alpha\beta} \nu_{\beta L} + \text{H.c.}$$

$$\mathscr{L}^{R}_{\text{mass}} = \frac{1}{2} \sum_{a,b=1}^{N_S} \nu^{T}_{s_a R} \mathcal{C}^{\dagger} M^{R}_{ab} \nu_{s_b R} + \text{H.c.}$$

$$\nu^{(\text{F})}_{L} = \begin{pmatrix} \nu^{(\text{a})}_{L} \\ \nu^{(\text{s})c}_{R} \end{pmatrix} \qquad \nu^{(\text{a})}_{L} = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \qquad \nu^{(\text{s})}_{R} = \begin{pmatrix} \nu_{s_1 R} \\ \vdots \\ \nu_{s_{N_s} R} \end{pmatrix}$$

$$\mathscr{L}_{\text{mass}} = \frac{1}{2} \nu^{(\text{F})T}_{L} \mathcal{C}^{\dagger} M \nu^{(\text{F})}_{L} + \text{H.c.} \qquad M = \begin{pmatrix} M^{L} & M^{\text{D}} \\ M^{\text{D}T} & M^{R} \end{pmatrix}$$

- Charge conjugation matrix: $\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu, \quad \mathcal{C}^\dagger = \mathcal{C}^{-1}, \quad \mathcal{C}^T = -\mathcal{C}$
- Useful property: $\mathcal{C}(\gamma^5)^T\mathcal{C}^{-1} = \gamma^5$
- Charge conjugation: $\nu_R^{(\mathrm{s})c} = \mathcal{C}\overline{\nu_R^{(\mathrm{s})}}^T$
- Left and right-handed chiral projectors: $P_L \equiv \dfrac{1-\gamma^5}{2}, \quad P_R \equiv \dfrac{1+\gamma^5}{2}$

$$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L + P_R = 1\,, \quad P_L P_R = P_R P_L = 0$$

- $P_L\nu_L^{(\mathrm{a})} = \nu_L^{(\mathrm{a})}, \qquad P_R\nu_L^{(\mathrm{a})} = 0, \qquad P_L\nu_R^{(\mathrm{s})} = 0, \qquad P_R\nu_R^{(\mathrm{s})} = \nu_R^{(\mathrm{s})}$
- $\nu_R^{(\mathrm{s})c}$ is left-handed:

$$\begin{aligned} P_L\nu_R^{(\mathrm{s})c} &= P_L\mathcal{C}\overline{\nu_R^{(\mathrm{s})}}^T = \mathcal{C}P_L^T\overline{\nu_R^{(\mathrm{s})}}^T = \mathcal{C}(\overline{\nu_R^{(\mathrm{s})}}P_L)^T \\ &= \mathcal{C}(\nu_R^{(\mathrm{s})\dagger}\gamma^0 P_L)^T = \mathcal{C}(\nu_R^{(\mathrm{s})\dagger}P_R\gamma^0)^T = \mathcal{C}\overline{\nu_R^{(\mathrm{s})}}^T = \nu_R^{(\mathrm{s})c} \end{aligned}$$

$$P_R\nu_R^{(\mathrm{s})c} = 0$$

- $\mathscr{L}_{\text{mass}}$ has the structure of a Majorana mass Lagrangian

$$\mathscr{L}_{\text{mass}} = \frac{1}{2} \left(\nu_L^{(\mathsf{F})\,T} \mathcal{C}^\dagger M \nu_L^{(\mathsf{F})} - \overline{\nu_L^{(\mathsf{F})}} M^\dagger \mathcal{C} \overline{\nu_L^{(\mathsf{F})}}^{\,T} \right)$$

$$\left(\nu_L^{(\mathsf{F})\,T} \mathcal{C}^\dagger M \nu_L^{(\mathsf{F})} \right)^\dagger = \nu_L^{(\mathsf{F})\dagger} M^\dagger \mathcal{C} \nu_L^{(\mathsf{F})\dagger\,T} = \overline{\nu_L^{(\mathsf{F})}} \gamma^0 M^\dagger \mathcal{C} \nu_L^{(\mathsf{F})\dagger\,T}$$

$$= \overline{\nu_L^{(\mathsf{F})}} M^\dagger \mathcal{C} \mathcal{C}^{-1} \gamma^0 \mathcal{C} \nu_L^{(\mathsf{F})\dagger\,T} = -\overline{\nu_L^{(\mathsf{F})}} M^\dagger \mathcal{C} \gamma^{0\,T} \nu_L^{(\mathsf{F})\dagger\,T} = -\overline{\nu_L^{(\mathsf{F})}} M^\dagger \mathcal{C} \overline{\nu_L^{(\mathsf{F})}}^{\,T}$$

- $\nu_L^{(\mathsf{F})c} = \mathcal{C} \overline{\nu_L}^{(\mathsf{F})\,T}, \qquad \overline{\nu_L^{(\mathsf{F})c}} = -\nu_L^{(\mathsf{F})\,T} \mathcal{C}^\dagger$

$$\mathscr{L}_{\text{mass}} = -\frac{1}{2} \left(\overline{\nu_L^{(\mathsf{F})c}} M \nu_L^{(\mathsf{F})} + \overline{\nu_L^{(\mathsf{F})}} M^\dagger \nu_L^{(\mathsf{F})c} \right)$$

- In general, M is a complex symmetric matrix: $M = \begin{pmatrix} M^{L} & M^{\mathrm{D}} \\ M^{\mathrm{D}\,T} & M^{R} \end{pmatrix}$

$$\begin{aligned} \nu_L^{(\mathrm{F})T} \mathcal{C}^{\dagger} M \nu_L^{(\mathrm{F})} &= \left(\nu_L^{(\mathrm{F})T} \mathcal{C}^{\dagger} M \nu_L^{(\mathrm{F})} \right)^{T} \\ &= -\nu_L^{(\mathrm{F})T} M^{T} (\mathcal{C}^{\dagger})^{T} \nu_L^{(\mathrm{F})} \\ &= \nu_L^{(\mathrm{F})T} \mathcal{C}^{\dagger} M^{T} \nu_L^{(\mathrm{F})} \quad \Longrightarrow \quad M = M^{T} \end{aligned}$$

- M can be diagonalized with the unitary transformation $\nu_L^{(\mathrm{F})} = \mathscr{U} \nu_L^{(\mathrm{M})}$

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 R} \\ \vdots \\ \nu_{s_{N_s} R} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{eN} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu N} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau N} \\ U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} & \cdots & U_{s_1 N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{s_{N_s} 1} & U_{s_{N_s} 2} & U_{s_{N_s} 3} & U_{s_{N_s} 4} & \cdots & U_{s_{N_s} N} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \\ \nu_{4L} \\ \vdots \\ \nu_{NL} \end{pmatrix}$$

- $\mathscr{U}$ is a unitary $N \times N$ mixing matrix with $N = 3 + N_s$.

- Diagonalization: $\mathscr{U}^T M \mathscr{U} = \text{diag}(m_1, \ldots, m_N)$ with real and positive masses $m_1, \ldots, m_N$.
- Mass Lagrangian in the mass basis:

$$\begin{aligned}\mathscr{L}_{\text{mass}} &= \frac{1}{2}\sum_{k=1}^{N} m_k \left(\nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} - \overline{\nu_{kL}} \mathcal{C} \overline{\nu_{kL}}^T \right) \\ &= -\frac{1}{2}\sum_{k=1}^{N} m_k \left(\overline{\nu_{kL}^c} \nu_{kL} + \overline{\nu_{kL}} \nu_{kL}^c \right) \\ &= -\frac{1}{2}\sum_{k=1}^{N} m_k \overline{\nu_k} \nu_k\end{aligned}$$

- Massive Majorana neutrino fields: $\nu_k = \nu_{kL} + \nu_{kL}^c \qquad \nu_k = \nu_k^c$
- In the general case of active-sterile neutrino mixing the massive neutrinos are Majorana particles.
- However, it is not excluded that the mixing is such that there are pairs of Majorana neutrino fields with exactly the same mass which form Dirac neutrino fields.

Charged-Current Weak Interactions

- The physical effects of neutrino mixing appear in Weak Interactions.

- In the flavor basis where the mass matrix of the charged leptons is diagonal

$$\begin{aligned}\mathscr{L}_{\mathsf{CC}} &= -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \overline{\ell_{\alpha L}} \gamma^{\rho} \nu_{\alpha L} W_{\rho}^{\dagger} + \text{H.c.} \\ &= -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{N} \overline{\ell_{\alpha L}} \gamma^{\rho} \mathscr{U}_{\alpha k} \nu_{kL} W_{\rho}^{\dagger} + \text{H.c.}\end{aligned}$$

- In matrix form

$$\mathscr{L}_{\mathsf{CC}} = -\frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^{\rho} \nu_L^{(\mathrm{a})} W_{\rho}^{\dagger} + \text{H.c.} = -\frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^{\rho} U \nu_L^{(\mathrm{M})} W_{\rho}^{\dagger} + \text{H.c.}$$

- Effective rectangular $3 \times N$ mixing matrix:

$$\nu_L^{(\mathrm{a})} = U \nu_L^{(\mathrm{M})} \qquad\qquad U = \mathscr{U}\big|_{3\times N}$$

- Effective rectangular $3 \times N$ mixing matrix:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{eN} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu N} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau N} \end{pmatrix}$$

- The number of physical mixing parameters is smaller than the number necessary to parameterize the $N \times N$ unitary matrix $\mathscr{U}$.

- This is due to the arbitrariness of the mixing in the sterile sector, which does not affect weak interactions. Any linear combination of the sterile neutrinos is equivalent.

- The effective rectangular $3 \times N$ mixing matrix is not unitary:

$$UU^{\dagger} = \mathbf{1}_{3\times 3}, \qquad \text{but} \qquad U^{\dagger}U \neq \mathbf{1}_{N\times N}$$

- How many mixing parameters?

- A rectangular $3 \times N$ matrix depends on $6N$ real parameters, but

$$UU^{\dagger} = \mathbf{1}_{3\times 3} \quad \Longrightarrow \quad 9 \quad \text{constraints}$$

$$N_{\text{real parameters}} = 6N - 9 = 6\,(3 + N_s) - 9 = 9 + 6N_s$$

- But how many mixing angles and physical CP-violating phases?

- For example, we know that for $N_s = 0$ three phases can be eliminated by rephasing the charged lepton fields and we have

  3 mixing angles

  3 physical CP-violating phases (one Dirac and 2 Majorana)

- Standard parameterization of the mixing matrix in three-neutrino mixing:

$$U^{(3\nu)} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- The unitary $N \times N$ matrix $\mathscr{U}$ can be written as

$$\mathscr{U} = \text{diag}\left(e^{i\omega_e}, e^{i\omega_\mu}, e^{i\omega_\tau}, e^{i\omega_{s_1}}, \ldots, e^{i\omega_{s_{N_s}}}\right) \left[\prod_{a=1}^{N} \prod_{b=a+1}^{N} W^{ab}(\vartheta_{ab}, \delta_{ab})\right]$$

- Complex rotation in the $a - b$ plane:

$$\begin{aligned}\left[W^{ab}(\vartheta_{ab}, \delta_{ab})\right]_{rs} = \delta_{rs} & + (c_{ab} - 1)\left(\delta_{ra}\delta_{sa} + \delta_{rb}\delta_{sb}\right) \\ & + s_{ab}\left(e^{-i\delta_{ab}}\delta_{ra}\delta_{sb} - e^{i\delta_{ab}}\delta_{rb}\delta_{sa}\right)\end{aligned}$$

- Example:

$$W^{12}(\vartheta_{12}, \delta_{12}) = \begin{pmatrix} \cos\vartheta_{12} & \sin\vartheta_{12} e^{-i\delta_{12}} & 0 & 0 & \cdots & 0 \\ -\sin\vartheta_{12} e^{i\delta_{12}} & \cos\vartheta_{12} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

- The effective $3 \times N$ mixing matrix U is made of the first 3 rows of $\mathscr{U}$:
  Truncation of the phases $e^{i\omega_{s_1}}, \ldots, e^{i\omega_{s_{N_s}}}$
  Truncation of the complex rotations $W^{ab}(\vartheta_{ab}, \delta_{ab})$ with $b > a > 3$

- Effective rectangular $3 \times N$ mixing matrix:

$$U = \text{diag}\left(e^{i\omega_e}, e^{i\omega_\mu}, e^{i\omega_\tau}\right) \left[\prod_{a=1}^{3} \prod_{b=a+1}^{N} W^{ab}(\vartheta_{ab}, \delta_{ab}) \right]_{3\times N}$$

- The three phases ω_1, ω_2, ω_3 can be eliminated by rephasing the charged lepton fields.

$$\mathscr{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{N} \overline{\ell_{\alpha L}} \gamma^\rho U_{\alpha k} \nu_{kL} W_\rho^\dagger + \text{H.c.}$$

$$\ell_{\alpha L} \to e^{i\omega_\alpha} \ell_{\alpha L}$$

$$\mathscr{L}_{\text{CC}} \to -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{N} \overline{\ell_{\alpha L}} \gamma^\rho e^{-i\omega_\alpha} U_{\alpha k} \nu_{kL} W_\rho^\dagger + \text{H.c.}$$

- Physical effective rectangular $3 \times N$ mixing matrix:

$$U = \left[\prod_{a=1}^{3} \prod_{b=a+1}^{N} W^{ab}(\vartheta_{ab}, \delta_{ab}) \right]_{3\times N}$$

- How many complex rotations?

- For each value of $a = 1, 2, 3$ there are $N - a$ values of b:

$$\begin{aligned} N_{\text{complex rotations}} &= (N-1) + (N-2) + (N-3) \\ &= 3N - 6 = 3\,(3 + N_s) = 3 + 3N_s \end{aligned}$$

$3 + 3N_s$ mixing angles

$3 + 3N_s$ physical CP-violating phases

$N - 1 = 2 + N_s$ phases are Majorana

$1 + 2N_s$ phases are Dirac

- Note that in the case under consideration none of the phases of the complex rotations can be eliminated, because the Majorana mass Lagrangian

$$\mathscr{L}_{\text{mass}} = \frac{1}{2} \sum_{k=1}^{N} m_k \left(\nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} - \overline{\nu_{kL}} \mathcal{C} \overline{\nu_{kL}}^T \right)$$

  is not invariant under rephasing of the neutrino fields

$$\nu_{kL} \to e^{i\varphi_k} \nu_{kL}$$

- We distinguish the Majorana phases as those that could be eliminated by rephasing the neutrino fields when the Majorana neutrino masses can be neglected.

- Therefore the physical effects of the Majorana phases appear only in $|\Delta L| = 2$ processes that are induced by the Majorana mass Lagrangian.

- Why there are only $N - 1$ Majorana phases when there are N massive neutrino fields?

