

Theory and Phenomenology of Massive Neutrinos

Part II: Neutrino Oscillations

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and

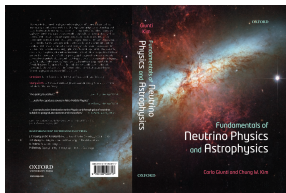
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Neutrino Unbound: <http://www.nu.to.infn.it>

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<http://personalpages.to.infn.it/~giunti/slides/2017/giunti-170104-LAL.pdf>



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and
Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part II: Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

Neutrino Mixing

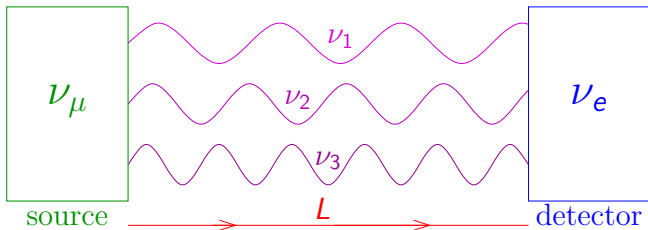
- ▶ Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- ▶ Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- ▶ Neutrino Mixing: a Flavor Neutrino is a superposition of Massive Neutrinos

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

- ▶ U is the 3×3 unitary Neutrino Mixing Matrix

Neutrino Oscillations

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = U_{\mu 1} |\nu_1\rangle + U_{\mu 2} |\nu_2\rangle + U_{\mu 3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = U_{\mu 1} e^{-iE_1 t} |\nu_1\rangle + U_{\mu 2} e^{-iE_2 t} |\nu_2\rangle + U_{\mu 3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\mu\rangle$$

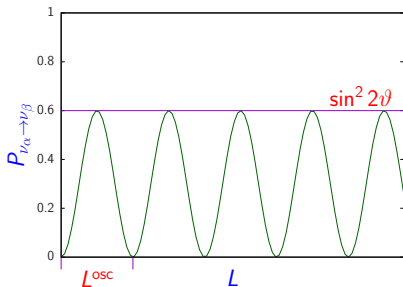
$$E_k^2 = p^2 + m_k^2$$

$$P_{\nu_\mu \rightarrow \nu_e}(t > 0) = |\langle \nu_e | \nu(t > 0) \rangle|^2 \sim \sum_{k>j} \text{Re}[U_{ek} U_{\mu k}^* U_{ej}^* U_{\mu j}] \sin^2 \left(\frac{\Delta m_{kj}^2 L}{4E} \right)$$

transition probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

$$\begin{array}{cccc} \nu_e \rightarrow \nu_\mu & \nu_e \rightarrow \nu_\tau & \nu_\mu \rightarrow \nu_e & \nu_\mu \rightarrow \nu_\tau \\ \bar{\nu}_e \rightarrow \bar{\nu}_\mu & \bar{\nu}_e \rightarrow \bar{\nu}_\tau & \bar{\nu}_\mu \rightarrow \bar{\nu}_e & \bar{\nu}_\mu \rightarrow \bar{\nu}_\tau \end{array}$$

$$2\nu\text{-mixing: } P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2 2\vartheta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \implies L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$



Tiny neutrino masses lead to observable macroscopic oscillation distances!

$$\frac{L}{E} \gtrsim \begin{cases} 10 \frac{\text{m}}{\text{MeV}} \left(\frac{\text{km}}{\text{GeV}} \right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1} \text{ eV}^2 \\ 10^3 \frac{\text{m}}{\text{MeV}} \left(\frac{\text{km}}{\text{GeV}} \right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3} \text{ eV}^2 \\ 10^4 \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4} \text{ eV}^2 \\ 10^{11} \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11} \text{ eV}^2 \end{cases}$$

Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

A Brief History of Neutrino Oscillations

- ▶ **1957:** Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrow \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\implies \nu \leftrightarrow \bar{\nu}$
- ▶ In **1957** only one neutrino type $\nu = \nu_e$ was known! The possible existence of ν_μ was discussed by several authors. Maybe the first have been Sakata and Inoue in **1946** and Konopinski and Mahmoud in **1953**. Maybe Pontecorvo did not know. He discussed the possibility to distinguish ν_μ from ν_e in **1959**.
- ▶ **1962:** Maki, Nakagawa, Sakata proposed a model with ν_e and ν_μ and Neutrino Mixing:
“weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrow \nu_\mu$ ”
- ▶ **1962:** Lederman, Schwartz and Steinberger discover ν_μ
- ▶ **1967:** Pontecorvo: intuitive $\nu_e \leftrightarrow \nu_\mu$ oscillations with maximal mixing. Applications to reactor and solar neutrinos (“prediction” of the solar neutrino problem).
- ▶ **1969:** Gribov and Pontecorvo: $\nu_e - \nu_\mu$ mixing and oscillations. But no clear derivation of oscillations with a factor of 2 mistake in the phase (misprint?).

- ▶ **1975-76:** Start of the “Modern Era” of Neutrino Oscillations with a general theory of neutrino mixing and a rigorous derivation of the oscillation probability by **Eliezer and Swift, Fritzsche and Minkowski, and Bilenky and Pontecorvo.** [Bilenky, Pontecorvo, Phys. Rep. (1978) 225]
- ▶ **1978:** **Wolfenstein** discovers the effect on neutrino oscillations of the matter potential (“**Matter Effect**”)
- ▶ **1985:** **Mikheev and Smirnov** discover the resonant amplification of solar $\nu_e \rightarrow \nu_\mu$ oscillations due to the Matter Effect (“**MSW Effect**”)
- ▶ **1998:** the **Super-Kamiokande** experiment observed in a model-independent way the Vacuum Oscillations of atmospheric neutrinos ($\nu_\mu \rightarrow \nu_\tau$).
- ▶ **2002:** the **SNO** experiment observed in a model-independent way the flavor transitions of solar neutrinos ($\nu_e \rightarrow \nu_\mu, \nu_\tau$), mainly due to adiabatic MSW transitions. [see: Smirnov, arXiv:1609.02386]
- ▶ **2015:** **Takaaki Kajita** (Super-Kamiokande) and **Arthur B. McDonald** (SNO) received the Physics Nobel Prize “for the discovery of neutrino oscillations, which shows that neutrinos have mass”

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 \text{ MeV}$ are detectable!

