Theory and Phenomenology of Massive Neutrinos

Part III: Phenomenology

Carlo Giunti

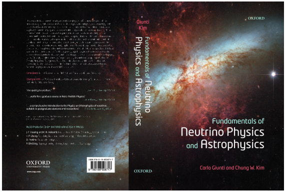
INFN, Sezione di Torino
and
Dipartimento di Fisica Teorica, Università di Torino

giunti@to.infn.it

Neutrino Unbound: http://www.nu.to.infn.it

Cours d'Hiver 2017 du LAL – 3-5 January 2017

http://personalpages.to.infn.it/~giunti/slides/2017/giunti-170105-LAL.pdf



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part III: Phenomenology

- Three-Neutrino Mixing Paradigm
- Absolute Scale of Neutrino Masses
- Neutrinoless Double-Beta Decay
- Light Sterile Neutrinos
- Cosmology
- Conclusions

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS

- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

Experimental Evidences of Neutrino Oscillations

Solar $\nu_e \to \nu_\mu, \nu_\tau$	(SNO, BOREXino, Super-Kamiokande, GALLEX/GNO, SAGE, Homestake, Kamiokande)	
VLBL Reactor $\bar{\nu}_e$ disappearance	(KamLAND)	

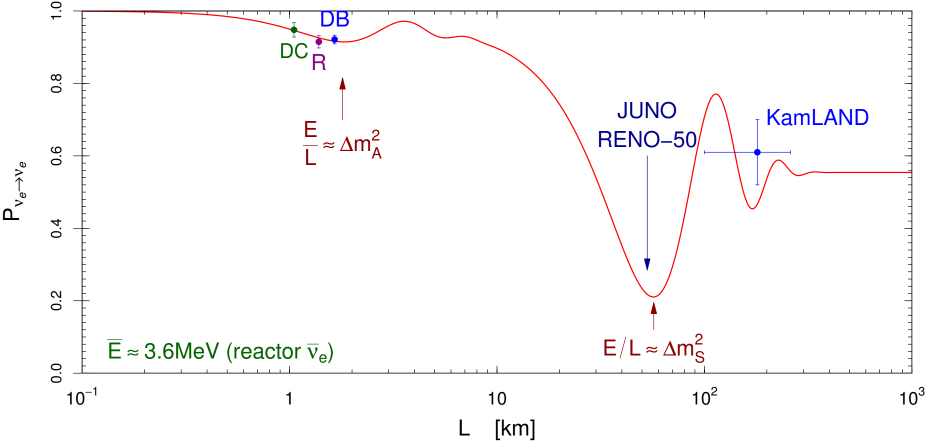
$$\to \begin{cases} \Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{S}} = \sin^2\vartheta_{12} \simeq 0.30 \end{cases}$$

Atmospheric $\nu_\mu \to \nu_\tau$	(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)
LBL Accelerator ν_μ disappearance	(K2K, MINOS, T2K, NOνA)
LBL Accelerator $\nu_\mu \to \nu_\tau$	(Opera)

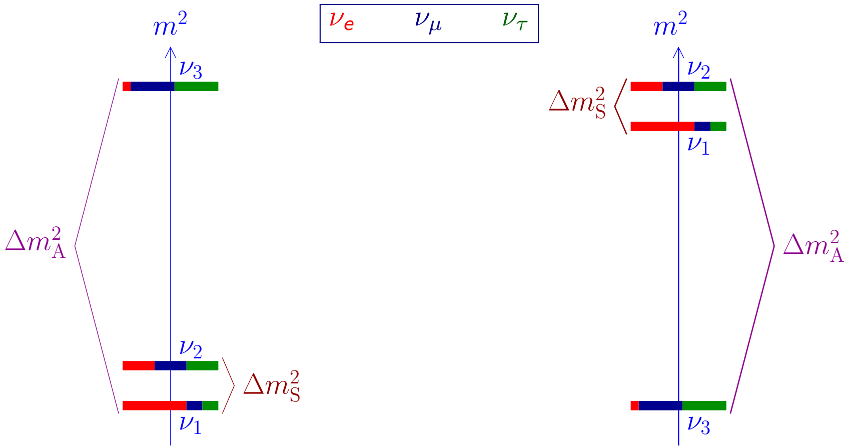
$$\to \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

LBL Accelerator $\nu_\mu \to \nu_e$	(T2K, MINOS, NOνA)
LBL Reactor $\bar{\nu}_e$ disappearance	(Daya Bay, RENO, Double Chooz)

$$\to \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31}| \\ \sin^2\vartheta_{13} \simeq 0.023 \end{cases}$$


1.0
0.8
0.6
0.4
0.2
0.0
$P_{\nu_e \to \nu_e}$
DB
DC
R
$\frac{E}{L} \approx \Delta m^2_A$
JUNO
RENO−50
KamLAND
$\overline{E} \approx 3.6\text{MeV}$ (reactor $\overline{\nu}_e$)
$E/L \approx \Delta m^2_S$
10^{-1}
1
10
10^2
10^3
L [km]

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

Three-Neutrino Mixing Around 2015

$$\Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.5 \pm 0.3 \times 10^{-5}\,\text{eV}^2 \qquad \text{uncertainty} \simeq 3\%$$

$$\Delta m^2_{\text{A}} = |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \pm 0.1 \times 10^{-3}\,\text{eV}^2 \qquad \text{uncertainty} \simeq 4\%$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$\vartheta_{23} = \vartheta_{\text{A}}$

$\sin^2\vartheta_{23} \simeq 0.4 - 0.6$

$P_{\text{osc}} \propto \sin^2 2\vartheta_{23}$

maximal and flat

at $\vartheta_{23} = 45^\circ$

Daya Bay, RENO

Double Chooz

T2K, MINOS

$\sin^2\vartheta_{13} \simeq 0.023 \pm 0.002$

$\delta_{13} \approx 3\pi/2?$

$\vartheta_{12} = \vartheta_{\text{S}}$

$\sin^2\vartheta_{12} \simeq 0.30 \pm 0.01$

$\beta\beta_{0\nu}$

$$\frac{\delta \sin^2\vartheta_{23}}{\sin^2\vartheta_{23}} \approx 40\% \qquad \frac{\delta \sin^2\vartheta_{13}}{\sin^2\vartheta_{13}} \approx 10\% \qquad \frac{\delta \sin^2\vartheta_{12}}{\sin^2\vartheta_{12}} \approx 5\%$$

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), ...
- CP violation ? $\delta_{13} \approx 3\pi/2$?
  - T2K (Japan), NOνA (USA), DUNE (USA), HyperK (Japan), ...
- Mass Ordering ?
  - JUNO (China), RENO-50 (Korea), PINGU (Antarctica), ORCA (EU), INO (India), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

Determination of Mass Ordering

1. Matter Effects: Atmospheric (PINGU, ORCA), Long-Baseline, Supernova Experiments

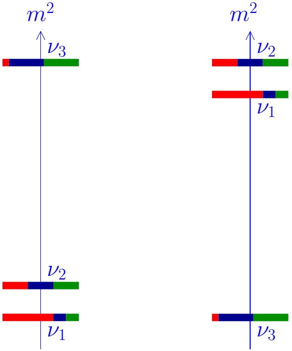
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} > 0$ NO
   - $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\dfrac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{13} < 0$ IO

2. Phase Difference: Reactor $\bar{\nu}_e \to \bar{\nu}_e$ (JUNO, RENO-50)

Normal Ordering

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$$

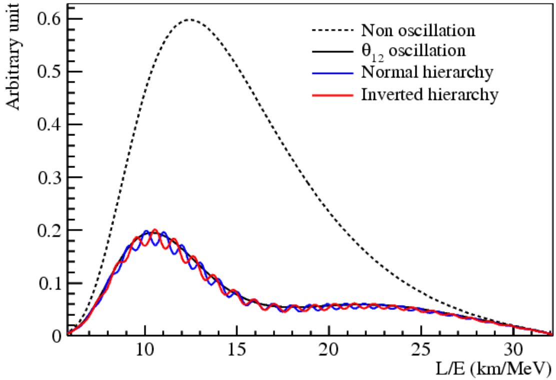
$$|\Delta m^2_{31}| > |\Delta m^2_{32}|$$



Inverted Ordering

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$$

$$|\Delta m^2_{31}| < |\Delta m^2_{32}|$$



Neutrino Physics with JUNO, arXiv:1507.05613

$$
\begin{aligned}
P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} = 1 & - \cos^4 \vartheta_{13} \sin^2 2\vartheta_{12} \sin^2 \left(\Delta m^2_{21} L / 4E \right) \\
& - \cos^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 \left(\Delta m^2_{31} L / 4E \right) \\
& - \sin^2 \vartheta_{12} \sin^2 2\vartheta_{13} \sin^2 \left(\Delta m^2_{32} L / 4E \right)
\end{aligned}
$$

[Petcov, Piai, PLB 533 (2002) 94; Choubey, Petcov, Piai, PRD 68 (2003) 113006; Learned, Dye, Pakvasa, Svoboda, PRD 78 (2008) 071302; Zhan, Wang, Cao, Wen, PRD 78 (2008) 111103, PRD 79 (2009) 073007]

CP Violation?

- In this approximation there is no observable CP-violation effect!
- CP-violation can be observed only with sensitivity to Δm^2_{21}: in vacuum

$$
\begin{aligned}
A^{\text{CP}}_{\alpha\beta} &= P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta} \\
&= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)
\end{aligned}
$$

$$J_{\alpha\beta} = \text{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

- Necessary conditions for observation of CP violation:
  - Sensitivity to all mixing angles, including small ϑ_{13}
  - Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

LBL Oscillation Probabilities

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad \alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \qquad A = \frac{2EV}{\Delta m^2_{31}} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad \alpha \ll 1$$

$$P^{\text{LBL}}_{\nu_e \to \nu_e} \simeq 1 - \sin^2 2\vartheta_{13} \sin^2 \Delta - \alpha^2 \Delta^2 \sin^2 2\vartheta_{12}$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu \to \nu_e} \simeq{} & \sin^2 2\vartheta_{13} \sin^2 \vartheta_{23} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ & + \alpha \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \delta_{13}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ & + \alpha^2 \sin^2 2\vartheta_{12} \cos^2 \vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

NO: $\Delta m^2_{31} > 0$ IO: $\Delta m^2_{31} < 0$

for antineutrinos: $\delta_{13} \to -\delta_{13}$ (CPV) and $A \to -A$ (Fake CPV!)