- In general only $3+N-1$ of the $3+N$ phases of the 3 charged lepton fields and N massive neutrino fields can be used to eliminate phases in the neutrino mixing matrix.

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{N} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$

$$\ell_{\alpha} \to e^{i\varphi_{\alpha}} \, \ell_{\alpha} \quad (\alpha = e, \mu, \tau) \qquad\qquad \nu_{k} \to e^{i\varphi_{k}} \, \nu_{k} \quad (k = 1, 2, 3)$$

$$j^{\rho\dagger}_{W,\mathrm{L}} \to 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{N} \overline{\ell_{\alpha L}} \, e^{-i\varphi_{\alpha}} \, \gamma^{\rho} \, U_{\alpha k} \, e^{i\varphi_{k}} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} \to 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{N} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_{\alpha}-\varphi_{1})}}_{3} \, \gamma^{\rho} \, U_{\alpha k} \, \underbrace{e^{i(\varphi_{k}-\varphi_{1})}}_{N-1} \, \nu_{kL}$$

- A common rephasing of the massive neutrino fields is equivalent to a common rephasing of the charged lepton fields, which can only eliminate an overall phase in $\mathrm{diag}\left(e^{i\omega_{e}}, e^{i\omega_{\mu}}, e^{i\omega_{\tau}}\right)$, which has already been eliminated.

- Convenient parameterization scheme:

$$U = \left[\left(\prod_{a=1}^{3} \prod_{b=4}^{N} W^{ab} \right) R^{23} W^{13} R^{12} \right]_{3\times N} \mathrm{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{N1}}\right)$$

- Real rotation in the $a-b$ plane: $R^{ab} = W^{ab}(\theta_{ab}, 0)$.
- In the product of $W^{ab}(\vartheta_{ab}, \delta_{ab})$ matrices one can eliminate an unphysical phase δ_{ab} for each value of the index $b = 4, \ldots, N$.
- For $N_s = 0$ we recover the standard parameterization in three-neutrino mixing:

$$\begin{aligned} U^{(3\nu)} &= \left[R^{23} W^{13} R^{12}\right]_{3\times 3} \mathrm{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}\right) \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} \end{aligned}$$

- It is convenient to choose the order of the real or complex rotations for each index $b \geq 4$ such that the rotations in the $3-b$, $2-b$ and $1-b$ planes are ordered from left to right.
- In this way, the first two lines, which are relevant for the study of the oscillations of the experimentally more accessible flavor neutrinos ν_e and ν_μ, are independent of the mixing angles and Dirac phases corresponding to the rotations in all the $3-b$ planes for $b \geq 4$.
- Moreover, the first line, which is relevant for the study of ν_e disappearance, is independent also of the mixing angles and Dirac phases corresponding to the rotations in the $2-b$ planes for $b \geq 3$.
- Example:

$$U = \left[W^{3N} R^{2N} W^{1N} \cdots W^{34} R^{24} W^{14} R^{23} W^{13} R^{12} \right]_{3\times N} \times \text{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{N1}}\right)$$

- Another example:

$$U = \left[W^{3N} \cdots W^{34} W^{2N} \cdots W^{24} R^{1N} \cdots R^{14} R^{23} W^{13} R^{12} \right]_{3\times N} \times \text{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{N1}}\right)$$

- $3+1$ mixing:

$$U = \left[W^{34} R^{24} W^{14} R^{23} W^{13} R^{12} \right]_{3\times4} \operatorname{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}, e^{i\lambda_{41}}\right)$$

$$= \begin{pmatrix} c_{12} c_{13} c_{14} & s_{12} c_{13} c_{14} & c_{14} s_{13} e^{-i\delta_{13}} & s_{14} e^{-i\delta_{14}} \\ \cdots & \cdots & \begin{matrix} c_{13} c_{24} s_{23} \\ -s_{13} s_{14} s_{24} e^{i(\delta_{14}-\delta_{13})} \end{matrix} & c_{14} s_{24} \\ \cdots & \cdots & \cdots & c_{14} c_{24} s_{34} e^{-i\delta_{34}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 & 0 \\ 0 & 0 & e^{i\lambda_{31}} & 0 \\ 0 & 0 & 0 & e^{i\lambda_{41}} \end{pmatrix}$$

- $3+2$ mixing:

$$U = \left[W^{35} R^{25} W^{15} W^{34} R^{24} W^{14} R^{23} W^{13} R^{12} \right]_{3\times5} \cdots$$

$$= \begin{pmatrix} c_{12} c_{13} c_{14} c_{15} & s_{12} c_{13} c_{14} c_{15} & c_{14} c_{15} s_{13} e^{-i\delta_{13}} & c_{15} s_{14} e^{-i\delta_{14}} & s_{15} e^{-i\delta_{15}} \\ \cdots & \cdots & \cdots & \begin{matrix} c_{14} c_{25} s_{24} \\ -s_{14} s_{15} s_{25} e^{i(\delta_{15}-\delta_{14})} \end{matrix} & c_{15} s_{25} \\ \cdots & \cdots & \cdots & \cdots & c_{15} c_{25} s_{35} e^{-i\delta_{35}} \end{pmatrix} \cdots$$

No GIM with Sterile Neutrinos

[Lee, Shrock, PRD 16 (1977) 1444; Schechter, Valle PRD 22 (1980) 2227]

- Neutrino Neutral-Current Weak Interaction Lagrangian:

$$\mathscr{L}_{\text{I}}^{(\text{NC})} = -\frac{g}{2\cos\vartheta_{\text{W}}} Z_\rho \overline{\nu_L^{(\text{a})}} \gamma^\rho \nu_L^{(\text{a})} = -\frac{g}{2\cos\vartheta_{\text{W}}} Z_\rho \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \gamma^\rho \nu_{\alpha L}$$

- Mixing with sterile neutrinos: $\nu_{\alpha L} = \sum_{k=1}^{3+N_s} U_{\alpha k} \nu_{kL}$

- No GIM: $\mathscr{L}_{\text{I}}^{(\text{NC})} = -\frac{g}{2\cos\vartheta_{\text{W}}} Z_\rho \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \overline{\nu_{jL}} \gamma^\rho \nu_{kL} \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k}$

- $\sum_{\alpha=e,\mu,\tau,s_1,\ldots} U_{\alpha j}^* U_{\alpha k} = \delta_{jk}$ but $\sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \neq \delta_{jk}$

Effect on Invisible Width of Z Boson?

- Amplitude of $Z \to \nu_j \bar{\nu}_k$ decay:

$$
\begin{aligned}
A(Z \to \nu_j \bar{\nu}_k) &= \langle \nu_j \bar{\nu}_k | - \int d^4x \, \mathscr{L}_{\mathrm{I}}^{(\mathrm{NC})}(x) | Z \rangle \\
&= \frac{g}{2 \cos \vartheta_{\mathrm{W}}} \langle \nu_j \bar{\nu}_k | \int d^4x \, \overline{\nu_{jL}}(x) \gamma^{\rho} \nu_{kL}(x) Z_{\rho}(x) | Z \rangle \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^{*} U_{\alpha k}
\end{aligned}
$$

- If $m_k \ll m_Z/2$ for all k's, the neutrino masses are negligible in all the matrix elements and we can approximate

$$
\frac{g}{2 \cos \vartheta_{\mathrm{W}}} \langle \nu_j \bar{\nu}_k | \int d^4x \, \overline{\nu_{jL}}(x) \, \gamma^{\rho} \, \nu_{kL}(x) \, Z_{\rho}(x) | Z \rangle \simeq A_{\mathrm{SM}}(Z \to \nu_{\ell} \bar{\nu}_{\ell})
$$

- $A_{\mathrm{SM}}(Z \to \nu_{\ell} \bar{\nu}_{\ell})$ is the Standard Model amplitude of Z decay into a massless neutrino-antineutrino pair of any flavor $\ell = e, \mu, \tau$
- $A(Z \to \nu_j \bar{\nu}_k) \simeq A_{\mathrm{SM}}(Z \to \nu_{\ell} \bar{\nu}_{\ell}) \displaystyle\sum_{\alpha=e,\mu,\tau} U_{\alpha j}^{*} \, U_{\alpha k}$
- $P(Z \to \nu \bar{\nu}) = \displaystyle\sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} |A(Z \to \nu_j \bar{\nu}_k)|^2$

- $P(Z \to \nu\bar{\nu}) \simeq P_{\text{SM}}(Z \to \nu_\ell \bar{\nu}_\ell) \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* \, U_{\alpha k} \right|^2$
- Effective number of neutrinos in Z decay:

$$N_\nu^{(Z)} = \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* \, U_{\alpha k} \right|^2$$

- Using the unitarity relation $\displaystyle\sum_{k=1}^{3+N_s} U_{\alpha k} \, U_{\beta k}^* = \delta_{\alpha\beta}$ we obtain

$$\begin{aligned} N_\nu^{(Z)} &= \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* \, U_{\alpha k} \sum_{\beta=e,\mu,\tau} U_{\beta j} \, U_{\beta k}^* \\ &= \sum_{\alpha=e,\mu,\tau} \sum_{\beta=e,\mu,\tau} \underbrace{\sum_{j=1}^{3+N_s} U_{\alpha j}^* \, U_{\beta j}}_{\delta_{\alpha\beta}} \underbrace{\sum_{k=1}^{3+N_s} U_{\alpha k} \, U_{\beta k}^*}_{\delta_{\alpha\beta}} = \sum_{\alpha=e,\mu,\tau} 1 = 3 \end{aligned}$$

- $\boxed{N_\nu^{(Z)} = 3}$ independently of the number of light sterile neutrinos!

Effect of Heavy Sterile Neutrinos

[Jarlskog, PLB 241 (1990) 579; Bilenky, Grimus, Neufeld, PLB 252 (1990) 119]

- $N_{\nu}^{(Z)} = \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^{*} \, U_{\alpha k} \right|^2 R_{jk}$ with

$$R_{jk} = \left(1 - \frac{m_j^2 + m_k^2}{2m_Z^2} - \frac{(m_j^2 - m_k^2)^2}{2m_Z^4} \right) \frac{\lambda(m_Z^2, m_j^2, m_k^2)}{m_Z^2} \theta(m_Z - m_j - m_k)$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

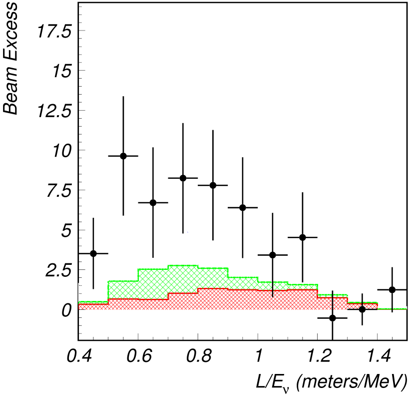
- $R_{jk} \leq 1 \quad \Longrightarrow \quad \boxed{N_{\nu}^{(Z)} \leq 3}$

Indications of SBL Oscillations Beyond 3ν

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $L \simeq 30\,\text{m}$ $20\,\text{MeV} \le E \le 60\,\text{MeV}$



- Well known source of $\bar{\nu}_\mu$:

$$\mu^+ \text{ at rest} \to e^+ + \nu_e + \bar{\nu}_\mu$$

- $\bar{\nu}_\mu \xrightarrow[L\simeq 30\,\text{m}]{} \bar{\nu}_e$
- Well known detection process of $\bar{\nu}_e$:

$$\bar{\nu}_e + p \to n + e^+$$

- But signal not seen by KARMEN with same method at $L \simeq 18\,\text{m}$

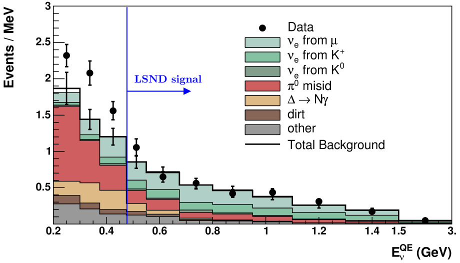
[PRD 65 (2002) 112001]

Nominal $\approx 3.8\sigma$ excess $\Delta m^2 \gtrsim 0.2\,\text{eV}^2$ $(\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

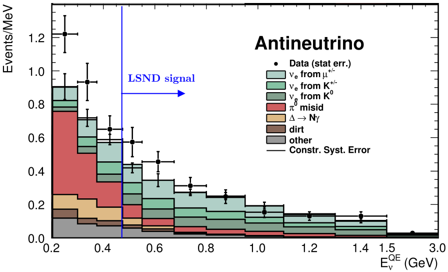
MiniBooNE

$L \simeq 541\,\mathrm{m}$ $200\,\mathrm{MeV} \leq E \lesssim 3\,\mathrm{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



- Purpose: check LSND signal.
- Different L and E.
- Similar L/E (oscillations).
- No money, no Near Detector.

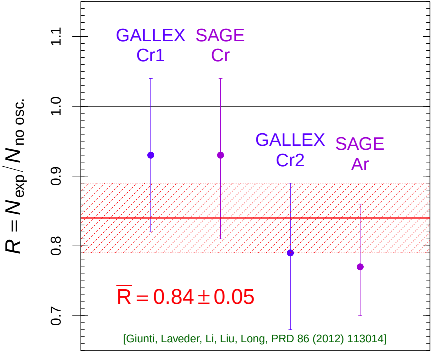
- LSND signal: $E > 475\,\mathrm{MeV}$.
- Agreement with LSND signal?
- CP violation?
- Low-energy anomaly!

Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$



[Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

$\bar{\nu}_e \to \bar{\nu}_e$ $\quad E \sim 0.7\,\text{MeV}$

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$

$\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

Nominal $\approx 2.9\sigma$ anomaly

$\Delta m^2 \gtrsim 1\,\text{eV}^2 \quad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807]

[Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344;
MPLA 22 (2007) 2499; PRD 78 (2008) 073009;
PRC 83 (2011) 065504]

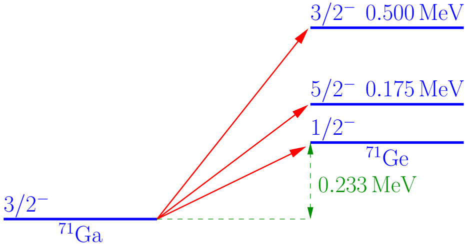
[Mention et al, PRD 83 (2011) 073006]

- ${}^{3}\text{He} + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + {}^{3}\text{H}$ cross section measurement [Frekers et al., PLB 706 (2011) 134]
- $E_{\text{th}}(\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-) = 233.5 \pm 1.2\,\text{keV}$ [Frekers et al., PLB 722 (2013) 233]

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\mathrm{G.S.}}$ from $T_{1/2}({}^{71}\mathrm{Ge}) = 11.43 \pm 0.03\,\mathrm{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) = 55.3 \times 10^{-46}\,\mathrm{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\mathrm{Cr}) = \sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) \left(1 + 0.669\,\frac{\mathrm{BGT}_{175}}{\mathrm{BGT}_{\mathrm{G.S.}}} + 0.220\,\frac{\mathrm{BGT}_{500}}{\mathrm{BGT}_{\mathrm{G.S.}}}\right)$

- Contribution of Excited States only 5%!