Charged-Current Processes: Threshold

$$\nu + A \rightarrow B + C$$



$$s = 2Em_A + m_A^2 \geq (m_B + m_C)^2$$



$$E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2}$$

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^- \quad E_{\text{th}} = 0.233 \text{ MeV}$$

$$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^- \quad E_{\text{th}} = 0.81 \text{ MeV}$$

$$\bar{\nu}_e + p \rightarrow n + e^+ \quad E_{\text{th}} = 1.8 \text{ MeV}$$

$$\nu_\mu + n \rightarrow p + \mu^- \quad E_{\text{th}} = 110 \text{ MeV}$$

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \quad E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9 \text{ GeV}$$

Elastic Scattering Processes: Cross Section \propto Energy

$$\nu + e^- \rightarrow \nu + e^- \quad \sigma(E) \sim \sigma_0 E/m_e \quad \sigma_0 \sim 10^{-44} \text{ cm}^2$$

Background $\implies E_{\text{th}} \simeq 5 \text{ MeV}$ (SK, SNO), 0.25 MeV (Borexino)

Laboratory and Astrophysical Limits $\implies m_\nu \lesssim 1 \text{ eV}$

Simple Example of Neutrino Production

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\nu_\mu = \sum_k U_{\mu k} \nu_k$$

two-body decay \implies fixed kinematics

$$E_k^2 = p_k^2 + m_k^2$$

$$\pi \text{ at rest: } \begin{cases} p_k^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_k^2}{2} \left(1 + \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4 m_\pi^2} \\ E_k^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_k^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4 m_\pi^2} \end{cases}$$

$$0^{\text{th}} \text{ order: } m_k = 0 \implies p_k = E_k = E = \frac{m_\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \text{ MeV}$$

$$1^{\text{st}} \text{ order: } E_k \simeq E + \xi \frac{m_k^2}{2E}$$

$$p_k \simeq E - (1 - \xi) \frac{m_k^2}{2E}$$

$$\xi = \frac{1}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{CC} \sim W_\rho (\bar{\nu}_{eL} \gamma^\rho e_L + \bar{\nu}_{\mu L} \gamma^\rho \mu_L + \bar{\nu}_{\tau L} \gamma^\rho \tau_L)$$

Fields $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Rightarrow \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$ States

initial flavor: $\alpha = e \text{ or } \mu \text{ or } \tau$

$$|\nu_k(t, x)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t, x)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t, x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t, x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(0, 0) = \sum_k U_{\alpha k}^* U_{\beta k} = \delta_{\alpha\beta} \quad \mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t > 0, x > 0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t, \mathbf{x}) = |\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t, \mathbf{x})|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + i p_k \cdot \mathbf{x}} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\implies t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) &= \left| \sum_k U_{\alpha k}^* e^{-im_k^2 L/2E} U_{\beta k} \right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

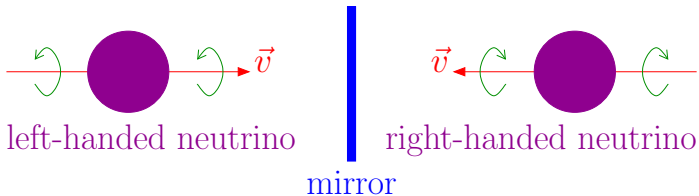
Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu_{\alpha L}^{\text{CP}} = \gamma^0 \mathcal{C} \overline{\nu_{\alpha L}}^T$$

C \implies Particle \iff Antiparticle

P \implies Left-Handed \iff Right-Handed



$$\text{Fields: } \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$$

$$\text{States: } |\nu_{\alpha}\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\text{CP}} |\bar{\nu}_{\alpha}\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$$

NEUTRINOS $U \Leftrightarrow U^*$ ANTINEUTRINOS

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT Symmetry

$$P_{\nu_\alpha \rightarrow \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$\text{CPT Asymmetries: } A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$\text{Local Quantum Field Theory} \implies A_{\alpha\beta}^{\text{CPT}} = 0 \quad \text{CPT Symmetry}$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\text{is invariant under CPT: } U \leftrightarrow U^* \quad \alpha \leftrightarrow \beta$$

$$P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha}$$

(solar ν_e , reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \rightarrow \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$$

$$\text{CP Asymmetries: } A_{\alpha\beta}^{\text{CP}} = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$$

$$A_{\alpha\beta}^{\text{CP}}(L, E) = 4 \sum_{k>j} \text{Im} [U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*] \sin \left(\frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$\text{Jarlskog rephasing invariant: } \text{Im} [U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*] = \pm J$$

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13}$$

$$J \neq 0 \iff \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

$$\begin{aligned}
\text{CPT} \quad \Rightarrow \quad 0 &= A_{\alpha\beta}^{\text{CPT}} \\
&= P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha} \\
&= P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} \leftarrow A_{\alpha\beta}^{\text{CP}} \\
&+ P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} - P_{\nu_\beta \rightarrow \nu_\alpha} \leftarrow -A_{\beta\alpha}^{\text{CPT}} = 0 \\
&+ P_{\nu_\beta \rightarrow \nu_\alpha} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha} \leftarrow A_{\beta\alpha}^{\text{CP}} \\
&= A_{\alpha\beta}^{\text{CP}} + A_{\beta\alpha}^{\text{CP}} \quad \Rightarrow \quad \boxed{A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}}
\end{aligned}$$

T Symmetry

$$P_{\nu_\alpha \rightarrow \nu_\beta} \xrightarrow{T} P_{\nu_\beta \rightarrow \nu_\alpha}$$

$$\text{T Asymmetries: } A_{\alpha\beta}^T = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\nu_\beta \rightarrow \nu_\alpha}$$

$$\text{CPT} \implies 0 = A_{\alpha\beta}^{\text{CPT}}$$

$$= P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$= P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\nu_\beta \rightarrow \nu_\alpha} \leftarrow A_{\alpha\beta}^T$$

$$+ P_{\nu_\beta \rightarrow \nu_\alpha} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha} \leftarrow A_{\beta\alpha}^{\text{CP}}$$

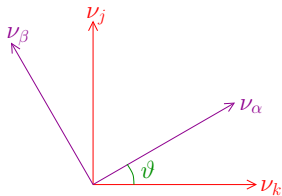
$$= A_{\alpha\beta}^T + A_{\beta\alpha}^{\text{CP}}$$

$$= A_{\alpha\beta}^T - A_{\alpha\beta}^{\text{CP}}$$

$$\implies \boxed{A_{\alpha\beta}^T = A_{\alpha\beta}^{\text{CP}}}$$

Two-Neutrino Oscillations

$$\begin{aligned} |\nu_\alpha\rangle &= \cos\vartheta |\nu_k\rangle + \sin\vartheta |\nu_j\rangle \\ |\nu_\beta\rangle &= -\sin\vartheta |\nu_k\rangle + \cos\vartheta |\nu_j\rangle \end{aligned}$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Transition Probability: $P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\nu_\beta \rightarrow \nu_\alpha} = \sin^2 2\vartheta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$

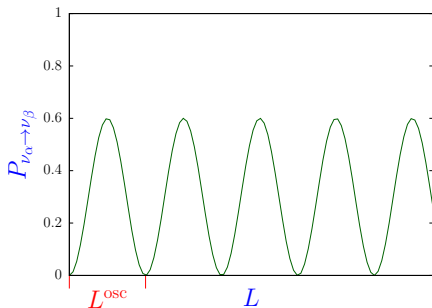
Survival Probabilities: $P_{\nu_\alpha \rightarrow \nu_\alpha} = P_{\nu_\beta \rightarrow \nu_\beta} = 1 - P_{\nu_\alpha \rightarrow \nu_\beta}$

oscillation phase

$$\frac{\Delta m^2 L}{4E} = 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{m}]}{E [\text{MeV}]} = 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \frac{E [\text{MeV}]}{\Delta m^2 [\text{eV}^2]} \text{ m} = 2.47 \frac{E [\text{GeV}]}{\Delta m^2 [\text{eV}^2]} \text{ km}$$



Types of Experiments

transitions due to Δm^2 observable only if $\frac{\Delta m^2 L}{E} \gtrsim 1 \Leftrightarrow \Delta m^2 \gtrsim \left(\frac{L}{E}\right)^{-1}$

SBL

$$L/E \lesssim 10 \text{ eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1 \text{ eV}^2$$