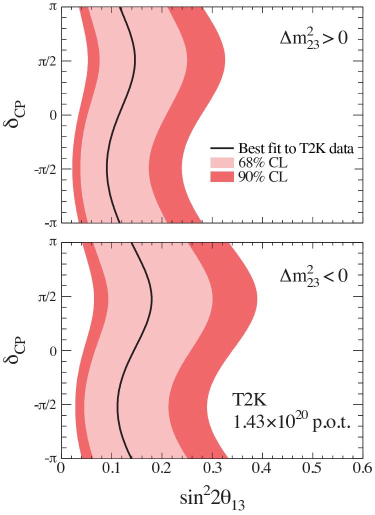
[see: Mezzetto, Schwetz, JPG 37 (2010) 103001]

T2K

[PRL 107 (2011) 041801, arXiv:1106.2822]

ND at 280 m FD at 295 km

2.5° off-axis $\Rightarrow$ NBB with $\langle E \rangle \simeq 0.6\,\text{GeV} \simeq |\Delta m^2_{31}|L/2\pi$



$\nu_\mu \to \nu_e$

6 ν_e events in FD

background: 1.5 ± 0.3

2.5σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.11^{+0.17}_{-0.08} & \text{(NO)} \\ 0.14^{+0.20}_{-0.10} & \text{(IO)} \end{cases}$$

90% C.L. $\delta_{13} = 0$

Assumptions

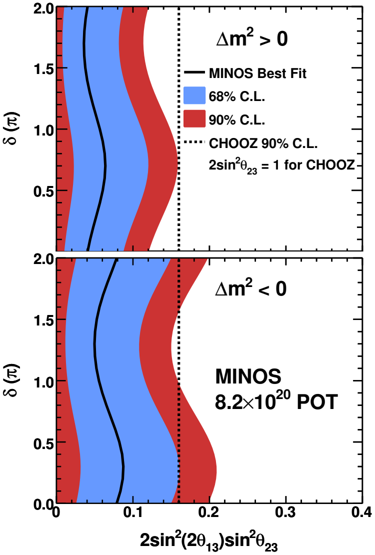
$\Delta m^2_{21} = 7.6 \times 10^{-5}\,\text{eV}$, $\sin^2 2\vartheta_{12} = 0.87$

$|\Delta m^2_{31}| = 2.4 \times 10^{-3}\,\text{eV}$, $\sin^2 2\vartheta_{23} = 1$

MINOS

[PRL 107 (2011) 181802, arXiv:1108.0015]

ND at 1.04 km   FD at 735 km $\langle E \rangle \simeq 3\,\text{GeV}$



$\nu_\mu \to \nu_e$

62 ν_e events in FD

background: 49.6 ± 7.5

1.6σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.041^{+0.047}_{-0.031} & \text{(NO)} \\ 0.079^{+0.071}_{-0.053} & \text{(IO)} \end{cases}$$

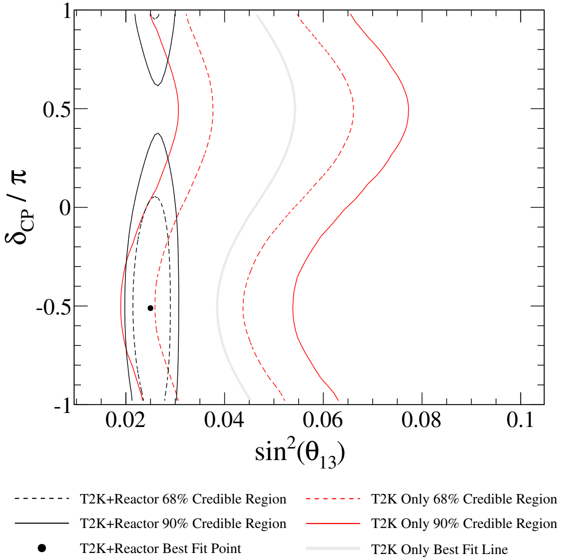
68% C.L. $\delta_{13} = 0$

Assumptions

$$\Delta m^2_{21} = 7.6 \times 10^{-5}\,\text{eV}\,,\ \sin^2 2\vartheta_{12} = 0.87$$

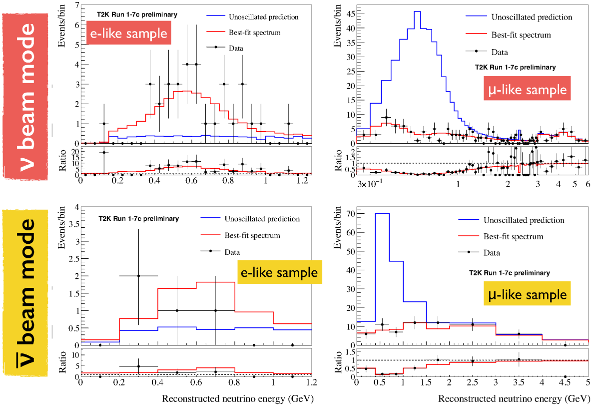
$$|\Delta m^2_{31}| = 2.3 \times 10^{-3}\,\text{eV}\,,\ \sin^2 2\vartheta_{23} = 1$$

Large CP Violation?



T2K, PRD 91 (2015) 072010, arXiv:1502.01550

T2K $\nu_e + \bar{\nu}_e$



Larger than expected ν_e appearance

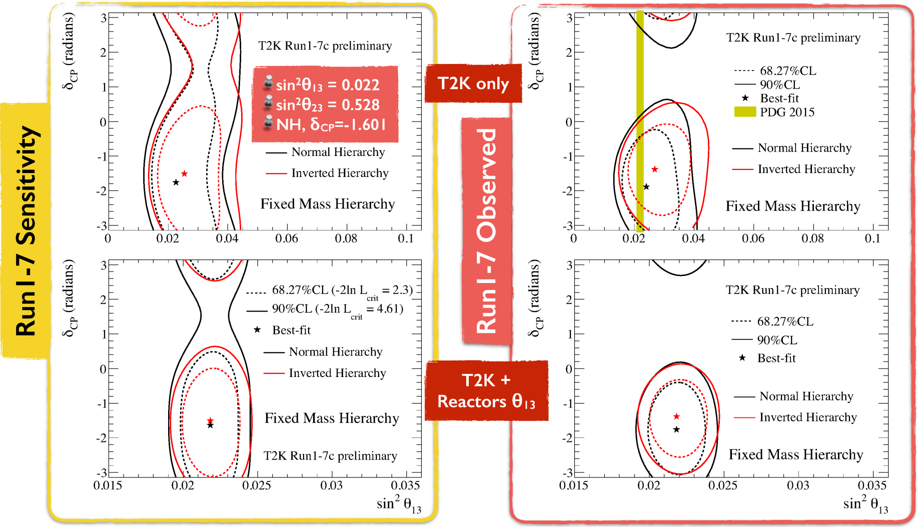
Smaller than expected $\bar{\nu}_e$ appearance

Data prefer the value of δ_{CP} inducing the largest ν-$\bar{\nu}$ asymmetry: $-\pi/2$

Normal Hierarchy						
Beam mode	Sample	$\delta_{CP} = -\pi/2$	$\delta_{CP} = 0$	$\delta_{CP} = +\pi/2$	$\delta_{CP} = \pi$	Observed
neutrino	μ-like	135.8	135.5	135.7	136.0	135
neutrino	e-like	28.7	24.2	19.6	24.1	32
anti-neutrino	μ-like	64.2	64.1	64.2	64.4	66
anti-neutrino	e-like	6	6.9	7.7	6.8	4

- Oscillation and systematic parameters are shared between the 4 samples
- Fit simultaneously the 4 samples to maximize the sensitivity to the oscillation parameters

[T2K @ NOW2016, September 2016]

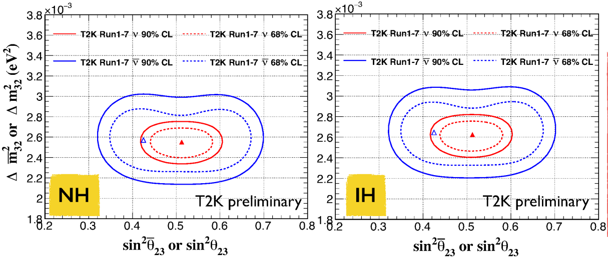


- T2K results consistent with reactor results
- Maximal CPV: data prefer $\delta_{CP}=-\pi/2$ ($\bar{\nu}_e$ data confirm the tendency observed for ν_e data)
- Favors the scenario of a small θ_{13} and large CPV

24

[T2K @ NOW2016, September 2016]

Constraints on the atmospheric parameters: θ_{23} and Δm^2_{31}

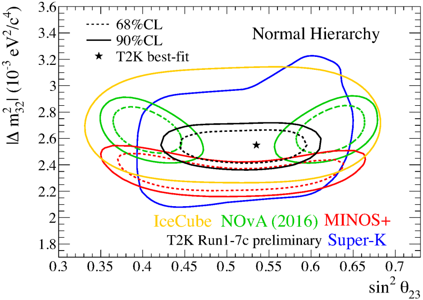


- **CPT theorem:**

  $P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$

  if $P(\nu_\mu \to \nu_\mu) \neq P(\bar{\nu}_\mu \to \bar{\nu}_\mu) \Rightarrow$

  CPT theorem is violated



- World-leading measurement of $\sin^2\theta_{23}$
- Results continue to be consistent with maximal mixing/oscillation
- No significant differences between ν and $\bar{\nu}$

