Neutrino Masses and Oscillations

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Neutrino Unbound: http://www.nu.to.infn.it

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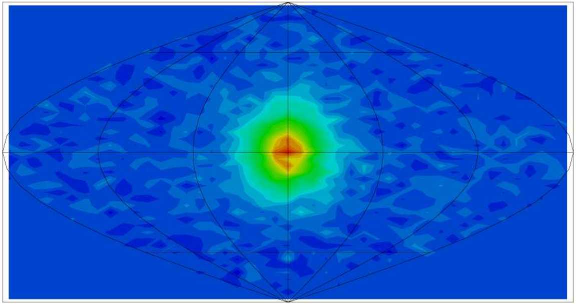
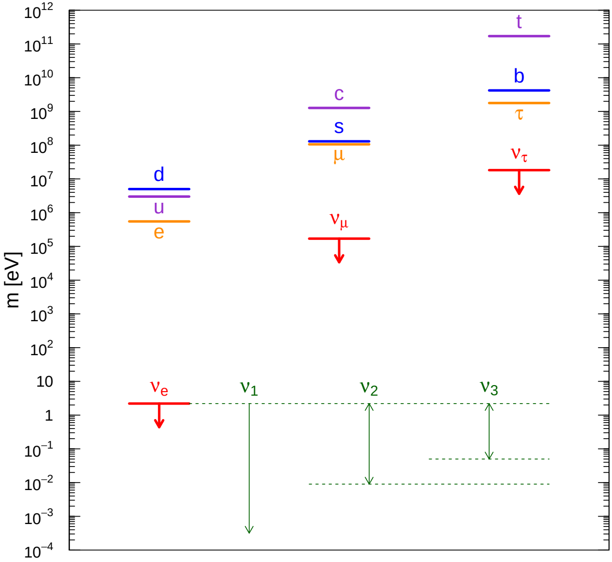


The sun observed through neutrinos by Super-Kamiokande

Fermion Mass Spectrum



Standard Model: Massless Neutrinos

	1st Generation	**2nd Generation**	**3rd Generation**
Quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \begin{matrix} t_R \\ b_R \end{matrix}$
Leptons	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{matrix} \xcancel{\nu_{eR}} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{matrix} \xcancel{\nu_{\mu R}} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \quad \begin{matrix} \xcancel{\nu_{\tau R}} \\ \tau_R \end{matrix}$

- No ν_R $\Longrightarrow$ No Dirac mass Lagrangian $\mathcal{L}_{\mathrm{D}} \sim m_{\mathrm{D}} \overline{\nu_L} \nu_R$

- Majorana Neutrinos: $\nu = \nu^c$ $\Longrightarrow$ $\nu_R = (\nu^c)_R = \nu_L^c$

  Majorana mass Lagrangian: $\mathcal{L}_{\mathrm{M}} \sim m_{\mathrm{M}} \overline{\nu_L} \nu_L^c$

  forbidden by Standard Model $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry!

- In Standard Model neutrinos are massless!

- Experimentally allowed until 1998, when the Super-Kamiokande atmospheric neutrino experiment obtained a model-independent proof of

  Neutrino Oscillations

SM Extension: Massive Dirac Neutrinos

	1st Generation	2nd Generation	3rd Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \begin{matrix} t_R \\ b_R \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{matrix} \nu_{eR} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{matrix} \nu_{\mu R} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \quad \begin{matrix} \nu_{\tau R} \\ \tau_R \end{matrix}$

- $\nu_R \implies$ Dirac mass Lagrangian $\mathcal{L}_\text{D} \sim m_\text{D} \overline{\nu_L} \nu_R$
- m_D is generated by the standard Higgs mechanism: $y \overline{L_L} \widetilde{\Phi} \nu_R \to y v \overline{\nu_L} \nu_R$
- Necessary assumption: lepton number conservation to forbid the Majorana mass terms

$$\mathcal{L}_\text{M} \sim m_\text{M} \overline{\nu_R} \nu_R^c \quad \text{singlet under SM symmetries!}$$

- Extremely small Yukawa couplings: $y \lesssim 10^{-11}$
- Not theoretically attractive.

Beyond the SM: Massive Majorana Neutrinos

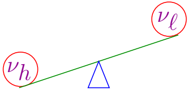
without the lepton number conservation assumption

$$\mathcal{L}^{\text{D+M}} = -\frac{1}{2} \left(\overline{\nu_L^c} \quad \overline{\nu_R} \right) \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_{\text{M}} \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

m_{M} can be arbitrarily large (not protected by SM symmetries)

$m_{\text{M}} \sim$ scale of new physics beyond Standard Model $\Rightarrow m_{\text{M}} \gg m_{\text{D}}$

diagonalization of $\begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_{\text{M}} \end{pmatrix} \Longrightarrow m_\ell \simeq \frac{m_{\text{D}}^2}{m_{\text{M}}} \qquad m_h \simeq m_{\text{M}}$



seesaw mechanism

natural explanation of smallness
of light neutrino masses

massive neutrinos are Majorana!

$$\nu_\ell \simeq -i \left(\nu_L - \nu_L^c \right) \qquad \nu_h \simeq \nu_R + \nu_R^c$$

3-GEN ⇒ effective low-energy 3-ν mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

- In general, if the SM is an effective low-energy theory

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]

- Only one dim-5 operator:

$$\mathscr{L}_5 = \frac{g_5}{2\mathcal{M}}\,(L_L^T\,\mathcal{C}^\dagger\,\sigma_2\,\vec{\sigma}\,L_L)\cdot(\Phi^T\,\sigma_2\,\vec{\sigma}\,\Phi) + \text{H.c.}$$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathscr{L}_{\text{M}} = \frac{1}{2}\,\frac{g_5\,v^2}{\mathcal{M}}\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.}$

  Massive Majorana Neutrinos

- $m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_{\text{D}}^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses

  (special case: See-Saw Mechanism)

- Example: $m_{\text{D}} \sim v \sim 10^2\,\text{GeV}$ and $\mathcal{M} \sim 10^{15}\,\text{GeV} \implies m \sim 10^{-2}\,\text{eV}$

