Theory and Phenomenology of Massive Neutrinos

Part III: Phenomenology

Carlo Giunti

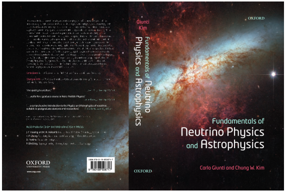
INFN, Sezione di Torino
and
Dipartimento di Fisica Teorica, Università di Torino

giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

IHEP – 23, 26, 29 September 2016

http://www.nu.to.infn.it/slides/2016/giunti-160929-ihep3.pdf



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

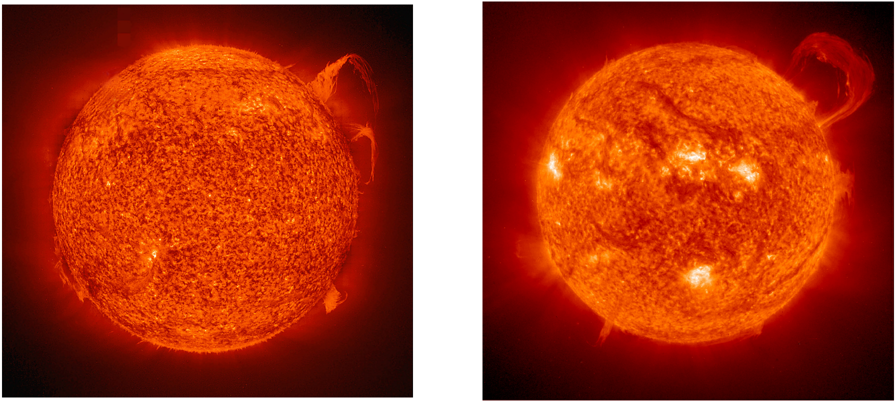
Part III: Phenomenology

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Absolute Scale of Neutrino Masses
- Light Sterile Neutrinos
- Cosmology
- Conclusions

Solar Neutrinos and KamLAND

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Absolute Scale of Neutrino Masses
- Light Sterile Neutrinos
- Cosmology
- Conclusions

The Sun



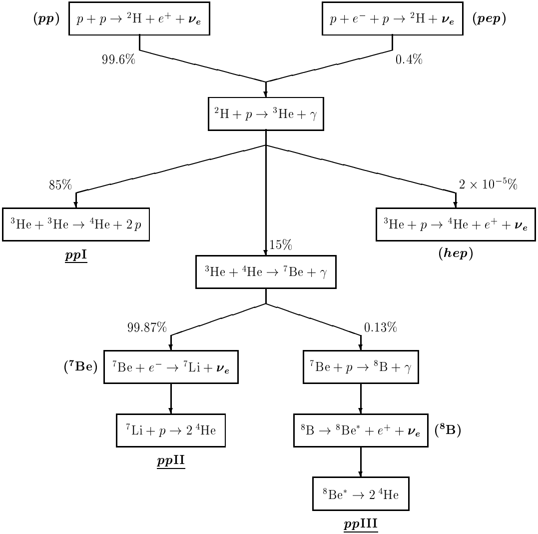
Extreme ultraviolet Imaging Telescope (EIT) 304 Å images of the Sun

emission in this spectral line (He II) shows the upper chromosphere

at a temperature of about 60,000 K

[The Solar and Heliospheric Observatory (SOHO), http://sohowww.nascom.nasa.gov/]

Standard Solar Model (SSM)



(*pp*) $p + p \to {}^{2}\mathrm{H} + e^{+} + \boldsymbol{\nu_e}$

(*pep*) $p + e^{-} + p \to {}^{2}\mathrm{H} + \boldsymbol{\nu_e}$

99.6%

0.4%

${}^{2}\mathrm{H} + p \to {}^{3}\mathrm{He} + \gamma$

85%

15%

$2 \times 10^{-5}\%$

${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \to {}^{4}\mathrm{He} + 2\,p$

***pp*I**

${}^{3}\mathrm{He} + {}^{4}\mathrm{He} \to {}^{7}\mathrm{Be} + \gamma$

${}^{3}\mathrm{He} + p \to {}^{4}\mathrm{He} + e^{+} + \boldsymbol{\nu_e}$

(*hep*)

99.87%

0.13%

(^{7}Be) ${}^{7}\mathrm{Be} + e^{-} \to {}^{7}\mathrm{Li} + \boldsymbol{\nu_e}$

${}^{7}\mathrm{Be} + p \to {}^{8}\mathrm{B} + \gamma$

${}^{7}\mathrm{Li} + p \to 2\,{}^{4}\mathrm{He}$

***pp*II**

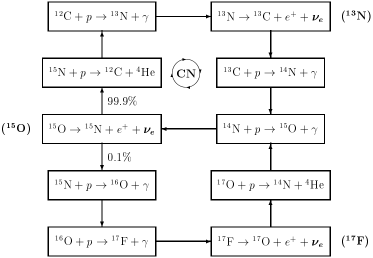
${}^{8}\mathrm{B} \to {}^{8}\mathrm{Be}^{*} + e^{+} + \boldsymbol{\nu_e}$ **(^{8}B)**

${}^{8}\mathrm{Be}^{*} \to 2\,{}^{4}\mathrm{He}$

***pp*III**

pp chain and CNO cycle

$4\,p + 2\,e^{-} \to {}^{4}\mathrm{He} + 2\,\nu_e + 26.731\ \mathrm{MeV}$



${}^{12}\mathrm{C} + p \to {}^{13}\mathrm{N} + \gamma$

${}^{13}\mathrm{N} \to {}^{13}\mathrm{C} + e^{+} + \boldsymbol{\nu_e}$ **(^{13}N)**

${}^{15}\mathrm{N} + p \to {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$

CN

${}^{13}\mathrm{C} + p \to {}^{14}\mathrm{N} + \gamma$

99.9%

(^{15}O) ${}^{15}\mathrm{O} \to {}^{15}\mathrm{N} + e^{+} + \boldsymbol{\nu_e}$

${}^{14}\mathrm{N} + p \to {}^{15}\mathrm{O} + \gamma$

0.1%

${}^{15}\mathrm{N} + p \to {}^{16}\mathrm{O} + \gamma$

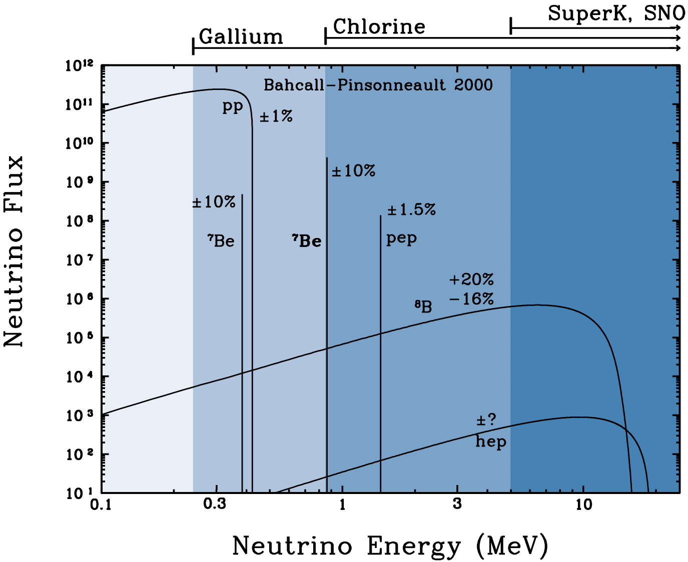
${}^{17}\mathrm{O} + p \to {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$

${}^{16}\mathrm{O} + p \to {}^{17}\mathrm{F} + \gamma$

${}^{17}\mathrm{F} \to {}^{17}\mathrm{O} + e^{+} + \boldsymbol{\nu_e}$ **(^{17}F)**

Bahcall SSMs

[J.N. Bahcall, http://www.sns.ias.edu/~jnb]



[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]

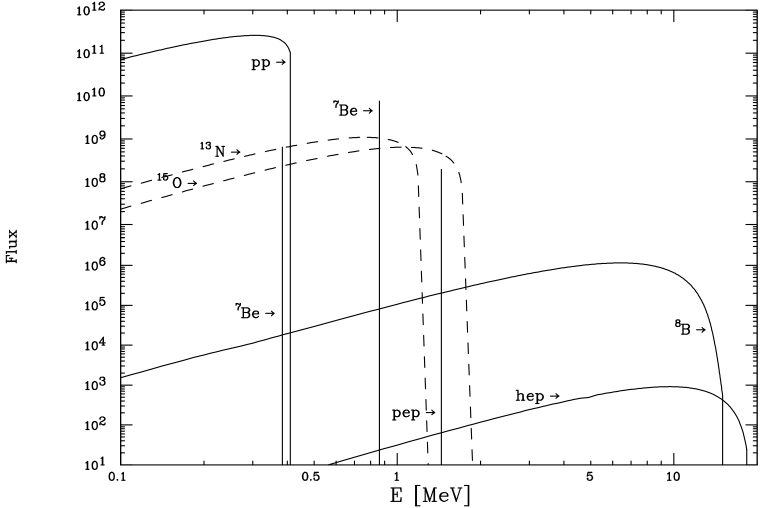


Fig 4

[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]

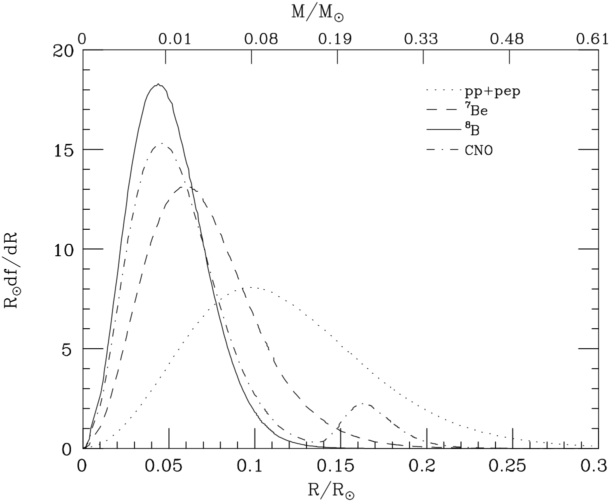
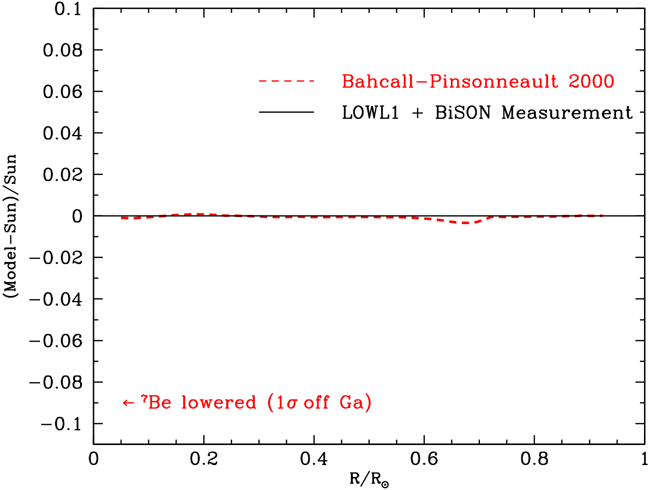


Fig. 5

[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[J.N. Bahcall, `http://www.sns.ias.edu/~jnb`]

predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between between $0.05\,R_\odot$ and $0.95\,R_\odot$ and is 0.08% for the deep interior region, $r < 0.25\,R_\odot$, in which neutrinos are produced

Homestake

$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$ [Pontecorvo (1946), Alvarez (1949)] radiochemical experiment

Homestake Gold Mine (South Dakota)

1478 m deep, 4200 m.w.e. $\Rightarrow \Phi_\mu \simeq 4\,\text{m}^{-2}\,\text{day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long (6×10^5 liters)

615 tons of tetrachloroethylene (C_2Cl_4), 2.16×10^{30} atoms of ^{37}Cl (133 tons)

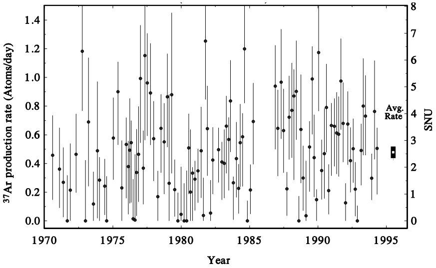
energy threshold: $E_{\text{th}}^{\text{Cl}} = 0.814\,\text{MeV} \Longrightarrow {}^{8}\text{B}\,,\ {}^{7}\text{Be}\,,\ pep\,,\ hep\,,\ {}^{13}\text{N}\,,\ {}^{15}\text{O}\,,\ {}^{17}\text{F}$

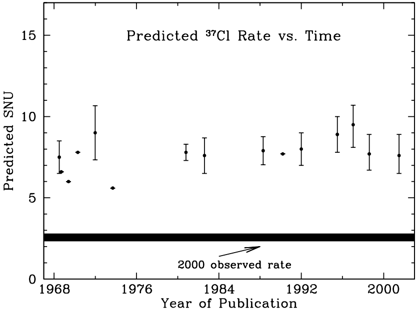
1970–1994, 108 extractions $\Longrightarrow \dfrac{R_{\text{Cl}}^{\text{exp}}}{R_{\text{Cl}}^{\text{BP00}}} = 0.34 \pm 0.03$

$R_{\text{Cl}}^{\text{exp}} = 2.56 \pm 0.23\,\text{SNU}$ [APJ 496 (1998) 505]

$1\ \text{SNU} = 10^{-36}\ \text{events atom}^{-1}\ \text{s}^{-1}$

$R_{\text{Cl}}^{\text{BP00}} = 7.6^{+1.3}_{-1.1}\,\text{SNU}$ [APJ 555 (2001) 990]





Gallium Experiments

SAGE, GALLEX, GNO

radiochemical experiments

$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ [Kuzmin (1965)]

threshold: $E_{\text{th}}^{\text{Ga}} = 0.233\,\text{MeV} \Longrightarrow pp\,,\ {}^{7}\text{Be}\,,\ {}^{8}\text{B}\,,\ pep\,,\ hep\,,\ {}^{13}\text{N}\,,\ {}^{15}\text{O}\,,\ {}^{17}\text{F}$

SAGE+GALLEX+GNO $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{exp}}}{R_{\text{Ga}}^{\text{BP00}}} = 0.56 \pm 0.03$

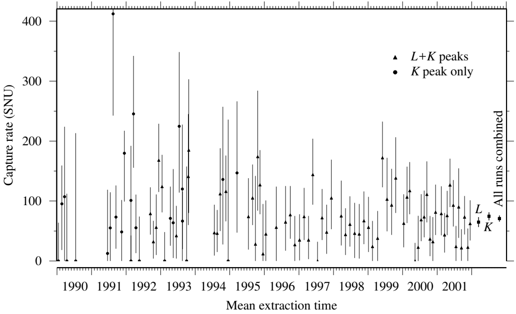
$R_{\text{Ga}}^{\text{exp}} = 72.4 \pm 4.7\,\text{SNU}$ $R_{\text{Ga}}^{\text{BP00}} = 128^{+9}_{-7}\,\text{SNU}$ [APJ 555 (2001) 990]

SAGE: Soviet-American Gallium Experiment

Baksan Neutrino Observatory, northern Caucasus

50 tons of metallic ^{71}Ga, 2000 m deep, 4700 m.w.e. $\Rightarrow \Phi_{\mu} \simeq 2.6\,\text{m}^{-2}\,\text{day}^{-1}$

1990 – 2001 $\Longrightarrow$ $\dfrac{R_{\text{Ga}}^{\text{SAGE}}}{R_{\text{Ga}}^{\text{BP00}}} = 0.54 \pm 0.05$ [JETP 95 (2002) 181, astro-ph/0204245]



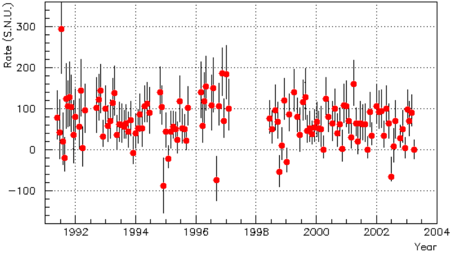
GALLEX + GNO

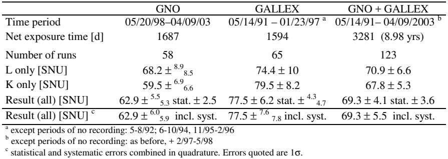
Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

30.3 tons of gallium in 101 tons of gallium chloride ($GaCl_3$-HCl) solution

May 1991 – Jan 1997 ⟹ GALLEX: GALLium EXperiment

May 1998 – Apr 2003 ⟹ GNO: Gallium Neutrino Observatory





	GNO	GALLEX	GNO + GALLEX
Time period	05/20/98–04/09/03	05/14/91 – 01/23/97 [a]	05/14/91– 04/09/2003 [b]
Net exposure time [d]	1687	1594	3281 (8.98 yrs)
Number of runs	58	65	123
L only [SNU]	$68.2 \pm^{8.9}_{8.5}$	74.4 ± 10	70.9 ± 6.6
K only [SNU]	$59.5 \pm^{6.9}_{6.6}$	79.5 ± 8.2	67.8 ± 5.3
Result (all) [SNU]	$62.9 \pm^{5.5}_{5.3}$ stat. ± 2.5	77.5 ± 6.2 stat. $\pm^{4.3}_{4.7}$	69.3 ± 4.1 stat. ± 3.6
Result (all) [SNU] [c]	$62.9 \pm^{6.0}_{5.9}$ incl. syst.	$77.5 \pm^{7.6}_{7.8}$ incl. syst.	69.3 ± 5.5 incl. syst.

[a] except periods of no recording: 5-8/92; 6-10/94, 11/95-2/96

[b] except periods of no recording: as before, + 2/97-5/98

[c] statistical and systematic errors combined in quadrature. Errors quoted are 1σ.

$$\frac{R_{\text{Ga}}^{\text{GALLEX+GNO}}}{R_{\text{Ga}}^{\text{BP00}}} = 0.53 \pm 0.04$$

[PLB 616 (2005) 174, hep-ex/0504037]

Kamiokande

water Cherenkov detector $\nu + e^- \to \nu + e^-$

Sensitive to ν_e, ν_μ, ν_τ, but $\sigma(\nu_e) \simeq 6\,\sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 6.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}\,,\ hep$

Jan 1987 – Feb 1995 (2079 days)

$$\frac{R_{\nu e}^{\text{Kam}}}{R_{\nu e}^{\text{BP00}}} = 0.55 \pm 0.08$$

[PRL 77 (1996) 1683]

Super-Kamiokande

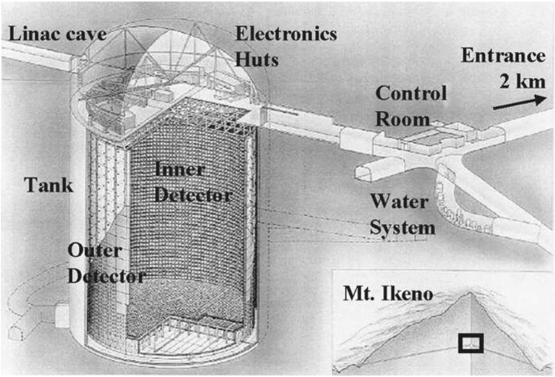
continuation of Kamiokande

50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold: $E_{\text{th}}^{\text{Kam}} \simeq 4.75\,\text{MeV} \Longrightarrow {}^{8}\text{B}\,,\ \textit{hep}$

1996 – 2001 (1496 days)

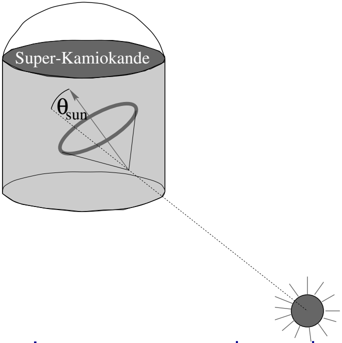
$$\frac{R_{\nu e}^{\text{SK}}}{R_{\nu e}^{\text{BP00}}} = 0.465 \pm 0.015$$ [SK, PLB 539 (2002) 179]

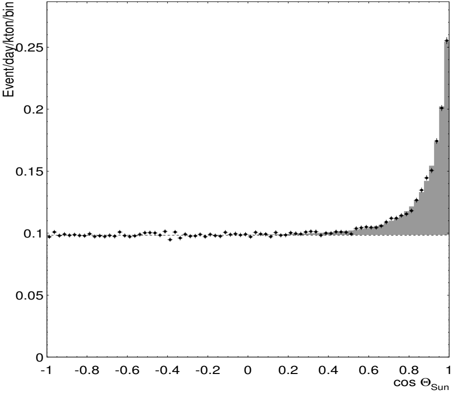


the Super-Kamiokande underground water Cherenkov detector
located near Higashi-Mozumi, Gifu Prefecture, Japan
access is via a 2 km long truck tunnel

[hep-ex/0212035]

Super-Kamiokande
$\cos\theta_{\text{sun}}$ distribution



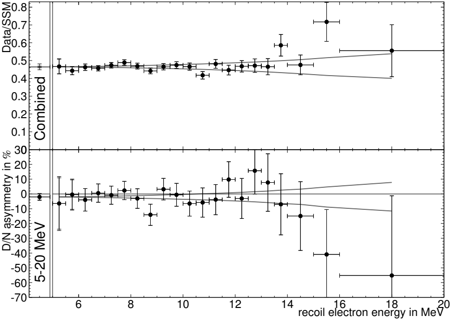


the points represent observed data, the histogram shows the best-fit signal (shaded) plus background, the horizontal dashed line shows the estimated background

the peak at $\cos\theta_{\text{sun}} = 1$ is due to solar neutrinos

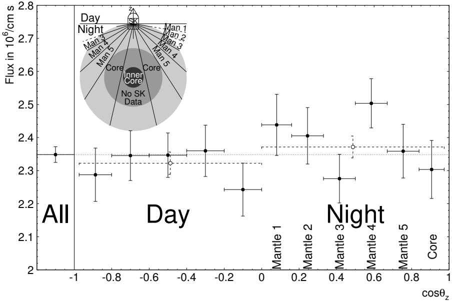
[hep-ex/0208004]

Super-Kamiokande energy spectrum
normalized to BP2000 SSM



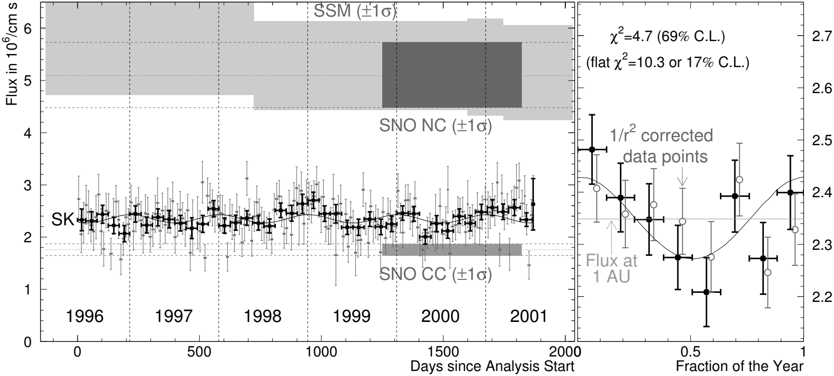
Day-Night asymmetry
as a function of energy

solar zenith angle (θ_z) dependence
of Super-Kamiokande data



[hep-ex/0208004]

Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days.