	NH	IH
$\sin^2\theta_{23}$	$0.532^{+0.046}_{-0.068}$	$0.534^{+0.043}_{-0.007}$
$\lvert\Delta m^2_{32}\rvert$ ($\times 10^{-5}$ eV^2/c^4)	$254.5^{+8.1}_{-8.4}$	$251.0^{+8.1}_{-8.3}$

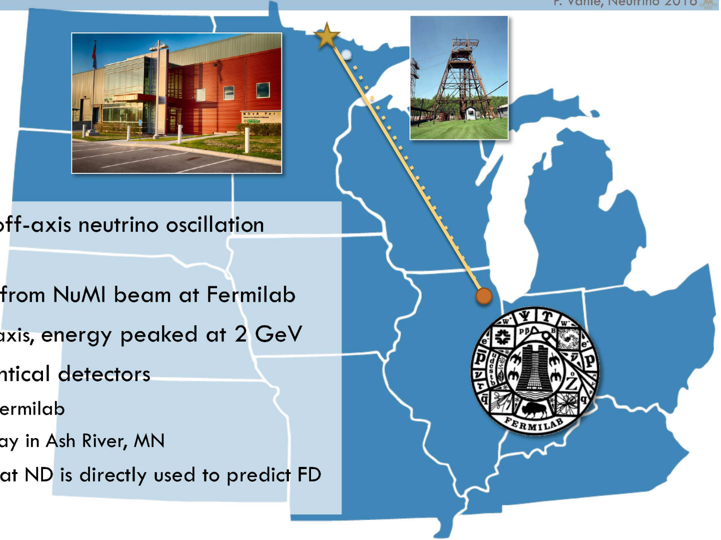
23

[T2K @ NOW2016, September 2016]

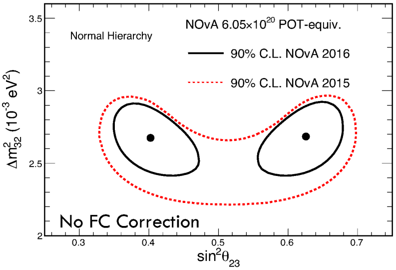
NOνA

2 P. Vahle, Neutrino 2016

- Long-baseline, off-axis neutrino oscillation experiment
- Study neutrinos from NuMI beam at Fermilab
- At 14 mrad off-axis, energy peaked at 2 GeV
- Functionally identical detectors
  - ND on site at Fermilab
  - FD 810 km away in Ash River, MN
  - Measurement at ND is directly used to predict FD



[NOνA @ Neutrino2016, July 2016]



- Fit for Δm^2 and $\sin^2\theta_{23}$
- Dominant systematic effects included in fit:
  - Normalization
  - NC background
  - Flux
  - Muon and hadronic energy scales
  - Cross section
  - Detector response and noise

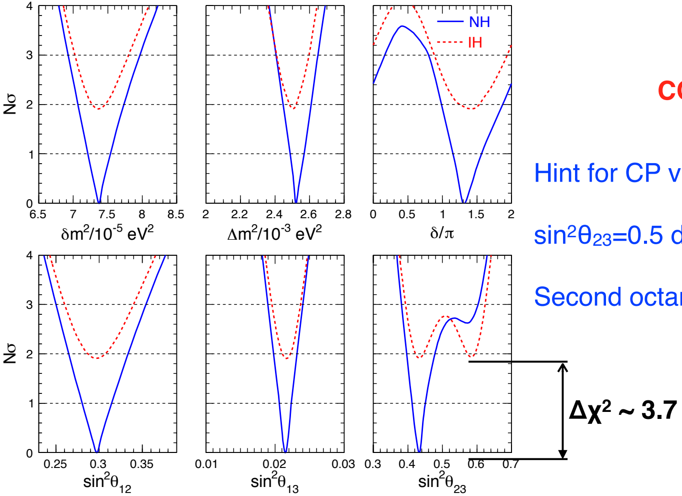
Best Fit (in NH):

$$\left|\Delta m^2_{32}\right| = 2.67 \pm 0.12 \times 10^{-3} \mathrm{eV}^2$$
$$\sin^2\theta_{23} = 0.40^{+0.03}_{-0.02} (0.63^{+0.02}_{-0.03})$$

Maximal mixing excluded at 2.5σ

[NOνA @ Neutrino2016, July 2016]

September 2016 Global Fit



COMMENTS

Hint for CP violation at ~2σ

$\sin^2\theta_{23}$=0.5 disfavoured at ~2.8σ

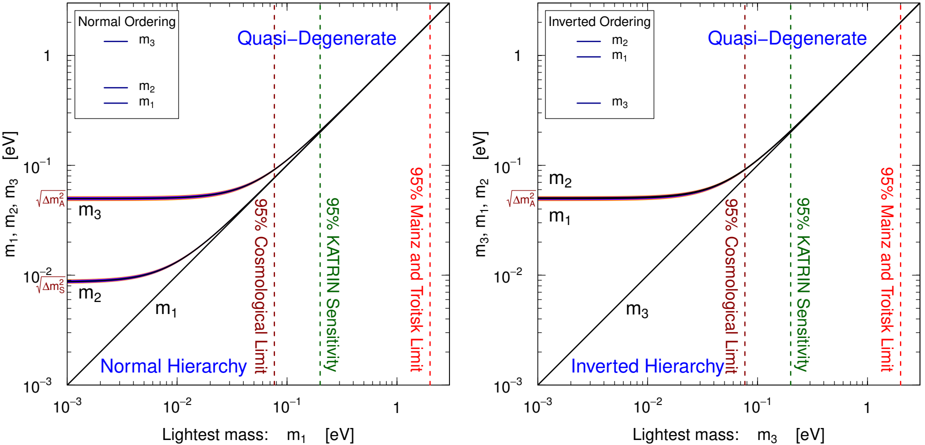
Second octant disfavoured at ~2σ

[Capozzi, Lisi, Marrone, Montanino, Palazzo @ NOW2016, September 2016]

[See also: Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Schwetz, arXiv:1611.01514]

Absolute Scale of Neutrino Masses

Mass Hierarchy or Degeneracy?



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{S}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{A}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{A}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{A}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_{\mathrm{A}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

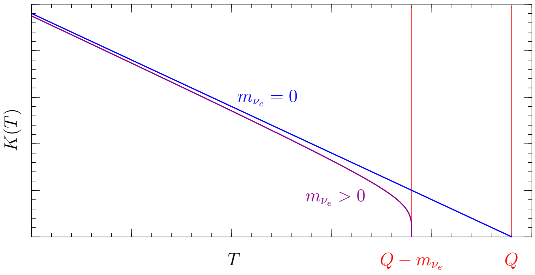
Tritium Beta-Decay

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

Kurie function: $$K(T) = \left[(Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2} \right]^{1/2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$



$m_{\nu_e} < 2.2\,\mathrm{eV}$ (95% C.L.)

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

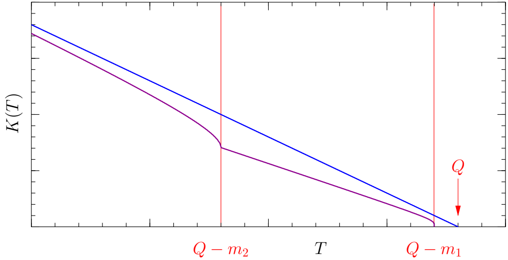
future: KATRIN

[www.katrin.kit.edu]

start data taking 2016?

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses ($m_k \ll Q-T$)

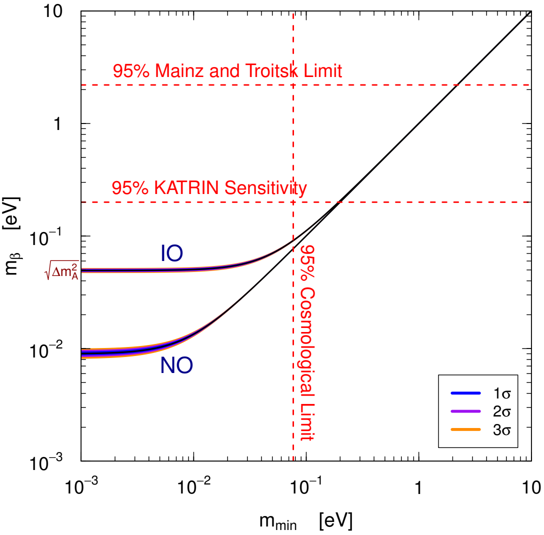
effective mass: $$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

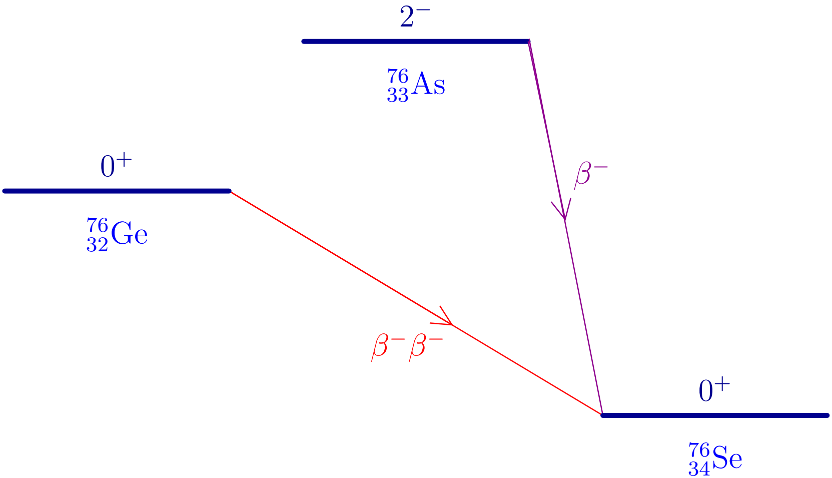
Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