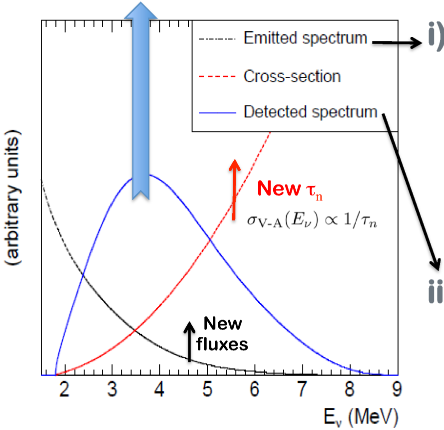
		$\frac{\mathrm{BGT}_{175}}{\mathrm{BGT}_{\mathrm{G.S.}}}$	$\frac{\mathrm{BGT}_{500}}{\mathrm{BGT}_{\mathrm{G.S.}}}$
Krofcheck et al. PRL 55 (1985) 1051	${}^{71}\mathrm{Ga}(p,n){}^{71}\mathrm{Ge}$	< 0.056	0.126 ± 0.023
Haxton PLB 431 (1998) 110	Shell Model	0.19 ± 0.18	
Frekers et al. PLB 706 (2011) 134	${}^{71}\mathrm{Ga}({}^{3}\mathrm{He},{}^{3}\mathrm{H}){}^{71}\mathrm{Ge}$	0.039 ± 0.030	0.202 ± 0.016

- Haxton: [Haxton, PLB 431 (1998) 110]

  "a sophisticated shell model calculation is performed ... for the transition to the first excited state in ^{71}Ge. The calculation predicts destructive interference between the (p, n) spin and spin-tensor matrix elements"
- Does Haxton argument apply also to $({}^{3}\mathrm{He}, {}^{3}\mathrm{H})$ measurements?
- 2.7σ discrepancy of $\mathrm{BGT}_{500}/\mathrm{BGT}_{\mathrm{G.S.}}$ measurements!
- Anyhow, new ${}^{71}\mathrm{Ga}({}^{3}\mathrm{He},{}^{3}\mathrm{H}){}^{71}\mathrm{Ge}$ data support Gallium Anomaly!

New Reactor $\bar{\nu}_e$ Fluxes

Increased prediction of detected flux by 6.5%



i) Neutrino Emission:

- **Improved reactor neutrino spectra → <u>+3.5%</u>**
- **Accounting for long-lived isotopes in reactors → <u>+1%</u>**

ii) Neutrino Detection:

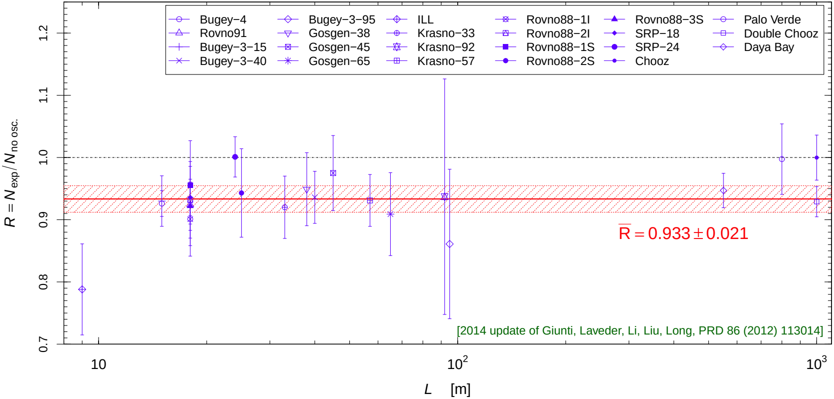
- **Reevaluation of σ_{IBD} → <u>+1.5%</u> (evolution of the neutron life time)**
- **Reanalysis of all SBL experiments**

[T. Lasserre, TAUP 2013]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006; update in White Paper, arXiv:1204.5379]

New reactor $\bar{\nu}_e$ fluxes [Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]



[2014 update of Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

$\bar{\nu}_e \to \bar{\nu}_e$ $\quad L \sim 10 - 100\,\text{m}$ $\quad E \sim 4\,\text{MeV}$

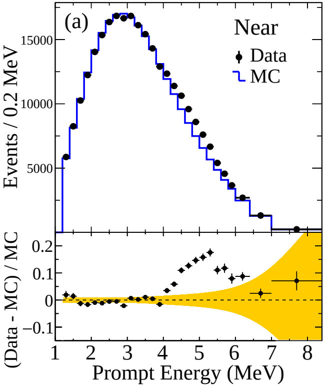
Nominal $\approx 3.1\sigma$ deficit $\quad \Delta m^2 \gtrsim 0.5\,\text{eV}^2$ $\quad (\gg \Delta m^2_{\text{A}} \gg \Delta m^2_{\text{S}})$

[see also: Sinev, arXiv:1103.2452; Ciuffoli, Evslin, Li, JHEP 12 (2012) 110; Zhang, Qian, Vogel, PRD 87 (2013) 073018; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Ivanov et al, PRC 88 (2013) 055501]

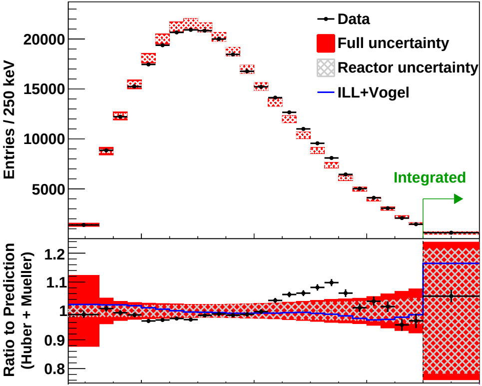
Problem: unknown $\bar{\nu}_e$ flux uncertainties?

[Hayes, Friar, Garvey, Jonkmans, PRL 112 (2014) 202501; Dwyer, Langford, PRL 114 (2015) 012502]

5 MeV Bump



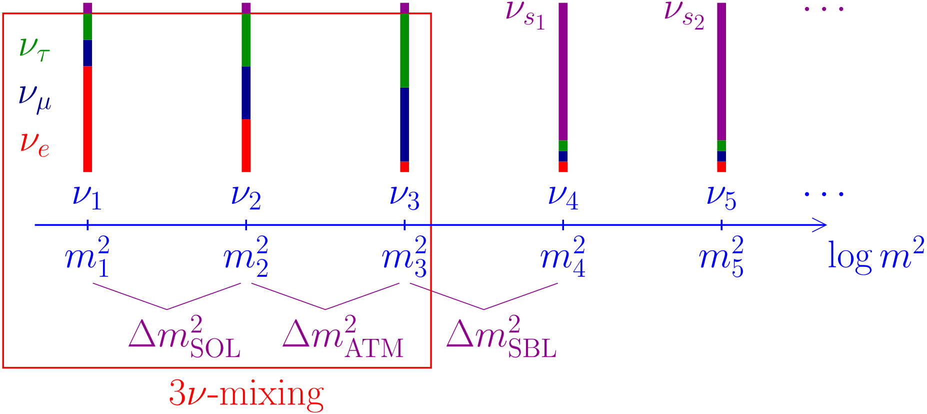
[RENO, arXiv:1511.05849]



[Daya Bay, arXiv:1508.04233]

- Local problem with $\sim 3\%$ effect on total flux.
- It is an excess!
- It occurs both for the new high Muller-Huber fluxes and the old low Schreckenbach-Vogel fluxes.
- Real problem: apparent incompatibility of the bump with the β spectra from ^{235}U and ^{239}Pu measured by Schreckenbach et al. at ILL in 1982-1985.

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

- Here I consider sterile neutrinos with mass scale $\sim 1\,\mathrm{eV}$ in light of short-baseline Reactor Anomaly, Gallium Anomaly, LSND.

- Other possibilities (not incompatible):
  - Very light sterile neutrinos with mass scale $\ll 1\,\mathrm{eV}$: important for solar neutrino phenomenology

    [de Holanda, Smirnov, PRD 69 (2004) 113002; PRD 83 (2011) 113011]

    [Das, Pulido, Picariello, PRD 79 (2009) 073010]

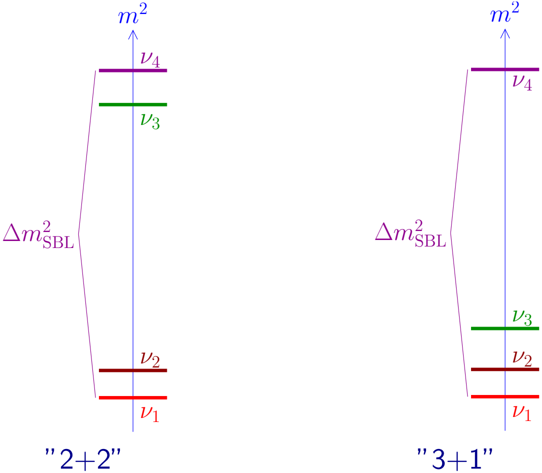
    Recent Daya Bay constraints for $10^{-3} \lesssim \Delta m^2 \lesssim 10^{-1}\,\mathrm{eV}^2$ [PRL 113 (2014) 141802]

  - Heavy sterile neutrinos with mass scale $\gg 1\,\mathrm{eV}$: could be Warm Dark Matter

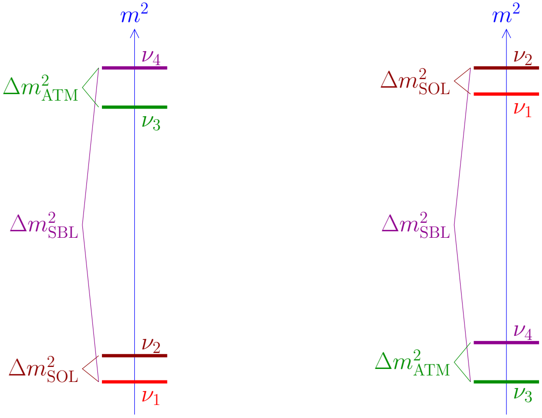
    [Asaka, Blanchet, Shaposhnikov, PLB 631 (2005) 151; Asaka, Shaposhnikov, PLB 620 (2005) 17; Asaka, Shaposhnikov, Kusenko, PLB 638 (2006) 401; Asaka, Laine, Shaposhnikov, JHEP 0606 (2006) 053, JHEP 0701 (2007) 091]

    [Reviews: Kusenko, Phys. Rept. 481 (2009) 1; Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191; Boyarsky, Iakubovskyi, Ruchayskiy, Phys. Dark Univ. 1 (2012) 136; Drewes, IJMPE, 22 (2013) 1330019]

Four-Neutrino Schemes: 2+2 and 3+1

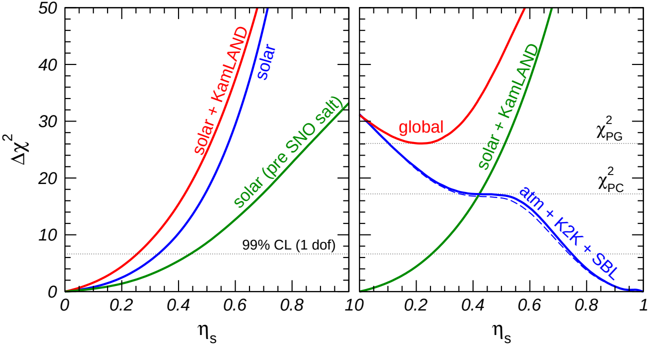


2+2 Four-Neutrino Schemes



"normal" "inverted"

2+2 Schemes are strongly disfavored by solar and atmospheric data



matter effects + SNO NC

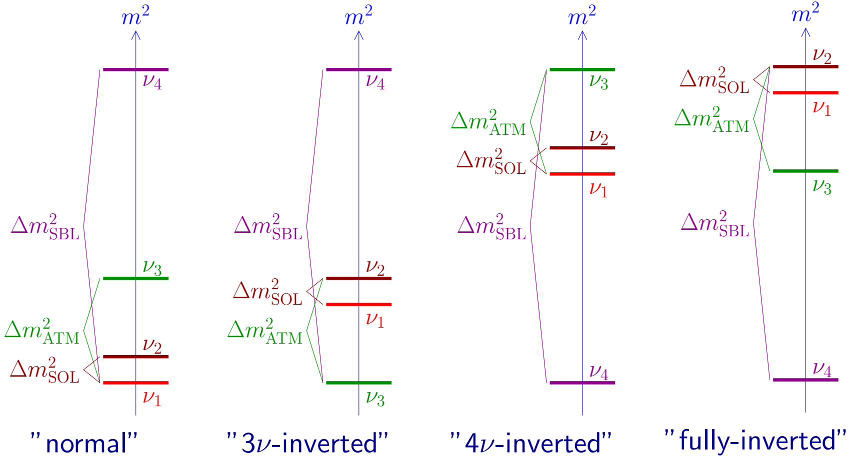
matter effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2$$
$$1 - \eta_s = |U_{s3}|^2 + |U_{s4}|^2$$

$$99\%\ \text{CL:} \quad \begin{cases} \eta_s < 0.25 & \text{(solar + KamLAND)} \\ \eta_s > 0.75 & \text{(atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122, arXiv:hep-ph/0405172]

3+1 Four-Neutrino Schemes



Perturbation of 3-ν Mixing

$$|U_{e4}|^2 \ll 1 \qquad |U_{\mu4}|^2 \ll 1 \qquad |U_{\tau4}|^2 \ll 1 \qquad |U_{s4}|^2 \simeq 1$$

Effective SBL Oscillation Probabilities

- General Bilenky formula of the probability of $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ oscillations:

$$P_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} = \delta_{\alpha\beta} - 4 \sum_{k \neq p} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2\right) \sin^2 \Delta_{kp}$$

$$+8 \sum_{\substack{j>k \\ j,k\neq p}} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{kp} \sin \Delta_{jp} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\Delta_{kp} = \frac{\Delta m^2_{kp} L}{4E} \qquad \eta_{\alpha\beta jk} = \arg\left[U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right]$$

- In SBL experiments $\Delta_{21} \ll \Delta_{31} \ll 1$. Choosing $p = 1$, we obtain

$$P^{(\text{SBL})}_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} \simeq \delta_{\alpha\beta} - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2\right) \sin^2 \Delta_{k1}$$

$$+8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

Survival Probabilities

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\alpha}} \simeq 1 - 4\sum_{k=4}^{N} |U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \sin^2\Delta_{k1}$$

$$+8\sum_{k=4}^{N}\sum_{j=k+1}^{N} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin\Delta_{j1} \sin\Delta_{k1} \cos\Delta_{jk}$$