The black data points are measured every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points by subtracting the expected 7% variation.

[hep-ex/0208004]

SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of D_2O, 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.

CC: $\nu_e + d \to p + p + e^-$
NC: $\nu + d \to p + n + \nu$
ES: $\nu + e^- \to \nu + e^-$

$$\left.\begin{array}{ll} \text{CC threshold:} & E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2\,\text{MeV} \\ \text{NC threshold:} & E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2\,\text{MeV} \\ \text{ES threshold:} & E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0\,\text{MeV} \end{array}\right\} \Longrightarrow {}^{8}\text{B},\ hep$$

D_2O phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{BP00}}} = 0.35 \pm 0.02 \quad \frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{BP00}}} = 1.01 \pm 0.13 \quad \frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{BP00}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

NaCl phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{BP00}}} = 0.31 \pm 0.02 \quad \frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{BP00}}} = 1.03 \pm 0.09 \quad \frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{BP00}}} = 0.44 \pm 0.06$$

[PRL 92 (2004) 181301]

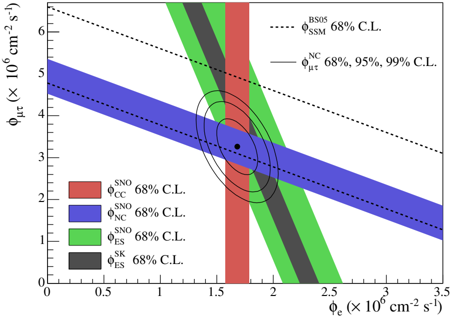
^{3}He phase: 2004 – 2006

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{BP00}}} = 0.33 \pm 0.02 \quad \frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{BP00}}} = 1.10 \pm 0.10 \quad \frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{BP00}}} = 0.35 \pm 0.05$$

[PRL 101 (2008) 111301]

$$\Phi^{\rm SNO}_{\nu_e} = 1.76 \pm 0.11 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$

$$\Phi^{\rm SNO}_{\nu_\mu,\nu_\tau} = 5.41 \pm 0.66 \times 10^6\,{\rm cm}^{-2}\,{\rm s}^{-1}$$



SNO solved
solar neutrino problem

$\Downarrow$

Neutrino Physics
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

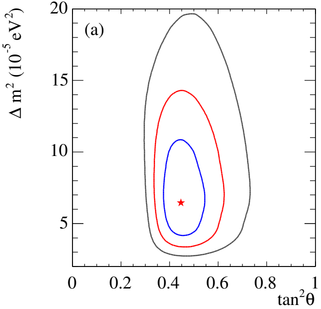
$\nu_e \to \nu_\mu, \nu_\tau$ oscillations

$\Downarrow$

Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5}\,{\rm eV}^2$$

$$\tan^2\vartheta \simeq 0.45$$



[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

average distance from reactors: 180 km
6.7% of flux from one reactor at 88 km
79% of flux from 26 reactors at 138–214 km
14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

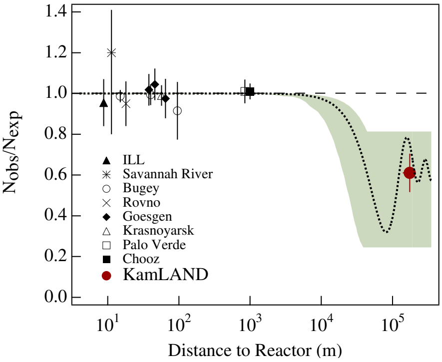
data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$
expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$
observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

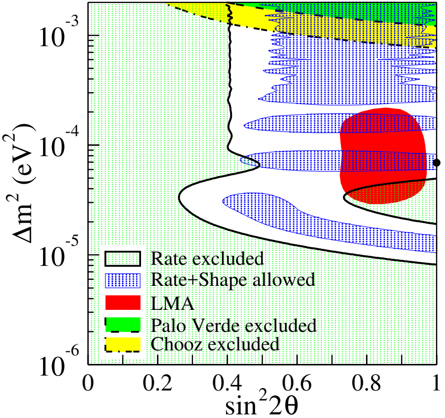
99.95% C.L. evidence
of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



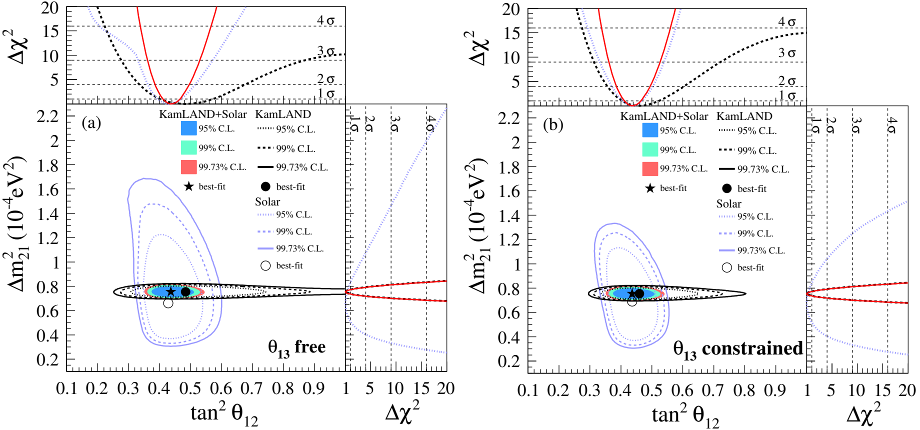
Shade: 95% C.L. LMA

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$



95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

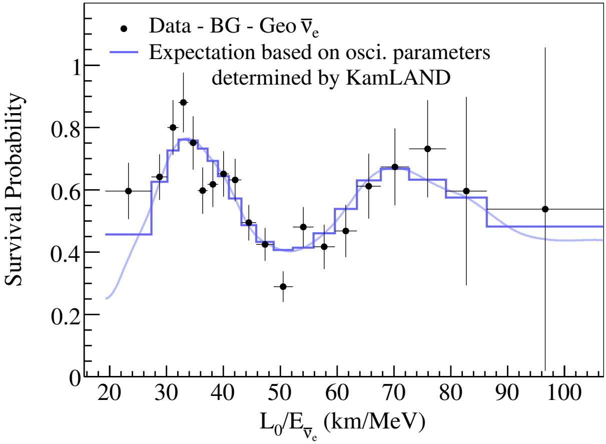


$$\Delta m^2_{21} = 7.53^{+0.19}_{-0.18} \times 10^{-5}\,\text{eV}^2 \qquad \Delta m^2_{21} = 7.53 \pm 0.18 \times 10^{-5}\,\text{eV}^2$$

$$\tan^2\vartheta_{12} = 0.437^{+0.029}_{-0.026} \qquad \tan^2\vartheta_{12} = 0.436^{+0.029}_{-0.025}$$

$$\sin^2\vartheta_{13} = 0.023 \pm 0.015 \qquad \sin^2\vartheta_{13} = 0.023 \pm 0.002$$

[KamLAND, PRD 88 (2013) 033001]



[KamLAND, PRL 100 (2008) 221803]

LMA Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\mathrm{eV}^2 \quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{c}}\right) \cos 2\vartheta^0_{\mathrm{M}} \, \cos 2\vartheta}$$

$$P_{\mathrm{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left|\frac{d\ln A}{dx}\right|_{\mathrm{R}}} \qquad F = 1 - \tan^2\vartheta$$

$$A_{\mathrm{CC}} \simeq 2\sqrt{2} E G_{\mathrm{F}} N_e^{\mathrm{c}} \exp\left(-\frac{x}{x_0}\right) \quad \Longrightarrow \quad \left|\frac{d\ln A}{dx}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15}\,\mathrm{eV}$$

$$\tan^2\vartheta \simeq 0.4 \quad \Longrightarrow \quad \sin^2 2\vartheta \simeq 0.82\,,\ \cos 2\vartheta \simeq 0.43 \qquad \gamma \simeq 2 \times 10^4 \left(\frac{E}{\mathrm{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \quad \Longrightarrow \quad P_{\mathrm{c}} \ll 1 \quad \Longrightarrow \quad \boxed{\overline{P}^{\mathrm{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2} \cos 2\vartheta^0_{\mathrm{M}} \, \cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta_{\mathrm{M}}^{0} = \frac{\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter [Bahcall, Peña-Garay, JHEP 0311 (2003) 004]

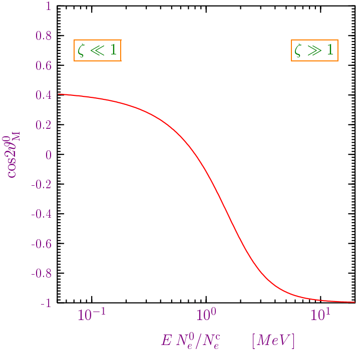
$$\zeta = \frac{A_{\mathrm{CC}}^{0}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}}\right) \left(\frac{N_e^0}{N_e^{\mathrm{c}}}\right)$$

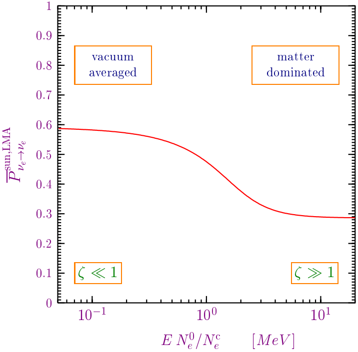
$$\zeta \ll 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \vartheta \implies \overline{P}_{\nu_e\to\nu_e}^{\mathrm{sun}} \simeq 1 - \tfrac{1}{2}\sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \pi/2 \implies \overline{P}_{\nu_e\to\nu_e}^{\mathrm{sun}} \simeq \sin^2\vartheta$$

matter dominated survival probability



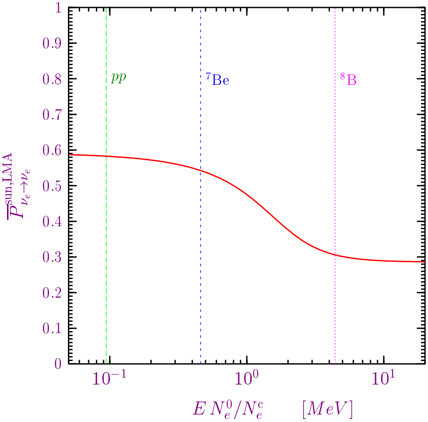


$$\zeta = \frac{A^0_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N^0_e}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}} \right) \left(\frac{N^0_e}{N^{\mathrm{c}}_e} \right)$$

$$\langle E \rangle_{pp} \simeq 0.27\,\mathrm{MeV}\,, \ \langle r_0 \rangle_{pp} \simeq 0.1\,R_\odot \quad \Longrightarrow \quad \langle E\,N^0_e / N^{\mathrm{c}}_e \rangle_{pp} \simeq 0.094\,\mathrm{MeV}$$

$$E_{^7\mathrm{Be}} \simeq 0.86\,\mathrm{MeV}\,, \ \langle r_0 \rangle_{^7\mathrm{Be}} \simeq 0.06\,R_\odot \quad \Longrightarrow \quad \langle E\,N^0_e / N^{\mathrm{c}}_e \rangle_{^7\mathrm{Be}} \simeq 0.46\,\mathrm{MeV}$$

$$\langle E \rangle_{^8\mathrm{B}} \simeq 6.7\,\mathrm{MeV}\,, \ \langle r_0 \rangle_{^8\mathrm{B}} \simeq 0.04\,R_\odot \quad \Longrightarrow \quad \langle E\,N^0_e / N^{\mathrm{c}}_e \rangle_{^8\mathrm{B}} \simeq 4.4\,\mathrm{MeV}$$

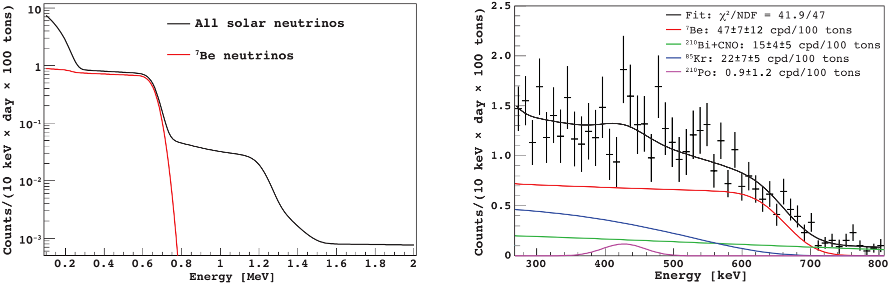


BOREXino

[BOREXino, PLB 658 (2008) 101]

Real-time measurement of ^{7}Be solar neutrinos (0.862 MeV)

$$\nu + e \to \nu + e \qquad E = 0.862\,\text{MeV} \quad \Longrightarrow \quad \sigma_{\nu_e} \simeq 5.5\,\sigma_{\nu_\mu,\nu_\tau}$$



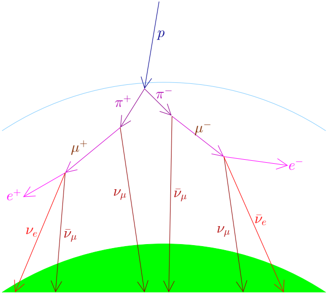
$$n_{\text{the}}^{\text{no-osc}} = 75 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1} \qquad n_{\text{exp}} = 47 \pm 7 \pm 12\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$$

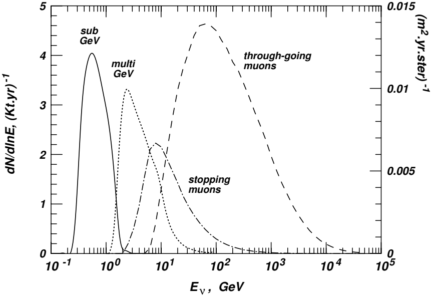
$$n_{\text{the}}^{\text{osc}} = 49 \pm 4\,\text{day}^{-1}\,(100\,\text{tons})^{-1}$$

Atmospheric and LBL Oscillation Experiments

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Absolute Scale of Neutrino Masses
- Light Sterile Neutrinos
- Cosmology
- Conclusions

Atmospheric Neutrinos





$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

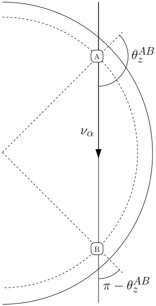
$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]

Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z^{AB}) \qquad \phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z) = \phi^{(A)}_{\nu_\alpha}(\pi - \theta_z)$$

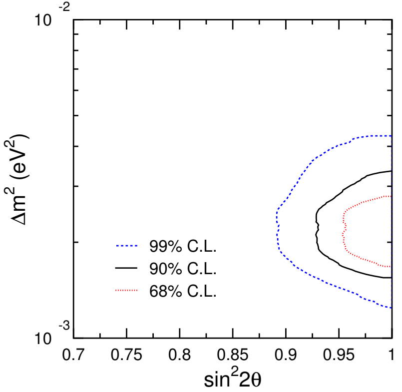
(December 1998)

$$\boxed{A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}} \right) = -0.296 \pm 0.048 \pm 0.01}$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



$$\underline{\nu_\mu \to \nu_\tau}$$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20$events/yr
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$$N^{\text{the}}_{\nu_\tau} = 78 \pm 26 \; @ \; \Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$$

$$\boxed{N^{\text{exp}}_{\nu_\tau} = 138^{+50}_{-58}}$$

$$N_{\nu_\tau} > 0 \quad @ \quad 2.4\sigma$$

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

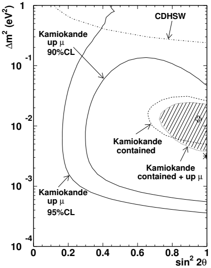
Check: OPERA ($\nu_\mu \to \nu_\tau$)

CERN to Gran Sasso (CNGS)

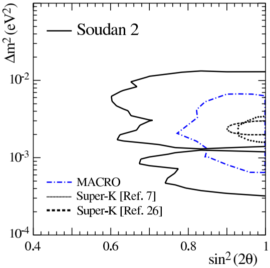
$L \simeq 732\,\text{km} \qquad \langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

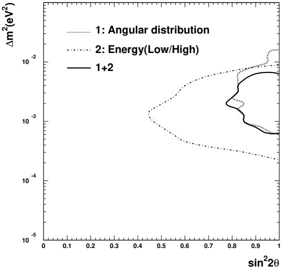
Kamiokande, Soudan-2, MACRO and MINOS



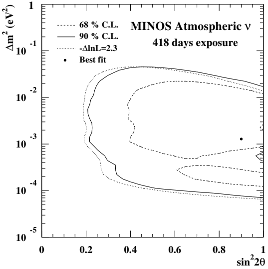
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



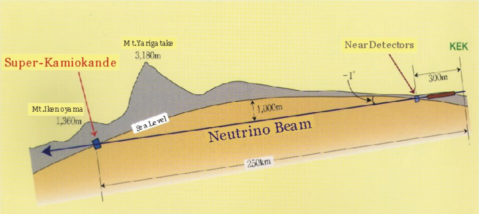
[MACRO, hep-ex/0304037]



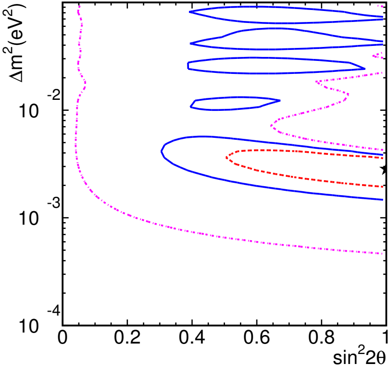
[MINOS, hep-ex/0512036]

K2K

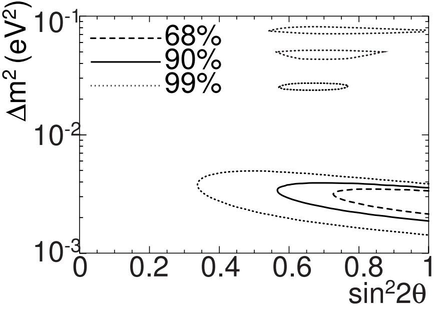
confirmation of atmospheric allowed region (June 2002)



KEK to Kamioka
(Super-Kamiokande)
250 km
$\nu_\mu \to \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]



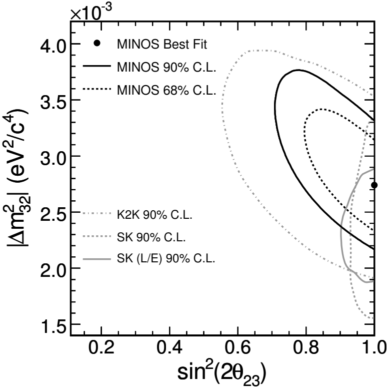
[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

MINOS

May 2005 – Feb 2006

`http://www-numi.fnal.gov/`

Near Detector: 1 km

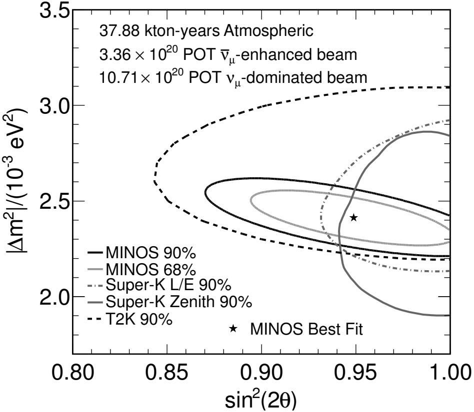


$$\nu_\mu \to \nu_\mu$$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\vartheta > 0.87 \text{ @ } 68\% CL$$

[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]

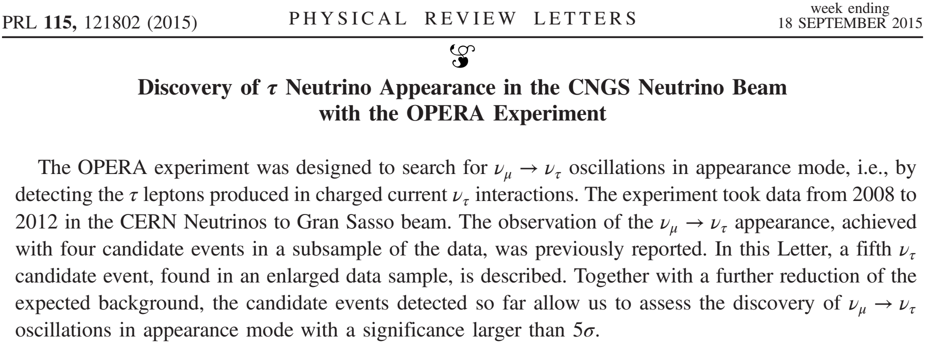


$$|\Delta m^2_{31}| = \left(2.41^{+0.09}_{-0.10}\right) \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\vartheta_{23} = 0.950^{+0.035}_{-0.036}$$

[MINOS, PRL 110 (2013) 251801]

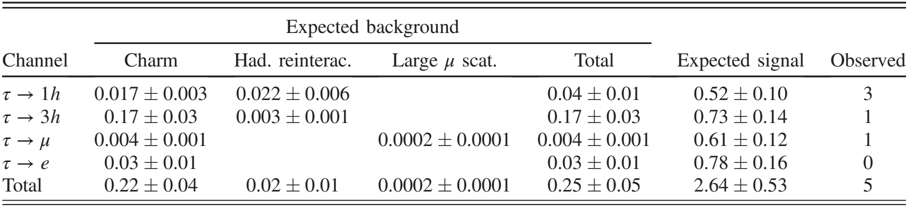
OPERA



PRL **115,** 121802 (2015) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2015

Discovery of τ Neutrino Appearance in the CNGS Neutrino Beam with the OPERA Experiment

The OPERA experiment was designed to search for $\nu_\mu \to \nu_\tau$ oscillations in appearance mode, i.e., by detecting the τ leptons produced in charged current ν_τ interactions. The experiment took data from 2008 to 2012 in the CERN Neutrinos to Gran Sasso beam. The observation of the $\nu_\mu \to \nu_\tau$ appearance, achieved with four candidate events in a subsample of the data, was previously reported. In this Letter, a fifth ν_τ candidate event, found in an enlarged data sample, is described. Together with a further reduction of the expected background, the candidate events detected so far allow us to assess the discovery of $\nu_\mu \to \nu_\tau$ oscillations in appearance mode with a significance larger than 5σ.