- Quasi-Degenerate:
  $$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$
- Inverted Hierarchy:
  $$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$$
- Normal Hierarchy:
  $$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2 \simeq 2 \times 10^{-5} + 6 \times 10^{-5}\,\text{eV}^2$$
- If $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV}$
  $\Downarrow$
  Normal Spectrum

Neutrinoless Double-Beta Decay



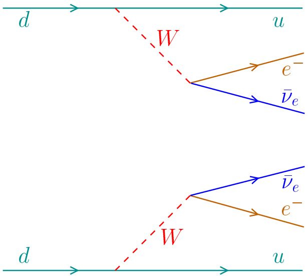
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

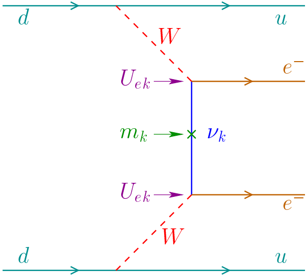


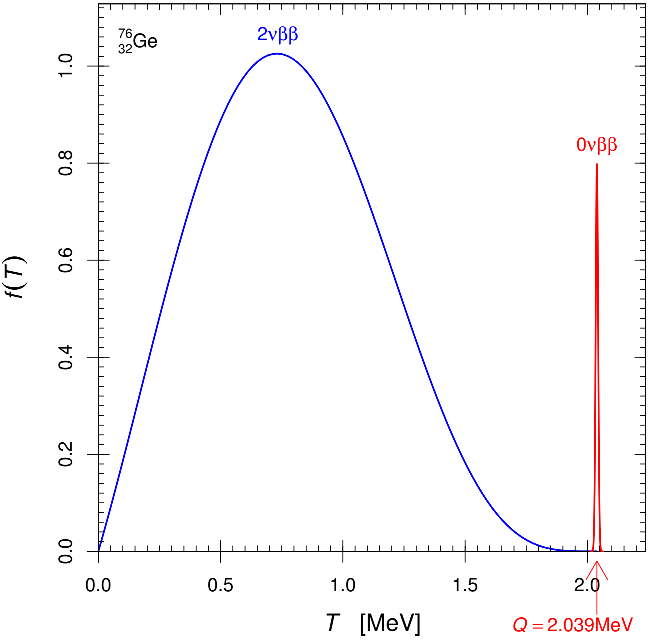
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$





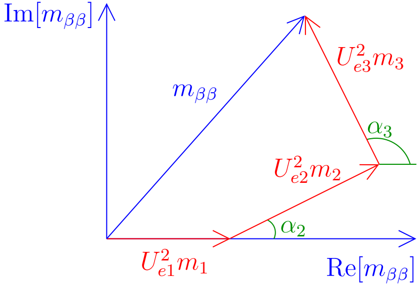
$^{76}_{32}$Ge
2νββ
0νββ
f(T)
T [MeV]
Q = 2.039MeV
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0

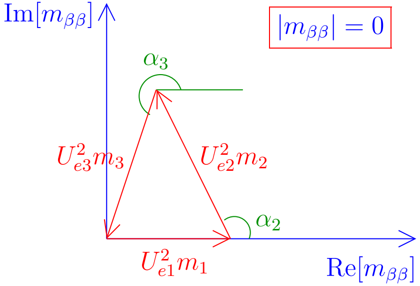
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

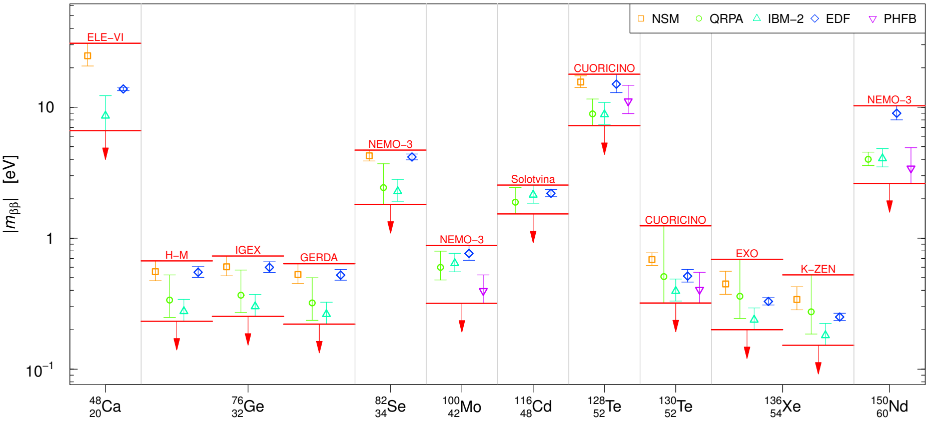
$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$





2015 90% C.L. Experimental Bounds

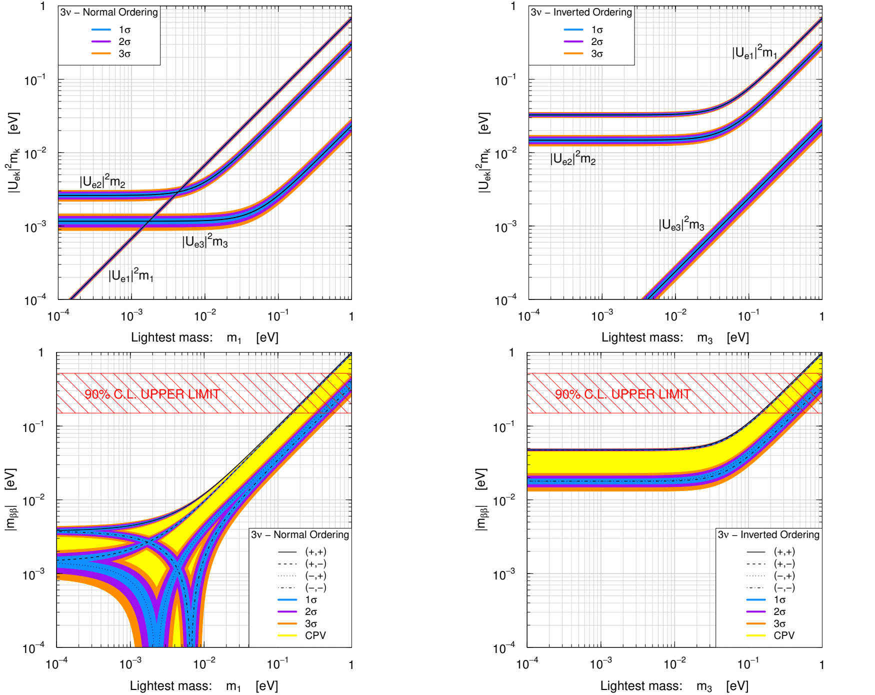
$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	GERDA	$> 2.1 \times 10^{25}$	$< 0.22 - 0.64$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORICINO	$> 2.8 \times 10^{24}$	$< 0.32 - 1.2$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.2 - 0.69$
	KamLAND-Zen	$> 1.9 \times 10^{25}$	$< 0.15 - 0.52$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



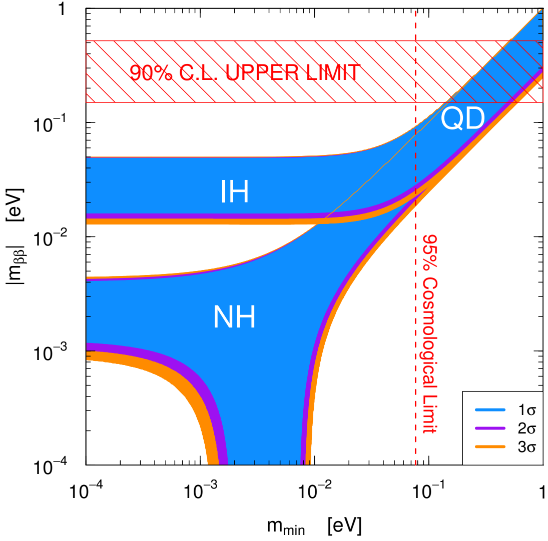
[Bilenky, Giunti, IJMPA 30 (2015) 0001]

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$


3ν – Normal Ordering
1σ
2σ
3σ
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_1 [eV]
3ν – Inverted Ordering
1σ
2σ
3σ
$|U_{e1}|^2 m_1$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_3 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν – Normal Ordering
(+,+)
(+,–)
(–,+)
(–,–)
1σ
2σ
3σ
CPV
Lightest mass: m_1 [eV]
90% C.L. UPPER LIMIT
$|m_{\beta\beta}|$ [eV]
3ν – Inverted Ordering
(+,+)
(+,–)
(–,+)
(–,–)
1σ
2σ
3σ
CPV
Lightest mass: m_3 [eV]

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

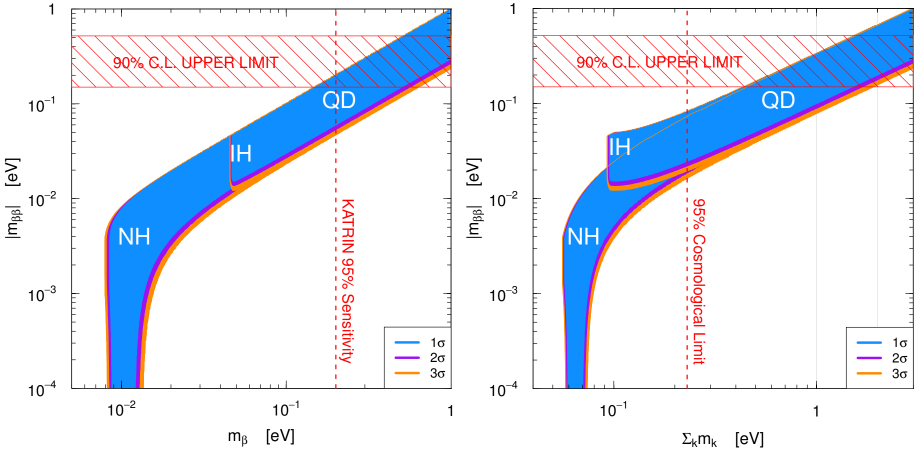
$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12}\sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13}\sqrt{\Delta m^2_{\text{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\ \text{eV}$$