Reactor: $L \sim 10 \text{ m}$, $E \sim 1 \text{ MeV}$

Accelerator: $L \sim 1 \text{ km}$, $E \gtrsim 0.1 \text{ GeV}$

ATM & LBL

$$L/E \lesssim 10^4 \text{ eV}^{-2}$$

Rea.: $L \sim 1 \text{ km}$, $E \sim 1 \text{ MeV}$ Daya Bay, RENO, Double Chooz

Acc.: $L \sim 10^3 \text{ km}$, $E \gtrsim 1 \text{ GeV}$ K2K, MINOS, OPERA, T2K, NO ν A

$$\Delta m^2 \gtrsim 10^{-4} \text{ eV}^2$$



Atmospheric: $L \sim 10^2 - 10^4 \text{ km}$, $E \sim 0.1 - 10^2 \text{ GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan-2, MACRO

Solar

$$L \sim 10^8 \text{ km}, \quad E \sim 0.1 - 10 \text{ MeV}$$

$$\frac{L}{E} \sim 10^{11} \text{ eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11} \text{ eV}^2$$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1$, $10^{-8} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4} \text{ eV}^2$

VLBL

$$L/E \lesssim 10^5 \text{ eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5} \text{ eV}^2$$

Reactor: $L \sim 10^2 \text{ km}$, $E \sim 1 \text{ MeV}$

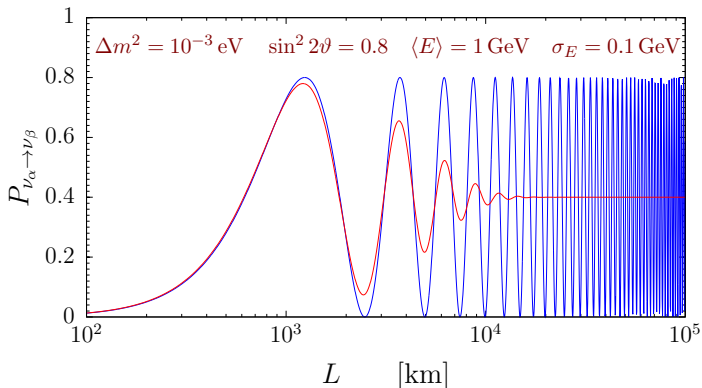
KamLAND

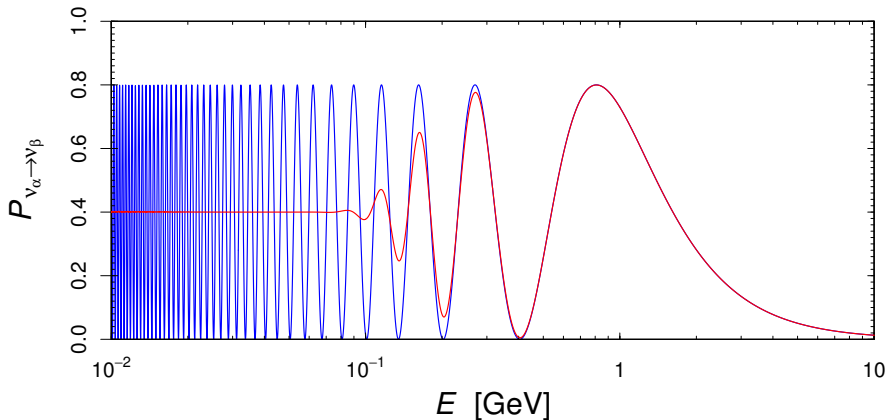
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos \left(\frac{\Delta m^2 L}{2E} \right) \right]$$



$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos \left(\frac{\Delta m^2 L}{2E} \right) \phi(E) dE \right] \quad (\alpha \neq \beta)$$

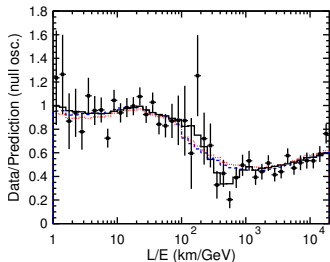




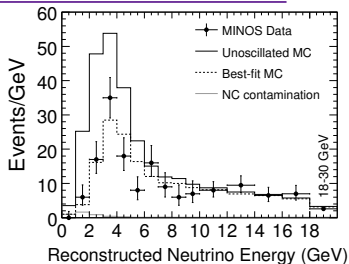
$$\Delta m^2 = 10^{-3} \text{ eV} \quad \sin^2 2\vartheta = 0.8 \quad L = 10^3 \text{ km} \quad \sigma_E = 0.01 \text{ GeV}$$

$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) dE \right] \quad (\alpha \neq \beta)$$

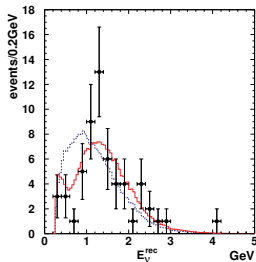
Observations of Neutrino Oscillations



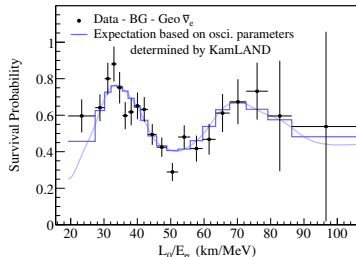
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



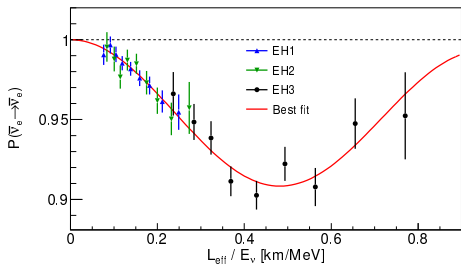
[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]



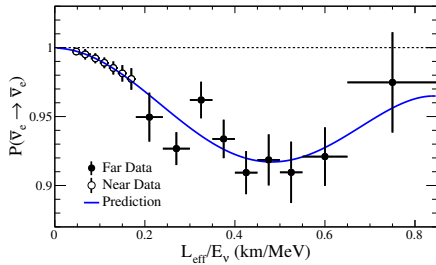
[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]



[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]



[Daya Bay, PRL, 112 (2014) 061801, arXiv:1310.6732]

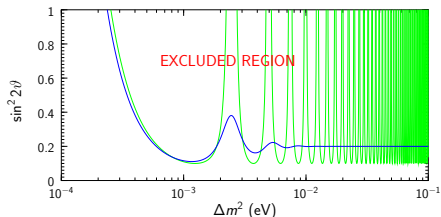


[RENO, arXiv:1511.05849]

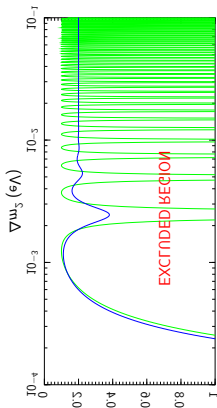
Exclusion Curves

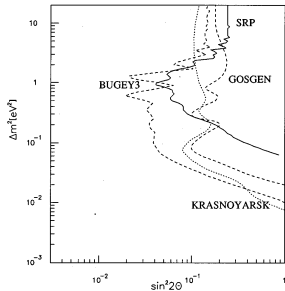
$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) dE \right] \quad (\alpha \neq \beta)$$

$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle \leq P_{\nu_\alpha \rightarrow \nu_\beta}^{\max} \implies \sin^2 2\vartheta \leq \frac{2 P_{\nu_\alpha \rightarrow \nu_\beta}^{\max}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) dE}$$