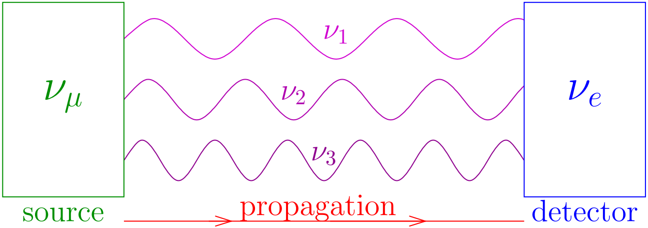
Neutrino Oscillations

- 1957: Bruno Pontecorvo proposed a form of neutrino oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955).
- Theoretical and experimental developments led to neutrino mixing [Maki, Nakagawa, Sakata, Prog. Theor. Phys. 28 (1962) 870] and the theory of neutrino oscillations as flavor transitions which oscillate with distance [Pontecorvo, Sov. Phys. JETP 26 (1968) 984; Gribov, Pontecorvo, PLB 28 (1969); Bilenky, Pontecorvo, Sov. J. Nucl. Phys. 24 (1976) 316, PLB 61 (1976) 248; Fritzsch, Minkowski, Phys. Lett. B62 (1976) 72; Eliezer, Swift, Nucl. Phys. B105 (1976) 45] .
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$
\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}
$$

- U is the 3×3 Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = U_{\mu 1}\,|\nu_1\rangle + U_{\mu 2}\,|\nu_2\rangle + U_{\mu 3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{\mu 1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{\mu 2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{\mu 3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\mu\rangle$$

$$E_k^2 = p^2 + m_k^2$$

$$P_{\nu_\mu\to\nu_e}(t>0) = |\langle\nu_e|\nu(t>0)\rangle|^2 \sim \sum_{k>j} \mathrm{Re}\big[U_{ek}\,U_{\mu k}^*\,U_{ej}^*\,U_{\mu j}\big]\,\sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)$$

transition probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

$$\begin{array}{cccc} \nu_e \to \nu_\mu & \nu_e \to \nu_\tau & \nu_\mu \to \nu_e & \nu_\mu \to \nu_\tau \\ \bar\nu_e \to \bar\nu_\mu & \bar\nu_e \to \bar\nu_\tau & \bar\nu_\mu \to \bar\nu_e & \bar\nu_\mu \to \bar\nu_\tau \end{array}$$

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $s_{ab} \equiv \sin\vartheta_{ab}$ $0 \le \vartheta_{ab} \le \frac{\pi}{2}$ $0 \le \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS
$$\begin{cases} \text{3 Mixing Angles: } \vartheta_{12},\ \vartheta_{23},\ \vartheta_{13} \\ \text{1 CPV Dirac Phase: } \delta_{13} \\ \text{2 independent } \Delta m^2_{kj} \equiv m^2_k - m^2_j\text{: } \Delta m^2_{21},\ \Delta m^2_{31} \end{cases}$$

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

Experimental Evidences of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

VLBL Reactor
$\bar{\nu}_e$ disappearance

(SNO, BOREXino
Super-Kamiokande
GALLEX/GNO, SAGE
Homestake, Kamiokande)

(KamLAND)

$$\to \begin{cases} \Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{S}} = \sin^2\vartheta_{12} \simeq 0.30 \end{cases}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

LBL Accelerator
ν_μ disappearance

LBL Accelerator
$\nu_\mu \to \nu_\tau$

(Super-Kamiokande
Kamiokande, IMB
MACRO, Soudan-2)

(K2K, MINOS
T2K, NOνA)

(Opera)

$$\to \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31}| \simeq 2.4 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

LBL Accelerator
$\nu_\mu \to \nu_e$

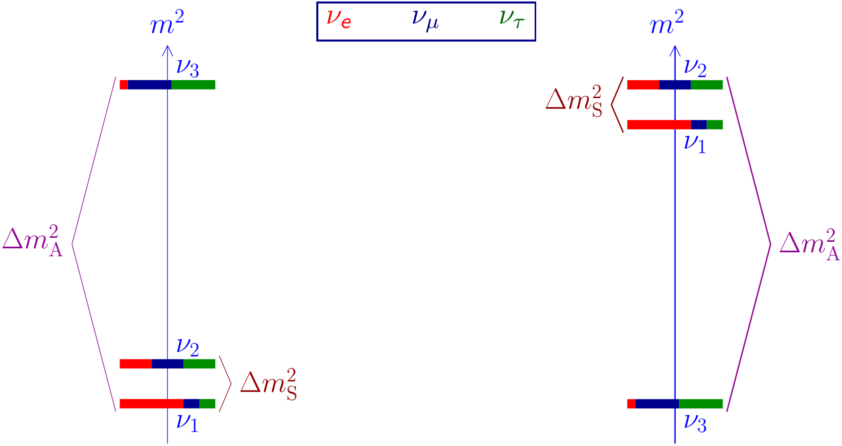
(T2K, MINOS, NOνA)

LBL Reactor
$\bar{\nu}_e$ disappearance

(Daya Bay, RENO
Double Chooz)

$$\to \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31}| \\ \sin^2\vartheta_{13} \simeq 0.023 \end{cases}$$

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

Recent Global Fits

- Capozzi, Lisi, Marrone, Montanino, Palazzo
  Neutrino masses and mixings: Status of known and unknown 3ν parameters
  arXiv:1601.07777

- Bergstrom, Gonzalez-Garcia, Maltoni, Schwetz
  Bayesian global analysis of neutrino oscillation data
  JHEP 09 (2015) 200, arXiv:1507.04366

www.nu-fit.org

- Forero, Tortola, Valle
  Neutrino oscillations refitted
  Phys.Rev. D90 (2014) 093006, arXiv:1405.7540

$$\Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.37 \pm 0.16 \times 10^{-5}\,\text{eV}^2 \qquad \text{uncertainty} \approx 2.4\%$$

$$\Delta m^2_{\text{A}} = \frac{1}{2}\left|\Delta m^2_{31} + \Delta m^2_{32}\right| \simeq \begin{cases} 2.50 \pm 0.04 \times 10^{-3}\,\text{eV}^2 & \text{(NO)} \\ 2.46 \pm 0.04 \times 10^{-3}\,\text{eV}^2 & \text{(IO)} \end{cases}$$

uncertainty $\approx 1.8\%$

[Capozzi, Lisi, Marrone, Montanino, Palazzo, arXiv:1601.07777]