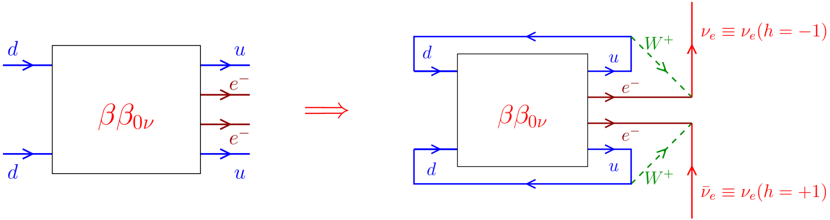
$$|m_{\beta\beta}| \lesssim 10^{-2}\ \text{eV} \Longrightarrow \text{Normal Spectrum}$$


90% C.L. UPPER LIMIT
QD
IH
NH
KATRIN 95% Sensitivity
1σ
2σ
3σ
$|m_{\beta\beta}|$ [eV]
m_β [eV]
95% Cosmological Limit
$\Sigma_k m_k$ [eV]

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

$|m_{\beta\beta}|$ can vanish because of unfortunate cancellations among m_1, m_2, m_3 contributions or because neutrinos are Dirac

$\beta\beta_{0\nu}$ decay can be generated by another mechanism beyond SM



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term: $\mathcal{L}^{\mathrm{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right)$

four-loop diagram calculation: $m_{ee} \sim 10^{-24}\,\mathrm{eV}$ [Duerr, Lindner, Merle, JHEP 06 (2011) 091]

- In any case finding $\beta\beta_{0\nu}$ decay is important information to solve the Dirac-Majorana question in favor of Majorana

- On the other hand, it is not possible to prove experimentally that neutrinos are Dirac.
  A Dirac neutrino is equivalent to 2 Majorana neutrinos with the same mass.
  Impossible to prove experimentally that mass splitting is exactly zero.

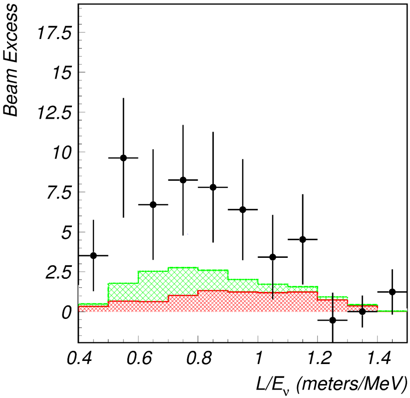
Light Sterile Neutrinos

Indications of SBL Oscillations Beyond 3ν

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad$ $20\,\text{MeV} \leq E \leq 60\,\text{MeV}$



- Well-known source of $\bar{\nu}_\mu$

  μ^+ at rest $\to e^+ + \nu_e + \bar{\nu}_\mu$

  $L \simeq 30\,\text{m}$

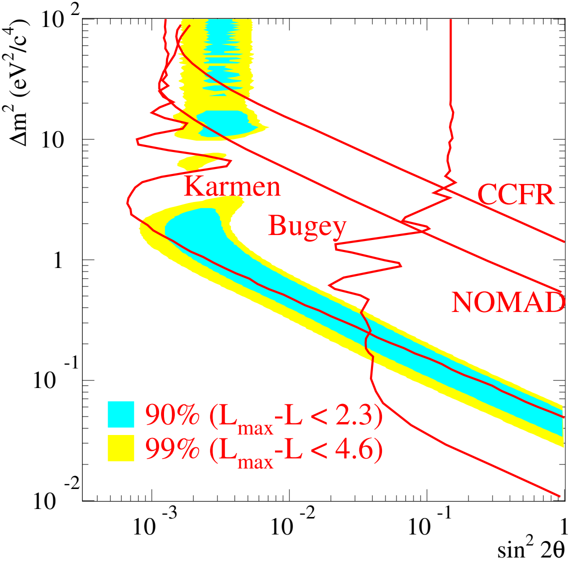
  $\bar{\nu}_e + p \to n + e^+$

  Well-known detection process of $\bar{\nu}_e$

- $\approx 3.8\sigma$ excess

- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

  [PRD 65 (2002) 112001]

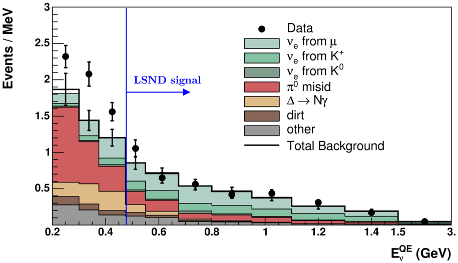


$$\Delta m^2_{\rm SBL} \gtrsim 3 \times 10^{-2}\,{\rm eV}^2 \gg \Delta m^2_{\rm ATM} \simeq 2.5 \times 10^{-3}\,{\rm eV}^2 \gg \Delta m^2_{\rm SOL}$$

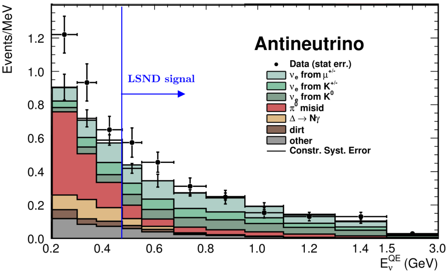
MiniBooNE

$L \simeq 541\,\text{m}$ $\qquad$ $200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$

$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]



$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



- Purpose: check LSND signal.
- Different L and E.
- Similar L/E (oscillations).
- No money, no Near Detector.
- LSND signal: $E > 475\,\text{MeV}$.
- Agreement with LSND signal?
- CP violation?
- Low-energy anomaly!

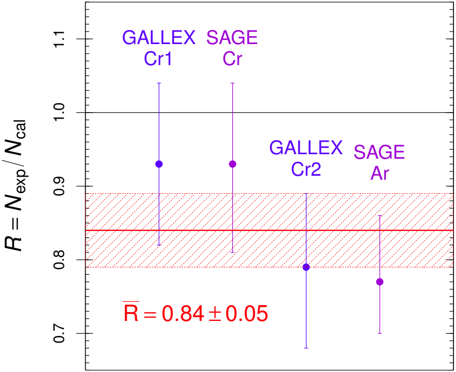
Gallium Anomaly

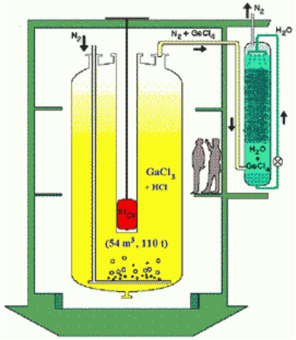
Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

$E \simeq 0.75\,\text{MeV}$ $\quad$ $E \simeq 0.81\,\text{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$





$\approx 2.9\sigma$ deficit

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$ $\quad$ $\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

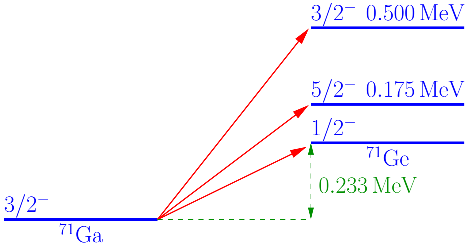
$\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}}$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807;
Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344,
MPLA 22 (2007) 2499, PRD 78 (2008) 073009,
PRC 83 (2011) 065504]

- Deficit could be due to overestimate of

$$\sigma(\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\text{G.S.}}$ from $T_{1/2}({}^{71}\text{Ge}) = 11.43 \pm 0.03\,\text{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\text{G.S.}}({}^{51}\text{Cr}) = 55.3 \times 10^{-46}\,\text{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\text{Cr}) = \sigma_{\text{G.S.}}({}^{51}\text{Cr}) \left(1 + 0.669\,\frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}} + 0.220\,\frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}}\right)$

- Contribution of excited states only 5%!

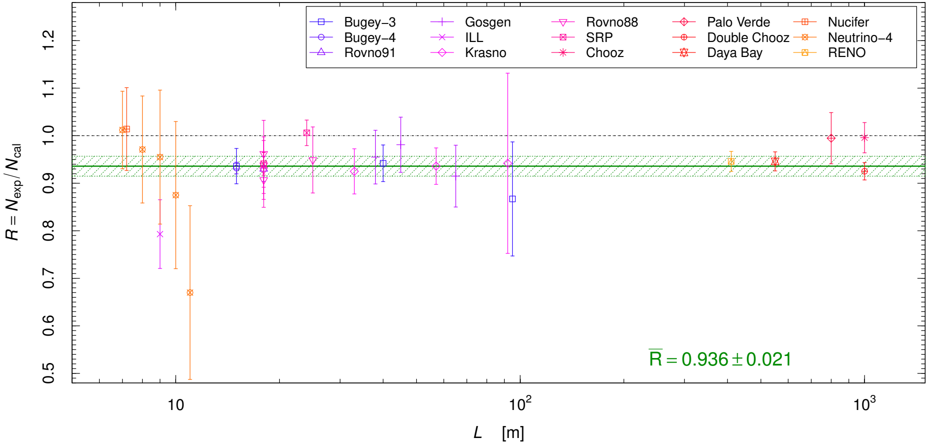
		$\frac{BGT_{175}}{BGT_{G.S.}}$	$\frac{BGT_{500}}{BGT_{G.S.}}$
Krofcheck et al. PRL 55 (1985) 1051	$^{71}Ga(p,n)^{71}Ge$	< 0.056	0.126 ± 0.023
Haxton PLB 431 (1998) 110	Shell Model	0.19 ± 0.18	
Frekers et al. PLB 706 (2011) 134	$^{71}Ga(^{3}He,^{3}H)^{71}Ge$	0.039 ± 0.030	0.202 ± 0.016

- The $^{71}Ga(^{3}He,^{3}H)^{71}Ge$ data confirm the contribution of the two excited states.
- Haxton: "The calculation predicts destructive interference between the (p,n) spin and spin-tensor matrix elements"
- It is unlikely that the deficit is caused by an overestimate of the cross section.
- Possible explanations:
  - Statistical fluctuations.
  - Experimental faults.
  - Short-baseline oscillations.