	Expected background					
Channel	Charm	Had. reinterac.	Large μ scat.	Total	Expected signal	Observed
$\tau \to 1h$	0.017 ± 0.003	0.022 ± 0.006		0.04 ± 0.01	0.52 ± 0.10	3
$\tau \to 3h$	0.17 ± 0.03	0.003 ± 0.001		0.17 ± 0.03	0.73 ± 0.14	1
$\tau \to \mu$	0.004 ± 0.001		0.0002 ± 0.0001	0.004 ± 0.001	0.61 ± 0.12	1
$\tau \to e$	0.03 ± 0.01			0.03 ± 0.01	0.78 ± 0.16	0
Total	0.22 ± 0.04	0.02 ± 0.01	0.0002 ± 0.0001	0.25 ± 0.05	2.64 ± 0.53	5

Experimental Evidences of Neutrino Oscillations

Solar

$\nu_e \to \nu_\mu, \nu_\tau$

VLBL Reactor

$\bar{\nu}_e$ disappearance

(SNO, BOREXino
Super-Kamiokande
GALLEX/GNO, SAGE
Homestake, Kamiokande)

(KamLAND)

$$\rightarrow \begin{cases} \Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{S}} = \sin^2\vartheta_{12} \simeq 0.30 \end{cases}$$

Atmospheric

$\nu_\mu \to \nu_\tau$

LBL Accelerator

ν_μ disappearance

LBL Accelerator

$\nu_\mu \to \nu_\tau$

(Super-Kamiokande
Kamiokande, IMB
MACRO, Soudan-2)

(K2K, MINOS
T2K, NOνA)

(Opera)

$$\rightarrow \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31}| \simeq 2.4 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

LBL Accelerator

$\nu_\mu \to \nu_e$

LBL Reactor

$\bar{\nu}_e$ disappearance

(T2K, MINOS, NOνA)

(Daya Bay, RENO
Double Chooz)

$$\rightarrow \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31}| \\ \sin^2\vartheta_{13} \simeq 0.023 \end{cases}$$

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS

- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

Three-Neutrino Mixing Around 2010

$$U = \underset{\vartheta_{23} \simeq \vartheta_{\text{ATM}}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \underset{\vartheta_{12} \simeq \vartheta_{\text{SOL}}}{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underset{\beta\beta_{0\nu}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\Delta m^2_{21} = \left(7.65^{+0.23}_{-0.20}\right) \times 10^{-5}\,\text{eV}^2 \qquad |\Delta m^2_{31}| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2\vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

$$\sin^2\vartheta_{13} < 0.035 \quad (90\%\ \text{C.L.})$$

[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]

Small ϑ_{13}

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

$$U = \begin{array}{r} \text{SOL} \rightarrow \\ \\ \\ \end{array} \begin{pmatrix} \boxed{U_{e1} \quad U_{e2}} & \boxed{U_{e3}} \\ U_{\mu1} \quad U_{\mu2} & U_{\mu3} \\ U_{\tau1} \quad U_{\tau2} & U_{\tau3} \end{pmatrix}$$

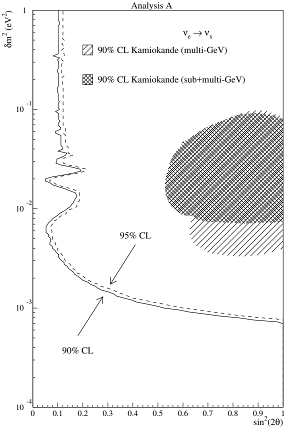
$$\begin{array}{r} \uparrow \\ \text{ATM \& LBL} \end{array}$$

$$\text{CHOOZ:} \begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$

[Bilenky, Giunti, PLB 444 (1998) 379]



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

Effective ATM and LBL Oscillation Probabilities

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i m_k^2 L / 2E} \right|^2 * \left| e^{i m_1^2 L / 2E} \right|^2$$

$$= \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}$$

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta} &= \left|\delta_{\alpha\beta} - U_{\alpha 3}^* U_{\beta 3}\left[1-\exp\left(-i\frac{\Delta m^2_{31}L}{2E}\right)\right]\right|^2 \\ &= \delta_{\alpha\beta} + |U_{\alpha 3}|^2|U_{\beta 3}|^2\left(2-2\cos\frac{\Delta m^2_{31}L}{2E}\right) \\ &\qquad - 2\delta_{\alpha\beta}|U_{\alpha 3}|^2\left(1-\cos\frac{\Delta m^2_{31}L}{2E}\right) \\ &= \delta_{\alpha\beta} - 2|U_{\alpha 3}|^2\left(\delta_{\alpha\beta}-|U_{\beta 3}|^2\right)\left(1-\cos\frac{\Delta m^2_{31}L}{2E}\right) \\ &= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2\left(\delta_{\alpha\beta}-|U_{\beta 3}|^2\right)\sin^2\frac{\Delta m^2_{31}L}{4E} \end{aligned}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\beta} = 4|U_{\alpha 3}|^2|U_{\beta 3}|^2\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha\to\nu_\alpha} = 1 - 4|U_{\alpha 3}|^2\left(1-|U_{\alpha 3}|^2\right)\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \qquad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \left(\begin{array}{cc|c|} \cline{3-3} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \\ \cline{3-3} \end{array}\right)$$

$\uparrow$

LBL

$$\sin^2 2\vartheta_{ee} \ll 1$$

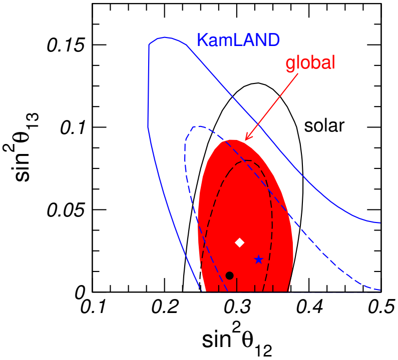
$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

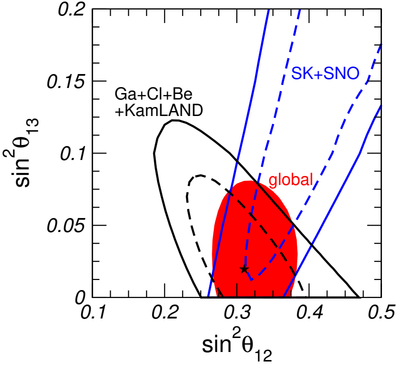
2008 Hint of $\sin^2\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$\sin^2\vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]



[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]



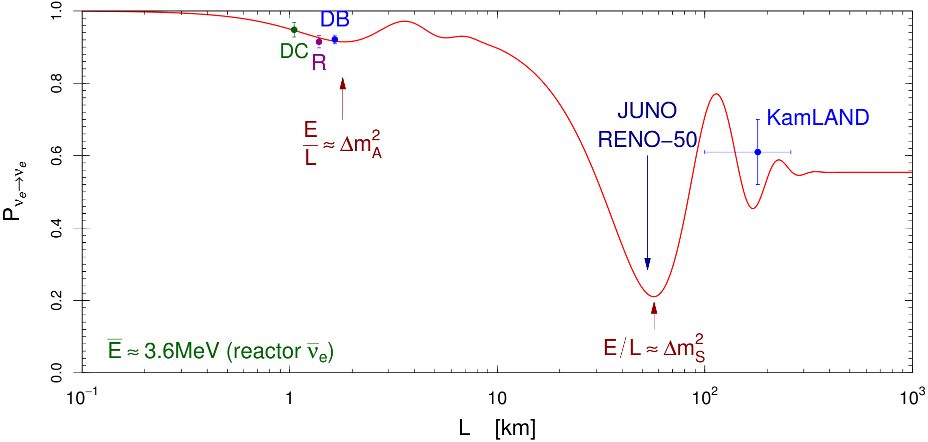
[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

$$P_{\overset{(-)}{\nu_e}\to\overset{(-)}{\nu_e}} \simeq \begin{cases} \left(1-\sin^2\vartheta_{13}\right)^2\left(1-0.5\sin^2\vartheta_{12}\right) & \text{SOL low-energy \& KamLAND} \\ \left(1-\sin^2\vartheta_{13}\right)^2\sin^2\vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

Measurements of ϑ_{13}

$0.03\,(0.04) < \sin^2 2\vartheta_{13} < 0.28\,(0.34)$	T2K, arXiv:1106.2822 (90% CL)
$\sin^2 2\vartheta_{13} = 0.041^{+0.047}_{-0.031}\,(0.079^{+0.071}_{-0.053})$	MINOS, arXiv:1108.0015
$\sin^2 \vartheta_{13} = 0.022 \pm 0.013$	Double Chooz, arXiv:1112.6353
$\sin^2 \vartheta_{13} = 0.024 \pm 0.004$	Daya Bay, arXiv:1203.1669 (6σ!)
$\sin^2 \vartheta_{13} = 0.029 \pm 0.006$	RENO, arXiv:1204.0626

$\sin^2 \vartheta_{13} > 0 \Longrightarrow$ CP violation, matter effects, mass ordering



$P_{\nu_e \to \nu_e}$
1.0
0.8
0.6
0.4
0.2
0.0
10^{-1}
1
10
10^2
10^3
L [km]
DB
DC
R
$\frac{E}{L} \approx \Delta m^2_A$
JUNO
RENO−50
KamLAND
$E/L \approx \Delta m^2_S$
$\overline{E} \approx 3.6\text{MeV}$ (reactor $\bar{\nu}_e$)

Three-Neutrino Mixing Around 2015

$$\Delta m^2_{\mathrm{S}} = \Delta m^2_{21} \simeq 7.5 \pm 0.3 \times 10^{-5}\,\mathrm{eV}^2$$ uncertainty $\simeq 3\%$

$$\Delta m^2_{\mathrm{A}} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \pm 0.1 \times 10^{-3}\,\mathrm{eV}^2$$ uncertainty $\simeq 4\%$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$\vartheta_{23} = \vartheta_{\mathrm{A}}$

$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$

$P_{\mathrm{osc}} \propto \sin^2 2\vartheta_{23}$

maximal and flat

at $\vartheta_{23} = 45^\circ$

Daya Bay, RENO

Double Chooz

T2K, MINOS

$\sin^2\vartheta_{13} \simeq 0.023 \pm 0.002$

$\boxed{\delta_{13} \approx 3\pi/2?}$

$\vartheta_{12} = \vartheta_{\mathrm{S}}$

$\sin^2\vartheta_{12} \simeq 0.30 \pm 0.01$

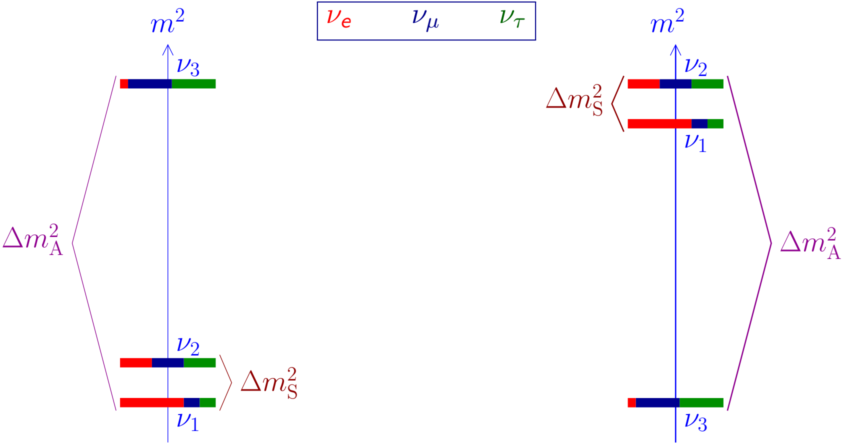
$\beta\beta_{0\nu}$

$$\frac{\delta \sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \approx 40\% \qquad \frac{\delta \sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \approx 10\% \qquad \frac{\delta \sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \approx 5\%$$

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), PINGU (Antarctica), ORCA (EU), INO (India), ...
- Mass Ordering ?
  - NOνA (USA), JUNO (China), RENO-50 (Korea), PINGU (Antarctica), ORCA (EU), INO (India), ...
- CP violation ? $\delta_{13} \approx 3\pi/2$?
  - T2K (Japan), NOνA (USA), DUNE (USA), HyperK (Japan), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

Determination of Mass Ordering

1. Matter Effects: Atmospheric (PINGU, ORCA), Long-Baseline, Supernova Experiments
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} > 0$ NO
   - $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} < 0$ IO
2. Phase Difference: Reactor $\bar{\nu}_e \to \bar{\nu}_e$ (JUNO, RENO-50)

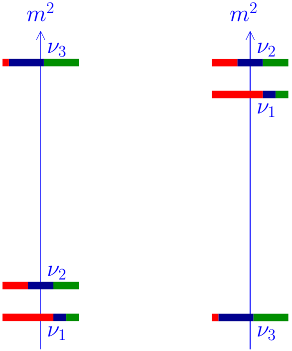
Normal Ordering

$|\Delta m^2_{31}|$

$\shortparallel$

$|\Delta m^2_{32}| + |\Delta m^2_{21}|$

$|\Delta m^2_{31}| > |\Delta m^2_{32}|$



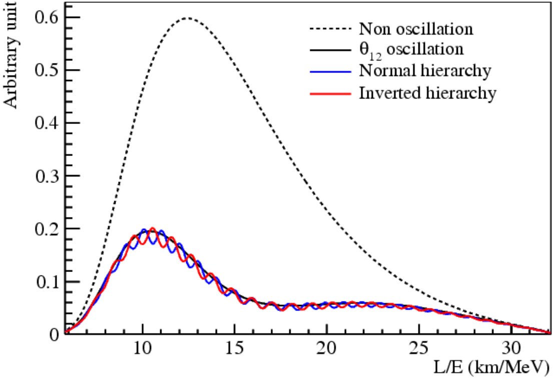
Inverted Ordering

$|\Delta m^2_{31}|$

$\shortparallel$

$|\Delta m^2_{32}| - |\Delta m^2_{21}|$

$|\Delta m^2_{31}| < |\Delta m^2_{32}|$



Neutrino Physics with JUNO, arXiv:1507.05613

$$\begin{aligned} P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} = 1 & - \cos^4 \vartheta_{13} \sin^2 2\vartheta_{12} \sin^2 \left(\Delta m^2_{21} L / 4E \right) \\ & - \cos^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 \left(\Delta m^2_{31} L / 4E \right) \\ & - \sin^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 \left(\Delta m^2_{32} L / 4E \right) \end{aligned}$$

[Petcov, Piai, PLB 533 (2002) 94; Choubey, Petcov, Piai, PRD 68 (2003) 113006; Learned, Dye, Pakvasa, Svoboda, PRD 78 (2008) 071302; Zhan, Wang, Cao, Wen, PRD 78 (2008) 111103, PRD 79 (2009) 073007]

CP Violation?

- In this approximation there is no observable CP-violation effect!
- CP-violation can be observed only with sensitivity to Δm^2_{21}: in vacuum

$$\begin{aligned} A^{\mathrm{CP}}_{\alpha\beta} &= P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta} \\ &= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right) \end{aligned}$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

- Necessary conditions for observation of CP violation:
  - Sensitivity to all mixing angles, including small ϑ_{13}
  - Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

LBL Oscillation Probabilities

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad \alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \qquad A = \frac{2EV}{\Delta m^2_{31}} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad \alpha \ll 1$$

$$P^{\text{LBL}}_{\nu_e\to\nu_e} \simeq 1 - \sin^2 2\vartheta_{13} \sin^2\Delta - \alpha^2 \Delta^2 \sin^2 2\vartheta_{12}$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq{}& \sin^2 2\vartheta_{13} \sin^2\vartheta_{23} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ & +\alpha \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \delta_{13}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ & +\alpha^2 \sin^2 2\vartheta_{12} \cos^2\vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

NO: $\Delta m^2_{31} > 0$ IO: $\Delta m^2_{31} < 0$

for antineutrinos: $\delta_{13} \to -\delta_{13}$ (CPV) and $A \to -A$ (Fake CPV!)

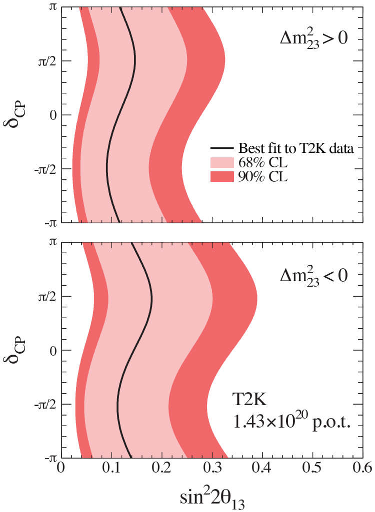
[see: Mezzetto, Schwetz, JPG 37 (2010) 103001]

T2K

[PRL 107 (2011) 041801, arXiv:1106.2822]

ND at 280 m FD at 295 km

2.5° off-axis ⇒ NBB with $\langle E \rangle \simeq 0.6\,\text{GeV} \simeq |\Delta m^2_{31}| L / 2\pi$



$\nu_\mu \to \nu_e$

6 ν_e events in FD

background: 1.5 ± 0.3

2.5σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.11^{+0.17}_{-0.08} & \text{(NO)} \\ 0.14^{+0.20}_{-0.10} & \text{(IO)} \end{cases}$$

90% C.L. $\delta_{13} = 0$

Assumptions

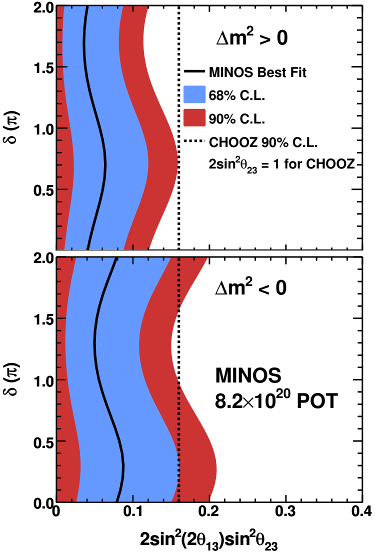
$\Delta m^2_{21} = 7.6 \times 10^{-5}\,\text{eV}$, $\sin^2 2\vartheta_{12} = 0.87$

$|\Delta m^2_{31}| = 2.4 \times 10^{-3}\,\text{eV}$, $\sin^2 2\vartheta_{23} = 1$

MINOS

[PRL 107 (2011) 181802, arXiv:1108.0015]

ND at 1.04 km FD at 735 km $\langle E \rangle \simeq 3\,\text{GeV}$



$$\nu_\mu \to \nu_e$$

62 ν_e events in FD

background: 49.6 ± 7.5

1.6σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.041^{+0.047}_{-0.031} & \text{(NO)} \\ 0.079^{+0.071}_{-0.053} & \text{(IO)} \end{cases}$$

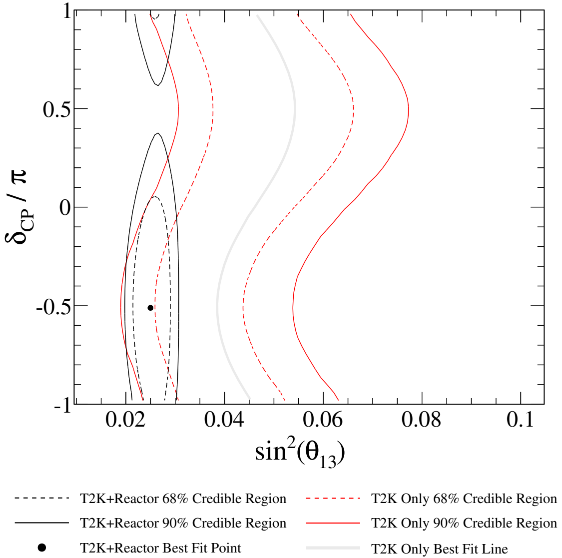
68% C.L. $\delta_{13} = 0$

Assumptions

$$\Delta m^2_{21} = 7.6 \times 10^{-5}\,\text{eV}\,,\ \sin^2 2\vartheta_{12} = 0.87$$

$$|\Delta m^2_{31}| = 2.3 \times 10^{-3}\,\text{eV}\,,\ \sin^2 2\vartheta_{23} = 1$$

Large CP Violation?

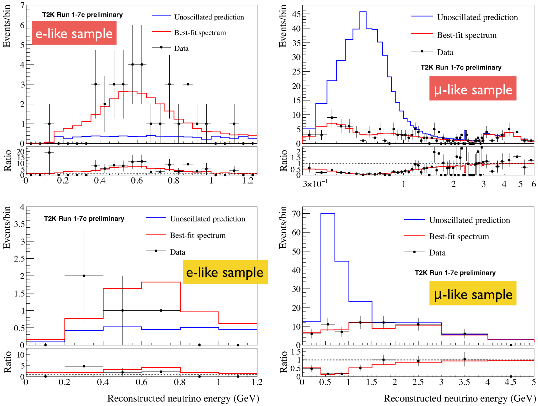


T2K, PRD 91 (2015) 072010, arXiv:1502.01550

T2K $\nu_e + \bar{\nu}_e$

ν beam mode

$\bar{\nu}$ beam mode



Larger than expected ν_e appearance

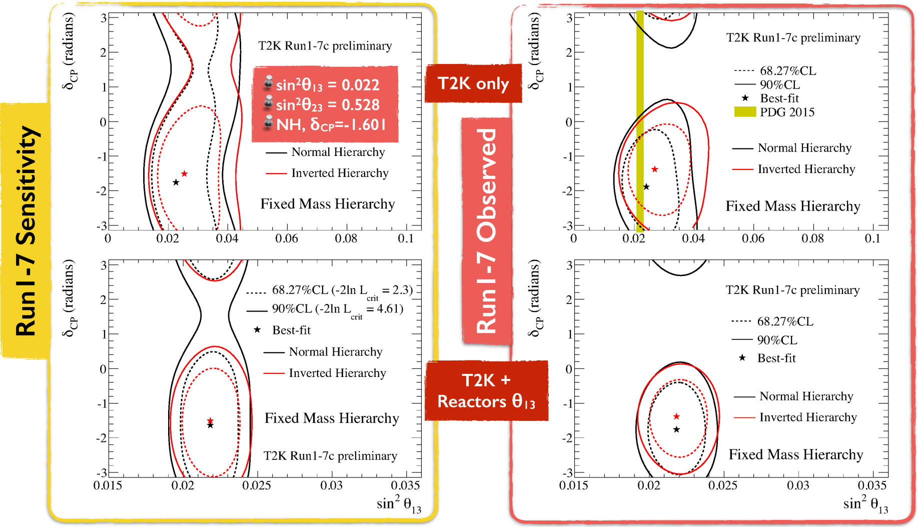
Smaller than expected $\bar{\nu}_e$ appearance

Data prefer the value of δ_{CP} inducing the largest ν-$\bar{\nu}$ asymmetry: -π/2

Normal Hierarchy						
Beam mode	**Sample**	**δ_{CP} = -π/2**	**δ_{CP} = 0**	**δ_{CP} = +π/2**	**δ_{CP} = π**	**Observed**
neutrino	μ-like	135.8	135.5	135.7	136.0	135
neutrino	e-like	28.7	24.2	19.6	24.1	32
anti-neutrino	μ-like	64.2	64.1	64.2	64.4	66
anti-neutrino	e-like	6	6.9	7.7	6.8	4

- Oscillation and systematic parameters are shared between the 4 samples
- Fit simultaneously the 4 samples to maximize the sensitivity to the oscillation parameters

[T2K @ NOW2016, September 2016]



Run1-7 Sensitivity

T2K only

Run1-7 Observed

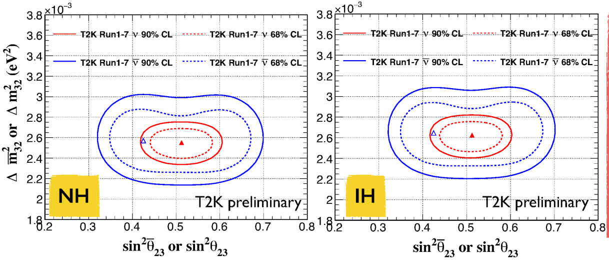
T2K + Reactors θ_{13}

- $\sin^2\theta_{13} = 0.022$
- $\sin^2\theta_{23} = 0.528$
- NH, δ_{CP}=-1.601

- T2K results consistent with reactor results
- Maximal CPV: data prefer δ_{CP}=$-\pi/2$ ($\bar{\nu}_e$ data confirm the tendency observed for ν_e data)
- Favors the scenario of a small θ_{13} and large CPV