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\vartheta_{23} = \vartheta_{\text{A}}$

$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$

$P_{\text{osc}} \propto \sin^2 2\vartheta_{23}$

maximal and flat

at $\vartheta_{23} = 45^\circ$

Daya Bay, RENO

Double Chooz

T2K, MINOS

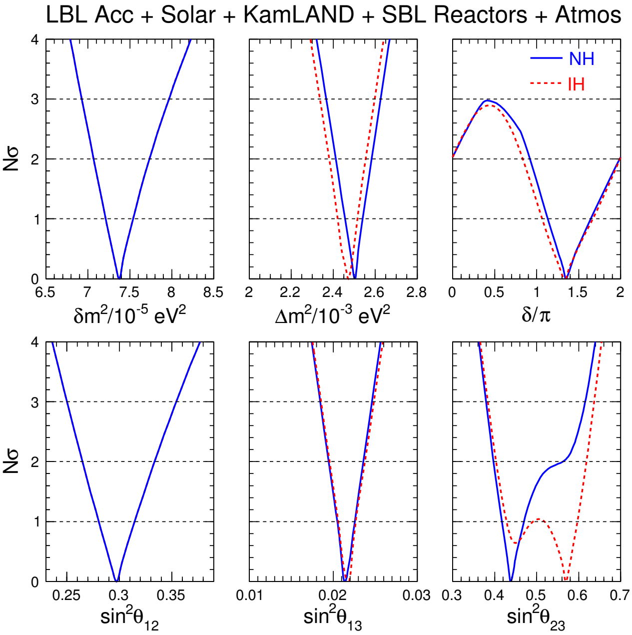
$$\sin^2\vartheta_{13} \simeq \begin{cases} 0.0214 \pm 0.0010 & \text{(NO)} \\ 0.0218 \pm 0.0011 & \text{(IO)} \end{cases}$$

$$\boxed{\delta_{13} \approx 3\pi/2?}$$

$\vartheta_{12} = \vartheta_{\text{S}}$

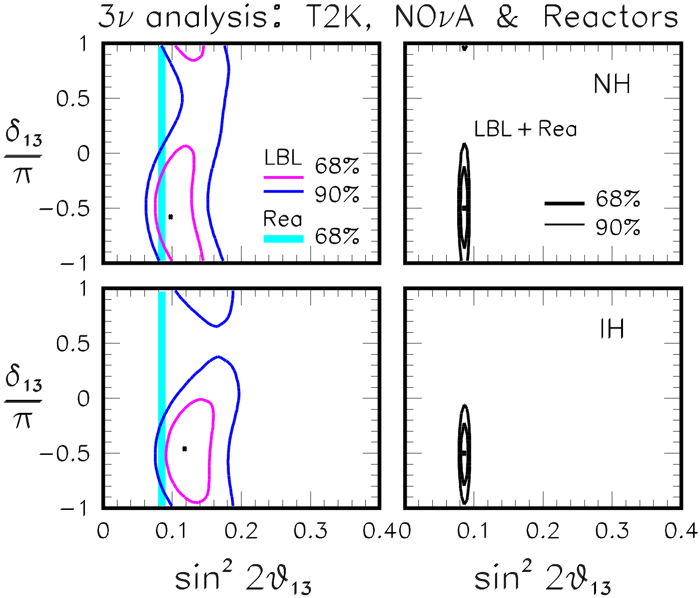
$\sin^2\vartheta_{12} \simeq 0.297 \pm 0.016$

$$\frac{\delta \sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \approx 9\% \qquad \frac{\delta \sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \approx 4.7\% \qquad \frac{\delta \sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \approx 5.8\%$$



[Capozzi, Lisi, Marrone, Montanino, Palazzo, arXiv:1601.07777]

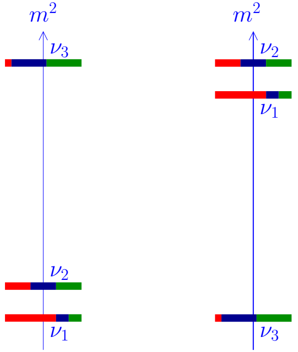
Maximal CP Violation?



[Palazzo, arXiv:1509.03148]

Determination of Mass Ordering

1. Matter Effects: Atmospheric (PINGU, ORCA), Long-Baseline, Supernova Experiments
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \frac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} > 0$ NO
   - $\bar\nu_e \leftrightarrows \bar\nu_\mu$ MSW resonance: $V = -\frac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} < 0$ IO
2. Phase Difference: Reactor $\bar\nu_e \to \bar\nu_e$ (JUNO, RENO-50)



Normal Ordering

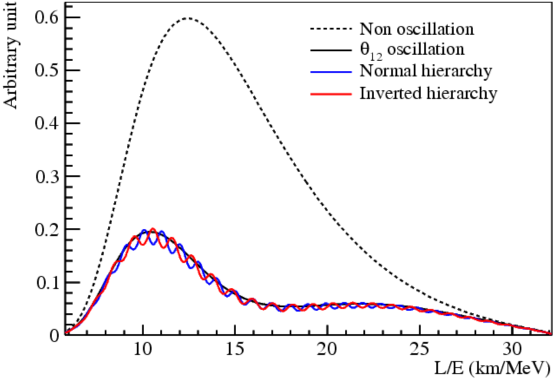
$|\Delta m^2_{31}|$
$=$
$|\Delta m^2_{32}| + |\Delta m^2_{21}|$