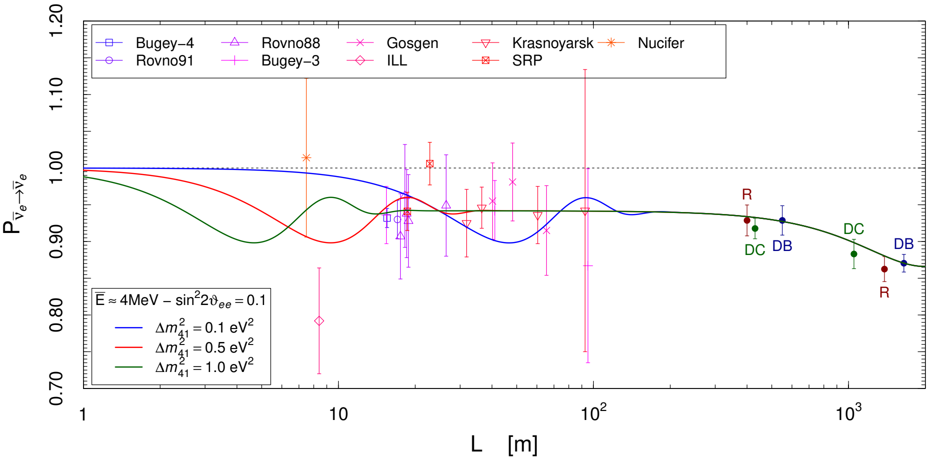
Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes [Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]

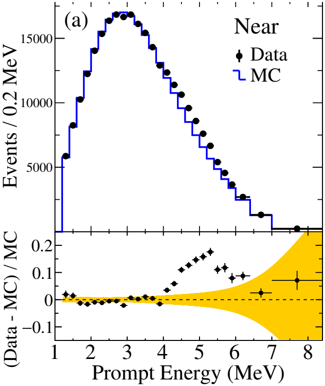


$\approx 3.1\sigma$ deficit

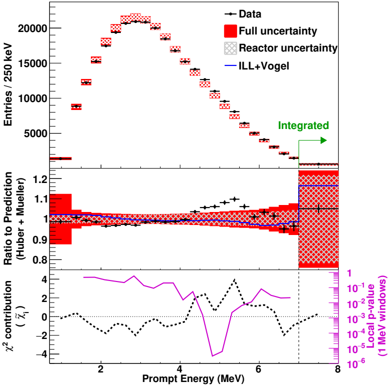


$$\Delta m^2_{\text{SBL}} \gtrsim 0.5\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}}$$

5 MeV Bump



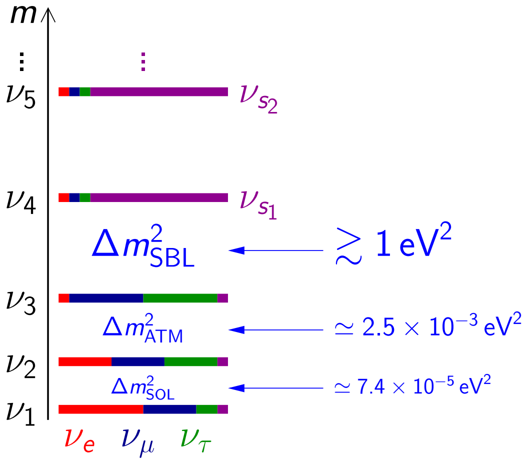
[RENO, arXiv:1511.05849]



[Daya Bay, arXiv:1508.04233]

- Local problem with $\sim 3\%$ effect on total flux.
- It is an excess!
- It occurs both for the new high Muller-Huber fluxes and the old low Schreckenbach-Vogel fluxes.
- Real problem: apparent incompatibility of the bump with the β spectra from ^{235}U and ^{239}Pu measured by Schreckenbach et al. at ILL in 1982-1985.

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

Sterile Neutrinos from Physics Beyond the SM

- Neutrinos are special in the Standard Model: the only neutral fermions
- Active left-handed neutrinos can mix with non-SM singlet fermions often called right-handed neutrinos   Neutrino Portal [A. Smirnov, arXiv:1502.04530]
- Light left-handed anti-ν_R are light sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions

[Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into light sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Short-baseline anomalies + 3ν-mixing:

$$\Delta m^2_{21} \ll |\Delta m^2_{31}| \ll |\Delta m^2_{41}| \leq \ldots$$

$$\begin{array}{ccccc} \nu_1 & \nu_2 & \nu_3 & \nu_4 & \ldots \\ \nu_e & \nu_\mu & \nu_\tau & \nu_{s_1} & \ldots \end{array}$$

- Here I consider sterile neutrinos with mass scale $\sim 1\,\text{eV}$ in light of short-baseline Reactor Anomaly, Gallium Anomaly, LSND.
- Other possibilities (not incompatible):
  - Very light sterile neutrinos with mass scale $\ll 1\,\text{eV}$: important for solar neutrino phenomenology

    [de Holanda, Smirnov, PRD 69 (2004) 113002; PRD 83 (2011) 113011]

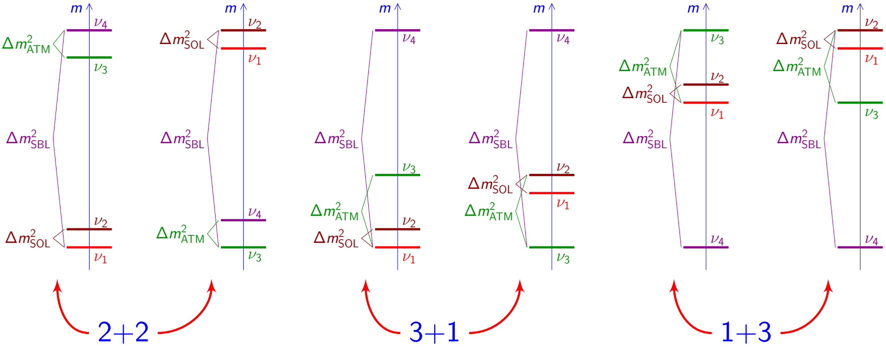
    [Das, Pulido, Picariello, PRD 79 (2009) 073010]

    Recent Daya Bay constraints for $10^{-3} \lesssim \Delta m^2 \lesssim 10^{-1}\,\text{eV}^2$ [PRL 113 (2014) 141802]
  - Heavy sterile neutrinos with mass scale $\gg 1\,\text{eV}$: could be Warm Dark Matter

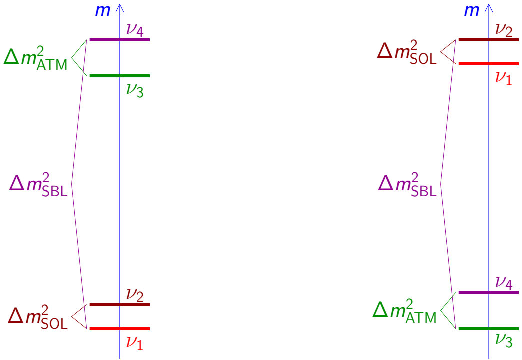
    [Asaka, Blanchet, Shaposhnikov, PLB 631 (2005) 151; Asaka, Shaposhnikov, PLB 620 (2005) 17; Asaka, Shaposhnikov, Kusenko, PLB 638 (2006) 401; Asaka, Laine, Shaposhnikov, JHEP 0606 (2006) 053, JHEP 0701 (2007) 091]

    [Reviews: Kusenko, Phys. Rept. 481 (2009) 1; Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191; Boyarsky, Iakubovskyi, Ruchayskiy, Phys. Dark Univ. 1 (2012) 136; Drewes, IJMPE, 22 (2013) 1330019]

Four-Neutrino Schemes: 2+2, 3+1 and 1+3



2+2 Four-Neutrino Schemes

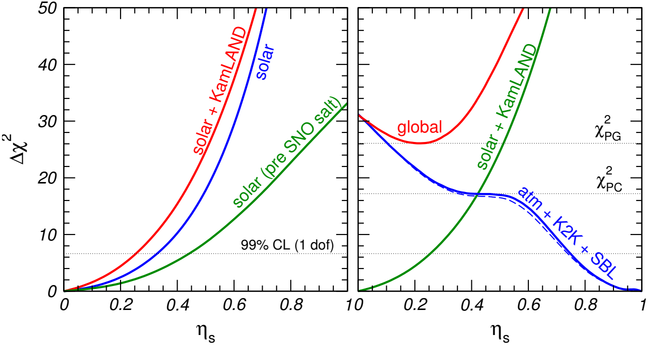


- After LSND (1995) 2+2 was preferred to 3+1, because of the 3+1 appearance-disappearance tension

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

- This is not a perturbation of 3-ν Mixing $\Longrightarrow$ Large active–sterile oscillations for solar or atmospheric neutrinos!