Effective amplitude of $\overset{(-)}{\nu}_{\alpha}$ disappearance due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

$$|U_{\alpha k}|^2 \ll 1 \qquad (\alpha = e, \mu, \tau; \quad k = 4, \ldots, N)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\alpha}} \simeq 1 - \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2\Delta_{k1}$$

Appearance Probabilities $(\alpha \neq \beta)$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 |U_{\beta k}|^2 \sin^2 \Delta_{k1}$$
$$+8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

Effective amplitude of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ transitions due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\beta} = 4 |U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\beta} \sin^2 \Delta_{k1}$$
$$+2 \sum_{k=4}^{N} \sum_{j=k+1}^{N} \sin 2\vartheta^{(k)}_{\alpha\beta} \sin 2\vartheta^{(j)}_{\alpha\beta} \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2\left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing: $|U_{e4}|^2 \ll 1\,,\ |U_{\mu 4}|^2 \ll 1\,,\ |U_{\tau 4}|^2 \ll 1\,,\ |U_{s4}|^2 \simeq 1$

$$U = \left(\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \\ U_{s1} & U_{s2} & U_{s3} \end{matrix} \; \underset{\substack{\uparrow \\ \text{SBL}}}{\boxed{\begin{matrix} U_{e4} \\ U_{\mu 4} \\ U_{\tau 4} \\ U_{s4} \end{matrix}}}\right)$$

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases
- But CP violation is not observable in current SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea, Kelly, Kobach, PRD 91 (2015) 053005; Klop, Palazzo, PRD 91 (2015) 073017; Berryman, de Gouvea, Kelly, Kobach, PRD 92 (2015) 073012, Palazzo, arXiv:1509.03148] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, Giunti, PRD 87, 113004 (2013) 113004]

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\Delta_{kj} = \Delta m^2_{kj} L/4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$\begin{aligned} P^{\text{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = {} & 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\Delta_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\Delta_{51} \\ & + 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin\Delta_{41} \sin\Delta_{51} \cos(\Delta_{54} \overset{(+)}{-} \eta) \end{aligned}$$

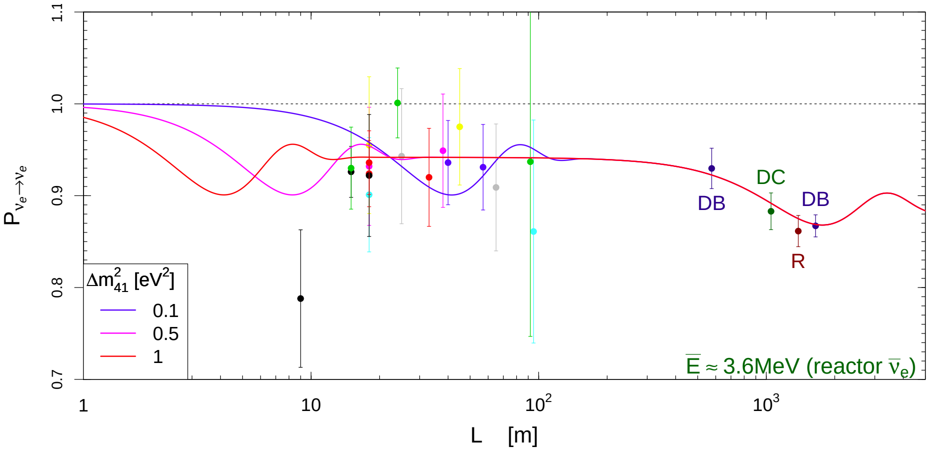
$$\begin{aligned} P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = {} & 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2\Delta_{41} + |U_{\alpha 5}|^2 \sin^2\Delta_{51}) \\ & - 4|U_{\alpha 4}|^2|U_{\alpha 5}|^2 \sin^2\Delta_{54} \end{aligned}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Archidiacono et al, PRD 86 (2012) 065028; Jacques, Krauss, Lunardini, PRD 87 (2013) 083515; Conrad et al, AHEP 2013 (2013) 163897; Archidiacono et al, PRD 87 (2013) 125034; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008; Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

- Good: CP violation
- Bad: Two massive sterile neutrinos at the eV scale!

  4 more parameters: $\underbrace{\Delta m^2_{41}, |U_{e4}|^2, |U_{\mu 4}|^2}_{3+1}, \Delta m^2_{51}, |U_{e5}|^2, |U_{\mu 5}|^2, \eta$

Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance



$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) = \sin^2 2\vartheta_{14}$$

$$P^{\text{SBL}}_{\nu_e\to\nu_e} \simeq 1 - \sin^2 2\vartheta_{14} \, \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$P^{\text{LBL}}_{\nu_e\to\nu_e} \simeq 1 - \frac{1}{2} \sin^2 2\vartheta_{14} - \cos^4\vartheta_{14} \, \sin^2 2\vartheta_{13} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Solar bound on $|U_{e4}|^2$

[Giunti, Li, PRD 80 (2009) 113007; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301]

$$P^{\text{SOL}}_{\nu_e \to \nu_e} \simeq \left(1 - \sum_{k \geq 3} |U_{ek}|^2 \right)^2 P^{\text{SOL},2\nu}_{\nu_e \to \nu_e} + \sum_{k \geq 3} |U_{ek}|^4$$

$$P^{\text{SOL}}_{\nu_e \to \nu_s} \simeq \left(1 - \sum_{k \geq 3} |U_{ek}|^2 \right) \left(1 - \sum_{k \geq 3} |U_{sk}|^2 \right) P^{\text{SOL},2\nu}_{\nu_e \to \nu_s} + \sum_{k \geq 3} |U_{ek}|^2 |U_{sk}|^2$$

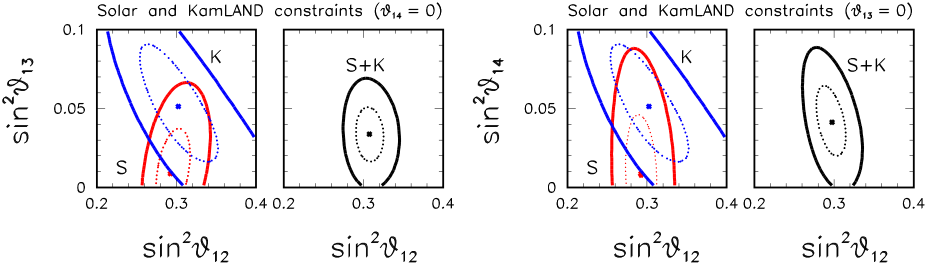
3+1 with simplifying assumptions: $U_{\mu 4} = U_{\tau 4} = 0$, no CP violation

$$\begin{array}{llll} U_{e1} = c_{12} c_{13} c_{14} & U_{e2} = s_{12} c_{13} c_{14} & U_{e3} = s_{13} c_{14} & U_{e4} = s_{14} \\ U_{s1} = -c_{12} c_{13} s_{14} & U_{s2} = -s_{12} c_{13} s_{14} & U_{s3} = -s_{13} s_{14} & U_{s4} = c_{14} \end{array}$$

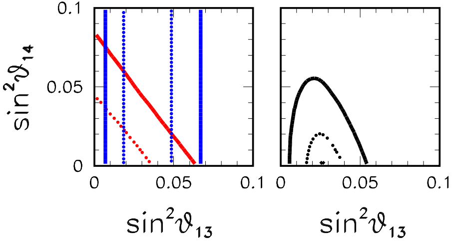
$$P^{\text{SOL}}_{\nu_e \to \nu_e} \simeq c_{13}^4 c_{14}^4 P^{\text{SOL},2\nu}_{\nu_e \to \nu_e} + s_{13}^4 c_{14}^4 + s_{14}^4$$

$$P^{\text{SOL}}_{\nu_e \to \nu_s} \simeq c_{14}^2 s_{14}^2 \left(c_{13}^4 P^{\text{SOL},2\nu}_{\nu_e \to \nu_s} + s_{13}^4 + 1 \right)$$

$$\begin{aligned} V &= c_{13}^2 c_{14}^2 V_{\text{CC}} - c_{13}^2 s_{14}^2 V_{\text{NC}} \\ &= \left(|U_{e1}|^2 + |U_{e2}|^2 \right) V_{\text{CC}} - \left(|U_{s1}|^2 + |U_{s2}|^2 \right) V_{\text{NC}} \end{aligned}$$



[Palazzo, PRD 83 (2011) 113013]



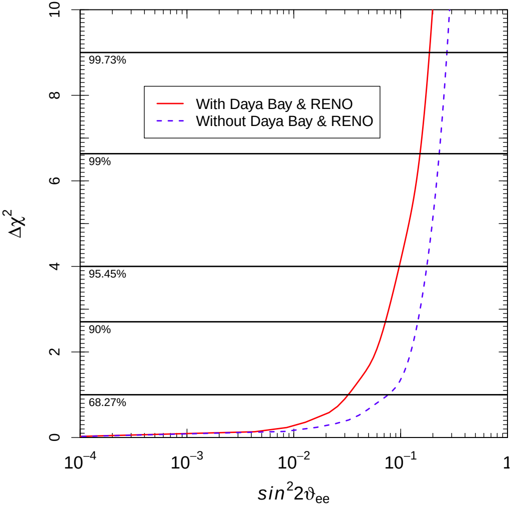
[Palazzo, PRD 85 (2012) 077301]

Daya Bay and RENO

$$\sin^2\vartheta_{13} = 0.025 \pm 0.004$$

$$|U_{e4}|^2 = \sin^2\vartheta_{14} \lesssim 0.02\,(1\sigma)$$

Fit of solar and KamLAND data with Daya Bay and RENO constraint $\sin^2\vartheta_{13} = 0.025 \pm 0.004$ and free $|U_{\mu 4}|$ and $|U_{\tau 4}|$ (neglecting small CP violation effects)



[Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

Tritium Beta-Decay

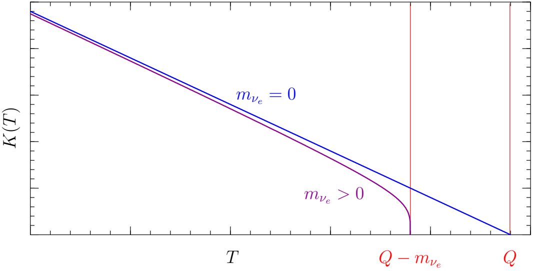
$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{\mathrm{d}\Gamma/\mathrm{d}T}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\text{eV} \quad (95\%\ \text{C.L.})}$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

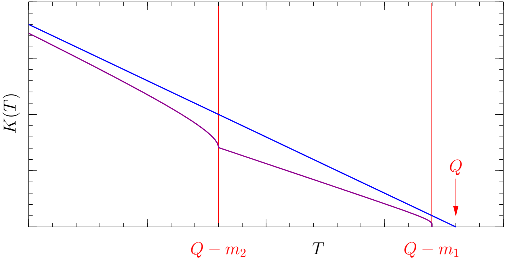
future: KATRIN

[`www.katrin.kit.edu`]

start data taking 2016?

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

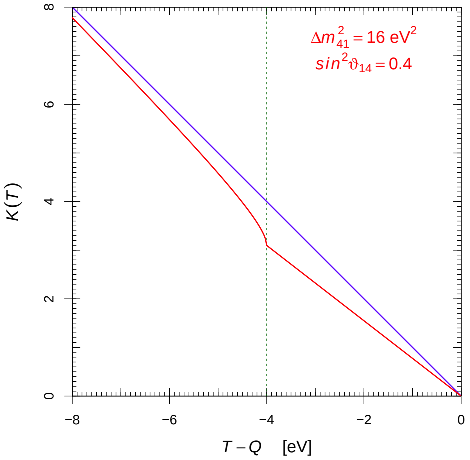
if experiment is not sensitive to masses ($m_k \ll Q-T$)

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

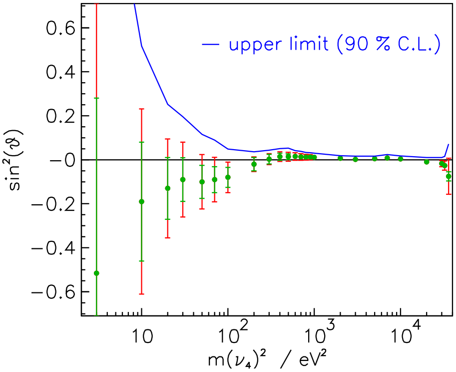
$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

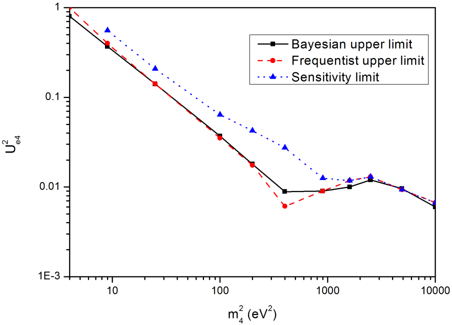
3+1 Mixing



$$m_4 \gg m_1, m_2, m_3 \implies \Delta m^2_{41} \equiv m^2_4 - m^2_1 \simeq m^2_4$$

Mainz and Troitsk Limit on m_4^2

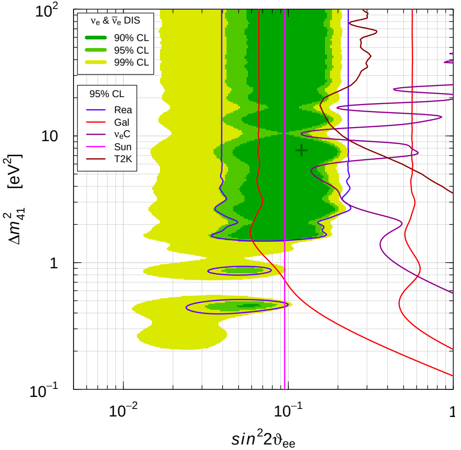




[Kraus, Singer, Valerius, Weinheimer, EPJC 73 (2013) 2323]

[Belesev et al, JPG 41 (2014) 015001]

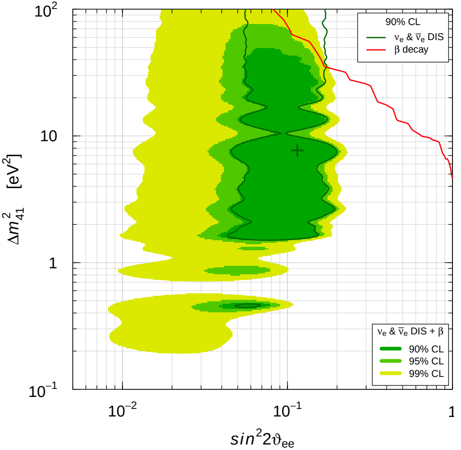
Global ν_e and $\bar{\nu}_e$ Disappearance



KARMEN + LSND $\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$
[Conrad, Shaevitz, PRD 85 (2012) 013017]
[Giunti, Laveder, PLB 706 (2011) 200]

solar ν_e + KamLAND $\bar{\nu}_e$ + ϑ_{13}
[Giunti, Li, PRD 80 (2009) 113007]
[Palazzo, PRD 83 (2011) 113013; PRD 85 (2012) 077301]
[Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