$|\Delta m^2_{31}| > |\Delta m^2_{32}|$

Inverted Ordering

$|\Delta m^2_{31}|$
$=$
$|\Delta m^2_{32}| - |\Delta m^2_{21}|$

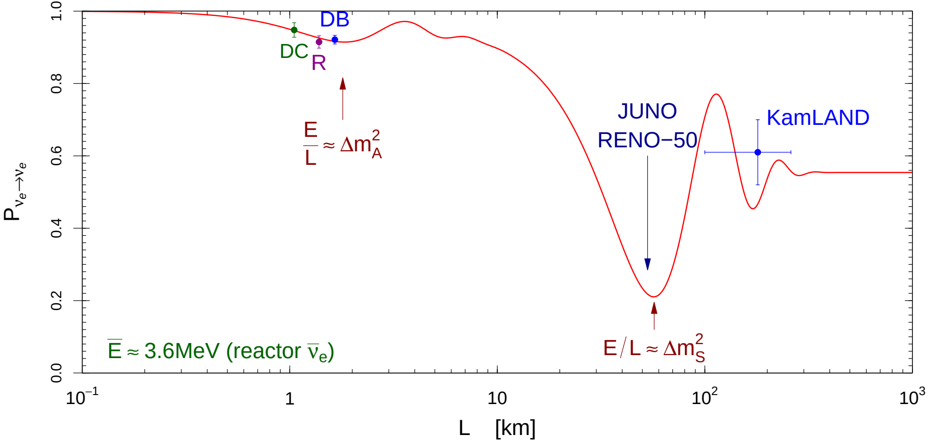
$|\Delta m^2_{31}| < |\Delta m^2_{32}|$



Neutrino Physics with JUNO, arXiv:1507.05613

$$
\begin{aligned}
P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} = 1 & - \cos^4 \vartheta_{13} \sin^2 2\vartheta_{12} \sin^2 \left(\Delta m^2_{21} L/4E \right) \\
& - \cos^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 \left(\Delta m^2_{31} L/4E \right) \\
& - \sin^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 \left(\Delta m^2_{32} L/4E \right)
\end{aligned}
$$

[Petcov, Piai, PLB 533 (2002) 94; Choubey, Petcov, Piai, PRD 68 (2003) 113006; Learned, Dye, Pakvasa, Svoboda, PRD 78 (2008) 071302; Zhan, Wang, Cao, Wen, PRD 78 (2008) 111103, PRD 79 (2009) 073007]

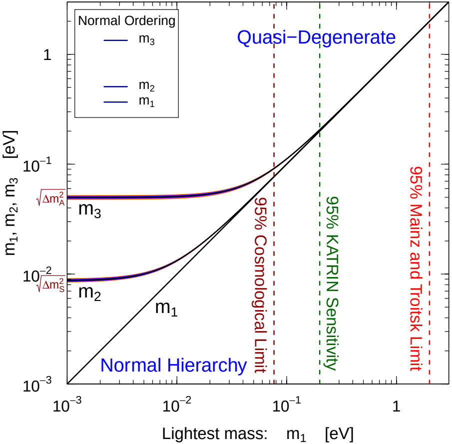


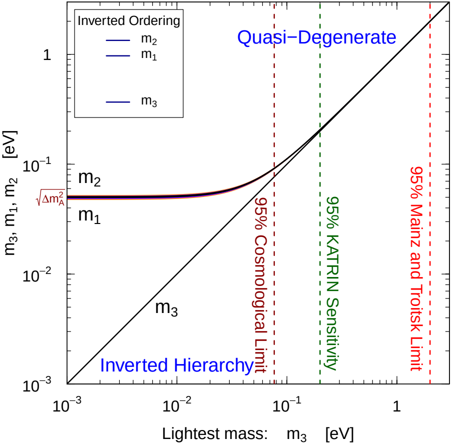
$P_{\nu_e \to \nu_e}$
1.0
0.8
0.6
0.4
0.2
0.0
DB
DC
R
$\frac{E}{L} \approx \Delta m^2_A$
JUNO
RENO−50
KamLAND
$E/L \approx \Delta m^2_S$
$\bar{E} \approx 3.6\text{MeV}$ (reactor $\bar{\nu}_e$)
10^{-1}
1
10
10^2
10^3
L [km]

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), PINGU (Antarctica), ORCA (EU), INO (India), ...
- Mass Ordering (Hierarchy) ?
  - NOνA (USA), JUNO (China), RENO-50 (Korea), PINGU (Antarctica), ORCA (EU), INO (India), ...
- CP violation ? $\delta_{13} \approx 3\pi/2$?
  - T2K (Japan), NOνA (USA), DUNE (USA), HyperK (Japan), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Absolute Scale of Neutrino Masses





$$m_2^2 = m_1^2 + \Delta m^2_{21} = m_1^2 + \Delta m^2_S$$

$$m_3^2 = m_1^2 + \Delta m^2_{31} = m_1^2 + \Delta m^2_A$$

$$m_1^2 = m_3^2 - \Delta m^2_{31} = m_3^2 + \Delta m^2_A$$

$$m_2^2 = m_1^2 + \Delta m^2_{21} \simeq m_3^2 + \Delta m^2_A$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m^2_A} \simeq 5 \times 10^{-2}\,\text{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

Tritium Beta-Decay

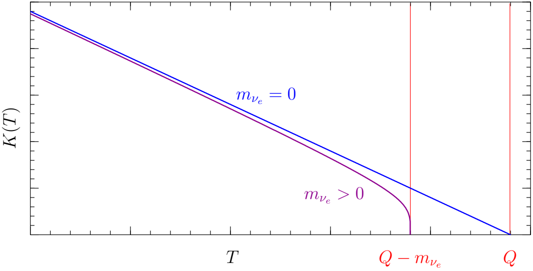
$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

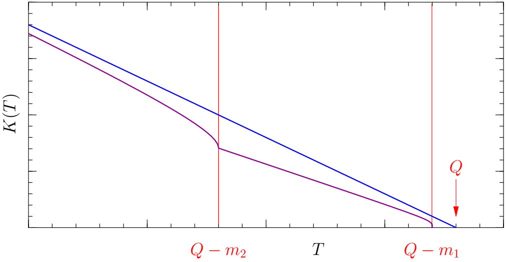
future: KATRIN

[`www.katrin.kit.edu`]

start data taking 2016?