2+2 Schemes are Strongly Disfavored



Solar: Matter Effects + SNO NC

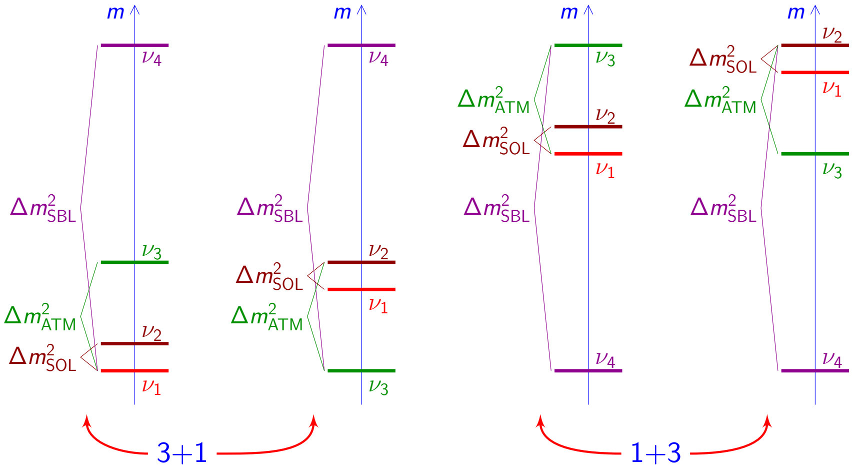
Atmospheric: Matter Effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 = 1 - |U_{s3}|^2 + |U_{s4}|^2$$

99% CL: $\begin{cases} \eta_s < 0.25 & \text{(Solar + KamLAND)} \\ \eta_s > 0.75 & \text{(Atmospheric + K2K)} \end{cases}$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122]

3+1 and 1+3 Four-Neutrino Schemes



- Perturbation of 3-ν Mixing: $|U_{e4}|^2, |U_{\mu 4}|^2, |U_{\tau 4}|^2 \ll 1 \quad |U_{s4}|^2 \simeq 1$
- 1+3 schemes are disfavored by cosmology (ΛCDM):

$$\sum_{k=1}^{3} m_k < 0.21\,\text{eV}$$ (95%, Planck TT + lowP + BAO) [arXiv:1502.01589]

Effective 3+1 SBL Oscillation Probabilities

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2$$

$$= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2 \to \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1 \qquad \Delta m_{41}^2 \to \Delta m^2$$

$$P_{\nu_\alpha \to \nu_\beta}^{\text{SBL}} \simeq \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(-i \frac{\Delta m^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}$$

$$
\begin{aligned}
P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} &\simeq \left|\delta_{\alpha\beta} - U^*_{\alpha4} U_{\beta4}\left[1-\exp\left(-i\frac{\Delta m^2 L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha4}|^2 |U_{\beta4}|^2 \left(2 - 2\cos\frac{\Delta m^2 L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha4}|^2 \left(1-\cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right)\left(1-\cos\frac{\Delta m^2 L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right)\sin^2\frac{\Delta m^2 L}{4E}
\end{aligned}
$$

$$
\alpha \neq \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq 4|U_{\alpha4}|^2|U_{\beta4}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)
$$

$$
\alpha = \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\alpha} \simeq 1 - 4|U_{\alpha4}|^2\left(1-|U_{\alpha4}|^2\right)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)
$$

Appearance ($\alpha \neq \beta$) — Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2\left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL (fourth column)

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Gandhi et al, JHEP 1511 (2015) 039] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, PRD 87, 113004 (2013) 113004]

Solar bound on $|U_{e4}|^2$

[Giunti, Li, PRD 80 (2009) 113007; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301]

$$P^{\text{SOL}}_{\nu_e\to\nu_e} \simeq \left(1 - \sum_{k\geq 3}|U_{ek}|^2\right)^2 P^{\text{SOL},2\nu}_{\nu_e\to\nu_e} + \sum_{k\geq 3}|U_{ek}|^4$$

$$P^{\text{SOL}}_{\nu_e\to\nu_s} \simeq \left(1 - \sum_{k\geq 3}|U_{ek}|^2\right)\left(1 - \sum_{k\geq 3}|U_{sk}|^2\right) P^{\text{SOL},2\nu}_{\nu_e\to\nu_s} + \sum_{k\geq 3}|U_{ek}|^2|U_{sk}|^2$$

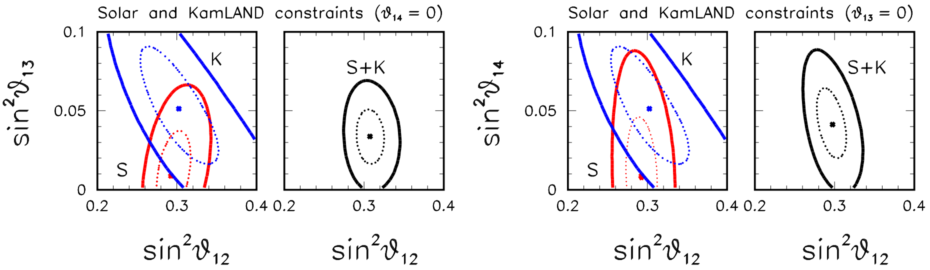
3+1 with simplifying assumptions: $U_{\mu 4} = U_{\tau 4} = 0$, no CP violation

$$\begin{array}{llll} U_{e1} = c_{12}c_{13}c_{14} & U_{e2} = s_{12}c_{13}c_{14} & U_{e3} = s_{13}c_{14} & U_{e4} = s_{14} \\ U_{s1} = -c_{12}c_{13}s_{14} & U_{s2} = -s_{12}c_{13}s_{14} & U_{s3} = -s_{13}s_{14} & U_{s4} = c_{14} \end{array}$$

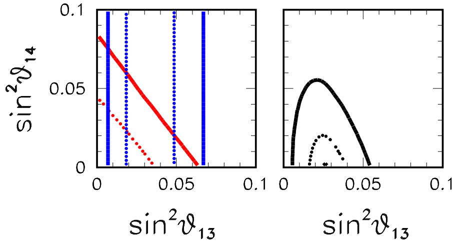
$$P^{\text{SOL}}_{\nu_e\to\nu_e} \simeq c_{13}^4 c_{14}^4 P^{\text{SOL},2\nu}_{\nu_e\to\nu_e} + s_{13}^4 c_{14}^4 + s_{14}^4$$

$$P^{\text{SOL}}_{\nu_e\to\nu_s} \simeq c_{14}^2 s_{14}^2 \left(c_{13}^4 P^{\text{SOL},2\nu}_{\nu_e\to\nu_s} + s_{13}^4 + 1\right)$$

$$\begin{aligned} V &= c_{13}^2 c_{14}^2 V_{\text{CC}} - c_{13}^2 s_{14}^2 V_{\text{NC}} \\ &= \left(|U_{e1}|^2 + |U_{e2}|^2\right) V_{\text{CC}} - \left(|U_{s1}|^2 + |U_{s2}|^2\right) V_{\text{NC}} \end{aligned}$$



[Palazzo, PRD 83 (2011) 113013]



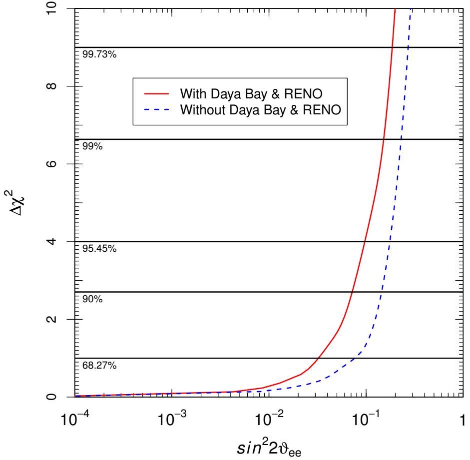
[Palazzo, PRD 85 (2012) 077301]

Daya Bay and RENO

$$\sin^2\vartheta_{13} = 0.025 \pm 0.004$$

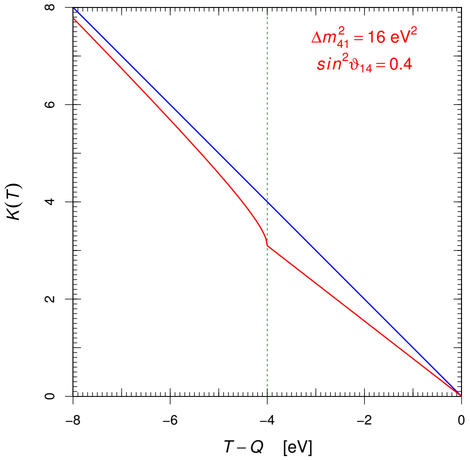
$$|U_{e4}|^2 = \sin^2\vartheta_{14} \lesssim 0.02\,(1\sigma)$$

Fit of solar and KamLAND data with Daya Bay and RENO constraint $\sin^2\vartheta_{13} = 0.025 \pm 0.004$ and free $|U_{\mu 4}|$ and $|U_{\tau 4}|$ (neglecting small CP violation effects)



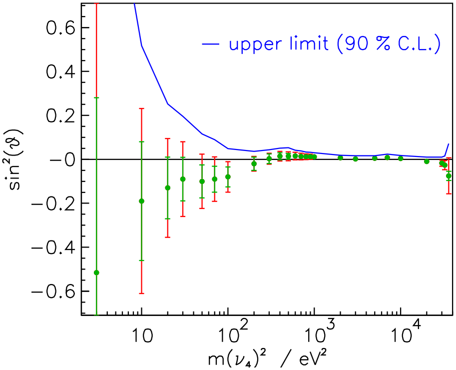
[Giunti, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

Tritium Beta-Decay

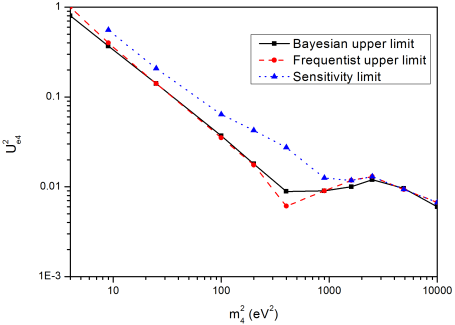


$$m_4 \gg m_1, m_2, m_3 \quad \Longrightarrow \quad \Delta m^2_{41} \equiv m^2_4 - m^2_1 \simeq m^2_4$$