[T2K @ NOW2016, September 2016]

Constraints on the atmospheric parameters: θ_{23} and Δm^2_{31}

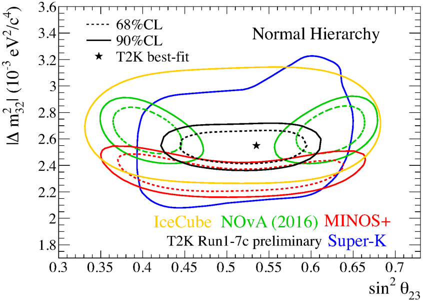


- **CPT theorem:**

$P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$

if $P(\nu_\mu \to \nu_\mu) \neq P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$ ⇒

CPT theorem is violated

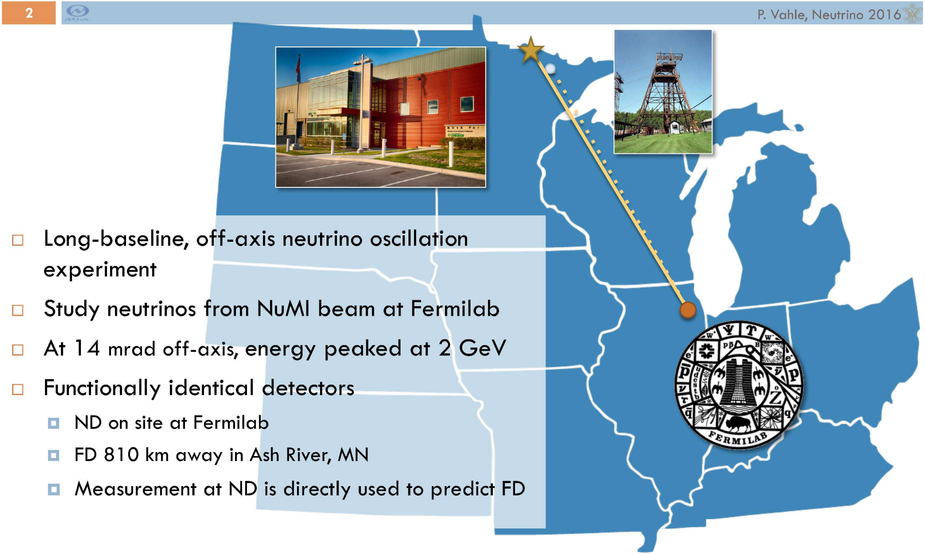


- World-leading measurement of $\sin^2\theta_{23}$
- Results continue to be consistent with maximal mixing/oscillation
- No significant differences between ν and $\bar{\nu}$

	NH	IH
$\sin^2\theta_{23}$	$0.532^{+0.046}_{-0.068}$	$0.534^{+0.043}_{-0.007}$
$\lvert\Delta m^2_{32}\rvert$ ($\times 10^{-5}$ eV^2/c^4)	$254.5^{+8.1}_{-8.4}$	$251.0^{+8.1}_{-8.3}$

[T2K @ NOW2016, September 2016]

NOνA

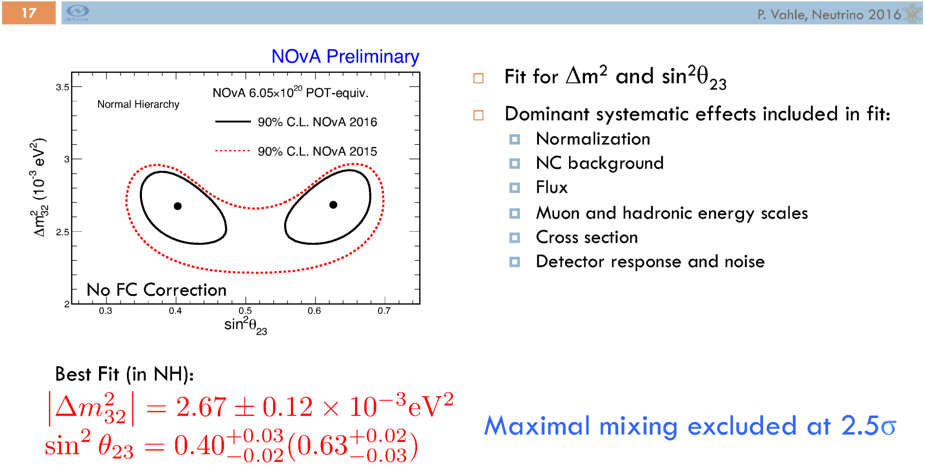


2

P. Vahle, Neutrino 2016

- Long-baseline, off-axis neutrino oscillation experiment
- Study neutrinos from NuMI beam at Fermilab
- At 14 mrad off-axis, energy peaked at 2 GeV
- Functionally identical detectors
  - ND on site at Fermilab
  - FD 810 km away in Ash River, MN
  - Measurement at ND is directly used to predict FD

[NOνA @ Neutrino2016, July 2016]



17

P. Vahle, Neutrino 2016

- Fit for Δm^2 and $\sin^2\theta_{23}$
- Dominant systematic effects included in fit:
  - Normalization
  - NC background
  - Flux
  - Muon and hadronic energy scales
  - Cross section
  - Detector response and noise

Best Fit (in NH):

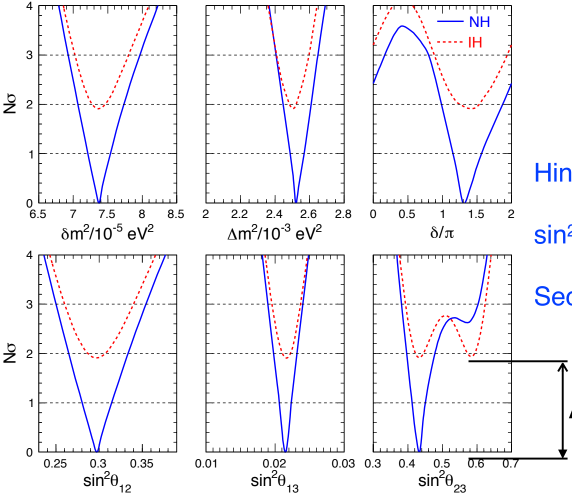
$$|\Delta m^2_{32}| = 2.67 \pm 0.12 \times 10^{-3} \mathrm{eV}^2$$

$$\sin^2\theta_{23} = 0.40^{+0.03}_{-0.02} (0.63^{+0.02}_{-0.03})$$

Maximal mixing excluded at 2.5σ

[NOνA @ Neutrino2016, July 2016]

September 2016 Global Fit



$\Delta\chi^2 \sim 3.7$

COMMENTS

Hint for CP violation at ~2σ

$\sin^2\theta_{23}=0.5$ disfavoured at ~2.8σ

Second octant disfavoured at ~2σ

[Capozzi, Lisi, Marrone, Montanino, Palazzo @ NOW2016, September 2016]

Absolute Scale of Neutrino Masses

- Solar Neutrinos and KamLAND

- Atmospheric and LBL Oscillation Experiments

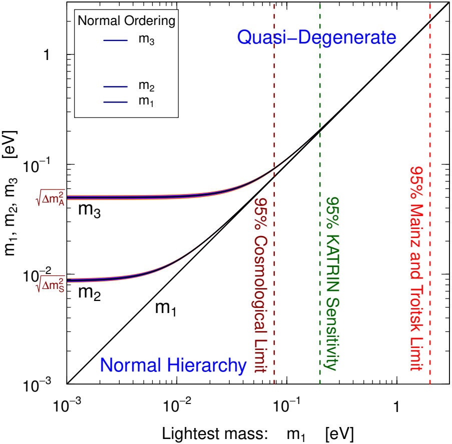
- Absolute Scale of Neutrino Masses
  - Tritium Beta-Decay
  - Neutrinoless Double-Beta Decay

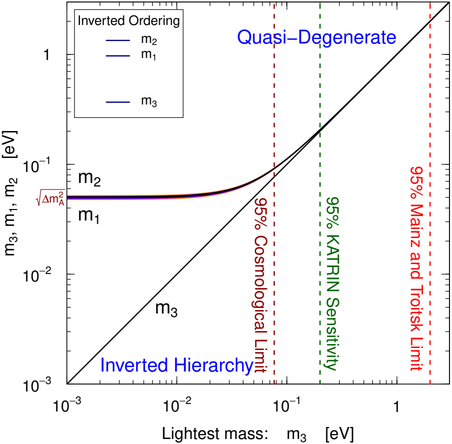
- Light Sterile Neutrinos

- Cosmology

- Conclusions

Mass Hierarchy or Degeneracy?





$$m_2^2 = m_1^2 + \Delta m^2_{21} = m_1^2 + \Delta m^2_S$$
$$m_3^2 = m_1^2 + \Delta m^2_{31} = m_1^2 + \Delta m^2_A$$

$$m_1^2 = m_3^2 - \Delta m^2_{31} = m_3^2 + \Delta m^2_A$$
$$m_2^2 = m_1^2 + \Delta m^2_{21} \simeq m_3^2 + \Delta m^2_A$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m^2_A} \simeq 5 \times 10^{-2}\,\text{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

Tritium Beta-Decay

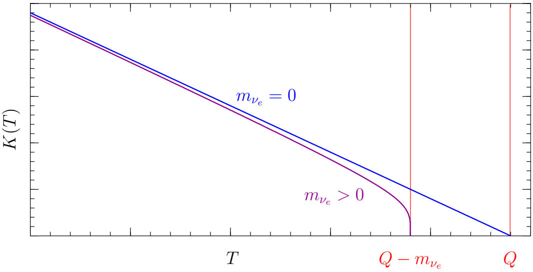
$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$$

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

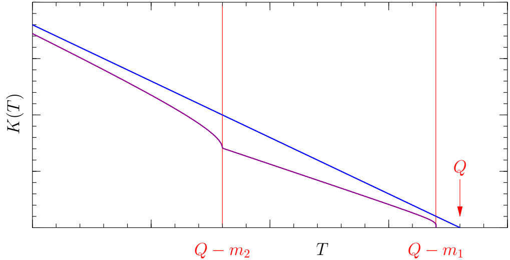
future: KATRIN

[`www.katrin.kit.edu`]

start data taking 2016?

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses $(m_k \ll Q - T)$

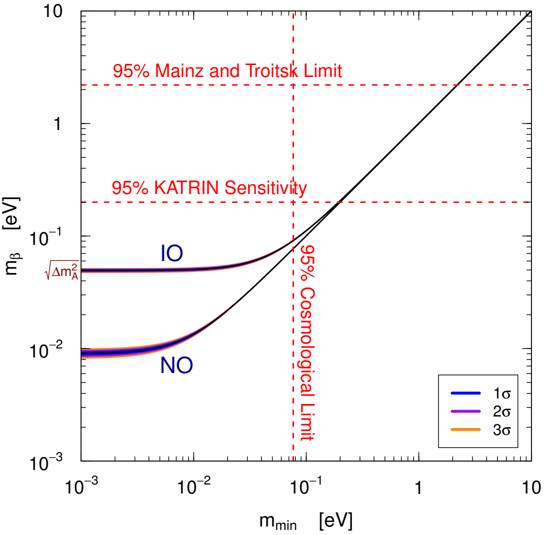
effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$

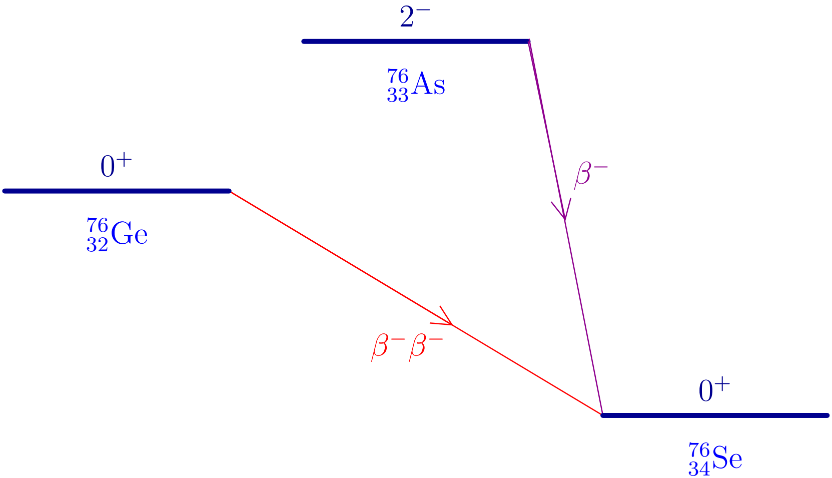


- Quasi-Degenerate:
  $m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$
- Inverted Hierarchy:
  $m_\beta^2 \simeq (1 - s_{13}^2) \Delta m_A^2 \simeq \Delta m_A^2$
- Normal Hierarchy:
  $m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$
  $\simeq 2 \times 10^{-5} + 6 \times 10^{-5} \, \text{eV}^2$
- If $m_\beta \lesssim 4 \times 10^{-2} \, \text{eV}$

$$\Downarrow$$

Normal Spectrum

Neutrinoless Double-Beta Decay



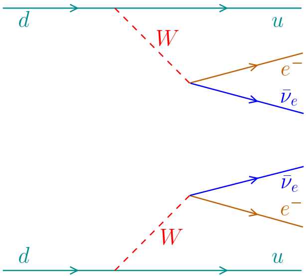
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

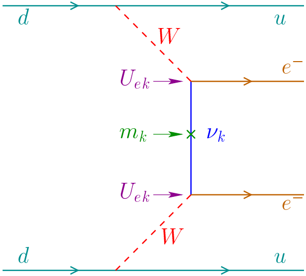


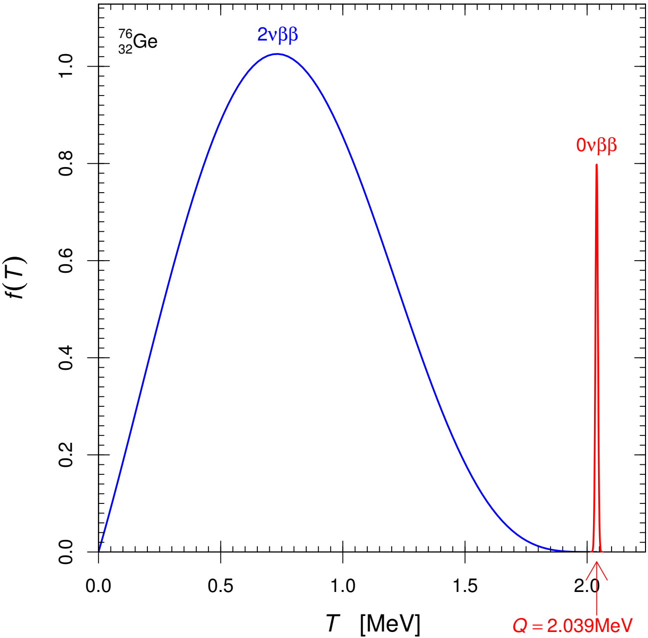
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$



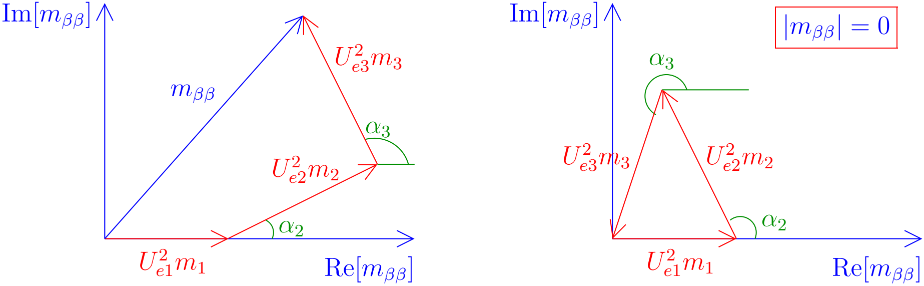

$^{76}_{32}$Ge
2νββ
0νββ
$f(T)$
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
T [MeV]
$Q = 2.039$MeV

Effective Majorana Neutrino Mass

$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$ complex U_{ek} ⇒ possible cancellations

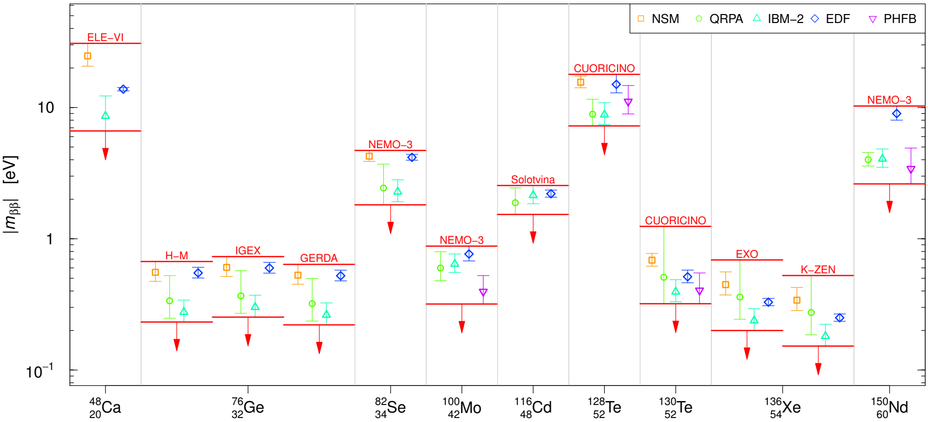
$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$



2015 90% C.L. Experimental Bounds

$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	GERDA	$> 2.1 \times 10^{25}$	$< 0.22 - 0.64$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORICINO	$> 2.8 \times 10^{24}$	$< 0.32 - 1.2$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.2 - 0.69$
	KamLAND-Zen	$> 1.9 \times 10^{25}$	$< 0.15 - 0.52$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$

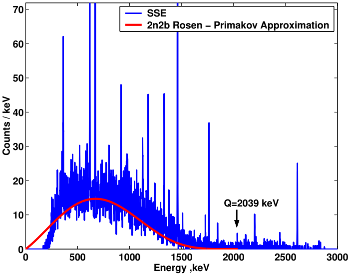


[Bilenky, Giunti, IJMPA 30 (2015) 0001]

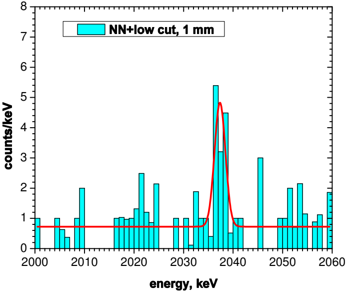
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409]

$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}\,\text{y}$ 6.5σ evidence [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]

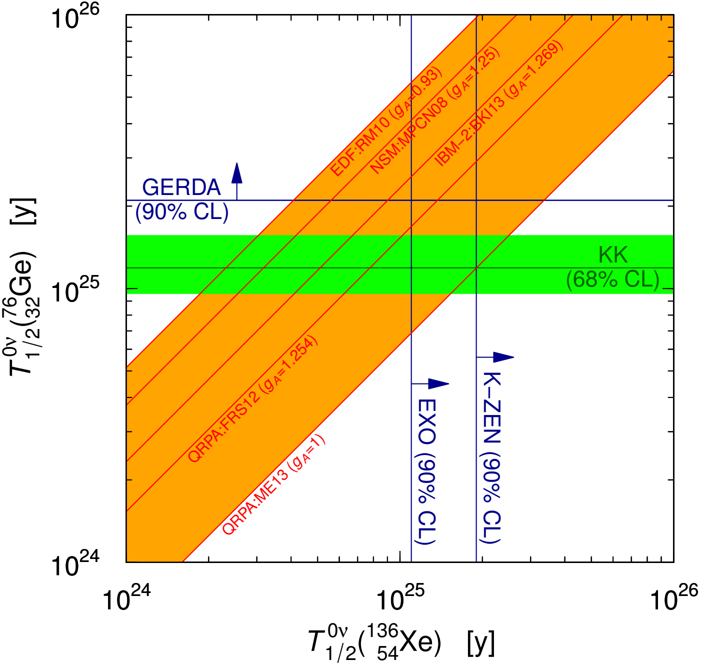


[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

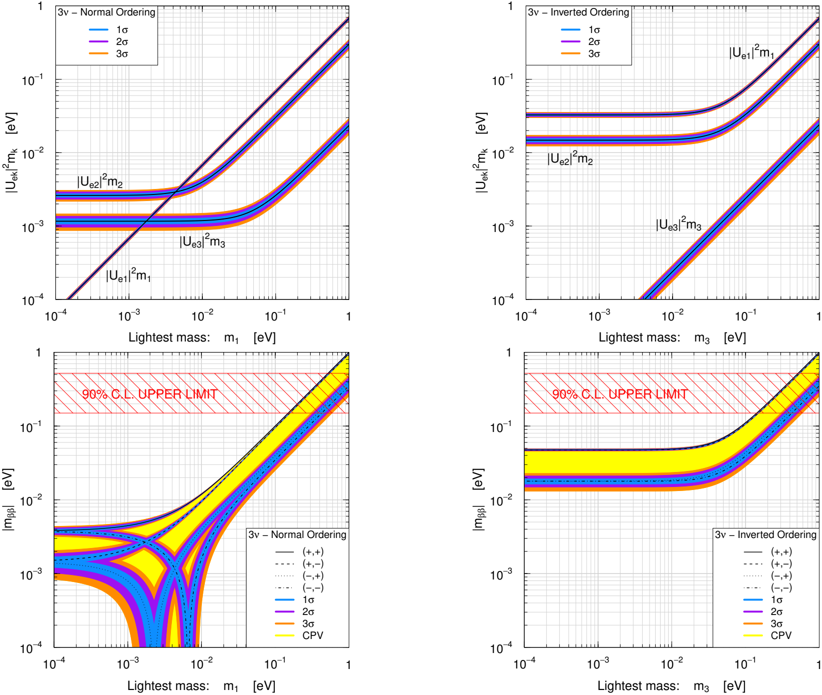
$|m_{\beta\beta}| = 0.32 \pm 0.03\,\text{eV}$ [MPLA 21 (2006) 1547]

if confirmed, very exciting: Majorana ν and large mass scale

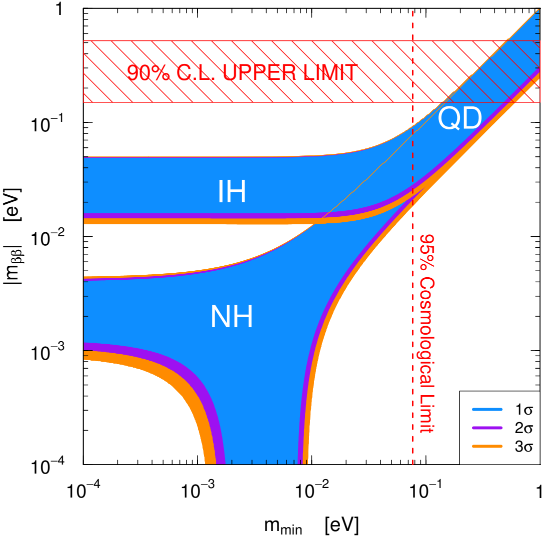

10^{26}
10^{25}
10^{24}
$T^{0\nu}_{1/2}({}^{76}_{32}\mathrm{Ge})$ [y]
10^{24}
10^{25}
10^{26}
$T^{0\nu}_{1/2}({}^{136}_{54}\mathrm{Xe})$ [y]
GERDA (90% CL)
KK (68% CL)
EXO (90% CL)
K–ZEN (90% CL)
EDF:RM10 (g_A=0.93)
NSM:MPCN08 (g_A=1.25)
IBM–2:BKI13 (g_A=1.269)
QRPA:FRS12 (g_A=1.254)
QRPA:ME13 (g_A=1)