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses ($m_k \ll Q-T$)

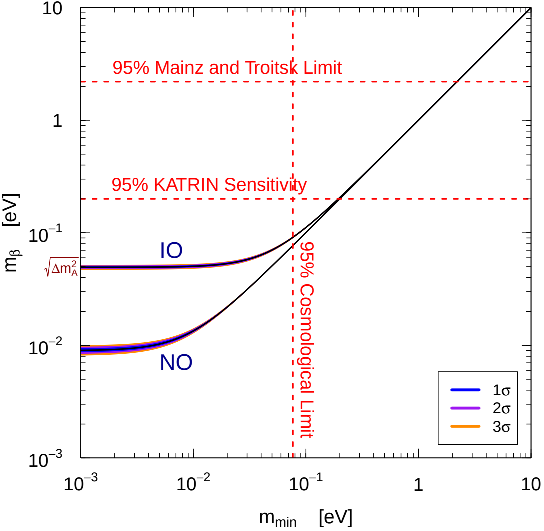
effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Neutrino Masses in β decay

$$m_\beta^2 = |U_{e1}|^2\,m_1^2 + |U_{e2}|^2\,m_2^2 + |U_{e3}|^2\,m_3^2$$



- Quasi-Degenerate:

  $$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- Inverted Hierarchy:

  $$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$$

- Normal Hierarchy:

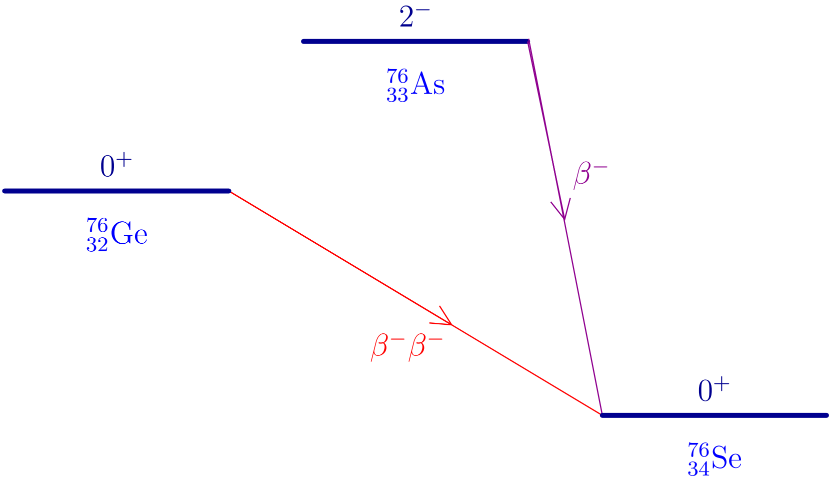
  $$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$$
  $$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\text{eV}^2$$

- $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV}$

$$\Downarrow$$

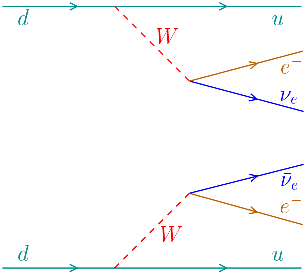
Normal Spectrum

Neutrinoless Double-Beta Decay



Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

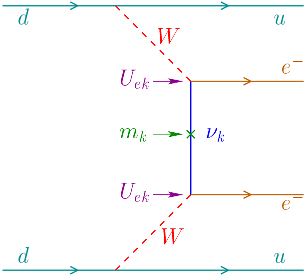


$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

Neutrinoless Double-β Decay: $\Delta L = 2$

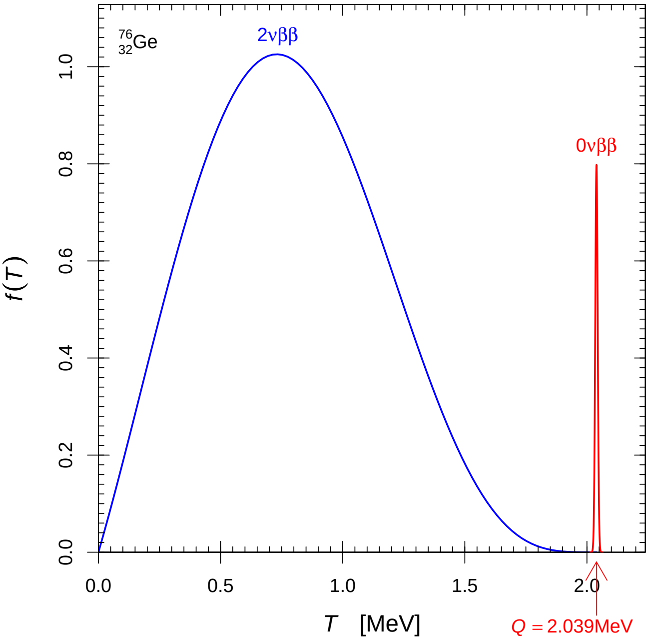


$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass

$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$$



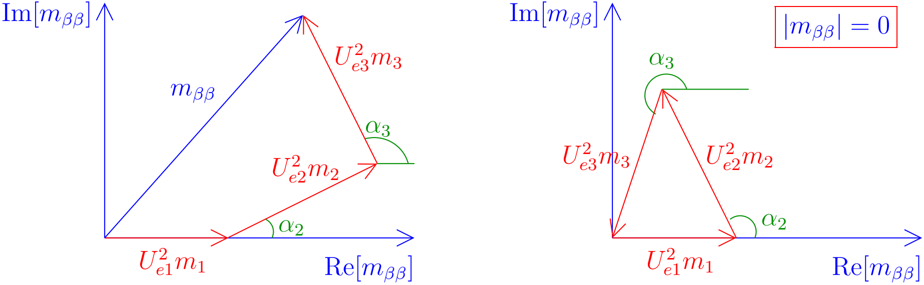
$^{76}_{32}\mathrm{Ge}$
2νββ
0νββ
f(T)
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
T [MeV]
$Q = 2.039\,\mathrm{MeV}$