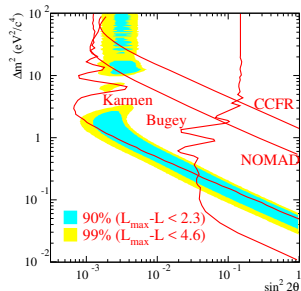


rotate
and
mirror

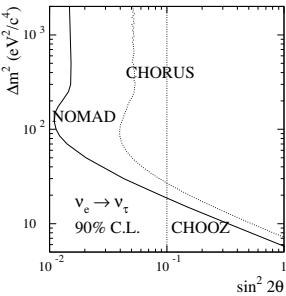
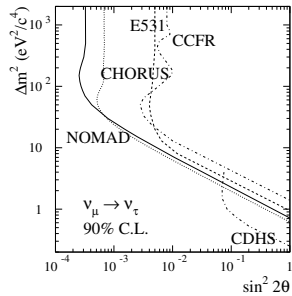




Reactor SBL Experiments: $\bar{\nu}_e \rightarrow \bar{\nu}_e$

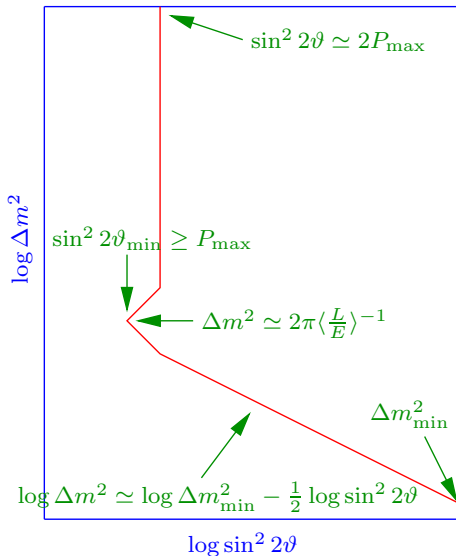


Accelerator SBL Experiments: $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$



Accelerator SBL Experiments: $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$

Anatomy of Exclusion Plots



▶ $\Delta m^2 \gg \langle L/E \rangle^{-1}$

$$P_{\max} \simeq \frac{1}{2} \sin^2 2\vartheta \Rightarrow \sin^2 2\vartheta \simeq 2P_{\max}$$

▶ $\Delta m^2 \simeq 2\pi \langle L/E \rangle^{-1}$

$$\text{Min} \left\langle \cos \left(\frac{\Delta m^2 L}{2E} \right) \right\rangle \geq -1$$

$$\sin^2 2\vartheta_{\min} = \frac{2P_{\max}}{1 - \text{Min} \left\langle \cos \left(\frac{\Delta m^2 L}{2E} \right) \right\rangle} \geq P_{\max}$$

▶ $\Delta m^2 \ll 2\pi \langle L/E \rangle^{-1}$

$$\cos \left(\frac{\Delta m^2 L}{2E} \right) \simeq 1 - \frac{1}{2} \left(\frac{\Delta m^2 L}{2E} \right)^2$$

$$\log \Delta m^2 \simeq \log \Delta m^2_{\min} - \frac{1}{2} \log \sin^2 2\vartheta$$

$$\frac{1}{2} \sin^2 2\vartheta \left[1 - \left\langle \cos \left(\frac{\Delta m^2 L}{2E} \right) \right\rangle \right] = P_{\max}$$

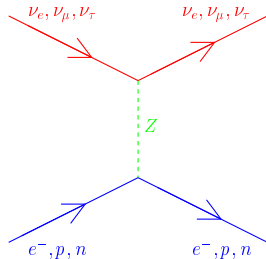
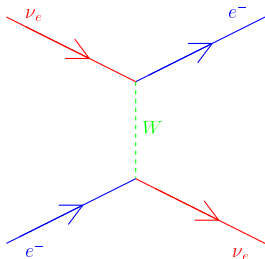
$$\Delta m^2_{\min} = 4\sqrt{P_{\max}} \left\langle \frac{L}{E} \right\rangle^{-1}$$

Neutrino Oscillations in Matter

- Neutrino Oscillations in Vacuum
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter
 - Effective Potentials in Matter
 - Evolution of Neutrino Flavors in Matter
 - Two-Neutrino Mixing
 - Constant Matter Density
 - MSW Effect (Resonant Transitions in Matter)

Effective Potentials in Matter

coherent interactions with medium: forward elastic CC and NC scattering



$$V_{CC} = \sqrt{2} G_F N_e$$

$$V_{NC}^{(e^-)} = -V_{NC}^{(p)} \Rightarrow$$

$$V_{NC} = V_{NC}^{(n)} = -\frac{\sqrt{2}}{2} G_F N_n$$

$$V_e = V_{CC} + V_{NC}$$

$$V_\mu = V_\tau = V_{NC}$$

only $V_{CC} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\bar{V}_{CC} = -V_{CC}$ $\bar{V}_{NC} = -V_{NC}$

Evolution of Neutrino Flavors in Matter

▶ Flavor neutrino ν_α with momentum p : $|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* |\nu_k(p)\rangle$

▶ Evolution is determined by Hamiltonian

▶ Hamiltonian in vacuum: $\mathcal{H} = \mathcal{H}_0$

$$\mathcal{H}_0 |\nu_k(p)\rangle = E_k |\nu_k(p)\rangle \quad E_k = \sqrt{p^2 + m_k^2}$$

▶ Hamiltonian in matter: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$ $\mathcal{H}_I |\nu_\alpha(p)\rangle = V_\alpha |\nu_\alpha(p)\rangle$

▶ Schrödinger evolution equation: $i \frac{d}{dt} |\nu(p, t)\rangle = \mathcal{H} |\nu(p, t)\rangle$

▶ Initial condition: $|\nu(p, 0)\rangle = |\nu_\alpha(p)\rangle$

▶ For $t > 0$ the state $|\nu(p, t)\rangle$ is a superposition of all flavors:

$$|\nu(p, t)\rangle = \sum_\beta \varphi_\beta(p, t) |\nu_\beta(p)\rangle$$

▶ Transition probability: $P_{\nu_\alpha \rightarrow \nu_\beta} = |\varphi_\beta|^2$

evolution equation of states

$$i \frac{d}{dt} |\nu(\mathbf{p}, t)\rangle = \mathcal{H} |\nu(\mathbf{p}, t)\rangle, \quad |\nu(\mathbf{p}, 0)\rangle = |\nu_\alpha(\mathbf{p})\rangle$$

flavor transition amplitudes

$$\varphi_\beta(\mathbf{p}, t) = \langle \nu_\beta(\mathbf{p}) | \nu(\mathbf{p}, t) \rangle, \quad \varphi_\beta(\mathbf{p}, 0) = \delta_{\alpha\beta}$$

evolution of flavor transition amplitudes

$$i \frac{d}{dt} \varphi_\beta(\mathbf{p}, t) = \langle \nu_\beta(\mathbf{p}) | \mathcal{H} | \nu(\mathbf{p}, t) \rangle$$

$$i \frac{d}{dt} \varphi_\beta(\mathbf{p}, t) = \langle \nu_\beta(\mathbf{p}) | \mathcal{H}_0 | \nu(\mathbf{p}, t) \rangle + \langle \nu_\beta(\mathbf{p}) | \mathcal{H}_I | \nu(\mathbf{p}, t) \rangle$$