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$


3ν – Normal Ordering
1σ
2σ
3σ
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
$|U_{e1}|^2m_1$
$|U_{ek}|^2m_k$ [eV]
Lightest mass: m_1 [eV]
3ν – Inverted Ordering
1σ
2σ
3σ
$|U_{e1}|^2m_1$
$|U_{e2}|^2m_2$
$|U_{e3}|^2m_3$
$|U_{ek}|^2m_k$ [eV]
Lightest mass: m_3 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν – Normal Ordering
(+,+)
(+,–)
(–,+)
(–,–)
1σ
2σ
3σ
CPV
Lightest mass: m_1 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν – Inverted Ordering
(+,+)
(+,–)
(–,+)
(–,–)
1σ
2σ
3σ
CPV
Lightest mass: m_3 [eV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

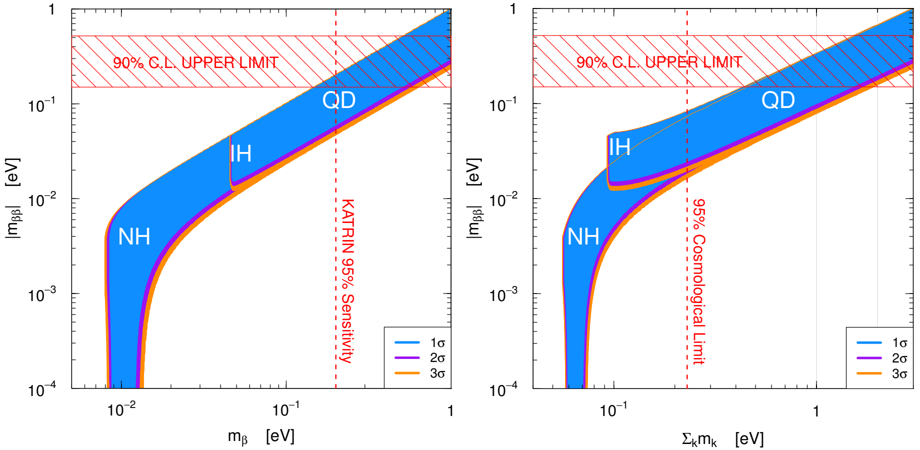
- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\rm A} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\rm S}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\rm A}}|$$
$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\,\text{eV}$$

$$|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow \text{Normal Spectrum}$$


1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|m_{\beta\beta}|$ [eV]
90% C.L. UPPER LIMIT
QD
IH
NH
KATRIN 95% Sensitivity
10^{-2}
10^{-1}
1
m_β [eV]
1σ
2σ
3σ
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|m_{\beta\beta}|$ [eV]
90% C.L. UPPER LIMIT
QD
IH
NH
95% Cosmological Limit
10^{-1}
1
$\Sigma_k m_k$ [eV]
1σ
2σ
3σ

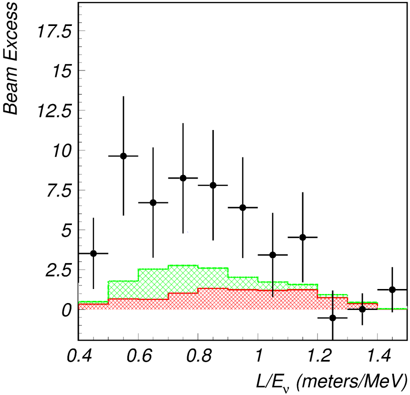
Light Sterile Neutrinos

Indications of SBL Oscillations Beyond 3ν

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad$ $20\,\text{MeV} \leq E \leq 60\,\text{MeV}$



- Well-known source of $\bar{\nu}_\mu$

μ^+ at rest $\to e^+ + \nu_e + \bar{\nu}_\mu$

$L \simeq 30\,\text{m}$

$\bar{\nu}_e + p \to n + e^+$

Well-known detection process of $\bar{\nu}_e$

- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

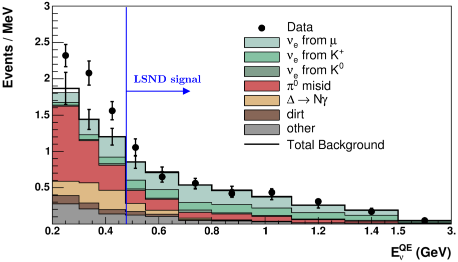
[PRD 65 (2002) 112001]

$\approx 3.8\sigma$ excess $\qquad$ $\Delta m^2_{\text{SBL}} \gtrsim 0.2\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}}$

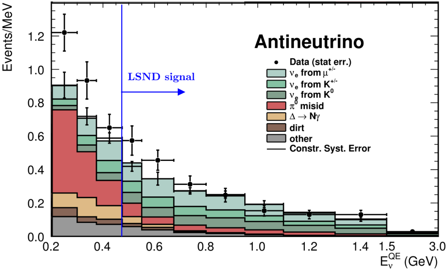
MiniBooNE

$L \simeq 541\,\text{m}$ $\qquad$ $200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



- Purpose: check LSND signal.
- Different L and E.
- Similar L/E (oscillations).
- No money, no Near Detector.

- LSND signal: $E > 475\,\text{MeV}$.
- Agreement with LSND signal?
- CP violation?
- Low-energy anomaly!

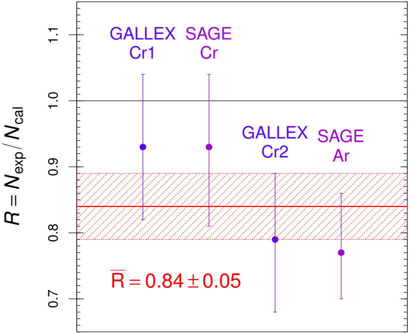
Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

$E \simeq 0.75\,\mathrm{MeV}$ $E \simeq 0.81\,\mathrm{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$



$$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$$

$$\langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$$

$\approx 2.9\sigma$ deficit

$$\Delta m^2_{\mathrm{SBL}} \gtrsim 1\,\mathrm{eV}^2 \gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SOL}}$$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807]

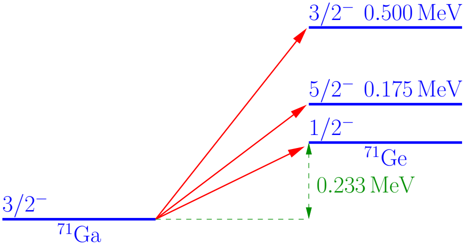
[Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344;
MPLA 22 (2007) 2499; PRD 78 (2008) 073009;
PRC 83 (2011) 065504]

- ${}^{3}\mathrm{He} + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + {}^{3}\mathrm{H}$ cross section measurement [Frekers et al., PLB 706 (2011) 134]
- $E_{\mathrm{th}}(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-) = 233.5 \pm 1.2\,\mathrm{keV}$ [Frekers et al., PLB 722 (2013) 233]

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\mathrm{G.S.}}$ from $T_{1/2}({}^{71}\mathrm{Ge}) = 11.43 \pm 0.03\,\mathrm{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) = 55.3 \times 10^{-46}\,\mathrm{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\mathrm{Cr}) = \sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) \left(1 + 0.669\,\frac{\mathrm{BGT}_{175}}{\mathrm{BGT}_{\mathrm{G.S.}}} + 0.220\,\frac{\mathrm{BGT}_{500}}{\mathrm{BGT}_{\mathrm{G.S.}}}\right)$

- Contribution of Excited States only 5%!

		$\frac{\mathrm{BGT}_{175}}{\mathrm{BGT}_{\mathrm{G.S.}}}$	$\frac{\mathrm{BGT}_{500}}{\mathrm{BGT}_{\mathrm{G.S.}}}$
Krofcheck et al. PRL 55 (1985) 1051	${}^{71}\mathrm{Ga}(p,n){}^{71}\mathrm{Ge}$	< 0.056	0.126 ± 0.023
Haxton PLB 431 (1998) 110	Shell Model	0.19 ± 0.18	
Frekers et al. PLB 706 (2011) 134	${}^{71}\mathrm{Ga}({}^{3}\mathrm{He},{}^{3}\mathrm{H}){}^{71}\mathrm{Ge}$	0.039 ± 0.030	0.202 ± 0.016

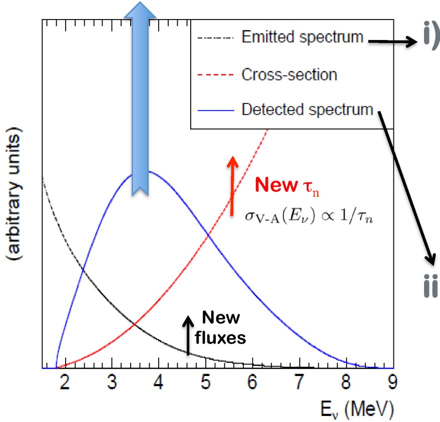
- Haxton: [Haxton, PLB 431 (1998) 110]

  "a sophisticated shell model calculation is performed ... for the transition to the first excited state in ${}^{71}\mathrm{Ge}$. The calculation predicts destructive interference between the (p,n) spin and spin-tensor matrix elements"
- Does Haxton argument apply also to $({}^{3}\mathrm{He},{}^{3}\mathrm{H})$ measurements?
- 2.7σ discrepancy of $\mathrm{BGT}_{500}/\mathrm{BGT}_{\mathrm{G.S.}}$ measurements!
- Anyhow, new ${}^{71}\mathrm{Ga}({}^{3}\mathrm{He},{}^{3}\mathrm{H}){}^{71}\mathrm{Ge}$ data support Gallium Anomaly!

New Reactor $\bar{\nu}_e$ Fluxes

New Reactor ν-Fluxes

Increased prediction of detected flux by 6.5%



i) **Neutrino Emission:**

- **Improved reactor neutrino spectra → +3.5%**
- **Accounting for long-lived isotopes in reactors → +1%**

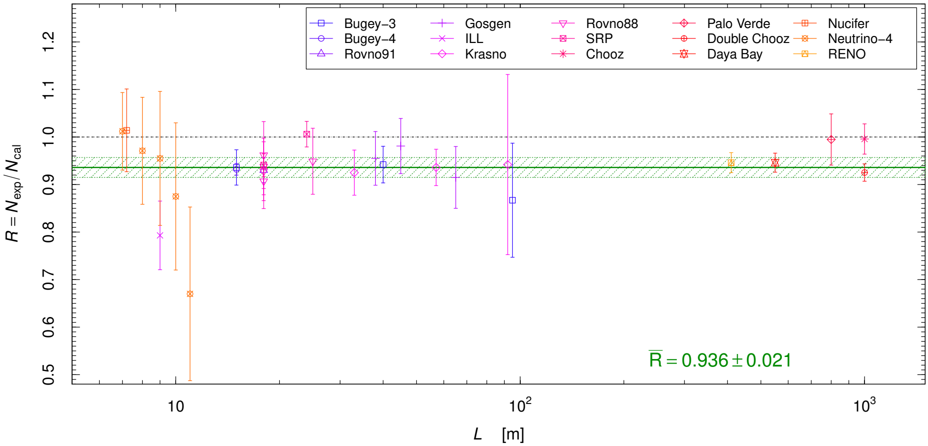
ii) **Neutrino Detection:**

- **Reevaluation of σ_{IBD} → +1.5% (evolution of the neutron life time)**
- **Reanalysis of all SBL experiments**

Reactor Electron Antineutrino Anomaly

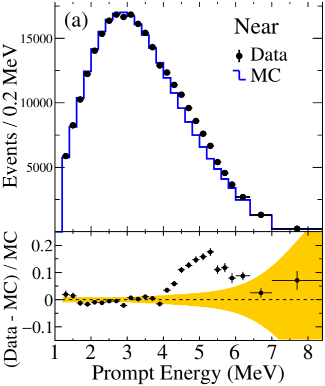
[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes [Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]

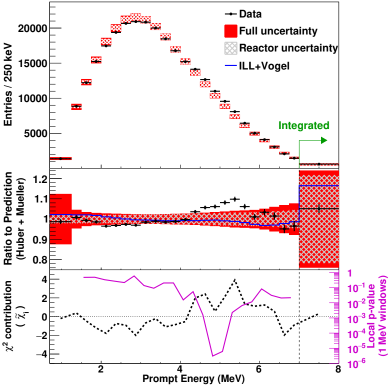


$\approx 3.1\sigma$ deficit $\Delta m^2_{\rm SBL} \gtrsim 0.5\,\mathrm{eV}^2 \gg \Delta m^2_{\rm ATM} \gg \Delta m^2_{\rm SOL}$

5 MeV Bump



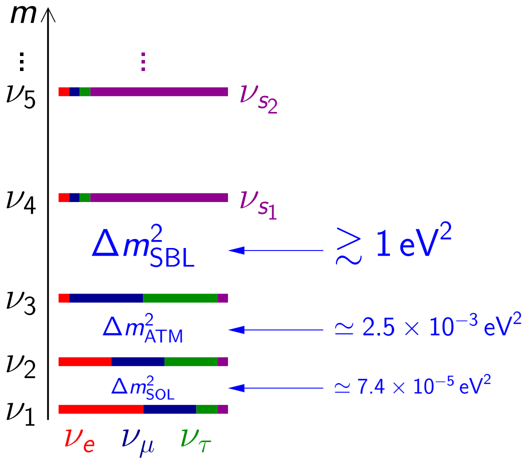
[RENO, arXiv:1511.05849]



[Daya Bay, arXiv:1508.04233]

- Local problem with $\sim 3\%$ effect on total flux.
- It is an excess!
- It occurs both for the new high Muller-Huber fluxes and the old low Schreckenbach-Vogel fluxes.
- Real problem: apparent incompatibility of the bump with the β spectra from ^{235}U and ^{239}Pu measured by Schreckenbach et al. at ILL in 1982-1985.

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Terminology: a eV-scale sterile neutrino
means: a eV-scale massive neutrino which is mainly sterile

Sterile Neutrinos from Physics Beyond the SM

- Here I consider sterile neutrinos with mass scale $\sim 1\,\text{eV}$ in light of short-baseline Reactor Anomaly, Gallium Anomaly, LSND.

- Other possibilities (not incompatible):
  - Very light sterile neutrinos with mass scale $\ll 1\,\text{eV}$: important for solar neutrino phenomenology

    [de Holanda, Smirnov, PRD 69 (2004) 113002; PRD 83 (2011) 113011]

    [Das, Pulido, Picariello, PRD 79 (2009) 073010]

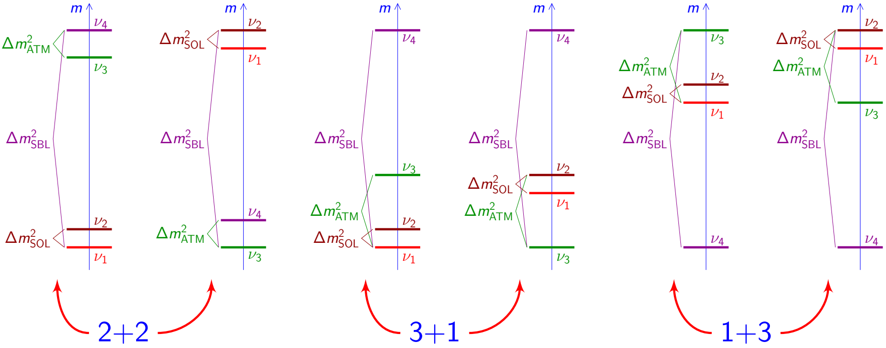
    Recent Daya Bay constraints for $10^{-3} \lesssim \Delta m^2 \lesssim 10^{-1}\,\text{eV}^2$ [PRL 113 (2014) 141802]

  - Heavy sterile neutrinos with mass scale $\gg 1\,\text{eV}$: could be Warm Dark Matter

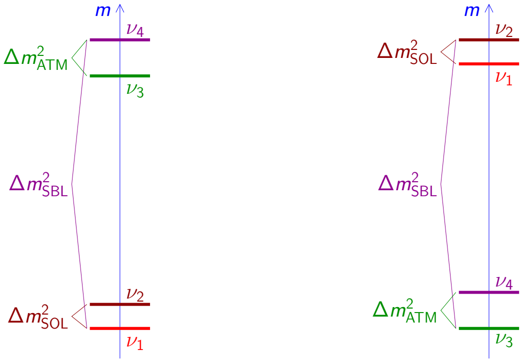
    [Asaka, Blanchet, Shaposhnikov, PLB 631 (2005) 151; Asaka, Shaposhnikov, PLB 620 (2005) 17; Asaka, Shaposhnikov, Kusenko, PLB 638 (2006) 401; Asaka, Laine, Shaposhnikov, JHEP 0606 (2006) 053, JHEP 0701 (2007) 091]

    [Reviews: Kusenko, Phys. Rept. 481 (2009) 1; Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191; Boyarsky, Iakubovskyi, Ruchayskiy, Phys. Dark Univ. 1 (2012) 136; Drewes, IJMPE, 22 (2013) 1330019]

Four-Neutrino Schemes: 2+2, 3+1 and 1+3



2+2 Four-Neutrino Schemes

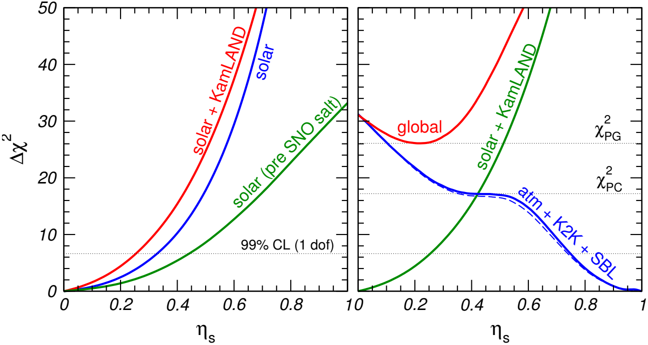


- After LSND (1995) 2+2 was preferred to 3+1, because of the 3+1 appearance-disappearance tension

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

- This is not a perturbation of 3-ν Mixing $\Longrightarrow$ Large active–sterile oscillations for solar or atmospheric neutrinos!

2+2 Schemes are Strongly Disfavored



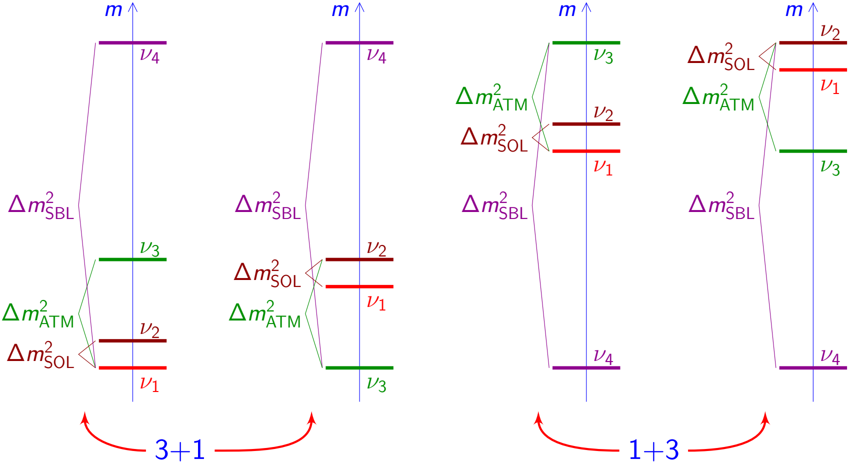
Solar: Matter Effects + SNO NC   Atmospheric: Matter Effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 = 1 - |U_{s3}|^2 + |U_{s4}|^2$$

$$99\%\ \text{CL:}\quad \begin{cases} \eta_s < 0.25 & \text{(Solar + KamLAND)} \\ \eta_s > 0.75 & \text{(Atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122]

3+1 and 1+3 Four-Neutrino Schemes



- Perturbation of 3-ν Mixing: $|U_{e4}|^2, |U_{\mu4}|^2, |U_{\tau4}|^2 \ll 1 \quad |U_{s4}|^2 \simeq 1$
- 1+3 schemes are disfavored by cosmology (ΛCDM):

$$\sum_{k=1}^{3} m_k < 0.21\,\text{eV} \text{ (95\%, Planck TT + lowP + BAO)}$$ [arXiv:1502.01589]

Effective 3+1 SBL Oscillation Probabilities

$$P_{\nu_\alpha\to\nu_\beta} = \left|\sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t}\right|^2 * \left|e^{iE_1 t}\right|^2$$

$$= \left|\sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t}\right|^2 \to \left|\sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i\frac{\Delta m_{k1}^2 L}{2E}\right)\right|^2$$

$$E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1 \qquad \Delta m_{41}^2 \to \Delta m^2$$

$$P_{\nu_\alpha\to\nu_\beta}^{\text{SBL}} \simeq \left|U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(-i\frac{\Delta m^2 L}{2E}\right)\right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}$$

$$\begin{aligned}
P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} &\simeq \left|\delta_{\alpha\beta} - U^*_{\alpha4}U_{\beta4}\left[1-\exp\left(-i\frac{\Delta m^2 L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha4}|^2|U_{\beta4}|^2\left(2-2\cos\frac{\Delta m^2 L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha4}|^2\left(1-\cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha4}|^2\left(\delta_{\alpha\beta}-|U_{\beta4}|^2\right)\left(1-\cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha4}|^2\left(\delta_{\alpha\beta}-|U_{\beta4}|^2\right)\sin^2\frac{\Delta m^2 L}{4E}
\end{aligned}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq 4|U_{\alpha4}|^2|U_{\beta4}|^2\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\alpha} \simeq 1 - 4|U_{\alpha4}|^2\left(1-|U_{\alpha4}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Appearance ($\alpha \neq \beta$)

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha4}|^2|U_{\beta4}|^2$$

Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

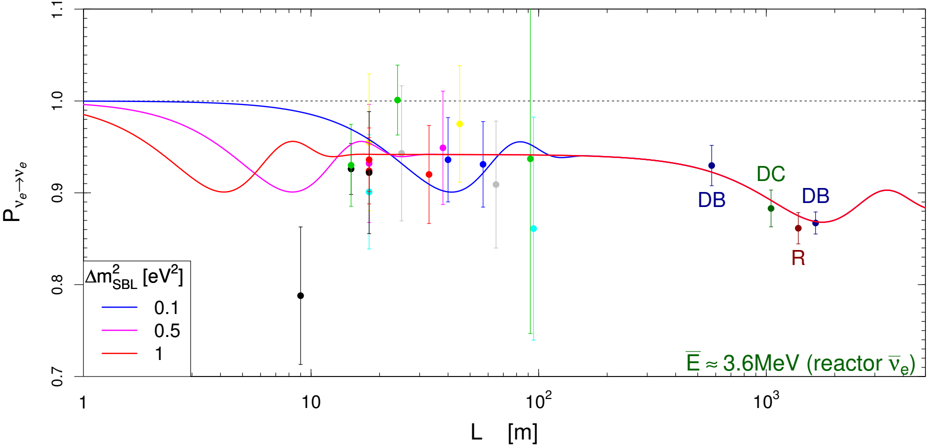
$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha4}|^2\left(1 - |U_{\alpha4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Gandhi et al, JHEP 1511 (2015) 039] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, PRD 87, 113004 (2013) 113004]