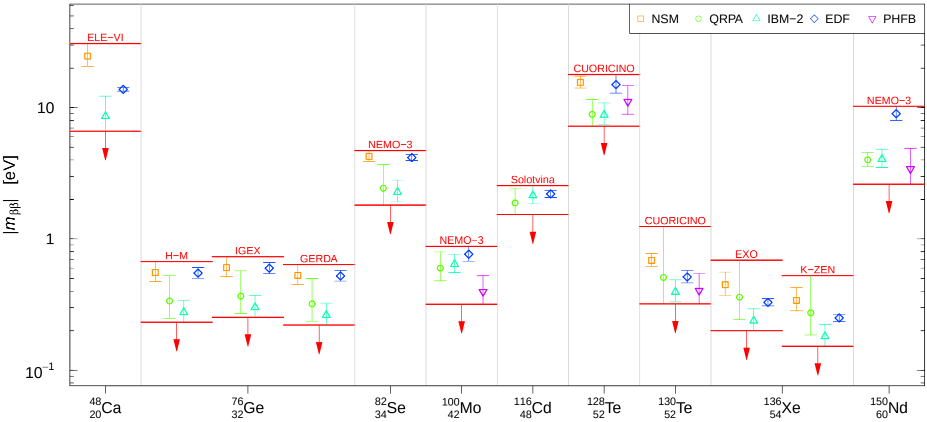
Effective Majorana Neutrino Mass

$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$ complex U_{ek} ⇒ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$

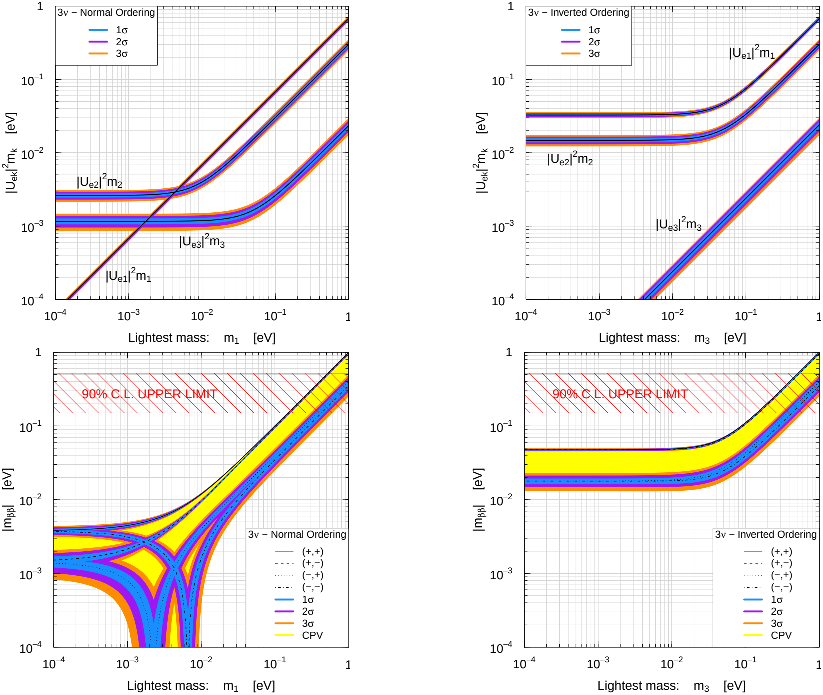




[Bilenky, Giunti, IJMPA 30 (2015) 0001]

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

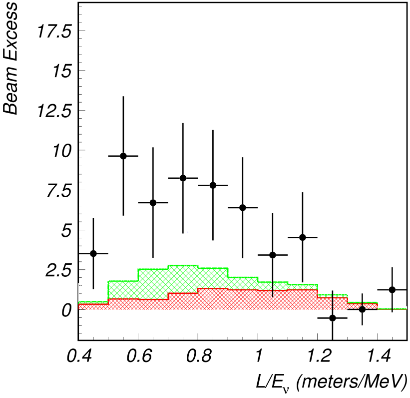
3ν − Normal Ordering
1σ
2σ
3σ
$|U_{ek}|^2 m_k$ [eV]
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
Lightest mass: m_1 [eV]
3ν − Inverted Ordering
1σ
2σ
3σ
$|U_{ek}|^2 m_k$ [eV]
$|U_{e1}|^2 m_1$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
Lightest mass: m_3 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν − Normal Ordering
(+,+)
(+,−)
(−,+)
(−,−)
1σ
2σ
3σ
CPV
Lightest mass: m_1 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν − Inverted Ordering
(+,+)
(+,−)
(−,+)
(−,−)
1σ
2σ
3σ
CPV
Lightest mass: m_3 [eV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Indications of SBL Oscillations Beyond 3ν

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad$ $L \simeq 30\,\text{m}$ $\qquad$ $20\,\text{MeV} \leq E \leq 60\,\text{MeV}$



- Well known source of $\bar{\nu}_\mu$:

$$\mu^+ \text{ at rest} \to e^+ + \nu_e + \bar{\nu}_\mu$$

- $\bar{\nu}_\mu \xrightarrow[L\simeq 30\,\text{m}]{} \bar{\nu}_e$
- Well known detection process of $\bar{\nu}_e$:

$$\bar{\nu}_e + p \to n + e^+$$

- But signal not seen by KARMEN with same method at $L \simeq 18\,\text{m}$

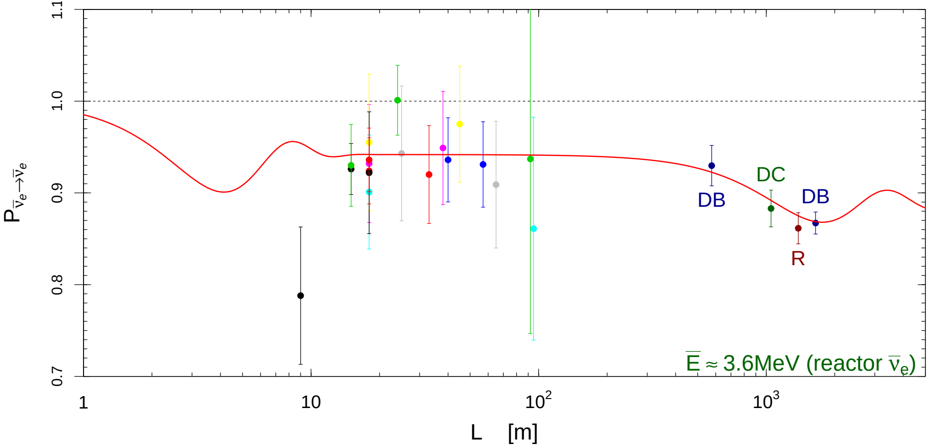
[PRD 65 (2002) 112001]