T2K Near Detector ν_e disappearance
[T2K, PRD 91 (2015) 051102]

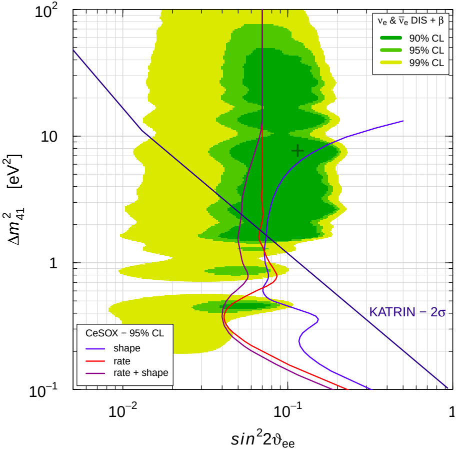


Mainz + Troitsk Tritium β decay
[Mainz, EPJC 73 (2013) 2323]
[Troitsk, JETPL 97 (2013) 67; JPG 41 (2014) 015001]

No Osc. excluded at 2.9σ
$(\Delta\chi^2/\text{NDF} = 11.2/2)$

$$7\,\text{cm} \lesssim \frac{L^{\text{osc}}_{41}}{E\,[\text{MeV}]} \lesssim 2\,\text{m} \qquad (2\sigma)$$

Near-Future Experiments



CeSOX (BOREXINO, Italy)

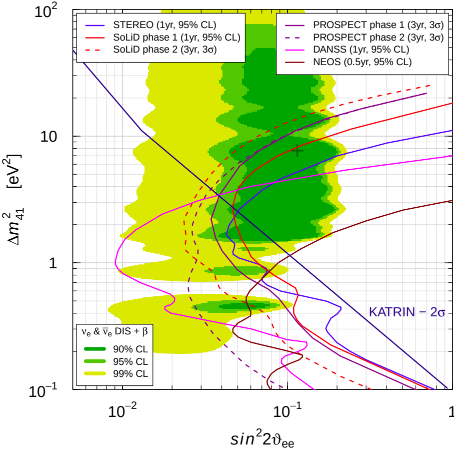
^{144}Ce – 100 kCi [Vivier@TAUP2015]

rate: 1% normalization uncertainty

8.5 m from detector center

KATRIN (Germany)

Tritium β decay [Mertens@TAUP2015]



STEREO (France) $L \simeq$ 8-12m [Sanchez@EPSHEP2015]

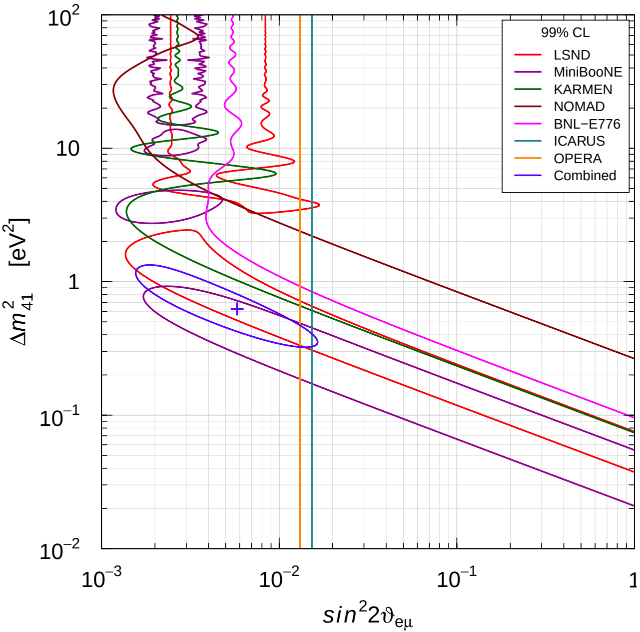
SoLid (Belgium) $L \simeq$ 5-8m [Yermia@TAUP2015]

PROSPECT (USA) $L \simeq$ 7-12m [Heeger@TAUP2015]

DANSS (Russia) $L \simeq$ 10-12m [arXiv:1412.0817]

NEOS (Korea) $L \simeq$ 25m [Oh@WIN2015]

$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ Appearance



3+1: Appearance vs Disappearance

- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

- Upper bounds on ν_e and ν_μ disappearance $\Rightarrow$ strong limit on $\nu_\mu \to \nu_e$

  [Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, Giunti, Grimus, EPJC 1 (1998) 247]

- Similar constraint in 3+2, 3+3, ..., 3+N_s! [Giunti, Zavanin, MPLA 31 (2015) 1650003]

$$\sin^2 2\vartheta_{\alpha\alpha}^{(k)} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

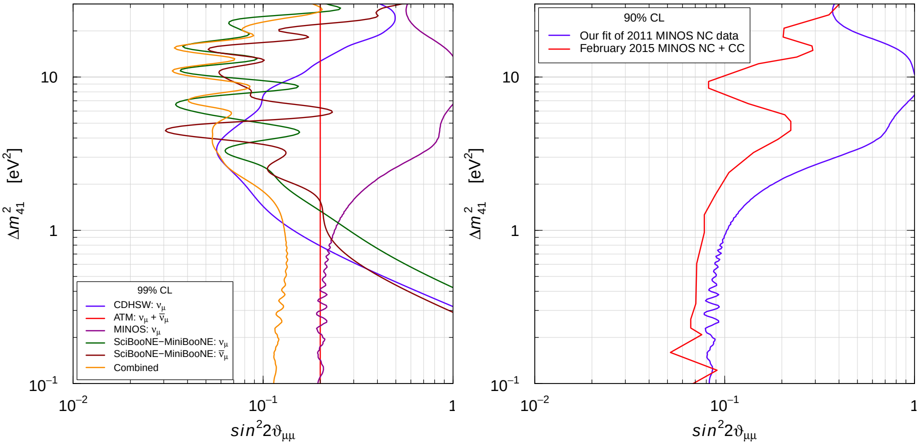
$$\sin^2 2\vartheta_{\alpha\beta}^{(k)} = 4|U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$\boxed{\sin^2 2\vartheta_{\alpha\beta}^{(k)} \simeq \frac{1}{4} \sin^2 2\vartheta_{\alpha\alpha}^{(k)} \sin^2 2\vartheta_{\beta\beta}^{(k)}}$$

$$\left.\begin{array}{l} \sin^2 2\vartheta_{ee}^{(k)} \ll 1 \\ \sin^2 2\vartheta_{\mu\mu}^{(k)} \ll 1 \end{array}\right\} \quad \Rightarrow \quad \sin^2 2\vartheta_{e\mu}^{(k)} \quad \text{is quadratically suppressed}$$

on the other hand, observation of $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ transitions due to Δm_{k1}^2 imply that the corresponding $\overset{(-)}{\nu}_{\alpha}$ and $\overset{(-)}{\nu}_{\beta}$ disappearances must be observed

ν_μ and $\bar{\nu}_\mu$ Disappearance

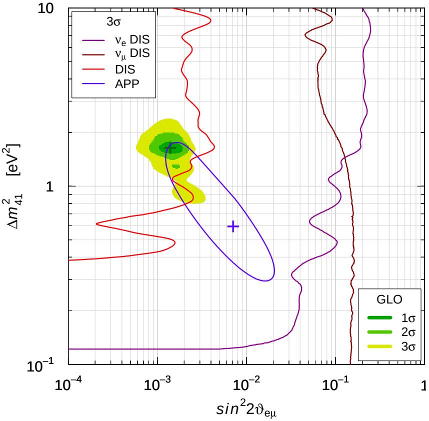


MINOS: $L_{\text{decay}} \simeq 0.675\,\text{km}$ $L_{\text{ND}} \simeq 1.04\,\text{km}$ $L_{\text{FD}} \simeq 735\,\text{km}$

$$E \approx 4\,\text{GeV} \Longrightarrow \frac{L_{\text{osc}}}{L_{\text{ND}}} \approx \frac{10}{\Delta m^2_{41}\,[\text{eV}^2]}$$

Global 3+1 Fit

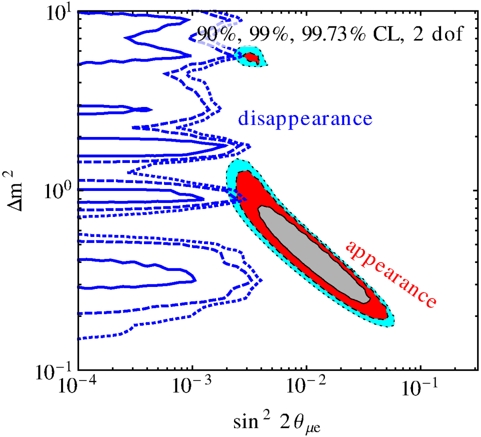
Our Fit



GoF = 5% PGoF = 0.1%

[Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001]

Kopp, Machado, Maltoni, Schwetz



GoF = 19% PGoF = 0.01%

[Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

There is no globally allowed region in this paper!

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

$$N_{\text{D}} = \text{Number of Data} \qquad N_{\text{P}} = \text{Number of Fitted Parameters}$$

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF} \qquad \text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, dz \qquad p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

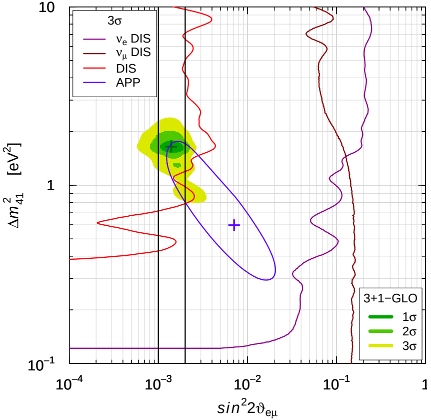
- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, dz$

Global 3+1 Fit

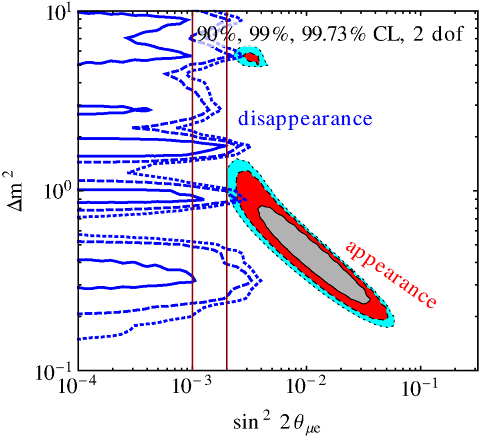
Our Fit



GoF = 5% PGoF = 0.1%

[Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001]

Kopp, Machado, Maltoni, Schwetz



GoF = 19% PGoF = 0.01%

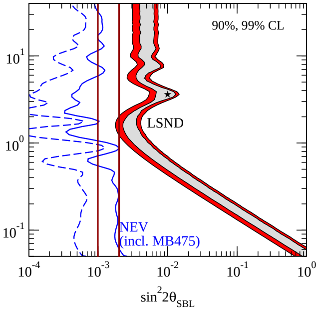
[Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

Different LSND Treatments

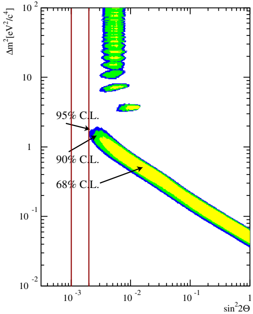
only LSND data from $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decay at rest

[Kopp, Machado, Maltoni, Schwetz]

[Maltoni, Schwetz, PRD 76 (2007) 093005]

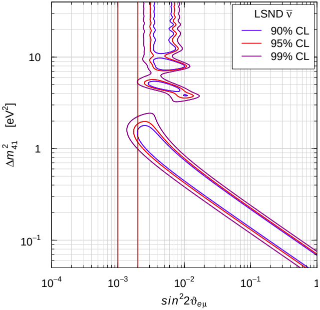


[Church, Eitel, Mills, Steidl, PRD 66 (2002) 013001]

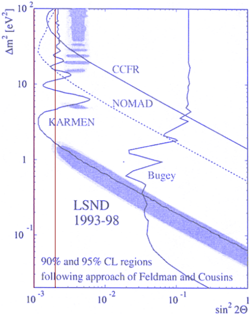


[Our Fit]

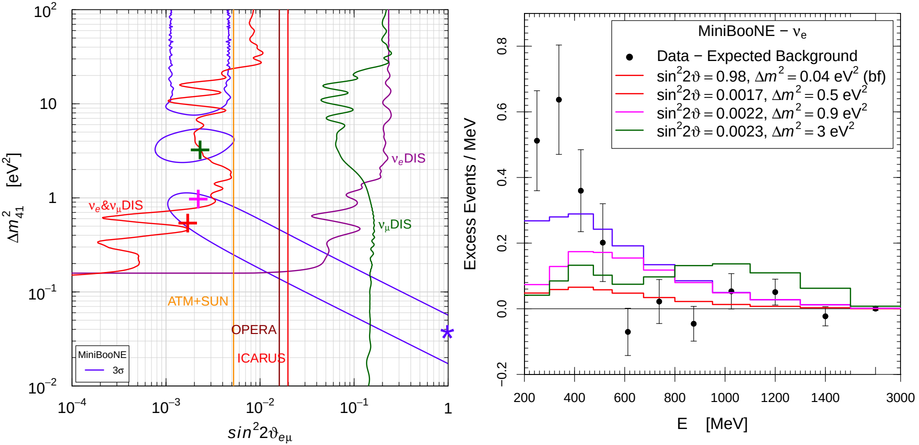
[improvement of Giunti, Laveder, PRD 82 (2010) 093016]



[Church (LSND), NPA 663 (2000) 799]



MiniBooNE Low-Energy Excess?



- No fit of low-energy excess for realistic $\sin^2 2\vartheta_{e\mu} \lesssim 3 \times 10^{-3}$
- MB low-energy excess is the main cause of bad APP-DIS PGoF $= 0.1\%$
- Pragmatic Approach: discard the Low-Energy Excess because it is very likely not due to oscillations

[Giunti, Laveder, Li, Long, PRD 88 (2013) 073008]

Neutrino energy reconstruction problem?