$P_{\nu_e\to\nu_e}$
1.1
1.0
0.9
0.8
0.7
1
10
10^2
10^3
L [m]
Δm^2_{SBL} [eV^2]
0.1
0.5
1
DB
DC
DB
R
$\overline{E} \approx 3.6$MeV (reactor $\overline{\nu}_e$)

Solar bound on $|U_{e4}|^2$

[Giunti, Li, PRD 80 (2009) 113007; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301]

$$P^{\mathrm{SOL}}_{\nu_e\to\nu_e} \simeq \left(1 - \sum_{k\geq3} |U_{ek}|^2\right)^2 P^{\mathrm{SOL},2\nu}_{\nu_e\to\nu_e} + \sum_{k\geq3} |U_{ek}|^4$$

$$P^{\mathrm{SOL}}_{\nu_e\to\nu_s} \simeq \left(1 - \sum_{k\geq3} |U_{ek}|^2\right)\left(1 - \sum_{k\geq3} |U_{sk}|^2\right) P^{\mathrm{SOL},2\nu}_{\nu_e\to\nu_s} + \sum_{k\geq3} |U_{ek}|^2 |U_{sk}|^2$$

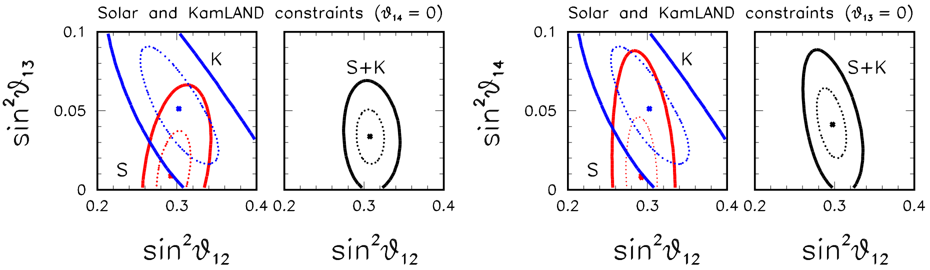
3+1 with simplifying assumptions: $U_{\mu4} = U_{\tau4} = 0$, no CP violation

$$\begin{array}{llll} U_{e1} = c_{12}c_{13}c_{14} & U_{e2} = s_{12}c_{13}c_{14} & U_{e3} = s_{13}c_{14} & U_{e4} = s_{14} \\ U_{s1} = -c_{12}c_{13}s_{14} & U_{s2} = -s_{12}c_{13}s_{14} & U_{s3} = -s_{13}s_{14} & U_{s4} = c_{14} \end{array}$$

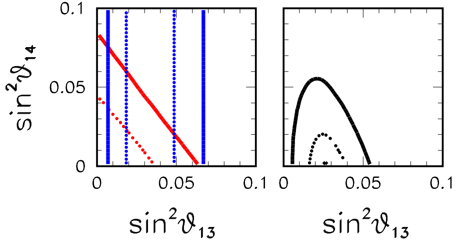
$$P^{\mathrm{SOL}}_{\nu_e\to\nu_e} \simeq c_{13}^4 c_{14}^4 P^{\mathrm{SOL},2\nu}_{\nu_e\to\nu_e} + s_{13}^4 c_{14}^4 + s_{14}^4$$

$$P^{\mathrm{SOL}}_{\nu_e\to\nu_s} \simeq c_{14}^2 s_{14}^2 \left(c_{13}^4 P^{\mathrm{SOL},2\nu}_{\nu_e\to\nu_s} + s_{13}^4 + 1\right)$$

$$\begin{aligned} V &= c_{13}^2 c_{14}^2 V_{\mathrm{CC}} - c_{13}^2 s_{14}^2 V_{\mathrm{NC}} \\ &= (|U_{e1}|^2 + |U_{e2}|^2) V_{\mathrm{CC}} - (|U_{s1}|^2 + |U_{s2}|^2) V_{\mathrm{NC}} \end{aligned}$$



[Palazzo, PRD 83 (2011) 113013]



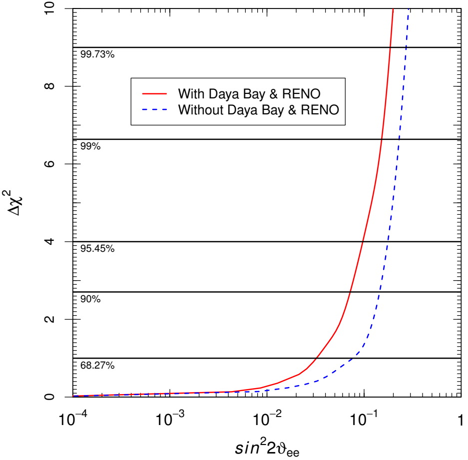
[Palazzo, PRD 85 (2012) 077301]

Daya Bay and RENO

$$\sin^2\vartheta_{13} = 0.025 \pm 0.004$$

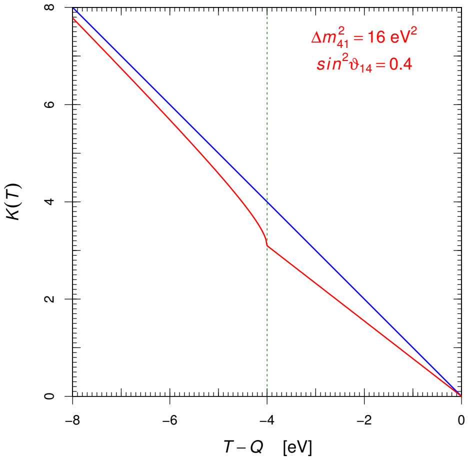
$$|U_{e4}|^2 = \sin^2\vartheta_{14} \lesssim 0.02\,(1\sigma)$$

Fit of solar and KamLAND data with Daya Bay and RENO constraint $\sin^2\vartheta_{13} = 0.025 \pm 0.004$ and free $|U_{\mu 4}|$ and $|U_{\tau 4}|$ (neglecting small CP violation effects)



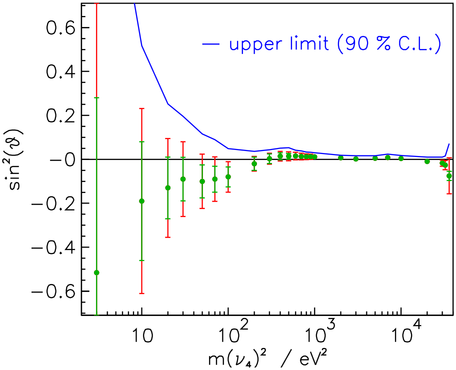
[Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

Tritium Beta-Decay

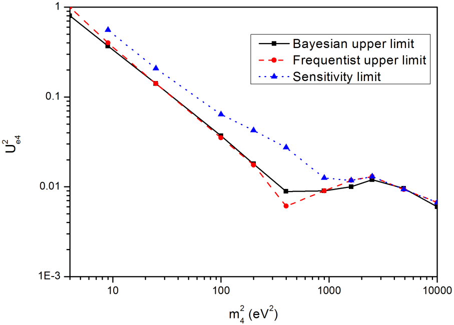


$$m_4 \gg m_1, m_2, m_3 \quad \Longrightarrow \quad \Delta m^2_{41} \equiv m^2_4 - m^2_1 \simeq m^2_4$$

Mainz and Troitsk Limit on m_4^2

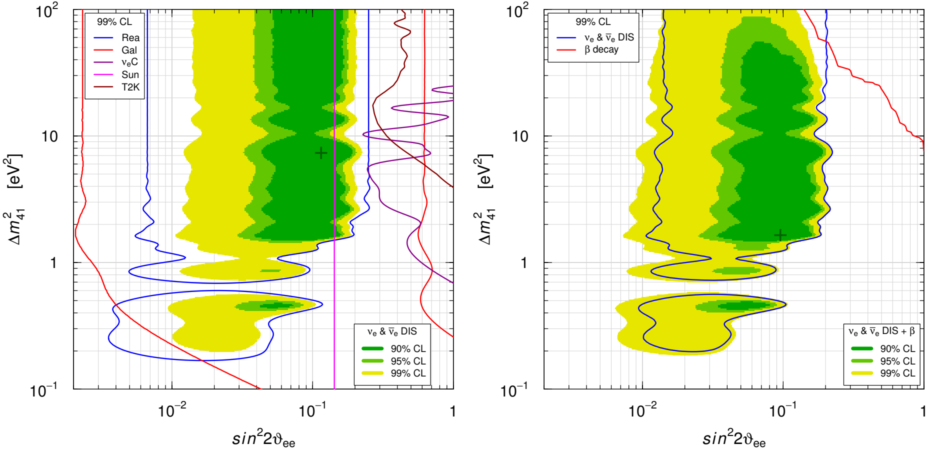


[Kraus, Singer, Valerius, Weinheimer, EPJC 73 (2013) 2323]



[Belesev et al, JPG 41 (2014) 015001]

Global ν_e and $\bar{\nu}_e$ Disappearance



KARMEN + LSND $\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$
[Conrad, Shaevitz, PRD 85 (2012) 013017]
[CG, Laveder, PLB 706 (2011) 200]

solar ν_e + KamLAND $\bar{\nu}_e$ + ϑ_{13}
[CG, Li, PRD 80 (2009) 113007]
[Palazzo, PRD 83 (2011) 113013; PRD 85 (2012) 077301]
[CG, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

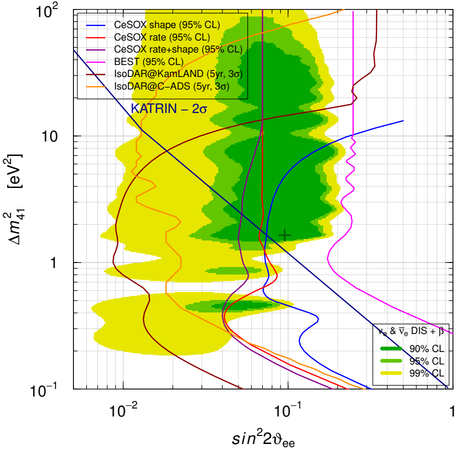
T2K Near Detector ν_e disappearance
[T2K, PRD 91 (2015) 051102]

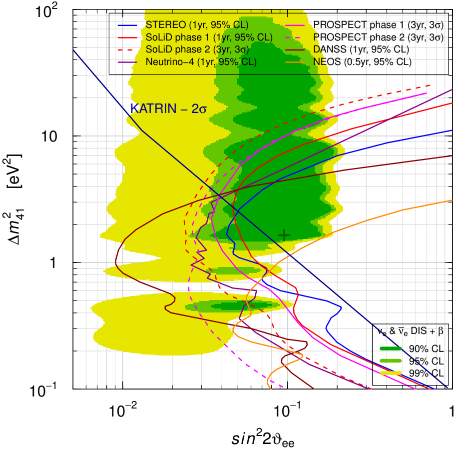
Mainz + Troitsk Tritium β decay
[Mainz, EPJC 73 (2013) 2323]
[Troitsk, JETPL 97 (2013) 67; JPG 41 (2014) 015001]

No Osc. excluded at 2.8σ
$(\Delta\chi^2/\text{NDF} = 10.8/2)$

$$6\,\text{cm} \lesssim \frac{L_{41}^{\text{osc}}}{E\,[\text{MeV}]} \lesssim 6\,\text{m} \qquad (2\sigma)$$

The Race for ν_e and $\bar{\nu}_e$ Disappearance





CeSOX (Gran Sasso, Italy) $^{144}\text{Ce} \to \bar{\nu}_e$
BOREXINO: $L \simeq$ 5-12m [Vivier@TAUP2015]

BEST (Baksan, Russia) $^{51}\text{Cr} \to \nu_e$
$L \simeq$ 5-12m [PRD 93 (2016) 073002]

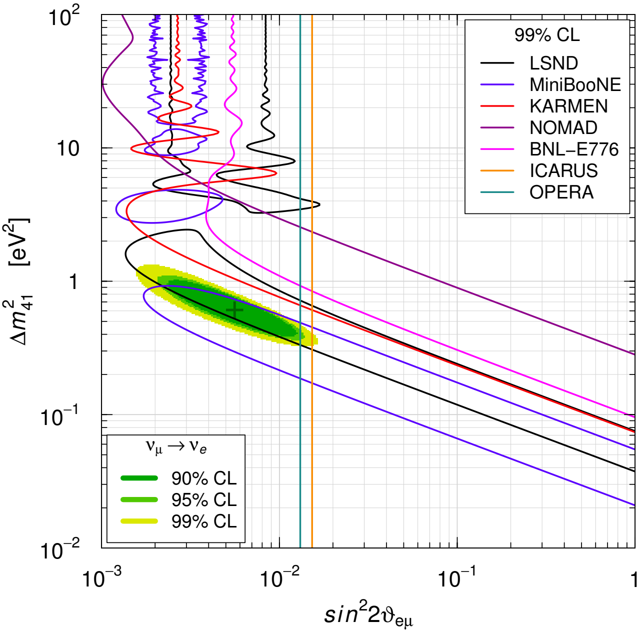
IsoDAR@KamLAND (Kamioka, Japan)
$^{8}\text{Li} \to \bar{\nu}_e$ $L \simeq$ 16m [arXiv:1511.05130]

IsoDAR@C-ADS (Guangdong, China)
$^{8}\text{Li} \to \bar{\nu}_e$ $L \simeq$ 15m [JHEP 1601 (2016) 004]

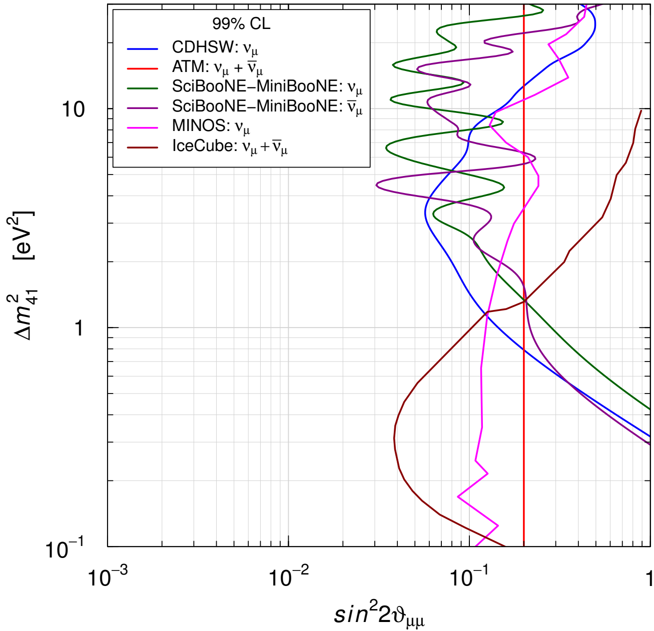
STEREO (ILL, France) $L \simeq$ 8-12m [arXiv:1602.00568]
SoLid (SCK-CEN, Belgium) $L \simeq$ 5-8m [arXiv:1510.07835]
Neutrino-4 (RIAR, Russia) $L \simeq$ 6-11m [JETP 121 (2015) 578]
PROSPECT (ORNL, USA) $L \simeq$ 7-12m [arXiv:1512.02202]
DANSS (Kalinin, Russia) $L \simeq$ 10-12m [arXiv:1606.02896]
NEOS (Hanbit, Korea) $L \simeq$ 24m [Oh@WIN2015]

KATRIN (Karlsruhe, Germany) $^{3}\text{H} \to \bar{\nu}_e$ [Mertens@TAUP2015]

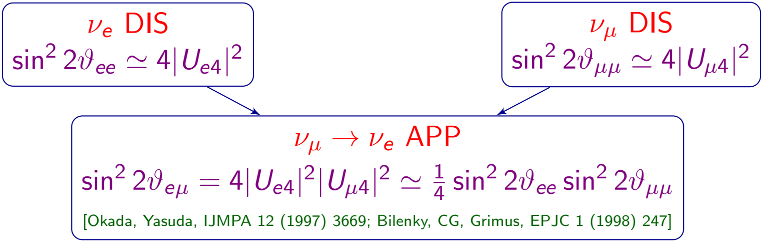
$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ Appearance



ν_μ and $\bar{\nu}_\mu$ Disappearance



3+1 Appearance-Disappearance Tension



ν_e DIS

$$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$$

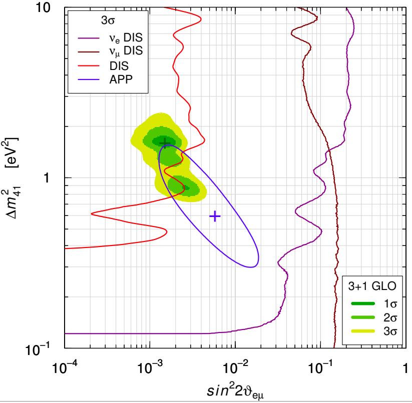
ν_μ DIS

$$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$$

$\nu_\mu \to \nu_e$ APP

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- Similar constraint in

$$3+2,\ 3+3,\ \ldots,\ 3+N_s$$

[CG, Zavanin, MPLA 31 (2015) 1650003]

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001] with improved treatment of the MiniBooNE background disappearance due to neutrino oscillations according to information from Bill Louis (thanks!)

Appearance vs Disappearance in $N = 3 + N_s$ Mixing

[Giunti, Zavanin, MPLA 31 (2015) 1650003]

$$\frac{\Delta m^2_{21} L}{4E} \ll \frac{\Delta m^2_{31} L}{4E} \ll 1$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \delta_{\alpha\beta} - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2 \right) \sin^2 \Delta_{k1}$$

$$+8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\Delta_{jk} = \frac{\Delta m^2_{jk} L}{4E} \qquad \eta_{\alpha\beta jk} = \arg\left[U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \right]$$

Survival Probabilities

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \sin^2 \Delta_{k1}$$
$$+ 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin \Delta_{j1} \sin \Delta_{k1} \cos \Delta_{jk}$$

Effective amplitude of $\overset{(-)}{\nu}_\alpha$ disappearance due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

$$|U_{\alpha k}|^2 \ll 1 \qquad (\alpha = e, \mu, \tau; \quad k = 4, \ldots, N)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2 \Delta_{k1}$$

Appearance Probabilities $(\alpha \neq \beta)$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} \simeq 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 |U_{\beta k}|^2 \sin^2 \Delta_{k1}$$

$$+8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

Effective amplitude of $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ transitions due to $\nu_{\alpha} - \nu_{k}$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\beta} = 4 |U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} \simeq \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\beta} \sin^2 \Delta_{k1}$$

$$+2 \sum_{k=4}^{N} \sum_{j=k+1}^{N} \sin 2\vartheta^{(k)}_{\alpha\beta} \sin 2\vartheta^{(j)}_{\alpha\beta} \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\sin^2 2\vartheta_{\alpha\alpha}^{(k)} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

$$\sin^2 2\vartheta_{\alpha\beta}^{(k)} = 4|U_{\alpha k}|^2|U_{\beta k}|^2$$

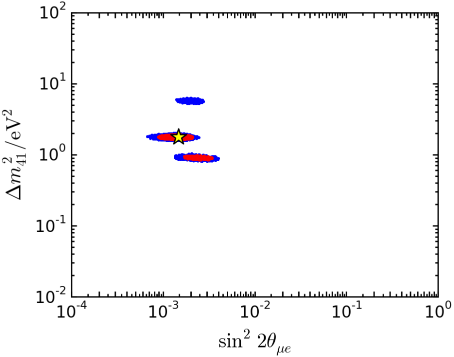
$$\boxed{\sin^2 2\vartheta_{\alpha\beta}^{(k)} \simeq \frac{1}{4} \sin^2 2\vartheta_{\alpha\alpha}^{(k)} \sin^2 2\vartheta_{\beta\beta}^{(k)}}$$

$$\left.\begin{array}{l} \sin^2 2\vartheta_{ee}^{(k)} \ll 1 \\ \sin^2 2\vartheta_{\mu\mu}^{(k)} \ll 1 \end{array}\right\} \quad \Rightarrow \quad \sin^2 2\vartheta_{e\mu}^{(k)} \quad \text{is quadratically suppressed}$$

on the other hand, observation of $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ transitions due to Δm^2_{k1} imply that the corresponding $\overset{(-)}{\nu}_{\alpha}$ and $\overset{(-)}{\nu}_{\beta}$ disappearances must be observed

Collin, Arguelles, Conrad, Shaevitz

[NPB 908 (2016) 354]



Best Fit: $\Delta m^2_{41} = 1.75\,\mathrm{eV}^2$

$|U_{e4}|^2 = 0.027$ $|U_{\mu 4}|^2 = 0.014$

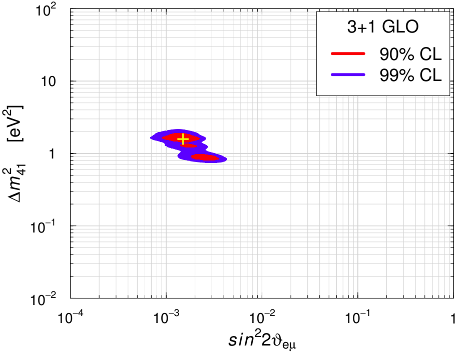
$\mathrm{GoF} = 57\%\,(\chi^2_{\mathrm{min}}/\mathrm{NDF} = 306.8/312)$

$\mathrm{GoF}_{\mathrm{null}} = 4.4\%\,(\chi^2/\mathrm{NDF} = 359.2/315)$

$\Delta\chi^2/\mathrm{NDF} = 52.3/3\ (\approx 6.7\sigma)$

Our Fit

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



Best Fit: $\Delta m^2_{41} = 1.6\,\mathrm{eV}^2$

$|U_{e4}|^2 = 0.028$ $|U_{\mu 4}|^2 = 0.014$

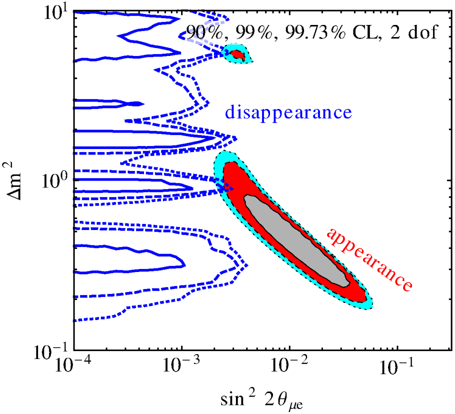
$\mathrm{GoF} = 6\%\,(\chi^2_{\mathrm{min}}/\mathrm{NDF} = 304.0/268)$

$\mathrm{GoF}_{\mathrm{null}} = 0.04\%\,(\chi^2/\mathrm{NDF} = 355.2/271)$

$\Delta\chi^2/\mathrm{NDF} = 51.2/3\ (\approx 6.6\sigma)$

Kopp, Machado, Maltoni, Schwetz

[JHEP 1305 (2013) 050]



Best Fit: $\Delta m^2_{41} = 0.93\,\text{eV}^2$

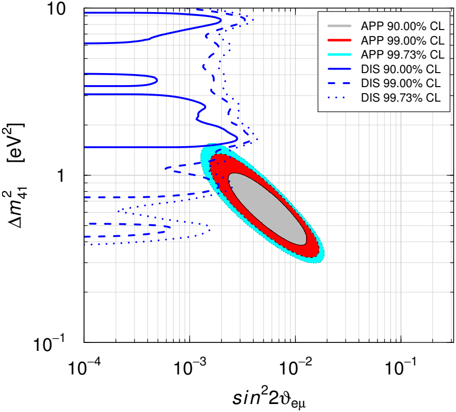
$|U_{e4}|^2 = 0.023$ $|U_{\mu 4}|^2 = 0.029$