$\approx 3.8\sigma$ excess $\qquad$ $\Delta m^2 \gtrsim 0.2\,\text{eV}^2$ $\qquad$ $(\gg \Delta m^2_\text{A} \gg \Delta m^2_\text{S})$

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes [Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]



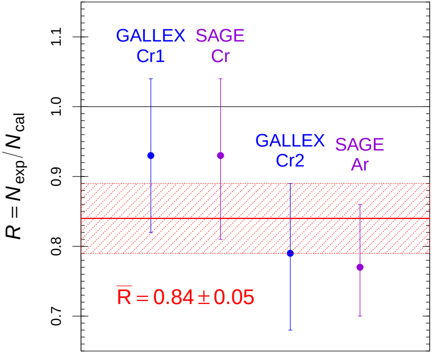
$\approx 3.1\sigma$ deficit $\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}}$

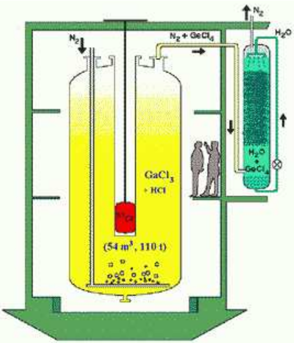
Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$





$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$ $\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

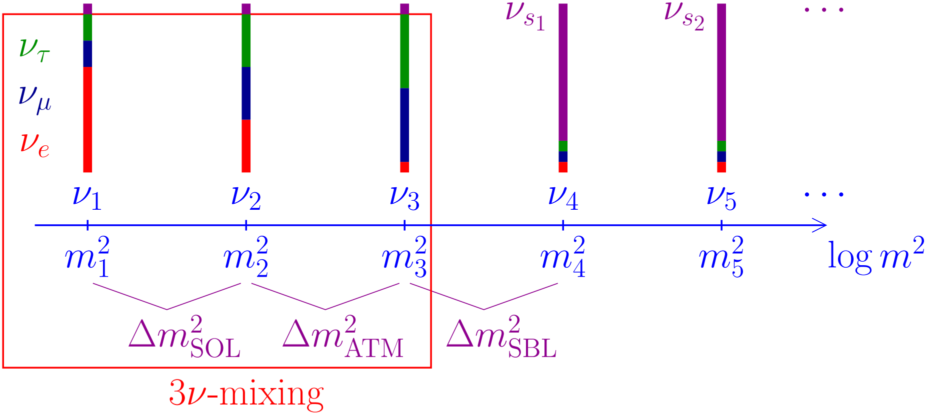
$\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}}$

$\approx 2.9\sigma$ deficit

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807]

[Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344; MPLA 22 (2007) 2499; PRD 78 (2008) 073009; PRC 83 (2011) 065504]

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Terminology:	a eV-scale sterile neutrino
means:	a eV-scale massive neutrino which is mainly sterile

New low-energy physics beyond the Standard Model!

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing: $|U_{e4}|^2 \ll 1\,,\ |U_{\mu 4}|^2 \ll 1\,,\ |U_{\tau 4}|^2 \ll 1\,,\ |U_{s4}|^2 \simeq 1$

$$U = \left(\begin{array}{ccc|c|} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{array}\right)$$

$$\uparrow$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases
- But CP violation is not observable in current SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea, Kelly, Kobach, PRD 91 (2015) 053005; Klop, Palazzo, PRD 91 (2015) 073017; Berryman, de Gouvea, Kelly, Kobach, PRD 92 (2015) 073012, Palazzo, arXiv:1509.03148] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, Giunti, PRD 87, 113004 (2013) 113004]

3+1: Appearance vs Disappearance

- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

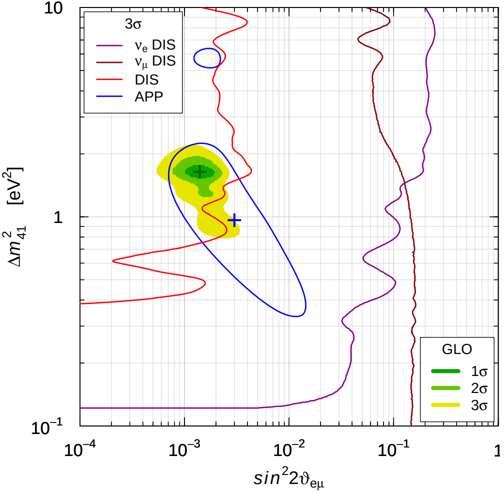
- Upper bounds on ν_e and ν_μ disappearance $\Rightarrow$ strong limit on $\nu_\mu \to \nu_e$

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, Giunti, Grimus, EPJC 1 (1998) 247]

- Similar constraint in 3+2, 3+3, . . . , $3+N_s$! [Giunti, Zavanin, MPLA 31 (2015) 1650003]

Global 3+1 Fit

[Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001]

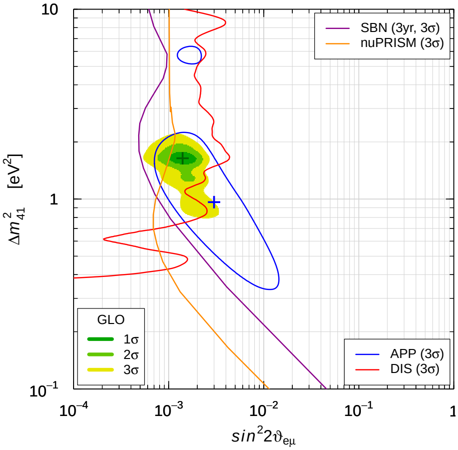


GoF = 26% PGoF = 7%

- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND ($\nu_s$), MiniBooNE (?), OPERA ($\cancel{\nu_s}$), ICARUS ($\cancel{\nu_s}$), KARMEN ($\cancel{\nu_s}$), NOMAD ($\cancel{\nu_s}$), BNL-E776 ($\cancel{\nu_s}$)
- DIS ν_e & $\bar{\nu}_e$: Reactors (ν_s), Gallium (ν_s), ν_eC ($\cancel{\nu_s}$), Solar ($\cancel{\nu_s}$)
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW ($\cancel{\nu_s}$), MINOS ($\cancel{\nu_s}$), Atmospheric ($\cancel{\nu_s}$), MiniBooNE/SciBooNE ($\cancel{\nu_s}$)