$$i \frac{d}{dt} \varphi_\beta(\mathbf{p}, t) = \langle \nu_\beta(\mathbf{p}) | \mathcal{H}_0 | \nu(\mathbf{p}, t) \rangle + \langle \nu_\beta(\mathbf{p}) | \mathcal{H}_I | \nu(\mathbf{p}, t) \rangle$$

$$\langle \nu_\beta(\mathbf{p}) | \mathcal{H}_0 | \nu(\mathbf{p}, t) \rangle =$$

$$\begin{aligned} & \sum_\rho \sum_{k,j} \underbrace{\langle \nu_\beta(\mathbf{p}) | \nu_k(\mathbf{p}) \rangle}_{U_{\beta k}} \underbrace{\langle \nu_k(\mathbf{p}) | \mathcal{H}_0 | \nu_j(\mathbf{p}) \rangle}_{\delta_{kj} E_k} \underbrace{\langle \nu_j(\mathbf{p}) | \nu_\rho(\mathbf{p}) \rangle}_{U_{\rho j}^*} \underbrace{\langle \nu_\rho(\mathbf{p}) | \nu(\mathbf{p}, t) \rangle}_{\varphi_\rho(\mathbf{p}, t)} \\ & = \sum_\rho \sum_k U_{\beta k} E_k U_{\rho k}^* \varphi_\rho(\mathbf{p}, t) \end{aligned}$$

$$\begin{aligned} \langle \nu_\beta(\mathbf{p}) | \mathcal{H}_I | \nu(\mathbf{p}, t) \rangle & = \sum_\rho \underbrace{\langle \nu_\beta(\mathbf{p}) | \mathcal{H}_I | \nu_\rho(\mathbf{p}) \rangle}_{\delta_{\beta\rho} V_\beta} \underbrace{\langle \nu_\rho(\mathbf{p}) | \nu(\mathbf{p}, t) \rangle}_{\varphi_\rho(\mathbf{p}, t)} \\ & = \sum_\rho \delta_{\beta\rho} V_\beta \varphi_\rho(\mathbf{p}, t) \end{aligned}$$

$$i \frac{d}{dt} \varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k} E_k U_{\rho k}^* + \delta_{\beta\rho} V_\beta \right) \varphi_\rho$$

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $E = p$ $t = x$

$$V_e = V_{CC} + V_{NC} \qquad V_\mu = V_\tau = V_{NC}$$

$$i \frac{d}{dx} \varphi_\beta(p, x) = (p + V_{NC}) \varphi_\beta(p, x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{CC} \right) \varphi_\rho(p, x)$$

$$\psi_\beta(p, x) = \varphi_\beta(p, x) e^{ipx + i \int_0^x V_{NC}(x') dx'}$$

$$i \frac{d}{dx} \psi_\beta = e^{ipx + i \int_0^x V_{NC}(x') dx'} \left(-p - V_{NC} + i \frac{d}{dx} \right) \varphi_\beta$$

$$i \frac{d}{dx} \psi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{CC} \right) \psi_\rho$$

$$P_{\nu_\alpha \rightarrow \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2$$

evolution of flavor transition amplitudes in matrix form

$$i \frac{d}{dx} \Psi_\alpha = \frac{1}{2E} \left(U M^2 U^\dagger + \mathbb{A} \right) \Psi_\alpha$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \quad M^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \quad \mathbb{A} = \begin{pmatrix} A_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{CC} = 2EV_{CC} = 2\sqrt{2}EG_F N_e$$

effective
mass-squared
matrix
in vacuum

$$M_{\text{VAC}}^2 = U M^2 U^\dagger \xrightarrow{\text{matter}} U M^2 U^\dagger + 2E \mathbb{V} = M_{\text{MAT}}^2$$

↑
potential due to coherent
forward elastic scattering

effective
mass-squared
matrix
in matter

Two-Neutrino Mixing

$\nu_e \rightarrow \nu_\mu$ transitions with $U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$

$$U M^2 U^\dagger = \begin{pmatrix} \cos^2 \vartheta m_1^2 + \sin^2 \vartheta m_2^2 & \cos \vartheta \sin \vartheta (m_2^2 - m_1^2) \\ \cos \vartheta \sin \vartheta (m_2^2 - m_1^2) & \sin^2 \vartheta m_1^2 + \cos^2 \vartheta m_2^2 \end{pmatrix}$$

$$= \frac{1}{2} \Sigma m^2 + \frac{1}{2} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}$$

↑

irrelevant common phase

$$\Sigma m^2 \equiv m_1^2 + m_2^2$$

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{\text{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

$$\text{initial } \nu_e \implies \begin{pmatrix} \psi_e(0) \\ \psi_\mu(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P_{\nu_e \rightarrow \nu_\mu}(x) = |\psi_\mu(x)|^2$$
$$P_{\nu_e \rightarrow \nu_e}(x) = |\psi_e(x)|^2 = 1 - P_{\nu_e \rightarrow \nu_\mu}(x)$$

Constant Matter Density

$$i \frac{d}{dx} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{CC} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

$$\frac{dA_{CC}}{dx} = 0$$

diagonalization of effective Hamiltonian: $\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$

$$\begin{pmatrix} \cos \vartheta_M & -\sin \vartheta_M \\ \sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{CC} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} = \\ = \begin{pmatrix} A_{CC} - \Delta m_M^2 & 0 \\ 0 & A_{CC} + \Delta m_M^2 \end{pmatrix}$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} = \left[\frac{A_{CC}}{4E} + \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} \right] \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$$

↑
irrelevant common phase

Effective Mixing Angle in Matter

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

Resonance ($\vartheta_M = \pi/4$)

$$A_{CC}^R = \Delta m^2 \cos 2\vartheta \quad \implies \quad N_e^R = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2}EG_F}$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$$

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} \Rightarrow \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M & -\sin \vartheta_M \\ \sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

$$\nu_e \rightarrow \nu_\mu \Rightarrow \begin{pmatrix} \psi_e(0) \\ \psi_\mu(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \psi_1^M(0) \\ \psi_2^M(0) \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M \\ \sin \vartheta_M \end{pmatrix}$$

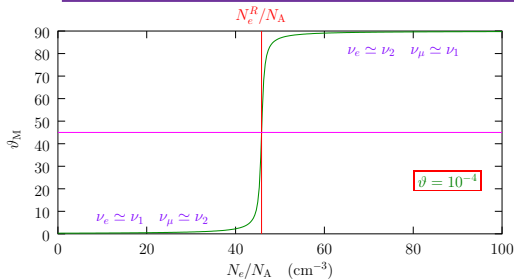
$$\psi_1^M(x) = \cos \vartheta_M \exp\left(i \frac{\Delta m_M^2 x}{4E}\right)$$

$$\psi_2^M(x) = \sin \vartheta_M \exp\left(-i \frac{\Delta m_M^2 x}{4E}\right)$$

$$P_{\nu_e \rightarrow \nu_\mu}(x) = |\psi_\mu(x)|^2 = \left| -\sin \vartheta_M \psi_1^M(x) + \cos \vartheta_M \psi_2^M(x) \right|^2$$

$$P_{\nu_e \rightarrow \nu_\mu}(x) = \sin^2 2\vartheta_M \sin^2 \left(\frac{\Delta m_M^2 x}{4E} \right)$$