$\text{GoF} = 19\%\,(\chi^2_{\text{min}}/\text{NDF} = 712/680)$

$\text{GoF}_{\text{PG}} = 0.01\%\,(\chi^2_{\text{PG}}/\text{NDF} = 18.0/2)$

Our Fit

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



Best Fit: $\Delta m^2_{41} = 1.6\,\text{eV}^2$

$|U_{e4}|^2 = 0.028$ $|U_{\mu 4}|^2 = 0.014$

$\text{GoF} = 6\%\,(\chi^2_{\text{min}}/\text{NDF} = 304.0/268)$

$\text{GoF}_{\text{PG}} = 0.06\%\,(\chi^2/\text{NDF} = 15.0/2)$

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

$$N_{\text{D}} = \text{Number of Data} \qquad N_{\text{P}} = \text{Number of Fitted Parameters}$$

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF}$ $\qquad$ $\text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, \text{d}z$ $\qquad$ $p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

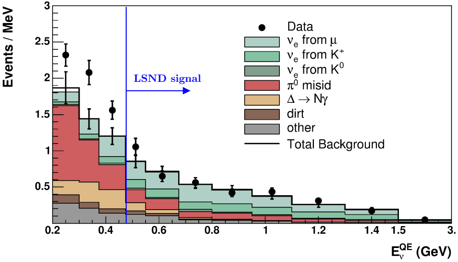
- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

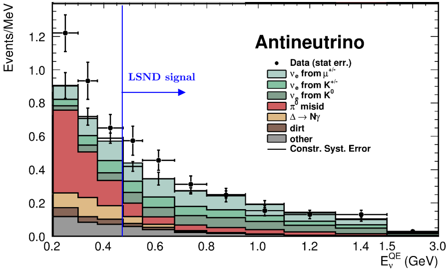
- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, \text{d}z$

MiniBooNE Low-Energy Anomaly

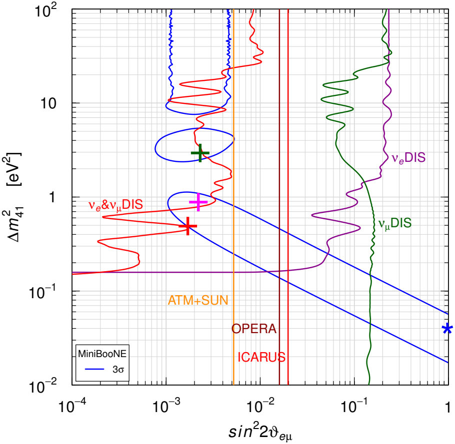
$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]

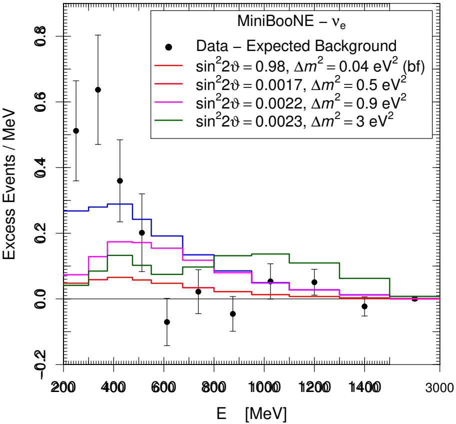


$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



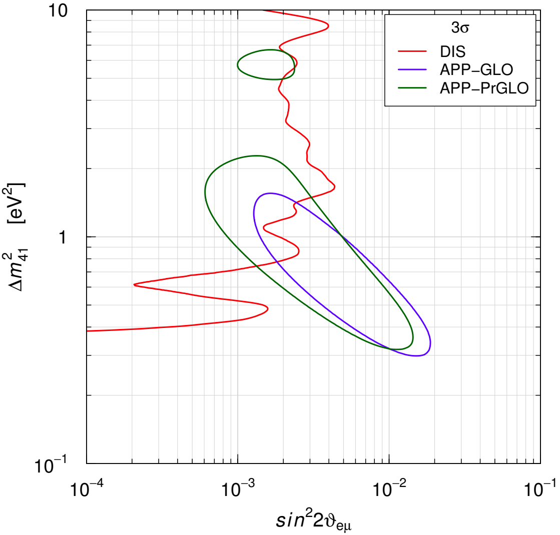
- Fit of MB Low-Energy Excess requires small Δm^2_{41} and large $\sin^2 2\vartheta_{e\mu}$, in contradiction with disappearance data
- MB low-energy excess is the main cause of bad APP-DIS $\text{GoF}_{\text{PG}} = 0.06\%$
- Multinucleon effects in neutrino energy reconstruction are not enough to solve the problem [Martini et al, PRD 85 (2012) 093012; PRD 87 (2013) 013009; PRD 93 (2016) 073008]
- Pragmatic Approach: discard the Low-Energy Excess because it is likely not due to oscillations [CG, Laveder, Li, Long, PRD 88 (2013) 073008]
- MicroBooNE is crucial for checking the MiniBooNE Low-Energy Anomaly and the consistency of different short-baseline data





No fit of low-energy excess for realistic $\sin^2 2\vartheta_{e\mu} \lesssim 3 \times 10^{-3}$

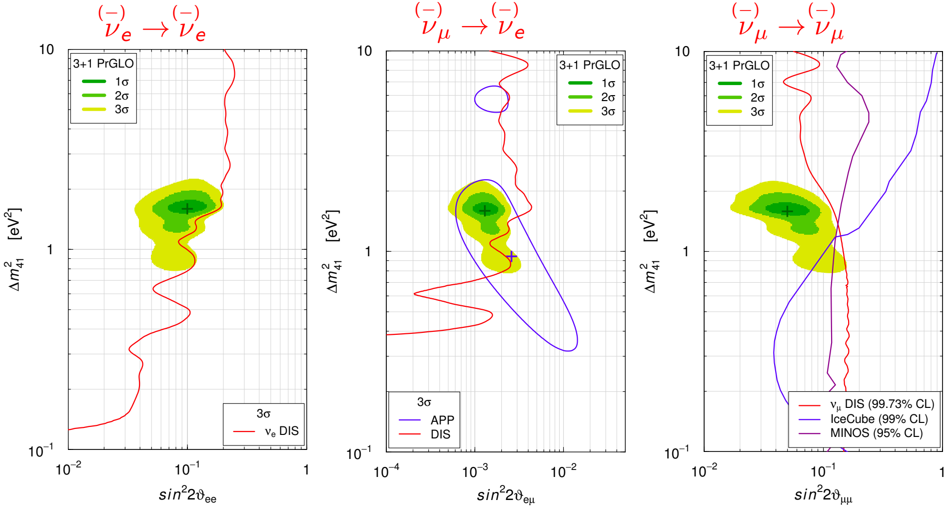
Global → Pragmatic



- APP-GLO: all MiniBooNE data
- APP-PrGLO: only MiniBooNE $E > 475\,\text{MeV}$ data (Pragmatic)

Pragmatic Global 3+1 Fit

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



$\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$ $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$

GoF = 24% PGoF = 7%

No Osc. disfavored at ≈ 6.2σ

$\Delta\chi^2$/NDF = 46.6/3

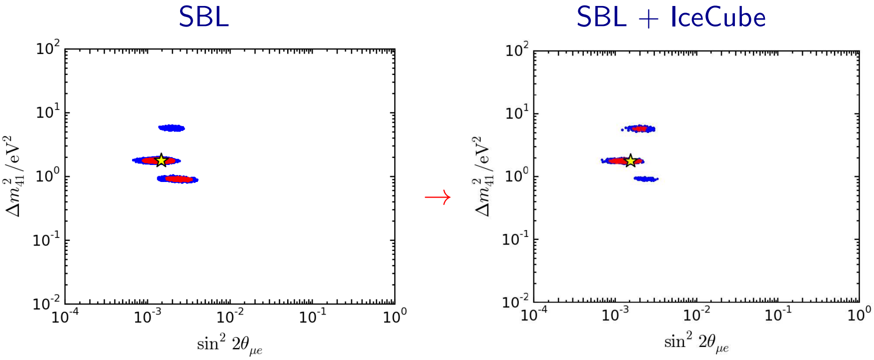
Not yet included:

– IceCube, arXiv:1605.01990

– MINOS, arXiv:1607.01176

SBL + IceCube

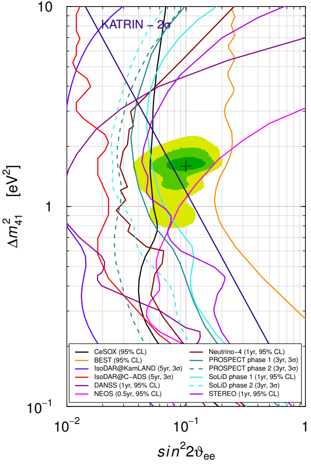
[Collin, Arguelles, Conrad, Shaevitz, arXiv:1607.00011]



Red: 90% CL   Blue: 99% CL

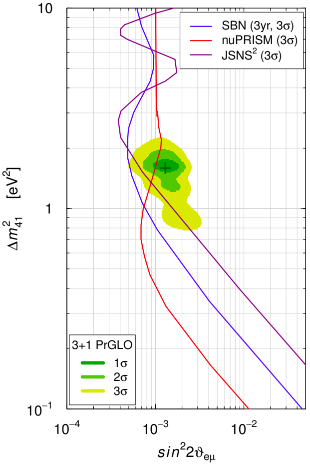
3+1	Δm^2_{41}	$\lvert U_{e4}\rvert$	$\lvert U_{\mu4}\rvert$	$\lvert U_{\tau4}\rvert$	N_{bins}	$\chi^2_{\rm min}$	$\chi^2_{\rm null}$	$\Delta\chi^2$ (dof)
SBL	1.75	0.163	0.117	-	315	306.81	359.15	52.34 (3)
SBL+IC	1.75	0.164	0.119	0.00	524	518.59	568.84	50.26 (4)
IC	5.62	-	0.314	-	209	207.11	209.69	2.58 (2)

The Race for the Light Sterile



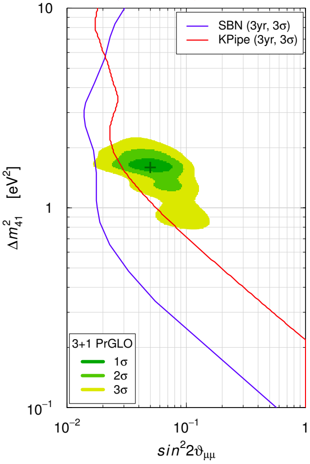
$\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$

SBN (FNAL, USA)
[arXiv:1503.01520]
3 Liquid Argon TPCs
LAr1-ND $L \simeq 100$ m
MicroBooNE $L \simeq 470$ m
ICARUS T600 $L \simeq 600$ m



$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$

nuPRISM (J-PARC, Japan)
[Wilking@NNN2015]
$L \simeq 1$ km
50 m tall water Cherenkov detector
$1^\circ - 4^\circ$ off-axis



$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$

KPipe (Japan) [arXiv:1510.06994]
KDAR: K Decay At Rest
$K^+ \to \mu^+ + \nu_\mu (E = 236 \text{ MeV})$
$L \simeq 30\text{-}150$m
120 m long detector!

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\Delta_{kj} = \Delta m^2_{kj} L / 4E$$

$$\eta = \arg[U^*_{e4} U_{\mu4} U_{e5} U^*_{\mu5}]$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = 4|U_{e4}|^2 |U_{\mu4}|^2 \sin^2\Delta_{41} + 4|U_{e5}|^2|U_{\mu5}|^2 \sin^2\Delta_{51} + 8|U_{\mu4}U_{e4}U_{\mu5}U_{e5}| \sin\Delta_{41} \sin\Delta_{51} \cos(\Delta_{54} \overset{(+)}{-} \eta)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - 4(1 - |U_{\alpha4}|^2 - |U_{\alpha5}|^2)(|U_{\alpha4}|^2 \sin^2\Delta_{41} + |U_{\alpha5}|^2 \sin^2\Delta_{51}) - 4|U_{\alpha4}|^2|U_{\alpha5}|^2 \sin^2\Delta_{54}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Archidiacono et al, PRD 86 (2012) 065028; Jacques, Krauss, Lunardini, PRD 87 (2013) 083515; Conrad et al, AHEP 2013 (2013) 163897; Archidiacono et al, PRD 87 (2013) 125034; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008; Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

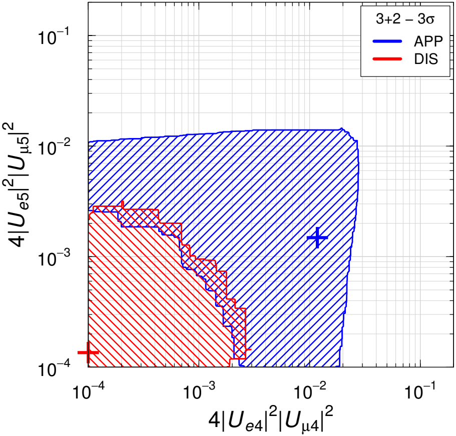
- Good: CP violation
- Bad: Two massive sterile neutrinos at the eV scale!

  4 more parameters: $\underbrace{\Delta m^2_{41}, |U_{e4}|^2, |U_{\mu4}|^2}_{3+1}, \Delta m^2_{51}, |U_{e5}|^2, |U_{\mu5}|^2, \eta$

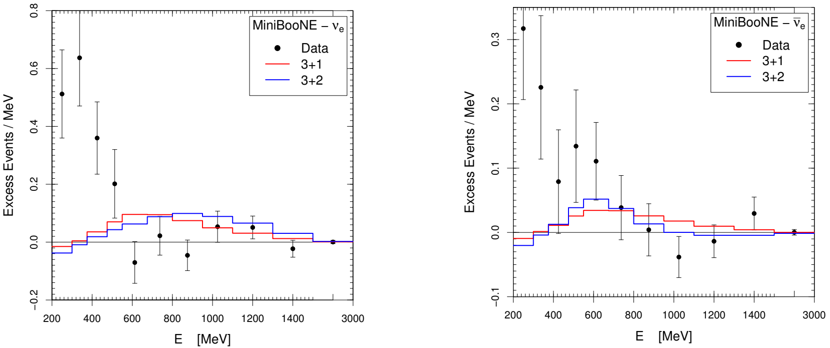
Global Fits	Our Fit		KMMS	
	3+1	3+2	3+1	3+2
GoF	5%	7%	19%	23%
PGoF	0.1%	0.04%	0.01%	0.003%

- Our Fit: Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001
- KMMS: Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050

APP-DIS 3+2 Tension:



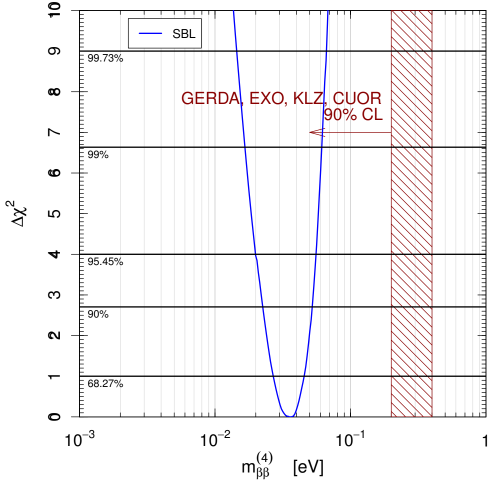
3+2 cannot fit MiniBooNE Low-Energy Excess



- Note difference between 3+2 ν_e and $\bar{\nu}_e$ histograms due to CP violation
- 3+2 can fit slightly better the small $\bar{\nu}_e$ excess at about 600 MeV
- 3+2 fit of low-energy excess as bad as 3+1
- Claims that 3+2 can fit low-energy excess do not take into account constraints from other data
- Conclusion: 3+2 is not needed

Neutrinoless Double-β Decay

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3 + |U_{e4}|^2 e^{i\alpha_{41}} m_4$$



Pragmatic 3+1 Fit

$$m_{\beta\beta}^{(k)} = |U_{ek}|^2 m_k$$

$$m_1 \ll m_4$$

$$\Downarrow$$

$$m_{\beta\beta}^{(4)} \simeq |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

surprise:

possible cancellation

with $m_{\beta\beta}^{(3\nu)}$

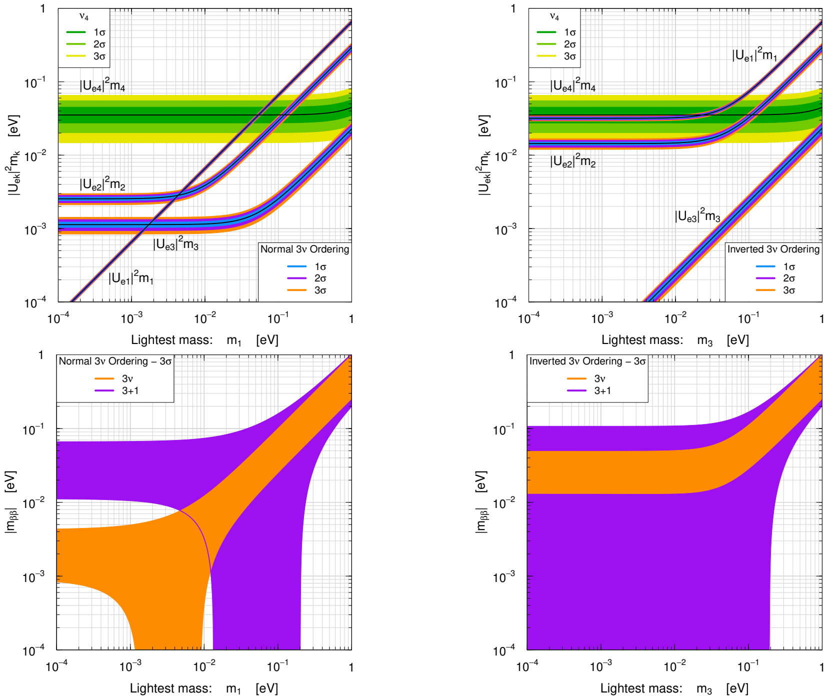
[Barry et al, JHEP 07 (2011) 091]

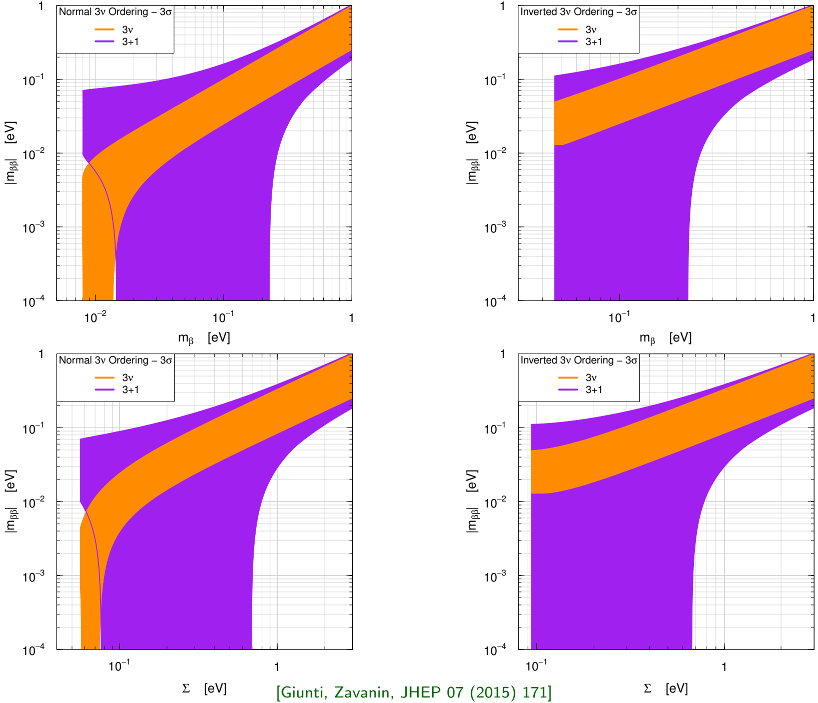
[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, JPG 39 (2012) 124008]

[Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

[Giunti, Zavanin, JHEP 07 (2015) 171]


ν4
1σ
2σ
3σ
$|U_{e4}|^2 m_4$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
Normal 3ν Ordering
1σ
2σ
3σ
$|U_{ek}|^2 m_k$ [eV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Lightest mass: m_1 [eV]
ν4
1σ
2σ
3σ
$|U_{e1}|^2 m_1$
$|U_{e4}|^2 m_4$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
Inverted 3ν Ordering
1σ
2σ
3σ
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_3 [eV]
Normal 3ν Ordering – 3σ
3ν
3+1
$|m_{\beta\beta}|$ [eV]
Lightest mass: m_1 [eV]
Inverted 3ν Ordering – 3σ
3ν
3+1
$|m_{\beta\beta}|$ [eV]
Lightest mass: m_3 [eV]



[Giunti, Zavanin, JHEP 07 (2015) 171]

Effects of light sterile neutrinos should also be seen in:

- β Decay Experiments
  [Hannestad et al, JCAP 1102 (2011) 011, PRC 84 (2011) 045503; Formaggio, Barrett, PLB 706 (2011) 68; Esmaili, Peres, PRD 85 (2012) 117301; Gastaldo et al, JHEP 1606 (2016) 061]
- Neutrinoless Double-β Decay Experiments
  [Rodejohann et al, JHEP 1107 (2011) 091; Li, Liu, PLB 706 (2012) 406; Meroni et al, JHEP 1311 (2013) 146, PRD 90 (2014) 053002; Pascoli et al, PRD 90 (2014) 093005; CG, Zavanin, JHEP 1507 (2015) 171; Guzowski et al, PRD 92 (2015) 012002]
- Long-baseline Neutrino Oscillation Experiments
  [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, arXiv:1601.05995, arXiv:1603.03759, arXiv:1605.04299; Gandhi et al, JHEP 1511 (2015) 039; Pant et al, arXiv:1509.04096, Choubey, Pramanik, arXiv:1604.04731]
- Solar neutrinos
  [Dooling et al, PRD 61 (2000) 073011, Gonzalez-Garcia et al, PRD 62 (2000) 013005; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301; Li et al, PRD 80 (2009) 113007, PRD 87, 113004 (2013), JHEP 1308 (2013) 056; Kopp et al, JHEP 1305 (2013) 050]
- Atmospheric neutrinos
  [Goswami, PRD 55 (1997) 2931; Bilenky et al, PRD 60 (1999) 073007; Maltoni et al, NPB 643 (2002) 321, PRD 67 (2003) 013011; Choubey, JHEP 0712 (2007) 014; Razzaque, Smirnov, JHEP 1107 (2011) 084, PRD 85 (2012) 093010; Gandhi, Ghoshal, PRD 86 (2012) 037301; Barger et al, PRD 85 (2012) 011302; Esmaili et al, JCAP 1211 (2012) 041, JCAP 1307 (2013) 048, JHEP 1312 (2013) 014; Rajpoot et al, EPJC 74 (2014) 2936; Lindner et al, JHEP 1601 (2016) 124; Behera et al, arXiv:1605.08607]
- Supernova neutrinos
  [Caldwell, Fuller, Qian, PRD 61 (2000) 123005; Peres, Smirnov, NPB 599 (2001); Sorel, Conrad, PRD 66 (2002) 033009; Tamborra et al, JCAP 1201 (2012) 013; Wu et al, PRD 89 (2014) 061303; Esmaili et al, PRD 90 (2014) 033013]
- Cosmic neutrinos
  [Cirelli et al, NPB 708 (2005) 215; Donini, Yasuda, arXiv:0806.3029; Barry et al, PRD 83 (2011) 113012]
- Indirect dark matter detection [Esmaili, Peres, JCAP 1205 (2012) 002]
- Cosmology [see: Wong, ARNPS 61 (2011) 69; Archidiacono et al, AHEP 2013 (2013) 191047]