[Martini, Ericson, Chanfray, PRD 85 (2012) 093012; PRD 87 (2013) 013009]

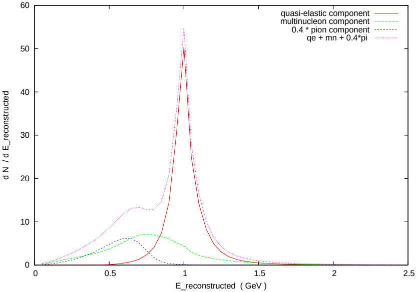
- Effect due to multinucleon interactions whose signal is indistinguishable from that due to quasielastic charged-current scattering

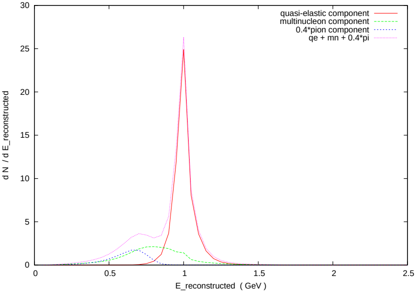
$$\nu_e + n \to p + e^- \qquad \bar{\nu}_e + p \to n + e^+$$

- In the MiniBooNE analysis the reconstructed neutrino energy is $(E_{\mathrm{B}} \simeq 25\,\mathrm{MeV})$

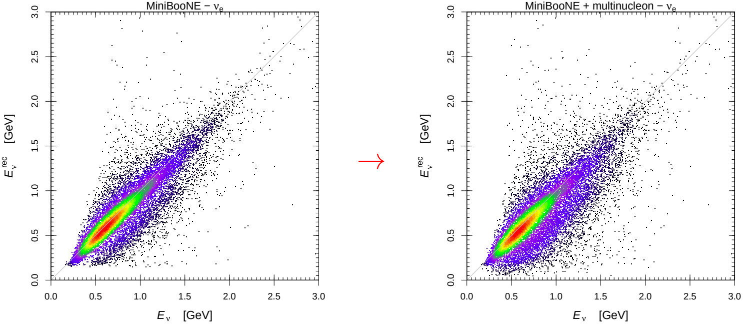
$$E_\nu^{\mathrm{QE}} = \frac{2\left(M_{\mathrm{i}} - E_{\mathrm{B}}\right) E_e - \left(m_e^2 - 2 M_{\mathrm{i}} E_{\mathrm{B}} + E_{\mathrm{B}}^2 + \Delta M_{\mathrm{if}}^2\right)}{2\left(M_{\mathrm{i}} - E_{\mathrm{B}} - E_e + p_e \cos\theta_e\right)}$$

- The MiniBooNE collaboration took into account:
  - Fermi motion of the initial nucleon
  - Charged-current single charged pion production events in which the pion is not observed (e.g. $\nu_e + n \to \Delta^+ + e^- \to n + \pi^+ + e^-$ with π^+ absorbed by a nucleus)

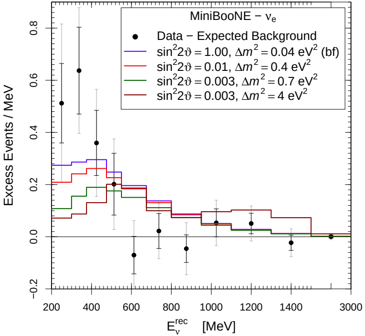




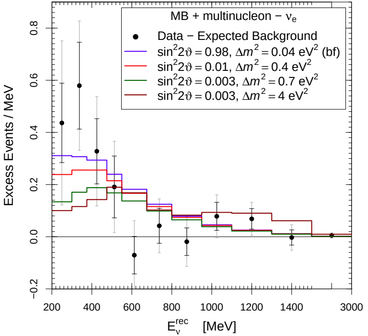
MiniBooNE $\nu_\mu \to \nu_e$ full transmutation Monte Carlo events

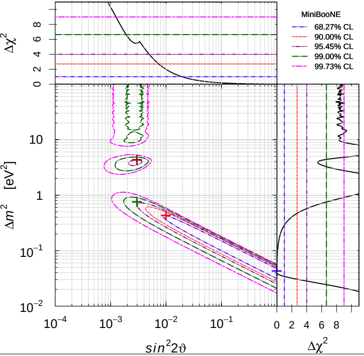


[Ericson, Garzelli, Giunti, Martini, in preparation]

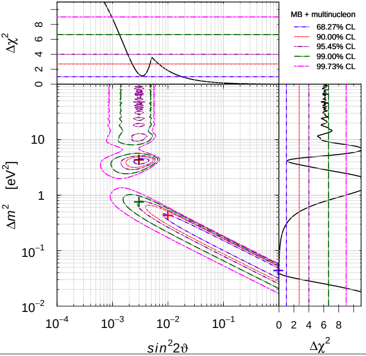

MiniBooNE − ν_e
Data − Expected Background
$\sin^2 2\vartheta = 1.00$, $\Delta m^2 = 0.04$ eV^2 (bf)
$\sin^2 2\vartheta = 0.01$, $\Delta m^2 = 0.4$ eV^2
$\sin^2 2\vartheta = 0.003$, $\Delta m^2 = 0.7$ eV^2
$\sin^2 2\vartheta = 0.003$, $\Delta m^2 = 4$ eV^2
Excess Events / MeV
−0.2
0.0
0.2
0.4
0.6
0.8
200
400
600
800
1000
1200
1400
3000
E_ν^{rec} [MeV]

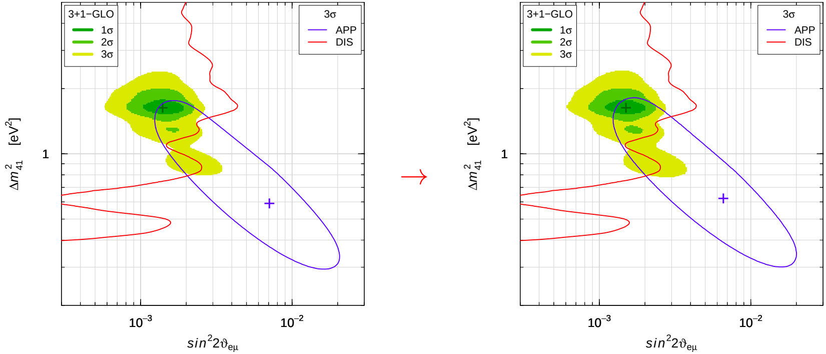
→


MB + multinucleon − ν_e
Data − Expected Background
$\sin^2 2\vartheta = 0.98$, $\Delta m^2 = 0.04$ eV^2 (bf)
$\sin^2 2\vartheta = 0.01$, $\Delta m^2 = 0.4$ eV^2
$\sin^2 2\vartheta = 0.003$, $\Delta m^2 = 0.7$ eV^2
$\sin^2 2\vartheta = 0.003$, $\Delta m^2 = 4$ eV^2
Excess Events / MeV
−0.2
0.0
0.2
0.4
0.6
0.8
200
400
600
800
1000
1200
1400
3000
E_ν^{rec} [MeV]


MiniBooNE
68.27% CL
90.00% CL
95.45% CL
99.00% CL
99.73% CL
$\Delta\chi^2$
0
2
4
6
8
Δm^2 [eV^2]
10^{-2}
10^{-1}
1
10
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$sin^2 2\vartheta$
0
2
4
6
8
$\Delta\chi^2$

→


MB + multinucleon
68.27% CL
90.00% CL
95.45% CL
99.00% CL
99.73% CL
$\Delta\chi^2$
0
2
4
6
8
Δm^2 [eV^2]
10^{-2}
10^{-1}
1
10
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$sin^2 2\vartheta$
0
2
4
6
8
$\Delta\chi^2$

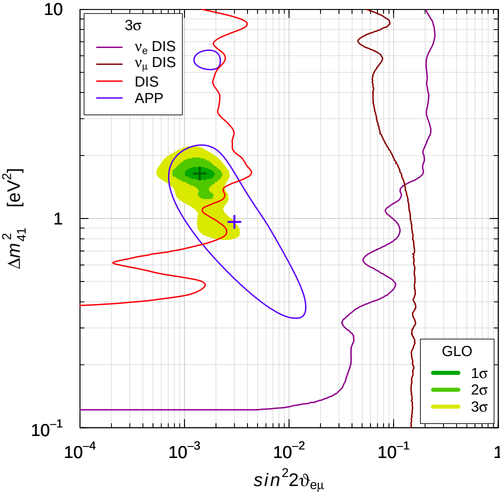


GoF = 5% PGoF = 0.1% GoF = 7% PGoF = 0.2%

- Multinucleon interactions can decrease slightly the MiniBooNE low-energy anomaly
- Multinucleon interactions cannot solve the APP-DIS tension
- MicroBooNE is crucial for checking the MiniBooNE low-energy anomaly
- If confirmed it is a real problem

Pragmatic Global 3+1 Fit

[Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



MiniBooNE $E > 475\,\text{MeV}$

GoF $= 26\%$ PGoF $= 7\%$

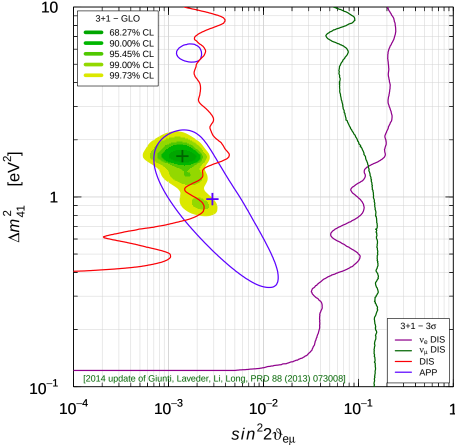
- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND (ν_s), MiniBooNE $(?)$, OPERA $(\cancel{\nu_s})$, ICARUS $(\cancel{\nu_s})$, KARMEN $(\cancel{\nu_s})$, NOMAD $(\cancel{\nu_s})$, BNL-E776 $(\cancel{\nu_s})$
- DIS ν_e & $\bar{\nu}_e$: Reactors (ν_s), Gallium (ν_s), ν_eC $(\cancel{\nu_s})$, Solar $(\cancel{\nu_s})$
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW $(\cancel{\nu_s})$, MINOS $(\cancel{\nu_s})$, Atmospheric $(\cancel{\nu_s})$, MiniBooNE/SciBooNE $(\cancel{\nu_s})$

No Osc. nominally disfavored at $\approx 6.3\sigma$

$\Delta\chi^2/\text{NDF} = 47.7/3$

MiniBooNE Impact in Pragmatic 3+1 Fit?

with MiniBooNE

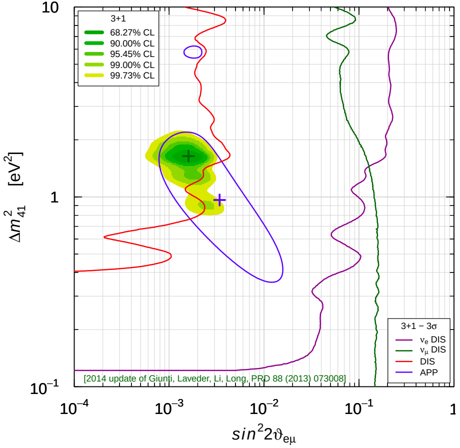


GoF $= 26\%$ PGoF $= 7\%$

No Osc. nominally disfavored

at $\approx 6.3\sigma$ $(\Delta\chi^2/\text{NDF} = 47.7/3)$

without MiniBooNE



GoF $= 16\%$ PGoF $= 5\%$

No Osc. nominally disfavored

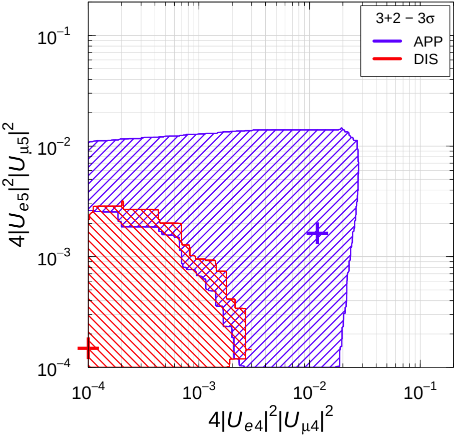
at $\approx 6.4\sigma$ $(\Delta\chi^2/\text{NDF} = 48.1/3)$

Without LSND: No Osc. nominally disfavored at $\approx 2.6\sigma$ $(\Delta\chi^2/\text{NDF} = 11.4/3)$

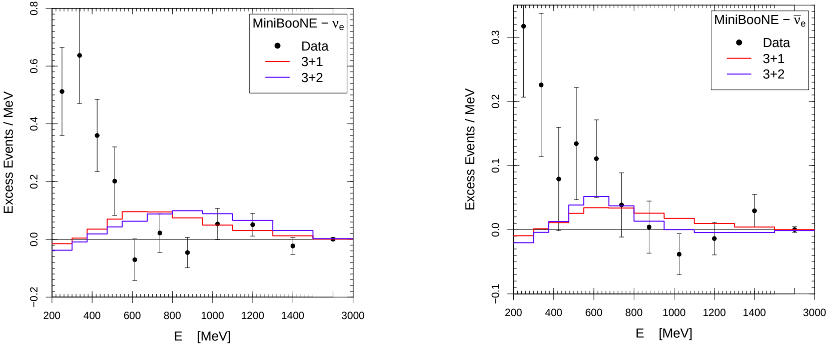
Global Fits	Our Fit		KMMS	
	3+1	3+2	3+1	3+2
GoF	5%	7%	19%	23%
PGoF	0.1%	0.04%	0.01%	0.003%

- Our Fit: Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001
- KMMS: Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050

APP-DIS 3+2 Tension:

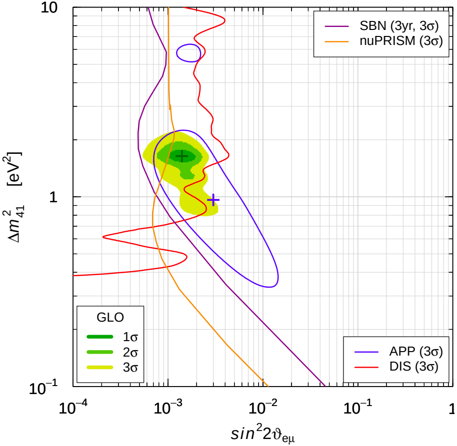


3+2 cannot fit MiniBooNE Low-Energy Excess



- Note difference between 3+2 ν_e and $\bar{\nu}_e$ histograms due to CP violation
- 3+2 can fit slightly better the small $\bar{\nu}_e$ excess at about 600 MeV
- 3+2 fit of low-energy excess as bad as 3+1
- Claims that 3+2 can fit low-energy excess do not take into account constraints from other data
- Conclusion: 3+2 is not needed

Future Experiments



SBN (FNAL, USA)

[arXiv:1503.01520]

3 Liquid Argon TPCs

LAr1-ND $L \simeq 100\,\mathrm{m}$

MicroBooNE $L \simeq 470\,\mathrm{m}$

ICARUS T600 $L \simeq 600\,\mathrm{m}$

nuPRISM (J-PARC, Japan)

[Wilking@NNN2015]