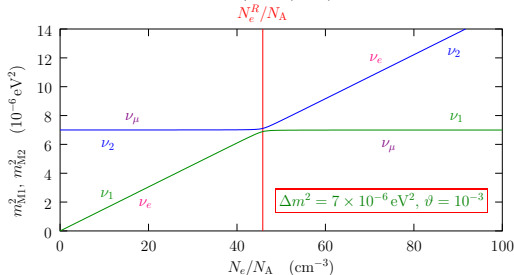
MSW Effect (Resonant Transitions in Matter)



$$\nu_e = \cos \vartheta_M \nu_1 + \sin \vartheta_M \nu_2$$

$$\nu_\mu = -\sin \vartheta_M \nu_1 + \cos \vartheta_M \nu_2$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \frac{A_{\text{CC}}}{\Delta m^2 \cos 2\vartheta}}$$



$$\Delta m_M^2 = \left[(\Delta m^2 \cos 2\vartheta - A_{\text{CC}})^2 + (\Delta m^2 \sin 2\vartheta)^2 \right]^{1/2}$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{CC} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

tentative diagonalization:
$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$$

$$i \frac{d}{dx} \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} =$$

$$= \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{CC} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$$

if matter density is not constant $d\vartheta_M/dx \neq 0$

$$i \frac{d}{dx} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} = \left[\underbrace{\frac{A_{CC}}{4E}}_{\uparrow} + \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} + \begin{pmatrix} 0 & -i \frac{d\vartheta_M}{dx} \\ i \frac{d\vartheta_M}{dx} & 0 \end{pmatrix} \right] \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$$

irrelevant common phase

$$i \frac{d}{dx} \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix} = \left[\frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} + \begin{pmatrix} 0 & -i \frac{d\vartheta_M}{dx} \\ i \frac{d\vartheta_M}{dx} & 0 \end{pmatrix} \right] \begin{pmatrix} \psi_1^M \\ \psi_2^M \end{pmatrix}$$

↑
adiabatic

↑
non-adiabatic
maximum at resonance

initial conditions:

$$\begin{pmatrix} \psi_1^M(0) \\ \psi_2^M(0) \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M^0 & -\sin \vartheta_M^0 \\ \sin \vartheta_M^0 & \cos \vartheta_M^0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M^0 \\ \sin \vartheta_M^0 \end{pmatrix}$$

solution approximating all non-adiabatic $\nu_1^M \leftrightarrow \nu_2^M$ transitions in resonance

$$\begin{aligned} \psi_1^M(x) &\simeq \left[\cos \vartheta_M^0 \exp \left(i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx' \right) \mathcal{A}_{11}^R + \sin \vartheta_M^0 \exp \left(-i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx' \right) \mathcal{A}_{21}^R \right] \\ &\quad \times \exp \left(i \int_{x_R}^x \frac{\Delta m_M^2(x')}{4E} dx' \right) \\ \psi_2^M(x) &\simeq \left[\cos \vartheta_M^0 \exp \left(i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx' \right) \mathcal{A}_{12}^R + \sin \vartheta_M^0 \exp \left(-i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx' \right) \mathcal{A}_{22}^R \right] \\ &\quad \times \exp \left(-i \int_{x_R}^x \frac{\Delta m_M^2(x')}{4E} dx' \right) \end{aligned}$$

Averaged ν_e Survival Probability on Earth

$$\psi_e(x) = \cos \vartheta \psi_1^M(x) + \sin \vartheta \psi_2^M(x)$$

neglect interference (averaged over energy spectrum)

$$\begin{aligned}\bar{P}_{\nu_e \rightarrow \nu_e}(x) = |\langle \psi_e(x) \rangle|^2 &= \cos^2 \vartheta \cos^2 \vartheta_M^0 |\mathcal{A}_{11}^R|^2 + \cos^2 \vartheta \sin^2 \vartheta_M^0 |\mathcal{A}_{21}^R|^2 \\ &+ \sin^2 \vartheta \cos^2 \vartheta_M^0 |\mathcal{A}_{12}^R|^2 + \sin^2 \vartheta \sin^2 \vartheta_M^0 |\mathcal{A}_{22}^R|^2\end{aligned}$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^R|^2 = |\mathcal{A}_{21}^R|^2 = P_c \quad |\mathcal{A}_{11}^R|^2 = |\mathcal{A}_{22}^R|^2 = 1 - P_c$$

$P_c \equiv$ crossing probability

$$\bar{P}_{\nu_e \rightarrow \nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_c \right) \cos 2\vartheta_M^0 \cos 2\vartheta$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter:
$$\gamma = \frac{\Delta m_M^2 / 2E}{2|d\vartheta_M/dx|} \Big|_R = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta} \Big| \frac{d \ln A_{CC}}{dx} \Big|_R$$

$A \propto x$ $F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $F = (1 - \tan^2 \vartheta)^2 / (1 + \tan^2 \vartheta)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

[Pizzochero, PRD 36 (1987) 2293]

$A \propto \exp(-x)$ $F = 1 - \tan^2 \vartheta$ [Toshev, PLB 196 (1987) 170]

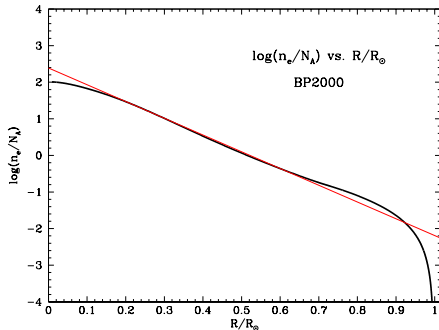
[Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

Solar Neutrinos

SUN: $N_e(x) \simeq N_e^c \exp\left(-\frac{x}{x_0}\right)$

$$N_e^c = 245 N_A / \text{cm}^3 \quad x_0 = \frac{R_\odot}{10.54}$$



$$\bar{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right) \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left| \frac{d \ln \text{Acc}}{dx} \right|_R}$$

$$F = 1 - \tan^2 \vartheta$$

$$\text{Acc} = 2\sqrt{2}EG_F N_e$$

practical prescription:

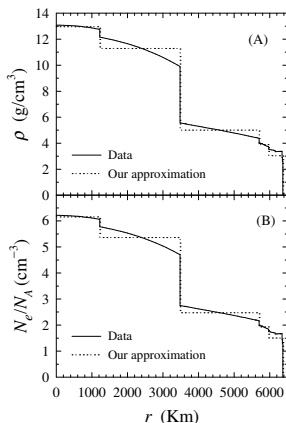
[Lisi et al., PRD 63 (2001) 093002]

$$\left\{ \begin{array}{ll} \text{numerical } |d \ln \text{Acc}/dx|_R & \text{for } x \leq 0.904R_\odot \\ |d \ln \text{Acc}/dx|_R \rightarrow \frac{18.9}{R_\odot} & \text{for } x > 0.904R_\odot \end{array} \right.$$

Electron Neutrino Regeneration in the Earth

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun+earth}} = \bar{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} + \frac{\left(1 - 2\bar{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}}\right) \left(P_{\nu_2 \rightarrow \nu_e}^{\text{earth}} - \sin^2 \vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



$P_{\nu_2 \rightarrow \nu_e}^{\text{earth}}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

Wave functions of flavor neutrinos are joined at the boundaries of steps.