Effective 3+1 LBL Oscillation Probabilities

[de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, arXiv:1601.05995, arXiv:1603.03759, arXiv:1605.04299; Gandhi et al, JHEP 1511 (2015) 039]

$$|U_{e3}| \simeq \sin\vartheta_{13} \simeq 0.15 \sim \varepsilon \quad \Longrightarrow \quad \varepsilon^2 \sim 0.03$$

$$|U_{e4}| \simeq \sin\vartheta_{14} \simeq 0.17 \sim \varepsilon$$

$$|U_{\mu4}| \simeq \sin\vartheta_{24} \simeq 0.11 \sim \varepsilon$$

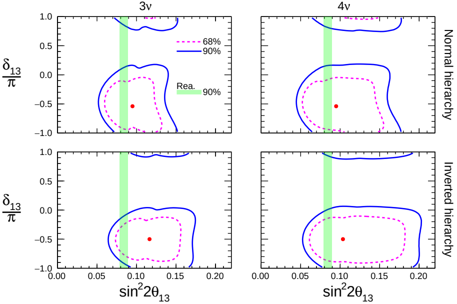
$$\alpha \equiv \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq \frac{7 \times 10^{-5}}{2.4 \times 10^{-3}} \simeq 0.031 \sim \varepsilon^2$$

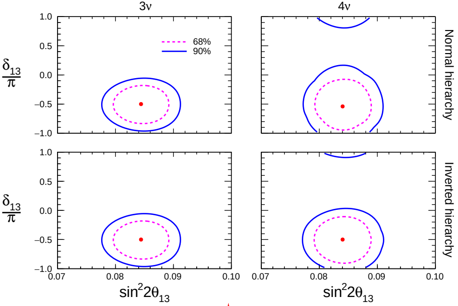
At order ε^3: [Klop, Palazzo, PRD 91 (2015) 073017] $\Delta_{kj} \equiv \Delta m^2_{kj} L/4E$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq\ & 4\sin^2\vartheta_{13}\sin^2\vartheta_{23}\sin^2\Delta_{31} & \sim \varepsilon^2 \\ & +2\sin\vartheta_{13}\sin 2\vartheta_{12}\sin 2\vartheta_{23}(\alpha\Delta_{31})\sin\Delta_{31}\cos(\Delta_{32}+\delta_{13}) & \sim \varepsilon^3 \\ & +4\sin\vartheta_{13}\sin\vartheta_{14}\sin\vartheta_{24}\sin\vartheta_{23}\sin\Delta_{31}\sin(\Delta_{31}+\delta_{13}-\delta_{14}) & \sim \varepsilon^3 \end{aligned}$$

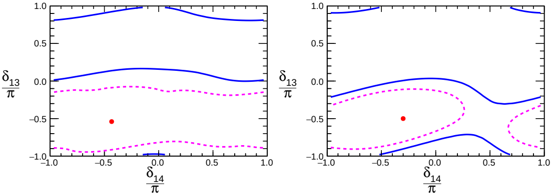
CP Violation in T2K and NOνA

[Capozzi, CG, Laveder, Palazzo, in preparation, with T2K and NOνA data presented at Neutrino 2016]





LBL + Reactors



Normal Ordering

Inverted Ordering

Inverted Ordering: Better agreement of LBL & Reactors for $\delta_{14} \approx -\pi/2$

Cosmology

- neutrinos in equilibrium in early Universe through weak interactions:

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N$$

$$\nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

- weak interactions freeze out $\Longrightarrow$ active $(\nu_e, \nu_\mu, \nu_\tau)$ neutrino decoupling

$$\Gamma_{\text{weak}} = N\sigma v \sim G_{\text{F}}^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H$$

$$T_{\nu\text{-dec}} \sim 1\,\text{MeV} \qquad t_{\nu\text{-dec}} \sim 1\,\text{s}$$

- relic neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$

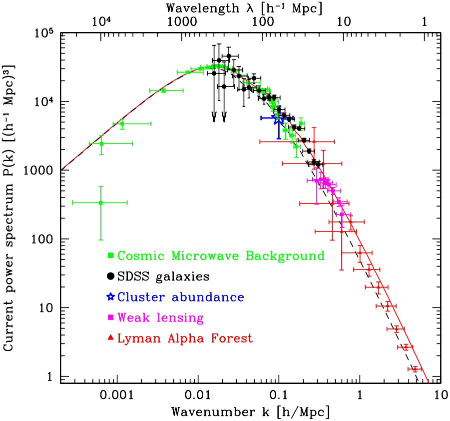
- number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\, T_\nu^3 \simeq 112\,\text{cm}^{-3}$

- density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.1\,\text{eV}} \Rightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.1\,\text{eV}}}$

$\left(\rho_c = \frac{3H^2}{8\pi G_{\text{N}}}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120; Cowsik, McClelland, PRL 29 (1972) 669]

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\mathrm{M}}^{0} = 0.28\,,\ h = 0.72\,,\ \Omega_{\mathrm{B}}^{0}/\Omega_{\mathrm{M}}^{0} = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\mathrm{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\mathrm{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\mathrm{nr}} \approx 0.026 \sqrt{\frac{m_\nu}{1\,\mathrm{eV}}}\sqrt{\Omega_m}\, h\,\mathrm{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

[PRD 78 (2008) 033010, hep-ph/0805.2517]

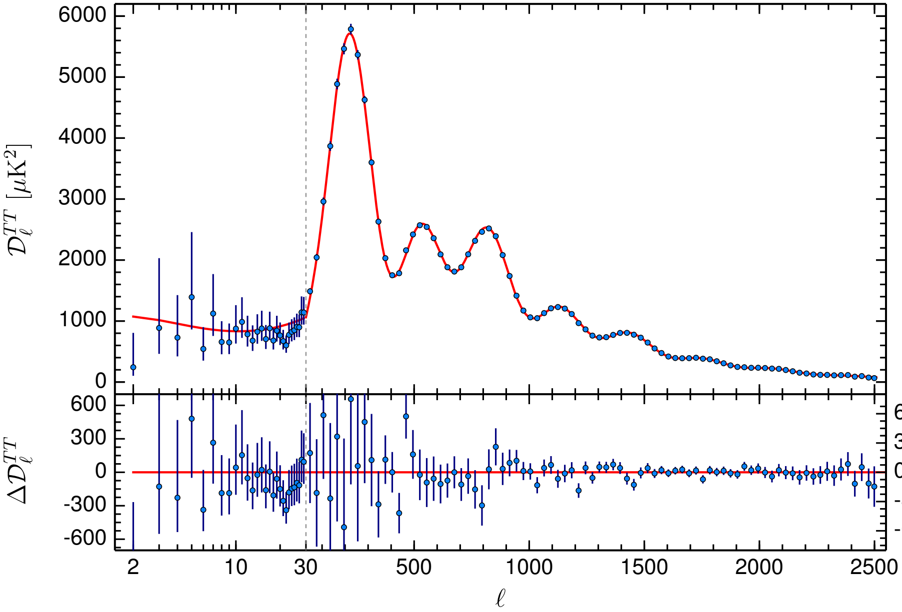
Flat ΛCDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

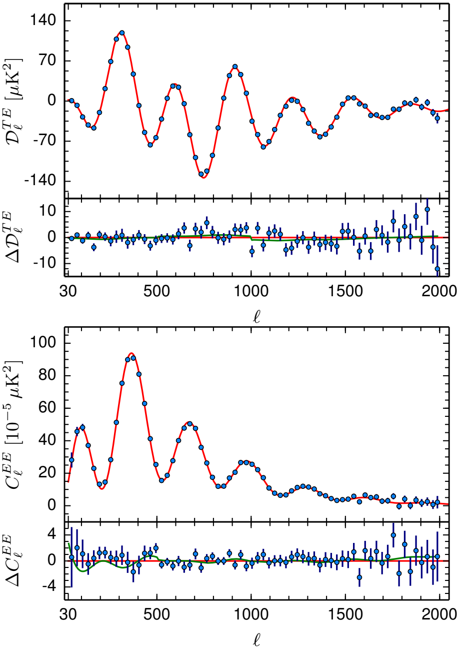
2σ (95% C.L.) constraints on the sum of ν masses Σ.

Planck

[arXiv:1502.01589]

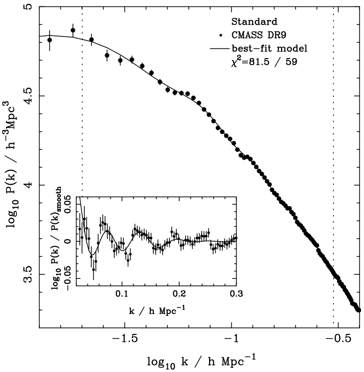


Planck Polarization Data



Planck Terminology

- TT denotes the Plank TT data (low-ℓ for $\ell < 30$ and high-ℓ for $\ell \geq 30$).
- lowP denotes the Planck polarization data at multipoles $\ell < 30$ (low-ℓ).
- TE denotes the Plank TE data at $\ell \geq 30$.
- EE denotes the Plank EE data at $\ell \geq 30$.
- Lensing denotes the Plank weak lensing data.
- BAO denotes the Baryon Acustic Oscillation data.

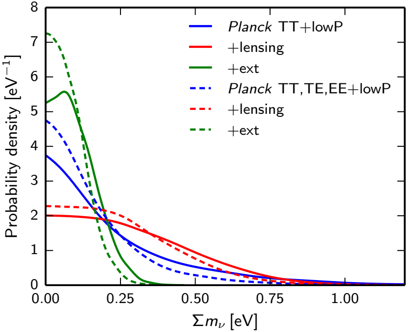


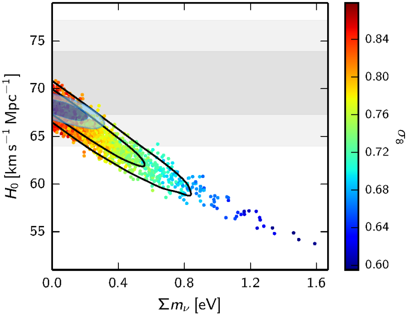
Baryon Oscillation Spectroscopic Survey
(BOSS)
part of the Sloan Digital Sky Survey III
(SDSS-III)
Data Release 9 (DR9) CMASS sample
[arXiv:1203.6594]

Limits on the Sum of Standard Light Neutrino Masses

[Planck, arXiv:1502.01589]

Cosmological data set	Σ (at 95% C.L.)
Plank TT + lowP	< 0.72 eV
Plank TT + lowP + BAO	< 0.21 eV
Plank TT,TE,EE + lowP	< 0.49 eV
Plank TT,TE,EE + lowP + BAO	< 0.17 eV
Plank TT + lowP + lensing	< 0.68 eV
Plank TT,TE,EE + lowP + lensing	< 0.59 eV
Plank TT + lowP + lensing + BAO + H_0	< 0.23 eV





Sterile Neutrinos in Cosmology

- sterile neutrinos can be produced by $\nu_{e,\mu,\tau} \to \nu_s$ oscillations before active neutrino decoupling $(t_{\nu\text{-dec}} \sim 1\,\text{s})$
- energy density of radiation before matter-radiation equality:

$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right]\rho_\gamma \qquad (t < t_{\text{eq}} \sim 6 \times 10^4\,\text{y})$$

$$N_{\text{eff}}^{\text{SM}} = 3.046 \qquad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$$

- sterile neutrino contribution:

$$\rho_s = (T_s/T_\nu)^4 \rho_\nu \quad \Longrightarrow \quad \Delta N_{\text{eff}} = (T_s/T_\nu)^4$$

- sterile neutrino $\nu_s \simeq \nu_4$ with mass $m_s = m_4 \simeq \sqrt{\Delta m^2_{41}} \sim 1\,\text{eV}$ becomes non-relativistic at $T_\nu \sim m_s/3$, that is at $t_{\nu_s\text{-nr}} \sim 2.0 \times 10^5\,\text{y}$, before recombination at $t_{\text{rec}} \sim 3.8 \times 10^5\,\text{y}$
- current energy density of sterile neutrinos:

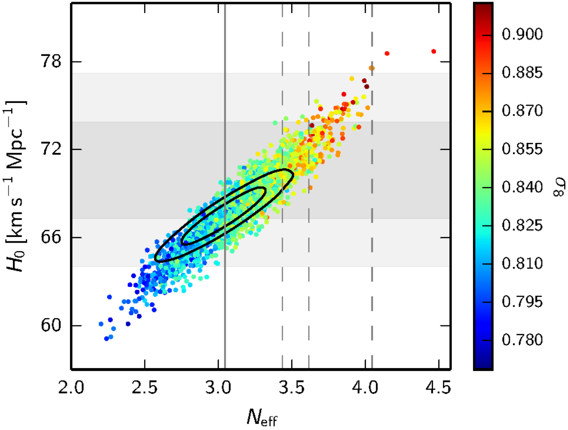
$$\Omega_s = \frac{n_s m_s}{\rho_c} \simeq \frac{1}{h^2}\,\frac{(T_s/T_\nu)^3 m_s}{94.1\,\text{eV}} = \frac{1}{h^2}\,\frac{\Delta N_{\text{eff}}^{3/4} m_s}{94.1\,\text{eV}} = \frac{1}{h^2}\,\frac{m_s^{\text{eff}}}{94.1\,\text{eV}}$$

$$m_s^{\text{eff}} = \Delta N_{\text{eff}}^{3/4} m_s = (T_s/T_\nu)^3 m_s$$

Limits on Dark Radiation

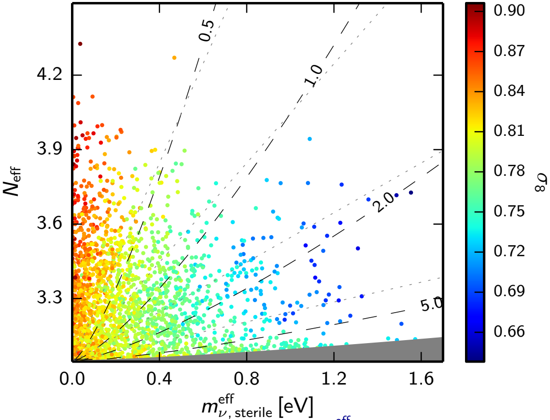
[Planck, arXiv:1502.01589]

Cosmological data set	N_{eff}
Plank TT + lowP	3.13 ± 0.32
Plank TT + lowP + BAO	3.15 ± 0.23
Plank TT,TE,EE + lowP	2.99 ± 0.20
Plank TT,TE,EE + lowP + BAO	3.04 ± 0.18



Limits on Massive Sterile Neutrinos

$N_{\text{eff}} < 3.7$ $\qquad m_s^{\text{eff}} < 0.52$ $\qquad$ (95%; Plank TT + lowP + lensing + BAO)



Samples from Plank TT + lowP in the N_{eff}–m_s^{eff} plane, colour-coded by σ_8, in models with one massive sterile neutrino family, with effective mass m_s^{eff}, and the three active neutrinos as in the base ΛCDM model. The physical mass of the sterile neutrino in the thermal scenario, m_s^{thermal}, is constant along the grey dashed lines, with the indicated mass in eV; the grey region shows the region excluded by our prior $m_s^{\text{thermal}} < 10\,\text{eV}$, which excludes most of the area where the neutrinos behave nearly like dark matter. The physical mass in the Dodelson-Widrow scenario, m_s^{DW}, is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

[arXiv:1502.01589]

- $m_s^{\text{eff}} \equiv 94.1 \Omega_s h^2\,\text{eV}$
- Thermally distributed:

$$f_s(E) = \frac{1}{e^{E/T_s}+1}$$

$$m_s^{\text{eff}} = \left(\frac{T_s}{T_\nu}\right)^3 m_4 = (\Delta N_{\text{eff}})^{3/4} m_4$$

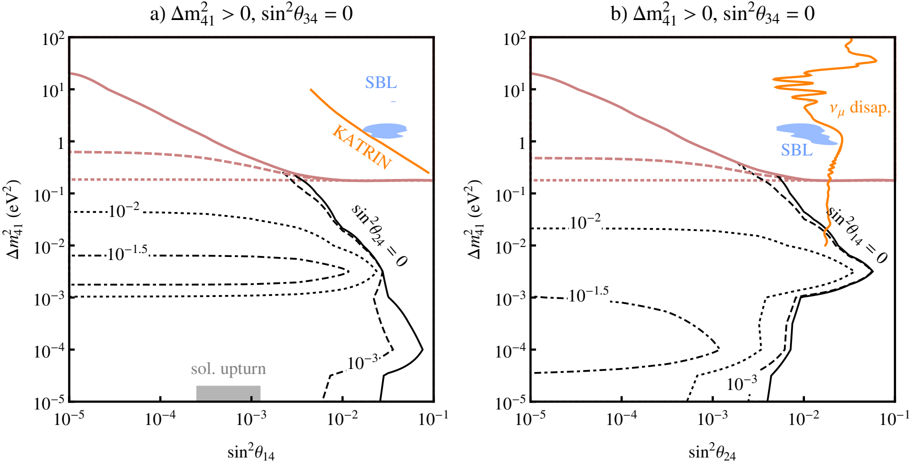
- Dodelson-Widrow:

$$f_s(E) = \frac{\chi}{e^{E/T_\nu}+1}$$

$$m_s^{\text{eff}} = \chi_s m_4$$

Standard Cosmological Scenario Mixing Bounds

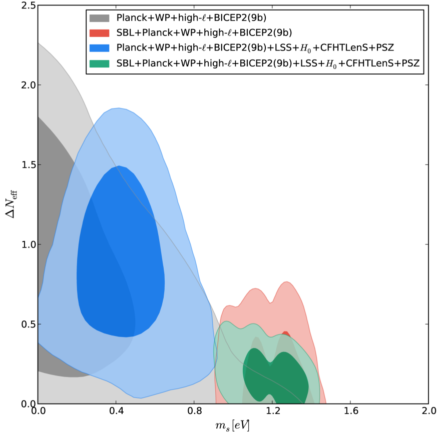
[Mirizzi, Mangano, Saviano, Borriello, Giunti, Miele, Pisanti, PLB 726 (2013) 8, arXiv:1303.5368]

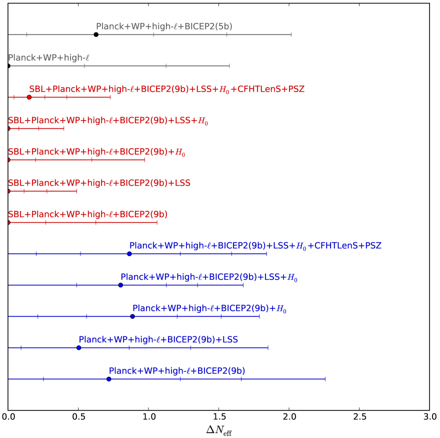


Non-standard mechanism for partial thermalization of ν_s is needed

Large primordial neutrino asymmetry?

[Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009; Saviano, Mirizzi, Pisanti, Serpico, Mangano, Miele, PRD 87 (2013) 073006]





[Archidiacono, Fornengo, Gariazzo, Giunti, Hannestad, Laveder, arXiv:1404.1794]

See also:
- [Gariazzo, Giunti, Laveder, JCAP 1504 (2015) 023]
- [Bergstrom, Gonzalez-Garcia, Niro, Salvado, JHEP 1410 (2014) 104]

Without oscillation data:
- [Giusarma, Di Valentino, Lattanzi, Melchiorri, Mena, arXiv:1403.4852]
- [Zhang, Li, Zhang, arXiv:1403.7028]
- [Dvorkin, Wyman, Rudd, Hu, arXiv:1403.8049]
- [Zhang, Li, Zhang, arXiv:1404.3598]

Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1\,\text{eV}$

Sterile neutrinos are thermalized $(\Delta N_{\text{eff}} = 1)$ by active-sterile oscillations before neutrino decoupling [Dolgov, Villante, NPB 679 (2004) 261]

Proposed mechanisms to avoid the tension:

- Large lepton asymmetry [Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009; Saviano et al., PRD 87 (2013) 073006; Hannestad, Hansen, Tram, JCAP 1304 (2013) 032]
- Interactions in the sterile sector [Hannestad, Hansen, Tram, PRL 112 (2014) 031802; Dasgupta, Kopp et al, PRL 112 (2014) 031803, JCAP 1510 (2015) 011; Bringmann, Hasenkamp, Kersten, JCAP 1407 (2014) 042; Ko, Tang, PLB 739 (2014) 62; Archidiacono, Hannestad et al, PRD 91 (2015) 065021, PRD 93 (2016) 045004, JCAP 1608 (2016) 067; Mirizzi, Mangano, Pisanti, Saviano, PRD 90 (2014) 113009, PRD 91 (2015) 025019; Tang, PLB 750 (2015) 201; Cherry, Friedland, Shoemaker, arXiv:1411.1071]
- A larger cosmic expansion rate at the time of sterile neutrino production [Rehagen, Gelmini JCAP 1406 (2014) 044]
- MeV dark matter annihilation [Ho, Scherrer, PRD 87 (2013) 065016]
- Invisible decay [Gariazzo, Giunti, Laveder, arXiv:1404.6160]
- Free primordial power spectrum of scalar fluctuations (Inflationary Freedom) [Gariazzo, Giunti, Laveder, JCAP 1504 (2015) 023]

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 7.6 \times 10^{-5}\,\text{eV}^2$ [SOL, KamLAND]

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ [ATM, K2K, MINOS]

$\sin^2\vartheta_{12} \simeq 0.3$ $\sin^2\vartheta_{23} \simeq 0.5$ $\sin^2\vartheta_{13} \simeq 0.02$ [Daya Bay]

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

Exp.&Pheno.: Measure mass ordering and CP violation.

Find absolute mass scale and Majorana or Dirac.

Find if sterile neutrinos exist.

Conclusions on Light Sterile Neutrinos

- Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance:
  - Experimental data agree on Reactor $\bar{\nu}_e$ and Gallium ν_e disappearance.
  - Problem: total rates may have unknown systematic uncertainties.
  - Many promising projects to test unambiguously short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources.
  - Because of 5 MeV bump we know that the calculated spectrum must be corrected: oscillations must be observed as a function of distance!
  - Independent tests through effect of m_4 in β-decay and $\beta\beta_{0\nu}$-decay.
- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - Not seen by other SBL $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ experiments.
  - Experiments with near detector are needed to check LSND signal!
  - Promising Fermilab program aimed at a conclusive solution of the mystery: a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-T600), all Liquid Argon Time Projection Chambers.
- Pragmatic 3+1 Fit is fine: moderate APP-DIS tension.
- 3+2 is not needed: same APP-DIS tension and no exp. CP violation.
- Cosmology:
  - Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1\,\text{eV}$.
  - Cosmological and oscillation data may be reconciled by a non-standard cosmological mechanism.