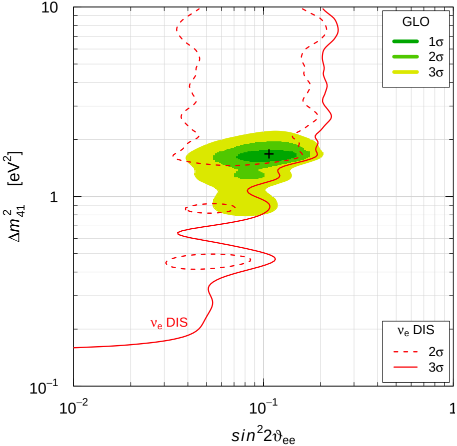
$L \simeq 1\,\mathrm{km}$

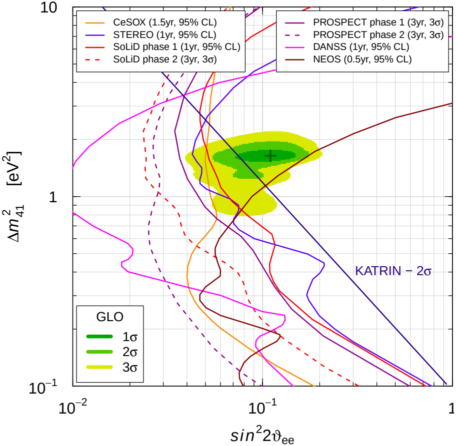
50 m tall water Cherenkov detector

1° – 4° off-axis

can be improved with T2K ND

ν_e Disappearance





CeSOX (BOREXINO, Italy)

^{144}Ce – 100 kCi [Vivier@TAUP2015]

rate: 1% normalization uncertainty

8.5 m from detector center

KATRIN (Germany)

Tritium β decay [Mertens@TAUP2015]

STEREO (France) $L \simeq$ 8-12m [Sanchez@EPSHEP2015]

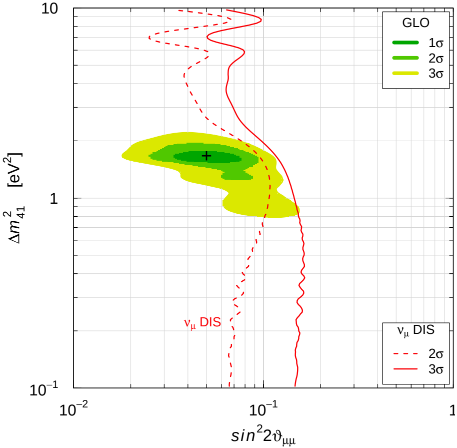
SoLid (Belgium) $L \simeq$ 5-8m [Yermia@TAUP2015]

PROSPECT (USA) $L \simeq$ 7-12m [Heeger@TAUP2015]

DANSS (Russia) $L \simeq$ 10-12m [arXiv:1412.0817]

NEOS (Korea) $L \simeq$ 25m [Oh@WIN2015]

ν_μ Disappearance

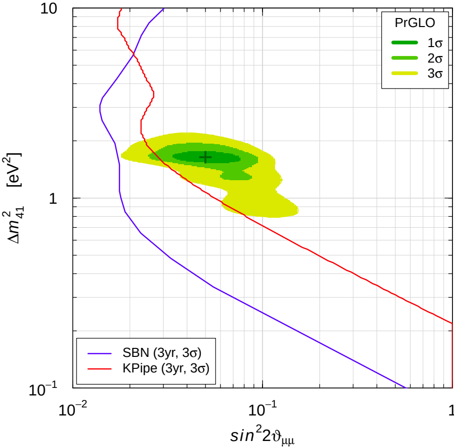


SBN (USA) [arXiv:1503.01520]

LAr1-ND $L \simeq 100$m

MicroBooNE $L \simeq 470$m

ICARUS T600 $L \simeq 600$m



KPipe (Japan) [arXiv:1510.06994]

$L \simeq 30$-150m

120 m long detector!

Effects of light sterile neutrinos should also be seen in:

- Long-baseline Neutrino Oscillation Experiments

  [de Gouvea, Kelly, Kobach, PRD 91 (2015) 053005; Klop, Palazzo, PRD 91 (2015) 073017; Berryman, de Gouvea, Kelly, Kobach, PRD 92 (2015) 073012; Gandhi, Kayser, Masud, Prakash, JHEP 1511 (2015) 039; Palazzo, arXiv:1509.03148; Agarwalla, Chatterjee, Dasgupta, Palazzo, arXiv:1601.05995]

- Solar neutrinos

  [Dooling et al, PRD 61 (2000) 073011, Gonzalez-Garcia et al, PRD 62 (2000) 013005; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301; Li et al, PRD 80 (2009) 113007, PRD 87, 113004 (2013), JHEP 1308 (2013) 056; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

- High-energy atmospheric neutrinos (IceCube, Km3Net)

  [Goswami, PRD 55 (1997) 2931; Bilenky, Giunti, Grimus, Schwetz, PRD 60 (1999) 073007; Maltoni, Schwetz, Tortola, Valle, NPB 643 (2002) 321, PRD 67 (2003) 013011; Choubey, JHEP 12 (2007) 014; Razzaque, Smirnov, JHEP 07 (2011) 084, PRD 85 (2012) 093010; Gandhi, Ghoshal, PRD 86 (2012) 037301; Esmaili, Halzen, Peres, JCAP 1211 (2012) 041; Esmaili, Smirnov, JHEP 1312 (2013) 014; Rajpoot, Sahu, Wang, EPJC 74 (2014) 2936; Collin, Arguelles, Conrad, Shaevitz, arXiv:1602.00671]

- Supernova neutrinos

  [Caldwell, Fuller, Qian, PRD 61 (2000) 123005; Peres, Smirnov, NPB 599 (2001); Sorel, Conrad, PRD 66 (2002) 033009; Tamborra, Raffelt, Huedepohl, Janka, JCAP 1201 (2012) 013; Wu, Fischer, Martinez-Pinedo, Qian, PRD 89 (2014) 061303; Esmaili, Peres, Serpico, PRD 90 (2014) 033013]

- High-energy cosmic neutrinos

  [Cirelli, Marandella, Strumia, Vissani, NPB 708 (2005) 215; Donini, Yasuda, arXiv:0806.3029; Barry, Mohapatra, Rodejohann,PRD 83 (2011) 113012]

- Indirect dark matter detection

  [Esmaili, Peres, JCAP 1205 (2012) 002]

- Cosmology

  [see Hannestad Lectures]

Effective LBL Oscillation Probabilities

General Bilenky formula of the probability of $\nu_\mu \to \nu_e$ oscillations:

$$P_{\nu_\mu \to \nu_e} = 4 \sum_{k \neq p} |U_{\mu k}|^2 |U_{ek}|^2 \sin^2 \Delta_{kp}$$

$$+8 \sum_{\substack{j>k \\ j,k \neq p}} |U_{\mu j} U_{ej} U_{\mu k} U_{ek}| \sin \Delta_{kp} \sin \Delta_{jp} \cos(\Delta_{jk} - \eta_{\mu e j k})$$

$$\Delta_{kp} = \frac{\Delta m^2_{kp} L}{4E} \qquad \eta_{\mu e j k} = \arg\left[U^*_{\mu j} U_{ej} U_{\mu k} U^*_{ek}\right]$$

$$|U_{e3}| \simeq \sin \vartheta_{13} \simeq 0.15 \sim \varepsilon \quad \Longrightarrow \quad \varepsilon^2 \sim 0.03$$

$$|U_{e4}| \simeq \sin \vartheta_{14} \simeq 0.17 \sim \varepsilon$$

$$|U_{\mu 4}| \simeq \sin \vartheta_{24} \simeq 0.11 \sim \varepsilon$$

$$\alpha \equiv \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq \frac{7 \times 10^{-5}}{2.4 \times 10^{-3}} \simeq 0.031 \sim \varepsilon^2$$

3ν mixing with $p = 1$:

$$\begin{aligned} P^{3\nu}_{\nu_\mu\to\nu_e} = {} & 4|U_{\mu2}|^2|U_{e2}|^2 \sin^2\Delta_{21} && \sim \varepsilon^4 \\ & +4|U_{\mu3}|^2|U_{e3}|^2 \sin^2\Delta_{31} && \sim \varepsilon^2 \\ & +8\,|U_{\mu3}U_{e3}U_{\mu2}U_{e2}| \sin\Delta_{21}\sin\Delta_{31}\cos(\Delta_{32}-\eta_{\mu e32}) && \sim \varepsilon^3 \end{aligned}$$

CP violation is observable in LBL experiments at order ε^3:

$$\begin{aligned} P^{\text{LBL};3\nu}_{\nu_\mu\to\nu_e} \simeq {} & 4|U_{\mu3}|^2|U_{e3}|^2 \sin^2\Delta_{31} \\ & +8\,|U_{\mu3}U_{e3}U_{\mu2}U_{e2}| \sin\Delta_{21}\sin\Delta_{31}\cos(\Delta_{32}-\eta_{\mu e32}) \\ \simeq {} & \sin^2 2\vartheta_{13}\sin^2\vartheta_{23}\sin^2\Delta_{31} \\ & +\sin 2\vartheta_{13}\sin 2\vartheta_{12}\sin^2\vartheta_{23}(\alpha\Delta_{31})\sin\Delta_{31}\cos(\Delta_{32}+\delta_{13}) \\ = {} & P^{\text{ATM}} + P^{\text{INT}} \end{aligned}$$

[Klop, Palazzo, PRD 91 (2015) 073017, arXiv:1412.7524]

3+1 mixing with $p = 1$:

$$
\begin{aligned}
P^{3+1}_{\nu_\mu \to \nu_e} = {} & 4|U_{\mu 2}|^2 |U_{e2}|^2 \sin^2 \Delta_{21} && \sim \varepsilon^4 \\
& + 4|U_{\mu 3}|^2 |U_{e3}|^2 \sin^2 \Delta_{31} && \sim \varepsilon^2 \\
& + 4|U_{\mu 4}|^2 |U_{e4}|^2 \sin^2 \Delta_{41} && \sim \varepsilon^4 \\
& + 8 \, |U_{\mu 3} U_{e3} U_{\mu 2} U_{e2}| \sin \Delta_{21} \sin \Delta_{31} \cos(\Delta_{32} - \eta_{\mu e 32}) && \sim \varepsilon^3 \\
& + 8 \, |U_{\mu 4} U_{e4} U_{\mu 2} U_{e2}| \sin \Delta_{21} \sin \Delta_{41} \cos(\Delta_{42} - \eta_{\mu e 42}) && \sim \varepsilon^4 \\
& + 8 \, |U_{\mu 4} U_{e4} U_{\mu 3} U_{e3}| \sin \Delta_{31} \sin \Delta_{41} \cos(\Delta_{43} - \eta_{\mu e 43}) && \sim \varepsilon^3
\end{aligned}
$$

At order ε^3:

$$
\begin{aligned}
P_{\nu_\mu\to\nu_e}^{\text{LBL;3+1}} \simeq\ & 4|U_{\mu 3}|^2|U_{e3}|^2 \sin^2\Delta_{31} \\
& +8\,|U_{\mu 3}U_{e3}U_{\mu 2}U_{e2}|\sin\Delta_{21}\sin\Delta_{31}\cos(\Delta_{32}-\eta_{\mu e32}) \\
& +8\,|U_{\mu 4}U_{e4}U_{\mu 3}U_{e3}|\sin\Delta_{31}\sin\Delta_{41}\cos(\Delta_{43}-\eta_{\mu e43}) \\
\simeq\ & \sin^2 2\vartheta_{13}\sin^2\vartheta_{23}\sin^2\Delta_{31} \\
& +\sin 2\vartheta_{13}\sin 2\vartheta_{12}\sin^2\vartheta_{23}(\alpha\Delta_{31})\sin\Delta_{31}\cos(\Delta_{32}+\delta_{13}) \\
& +\sin 2\vartheta_{13}\sin 2\vartheta_{14}\sin 2\vartheta_{24}\sin\vartheta_{23}\sin\Delta_{31}\sin\Delta_{41}\cos(\Delta_{43}-\delta_{13}+\delta_{14}) \\
=\ & P^{\text{ATM}}+P_{\text{I}}^{\text{INT}}+P_{\text{II}}^{\text{INT}}
\end{aligned}
$$

[Klop, Palazzo, PRD 91 (2015) 073017, arXiv:1412.7524]

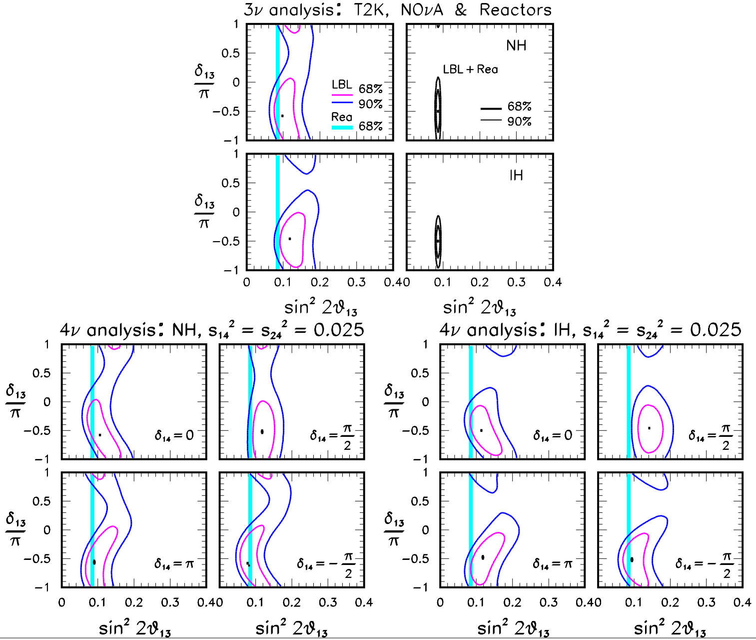
$$\sin\Delta_{41}\cos(\Delta_{43}-\delta)=\sin\Delta_{41}\cos(\Delta_{41}-\Delta_{31}-\delta) \qquad \Delta_{41}\gg 1$$

$$=\frac{1}{2}\sin 2\Delta_{41}\cos(\Delta_{31}+\delta)+\sin^2\Delta_{41}\sin(\Delta_{31}+\delta) \quad \to \quad \frac{1}{2}\sin(\Delta_{31}+\delta)$$

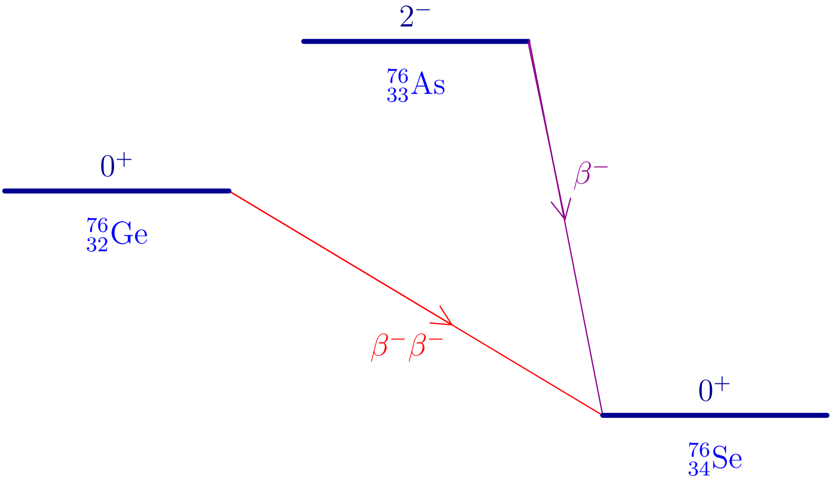
$$P_{\text{II}}^{\text{INT}}\simeq\sin 2\vartheta_{13}\sin 2\vartheta_{14}\sin 2\vartheta_{24}\sin\vartheta_{23}\sin\Delta_{31}\sin(\Delta_{31}+\delta_{13}-\delta_{14})$$