Solar Neutrino Oscillations

LMA (Large Mixing Angle):

LOW (LOW Δm^2):

SMA (Small Mixing Angle):

QVO (Quasi-Vacuum Oscillations):

VAC (VACuum oscillations):

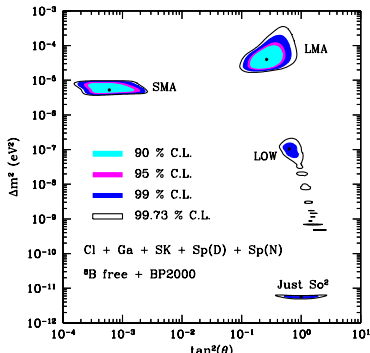
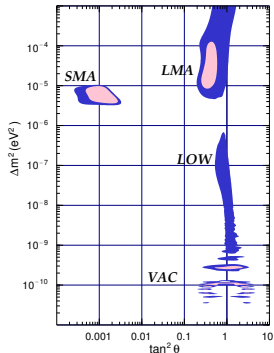
$$\Delta m^2 \sim 5 \times 10^{-5} \text{ eV}^2, \quad \tan^2 \vartheta \sim 0.8$$

$$\Delta m^2 \sim 7 \times 10^{-8} \text{ eV}^2, \quad \tan^2 \vartheta \sim 0.6$$

$$\Delta m^2 \sim 5 \times 10^{-6} \text{ eV}^2, \quad \tan^2 \vartheta \sim 10^{-3}$$

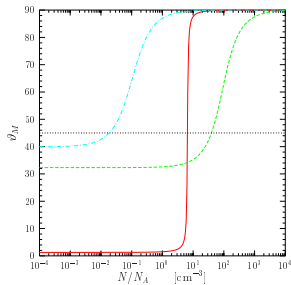
$$\Delta m^2 \sim 10^{-9} \text{ eV}^2, \quad \tan^2 \vartheta \sim 1$$

$$\Delta m^2 \lesssim 5 \times 10^{-10} \text{ eV}^2, \quad \tan^2 \vartheta \sim 1$$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]

[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



solid line:
(typical SMA)

$$\Delta m^2 = 5 \times 10^{-6} \text{ eV}^2$$

$$\tan^2 \vartheta = 5 \times 10^{-4}$$

dashed line:
(typical LMA)

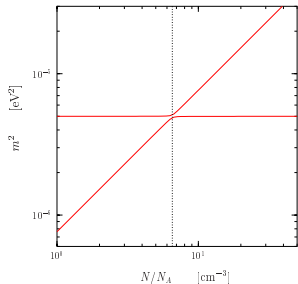
$$\Delta m^2 = 7 \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \vartheta = 0.4$$

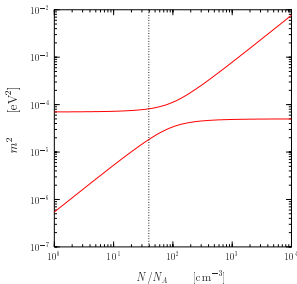
dash-dotted line:
(typical LOW)

$$\Delta m^2 = 8 \times 10^{-8} \text{ eV}^2$$

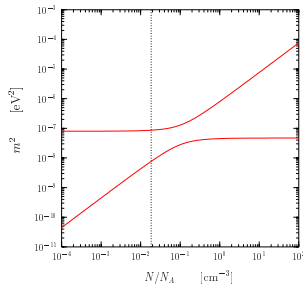
$$\tan^2 \vartheta = 0.7$$



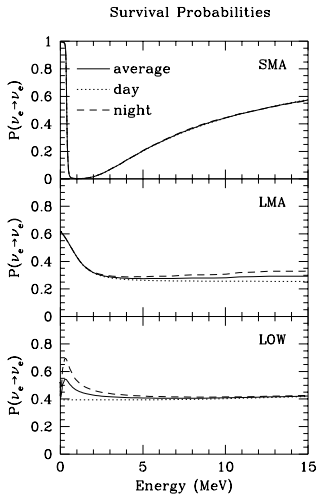
typical SMA



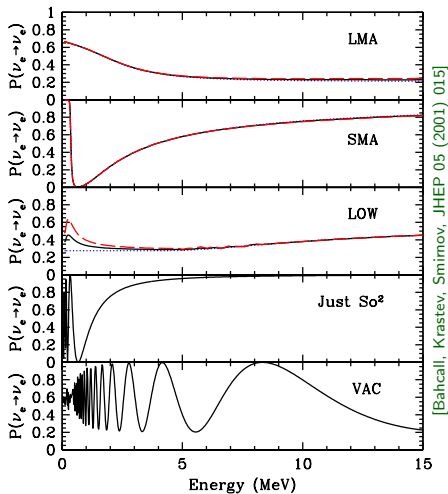
typical LMA



typical LOW



SMA: $\Delta m^2 = 5.0 \times 10^{-6} \text{ eV}^2 \quad \sin^2 2\vartheta = 3.5 \times 10^{-3}$
 LMA: $\Delta m^2 = 1.6 \times 10^{-5} \text{ eV}^2 \quad \sin^2 2\vartheta = 0.57$
 LOW: $\Delta m^2 = 7.9 \times 10^{-8} \text{ eV}^2 \quad \sin^2 2\vartheta = 0.95$



LMA: $\Delta m^2 = 4.2 \times 10^{-5} \text{ eV}^2 \quad \tan^2 \vartheta = 0.26$
 SMA: $\Delta m^2 = 5.2 \times 10^{-6} \text{ eV}^2 \quad \tan^2 \vartheta = 5.5 \times 10^{-4}$
 LOW: $\Delta m^2 = 7.6 \times 10^{-8} \text{ eV}^2 \quad \tan^2 \vartheta = 0.72$
 Just So²: $\Delta m^2 = 5.5 \times 10^{-12} \text{ eV}^2 \quad \tan^2 \vartheta = 1.0$
 VAC: $\Delta m^2 = 1.4 \times 10^{-10} \text{ eV}^2 \quad \tan^2 \vartheta = 0.38$

LMA Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5} \text{ eV}^2$ $\tan^2 \vartheta \simeq 0.4$

$$\overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} = \frac{1}{2} + \left(\frac{1}{2} - P_c \right) \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)} \quad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left| \frac{d \ln A}{dx} \right|_R} \quad F = 1 - \tan^2 \vartheta$$

$$A_{cc} \simeq 2\sqrt{2}EG_F N_e^c \exp\left(-\frac{x}{x_0}\right) \implies \left| \frac{d \ln A}{dx} \right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15} \text{ eV}$$

$$\tan^2 \vartheta \simeq 0.4 \implies \sin^2 2\vartheta \simeq 0.82, \cos 2\vartheta \simeq 0.43 \quad \gamma \simeq 2 \times 10^4 \left(\frac{E}{\text{MeV}} \right)^{-1}$$

$$\gamma \gg 1 \implies P_c \ll 1 \implies \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun,LMA}} \simeq \frac{1}{2} + \frac{1}{2} \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$\cos 2\vartheta_M^0 = \frac{\Delta m^2 \cos 2\vartheta - A_{CC}^0}{\sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC}^0)^2 + (\Delta m^2 \sin 2\vartheta)^2}}$$

critical parameter [Bahcall, Peña-Garay, JHEP 0311 (2003) 004]