No Osc. disfavored
at $\approx 6.3\sigma$
$\Delta\chi^2/\text{NDF} = 47.7/3$

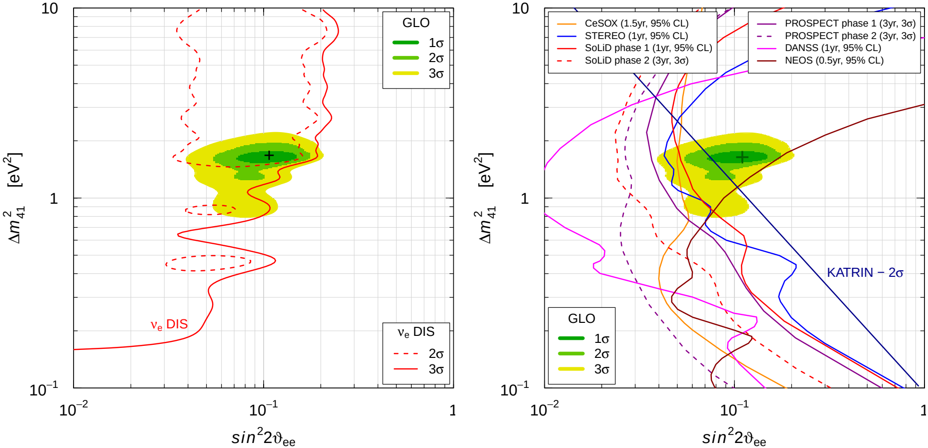
Future Experiments



SBN (FNAL, USA)
[arXiv:1503.01520]
3 Liquid Argon TPCs
LAr1-ND $L \simeq 100\,\mathrm{m}$
MicroBooNE $L \simeq 470\,\mathrm{m}$
ICARUS T600 $L \simeq 600\,\mathrm{m}$

nuPRISM (J-PARC, Japan)
[Wilking@NNN2015]
$L \simeq 1\,\mathrm{km}$
50 m tall water Cherenkov detector
$1^\circ - 4^\circ$ off-axis
can be improved with T2K ND

ν_e Disappearance



CeSOX (BOREXINO, Italy)

^{144}Ce − 100 kCi [Vivier@TAUP2015]

rate: 1% normalization uncertainty

8.5 m from detector center

KATRIN (Germany)

Tritium β decay [Mertens@TAUP2015]

STEREO (France) $L \simeq$ 8-12m [Sanchez@EPSHEP2015]

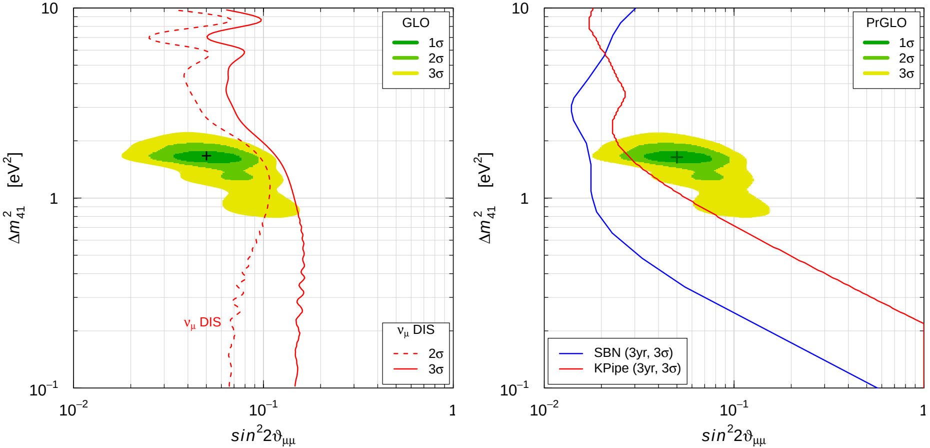
SoLid (Belgium) $L \simeq$ 5-8m [Yermia@TAUP2015]

PROSPECT (USA) $L \simeq$ 7-12m [Heeger@TAUP2015]

DANSS (Russia) $L \simeq$ 10-12m [arXiv:1412.0817]

NEOS (Korea) $L \simeq$ 25m [Oh@WIN2015]

ν_μ Disappearance



SBN (USA) [arXiv:1503.01520]

LAr1-ND $L \simeq 100$m

MicroBooNE $L \simeq 470$m

ICARUS T600 $L \simeq 600$m

KPipe (Japan) [arXiv:1510.06994]

$L \simeq 30$-150m

120 m long detector!

Conclusions

- Robust Three-Neutrino Mixing Paradigm.
  Open problems with exciting experimental program: $\vartheta_{23} \lesseqgtr 45^\circ$?, Mass Ordering, CP Violation, Absolute Mass Scale, Dirac or Majorana?
  Determination of Mass Ordering is very important!
- Theory: Why lepton mixing is not small and hierarchical as quark mixing? $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$
- Very interesting indications of light sterile neutrinos with $m_s \approx 1\,\text{eV}$:
  - LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal.
  - Reactor $\bar{\nu}_e$ disappearance.
  - Gallium ν_e disappearance.
- Many promising projects to check conclusively in a few years short-baseline ν_e and $\bar{\nu}_e$ disappearance with reactors and radioactive sources.
- Promising Fermilab program aimed at a conclusive solution of the LSND anomaly with Liquid Argon Time Projection Chamber detectors: a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-WA104).