CP Violation in T2K and NOνA

[Palazzo, arXiv:1509.03148]



Neutrinoless Double-Beta Decay



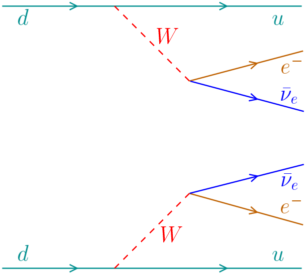
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



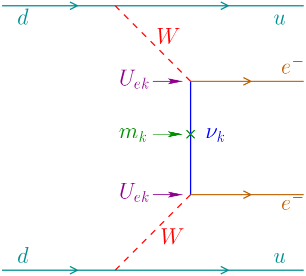
Neutrinoless Double-β Decay: $\Delta L = 2$

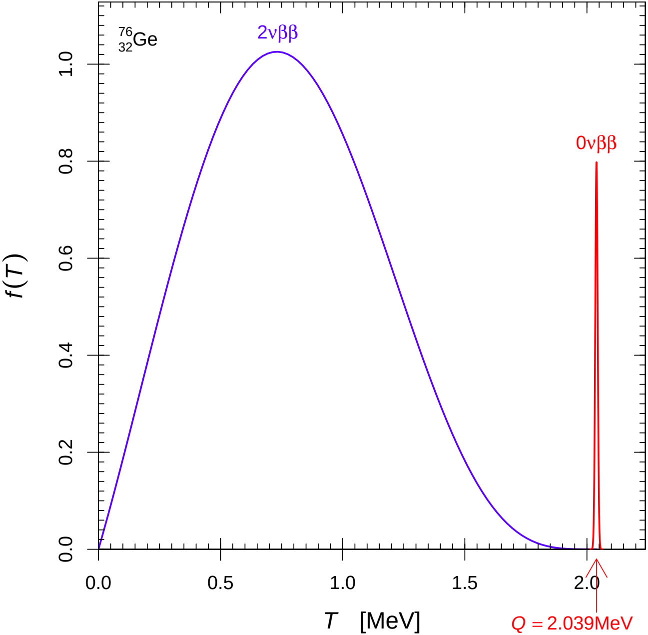
$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass

$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$$



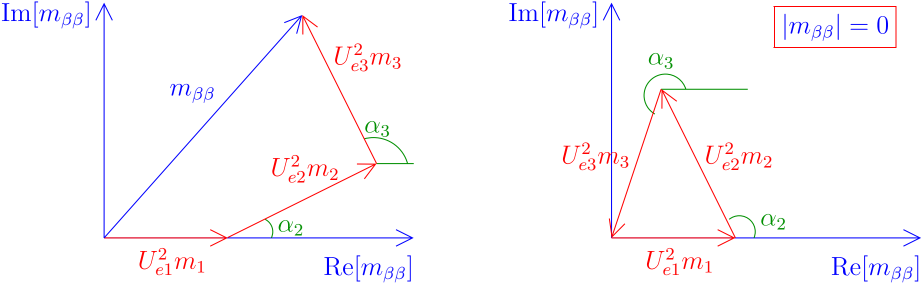

$^{76}_{32}$Ge
2νββ
0νββ
$f(T)$
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
T [MeV]
$Q = 2.039$MeV

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} ⇒ possible cancellations

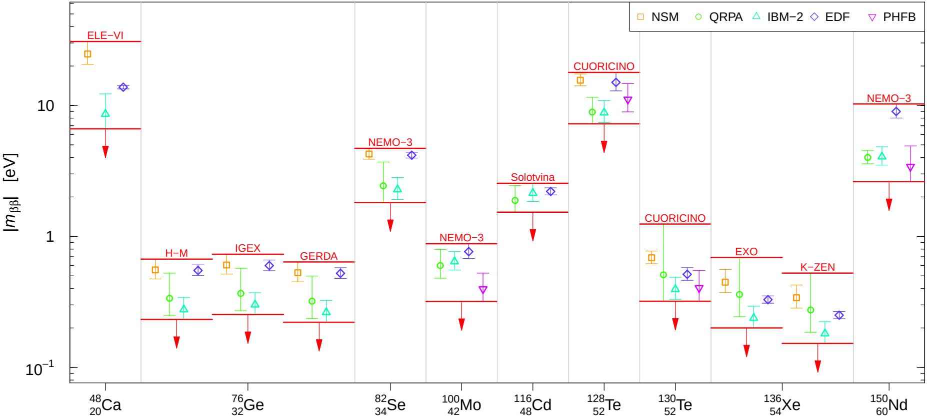
$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$



90% C.L. Experimental Bounds

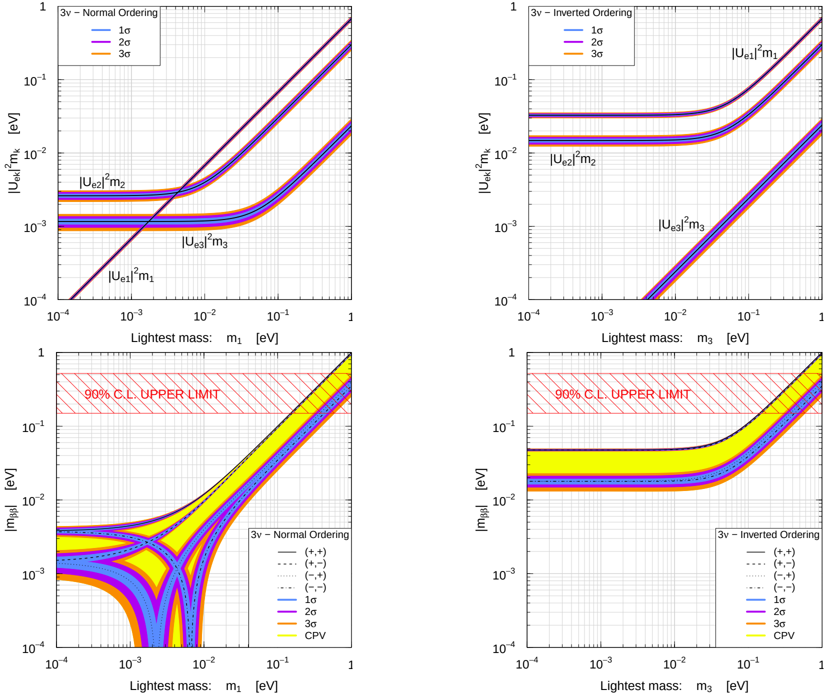
$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	GERDA	$> 2.1 \times 10^{25}$	$< 0.22 - 0.64$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORICINO	$> 2.8 \times 10^{24}$	$< 0.32 - 1.2$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.2 - 0.69$
	KamLAND-Zen	$> 1.9 \times 10^{25}$	$< 0.15 - 0.52$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



[Bilenky, Giunti, IJMPA 30 (2015) 0001]

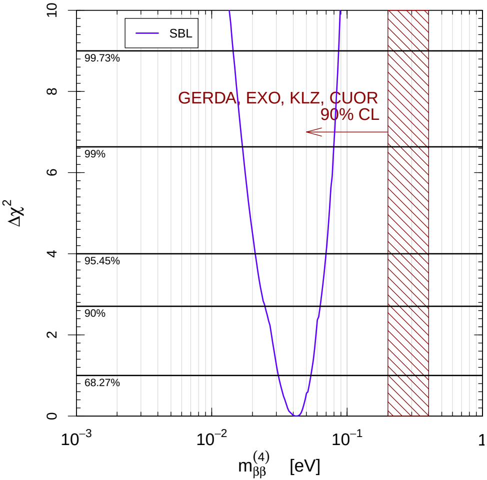
Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$


3ν − Normal Ordering
1σ
2σ
3σ
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
$|U_{e1}|^2m_1$
$|U_{ek}|^2m_k$ [eV]
Lightest mass: m_1 [eV]
3ν − Inverted Ordering
1σ
2σ
3σ
$|U_{e1}|^2m_1$
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
$|U_{ek}|^2m_k$ [eV]
Lightest mass: m_3 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν − Normal Ordering
(+,+)
(+,−)
(−,+)
(−,−)
1σ
2σ
3σ
CPV
Lightest mass: m_1 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν − Inverted Ordering
(+,+)
(+,−)
(−,+)
(−,−)
1σ
2σ
3σ
CPV
Lightest mass: m_3 [eV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}

3+1 Mixing

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_{21}}\, m_2 + |U_{e3}|^2\, e^{i\alpha_{31}}\, m_3 + |U_{e4}|^2\, e^{i\alpha_{41}}\, m_4$$



Pragmatic 3+1 Fit

$$m_{\beta\beta}^{(k)} = |U_{ek}|^2 m_k$$

$$m_1 \ll m_4$$

$$\Downarrow$$

$$m_{\beta\beta}^{(4)} \simeq |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

surprise:

possible cancellation

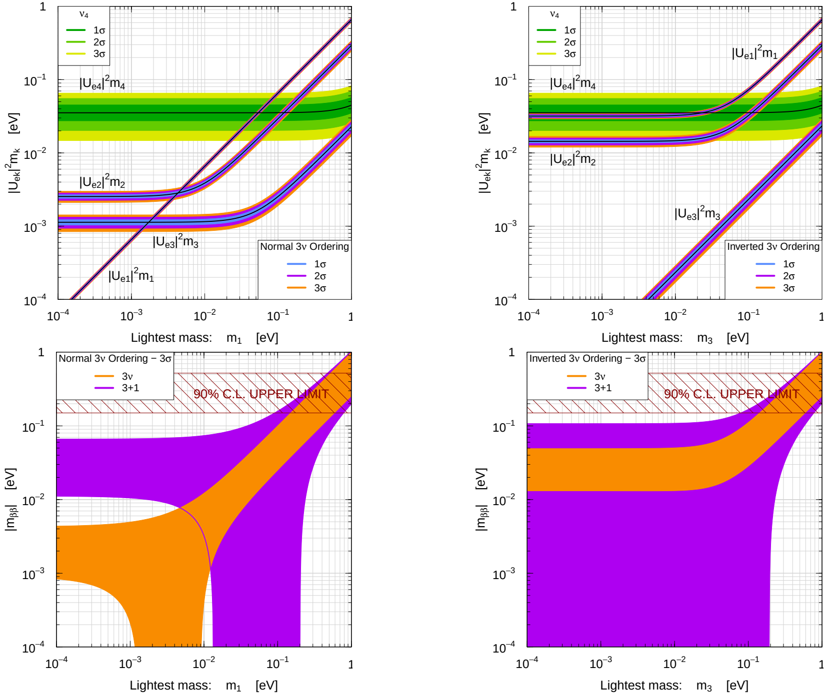
with $m_{\beta\beta}^{(3\nu)}$

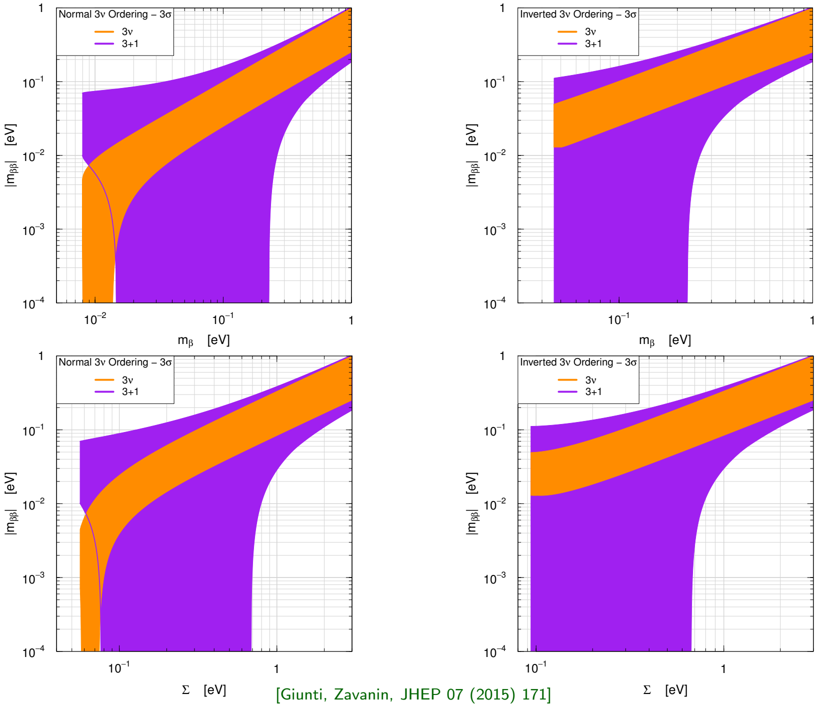
[Barry et al, JHEP 07 (2011) 091]

[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, JPG 39 (2012) 124008]

[Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]


ν_4
1σ
2σ
3σ
$|U_{e4}|^2m_4$
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
$|U_{e1}|^2m_1$
Normal 3ν Ordering
Inverted 3ν Ordering
$|U_{ek}|^2m_k$ [eV]
Lightest mass: m_1 [eV]
Lightest mass: m_3 [eV]
Normal 3ν Ordering − 3σ
Inverted 3ν Ordering − 3σ
3ν
3+1
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}



[Giunti, Zavanin, JHEP 07 (2015) 171]

Conclusions

- Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance:
  - Experimental data agree on Reactor $\bar{\nu}_e$ and Gallium ν_e disappearance.
  - Problem: total rates may have unknown systematic uncertainties.
  - Many promising projects to test unambiguously short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources.
  - Independent tests through effect of m_4 in β-decay and $\beta\beta_{0\nu}$-decay.
- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - Not seen by other SBL $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ experiments.
  - MiniBooNE experiment has been inconclusive.
  - Experiments with near detector are needed to check LSND signal!
  - Promising Fermilab program aimed at a conclusive solution of the mystery: a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-T600), all Liquid Argon Time Projection Chambers.
- Pragmatic 3+1 Fit is fine: moderate APP-DIS tension.
- 3+2 is not needed: same APP-DIS tension and no experimental evidence of CP violation.
- Cosmology:
  - Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1\,\text{eV}$.
  - Cosmological and oscillation data may be reconciled by a non-standard cosmological mechanism.