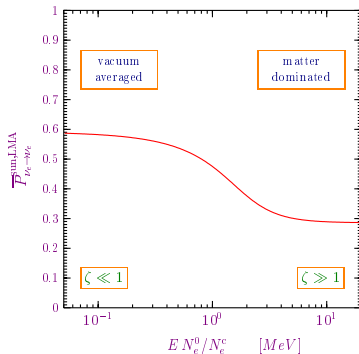
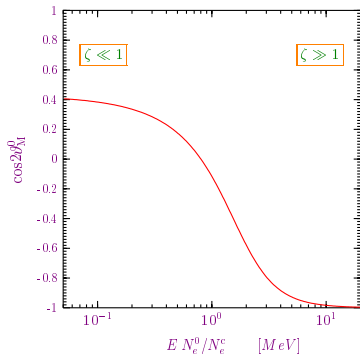
$$\zeta = \frac{A_{CC}^0}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2}EG_F N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\text{MeV}} \right) \left(\frac{N_e^0}{N_c^0} \right)$$

$$\zeta \ll 1 \implies \vartheta_M^0 \simeq \vartheta \implies \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} \simeq 1 - \frac{1}{2} \sin^2 2\vartheta$$

vacuum averaged
survival probability

$$\zeta \gg 1 \implies \vartheta_M^0 \simeq \pi/2 \implies \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} \simeq \sin^2 \vartheta$$

matter dominated
survival probability

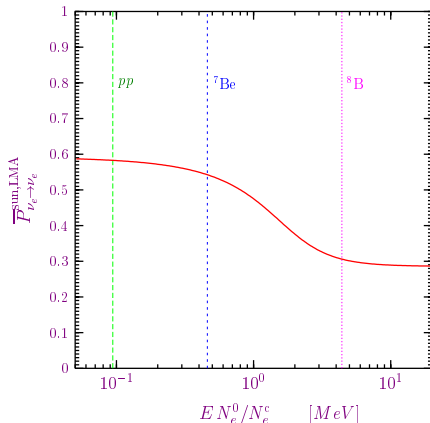


$$\zeta = \frac{A_{CC}^0}{\Delta m^2 \cos 2\theta} = \frac{2\sqrt{2}EG_F N_e^0}{\Delta m^2 \cos 2\theta} \simeq 1.2 \left(\frac{E}{\text{MeV}} \right) \left(\frac{N_e^0}{N_e^c} \right)$$

$$\langle E \rangle_{pp} \simeq 0.27 \text{ MeV}, \quad \langle r_0 \rangle_{pp} \simeq 0.1 R_\odot \quad \Rightarrow \quad \langle E N_e^0 / N_e^c \rangle_{pp} \simeq 0.094 \text{ MeV}$$

$$E_{7\text{Be}} \simeq 0.86 \text{ MeV}, \quad \langle r_0 \rangle_{7\text{Be}} \simeq 0.06 R_\odot \quad \Rightarrow \quad \langle E N_e^0 / N_e^c \rangle_{7\text{Be}} \simeq 0.46 \text{ MeV}$$

$$\langle E \rangle_{8\text{B}} \simeq 6.7 \text{ MeV}, \quad \langle r_0 \rangle_{8\text{B}} \simeq 0.04 R_\odot \quad \Rightarrow \quad \langle E N_e^0 / N_e^c \rangle_{8\text{B}} \simeq 4.4 \text{ MeV}$$



In Neutrino Oscillations Dirac = Majorana

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495; Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Evolution of Amplitudes:
$$i \frac{d\psi_\alpha}{dx} = \frac{1}{2E} \sum_\beta \left(UM^2U^\dagger + 2EV \right)_{\alpha\beta} \psi_\beta$$

difference:
$$\left\{ \begin{array}{ll} \text{Dirac:} & U^{(D)} \\ \text{Majorana:} & U^{(M)} = U^{(D)} D(\lambda) \end{array} \right.$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{i\lambda_{21}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{i\lambda_{N1}} \end{pmatrix} \Rightarrow D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \dots & 0 \\ 0 & m_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_N^2 \end{pmatrix} \Rightarrow DM^2 = M^2D \Rightarrow DM^2D^\dagger = M^2$$

$$U^{(M)} M^2 (U^{(M)})^\dagger = U^{(D)} D M^2 D^\dagger (U^{(D)})^\dagger = U^{(D)} M^2 (U^{(D)})^\dagger$$

Common Question: Do Charged Leptons Oscillate?

- ▶ Mass is the only property which distinguishes e , μ , τ .
- ▶ The flavor of a charged lepton is defined by its mass!
- ▶ By definition, the flavor of a charged lepton cannot change.

THE FLAVOR OF CHARGED LEPTONS DOES NOT OSCILLATE

[CG, Kim, FPL 14 (2001) 213] [CG, hep-ph/0409230] [Akhmedov, JHEP 09 (2007) 116]

a misleading argument

[Sassaroli, Srivastava, Widom, hep-ph/9509261, EPJC 2 (1998) 769] [Srivastava, Widom, hep-ph/9707268]

in $\pi^+ \rightarrow \mu^+ + \nu_\mu$ the final state of the antimuon and neutrino is entangled



if the probability to detect the neutrino oscillates as a function of distance,
also the probability to detect the muon must oscillate

WRONG!

the probability to detect the neutrino (as ν_μ or ν_τ or ν_e) does not oscillate
as a function of distance, because

$$\sum_{\beta=e,\mu,\tau} P_{\nu_\mu \rightarrow \nu_\beta} = 1 \quad \text{conservation of probability (unitarity)}$$

[Dolgov, Morozov, Okun, Shchepkin, NPB 502 (1997) 3] [CG, Kim, FPL 14 (2001) 213]

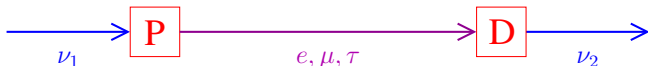
Λ oscillations from $\pi^- + p \rightarrow \Lambda + K^0$

[Widom, Srivastava, hep-ph/9605399] [Srivastava, Widom, Sassaroli, PLB 344 (1995) 436]

refuted in [Lowe et al., PLB 384 (1996) 288] [Burkhardt, Lowe, Stephenson, Goldman, PRD 59 (1999) 054018]

Correct definition of Charged Lepton Oscillations

[Pakvasa, Nuovo Cim. Lett. 31 (1981) 497]



Analogy

- ▶ **Neutrino Oscillations:** massive neutrinos propagate unchanged between production and detection, with a difference of mass (flavor) of the charged leptons involved in the production and detection processes.
- ▶ **Charged-Lepton Oscillations:** massive charged leptons propagate unchanged between production and detection, with a difference of mass of the neutrinos involved in the production and detection processes.

NO FLAVOR CONVERSION!

The propagating charged leptons must be ultrarelativistic, in order to be produced and detected coherently (if τ is not ultrarelativistic, only e and μ contribute to the phase).

Practical Problems

- ▶ The initial and final neutrinos must be massive neutrinos of known type: precise neutrino mass measurements.
- ▶ The energy of the propagating charged leptons must be extremely high, in order to have a measurable oscillation length

$$\frac{4\pi E}{(m_\mu^2 - m_e^2)} \simeq \frac{4\pi E}{m_\mu^2} \simeq 2 \times 10^{-11} \left(\frac{E}{\text{GeV}} \right) \text{ cm}$$

detailed discussion: [\[Akhmedov, JHEP 09 \(2007\) 116, arXiv:0706.1216\]](#)