Mainz and Troitsk Limit on m_4^2

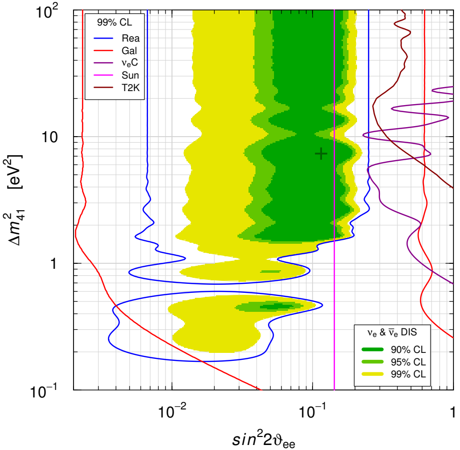


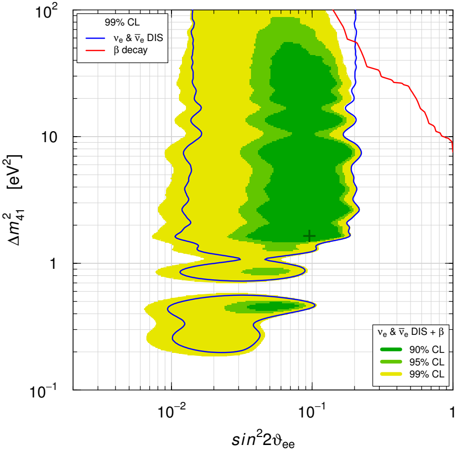
[Kraus, Singer, Valerius, Weinheimer, EPJC 73 (2013) 2323]



[Belesev et al, JPG 41 (2014) 015001]

Global ν_e and $\bar{\nu}_e$ Disappearance





KARMEN + LSND $\nu_e + {}^{12}\text{C} \to {}^{12}\text{N}_{\text{g.s.}} + e^-$
[Conrad, Shaevitz, PRD 85 (2012) 013017]
[CG, Laveder, PLB 706 (2011) 200]

solar ν_e + KamLAND $\bar{\nu}_e$ + ϑ_{13}
[CG, Li, PRD 80 (2009) 113007]
[Palazzo, PRD 83 (2011) 113013; PRD 85 (2012) 077301]
[CG, Laveder, Li, Liu, Long, PRD 86 (2012) 113014]

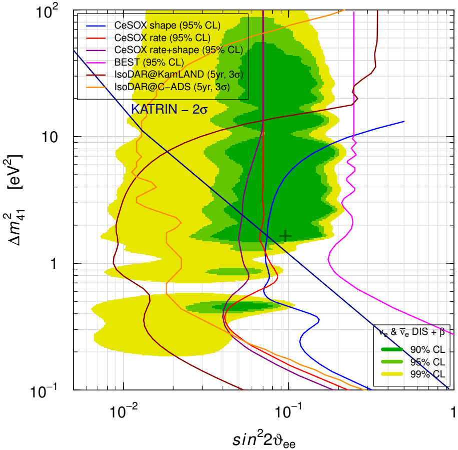
T2K Near Detector ν_e disappearance
[T2K, PRD 91 (2015) 051102]

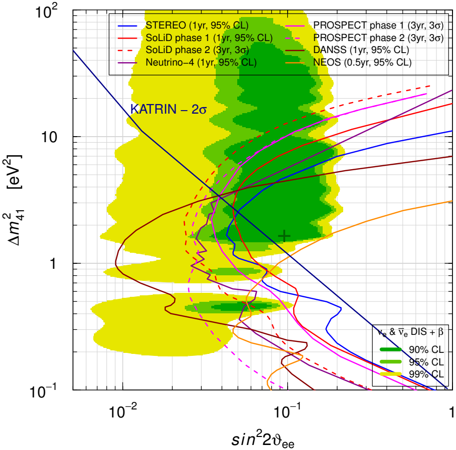
Mainz + Troitsk Tritium β decay
[Mainz, EPJC 73 (2013) 2323]
[Troitsk, JETPL 97 (2013) 67; JPG 41 (2014) 015001]

No Osc. excluded at 2.8σ
$(\Delta\chi^2/\text{NDF} = 10.8/2)$

$$6\,\text{cm} \lesssim \frac{L_{41}^{\text{osc}}}{E\,[\text{MeV}]} \lesssim 6\,\text{m} \qquad (2\sigma)$$

The Race for ν_e and $\bar{\nu}_e$ Disappearance





CeSOX (Gran Sasso, Italy) $^{144}\mathrm{Ce} \to \bar{\nu}_e$
BOREXINO: $L \simeq$ 5-12m [Vivier@TAUP2015]

BEST (Baksan, Russia) $^{51}\mathrm{Cr} \to \nu_e$
$L \simeq$ 5-12m [PRD 93 (2016) 073002]

IsoDAR@KamLAND (Kamioka, Japan)
$^{8}\mathrm{Li} \to \bar{\nu}_e$ $L \simeq$ 16m [arXiv:1511.05130]

IsoDAR@C-ADS (Guangdong, China)
$^{8}\mathrm{Li} \to \bar{\nu}_e$ $L \simeq$ 15m [JHEP 1601 (2016) 004]

STEREO (ILL, France) $L \simeq$ 8-12m [arXiv:1602.00568]

SoLid (SCK-CEN, Belgium) $L \simeq$ 5-8m [arXiv:1510.07835]

Neutrino-4 (RIAR, Russia) $L \simeq$ 6-11m [JETP 121 (2015) 578]

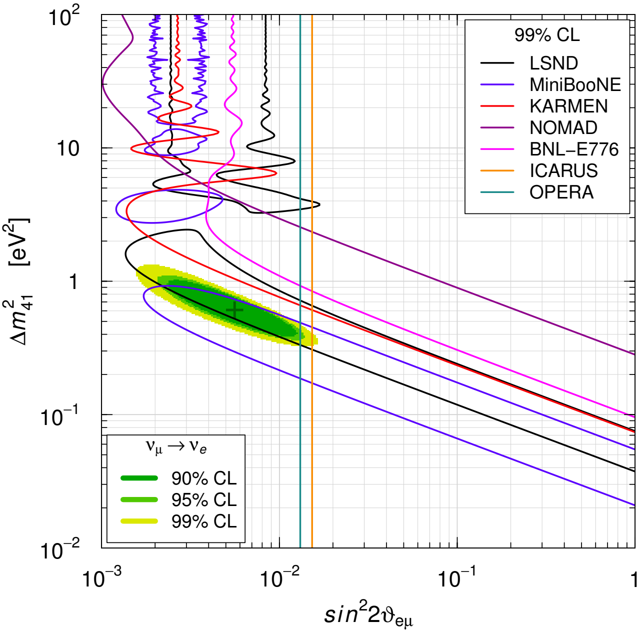
PROSPECT (ORNL, USA) $L \simeq$ 7-12m [arXiv:1512.02202]

DANSS (Kalinin, Russia) $L \simeq$ 10-12m [arXiv:1606.02896]

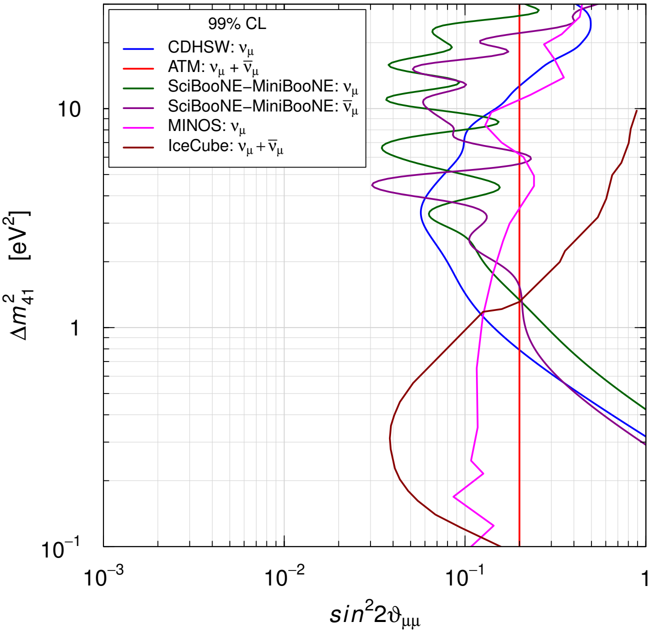
NEOS (Hanbit, Korea) $L \simeq$ 24m [Oh@WIN2015]

KATRIN (Karlsruhe, Germany) $^{3}\mathrm{H} \to \bar{\nu}_e$ [Mertens@TAUP2015]

$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ Appearance



ν_μ and $\bar{\nu}_\mu$ Disappearance



3+1 Appearance-Disappearance Tension

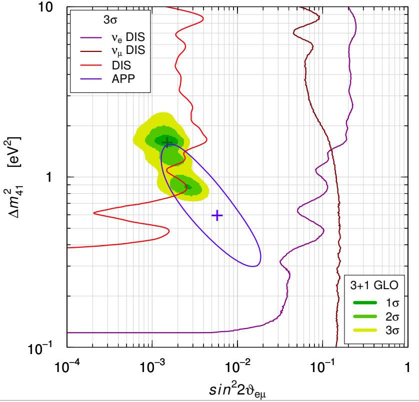
ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu 4}|^2$

$\nu_\mu \to \nu_e$ APP

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- Similar constraint in

  3+2, 3+3, ..., 3+N_s

  [CG, Zavanin, MPLA 31 (2015) 1650003]

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001] with improved treatment of the MiniBooNE background disappearance due to neutrino oscillations according to information from Bill Louis (thanks!)

Appearance vs Disappearance in $N = 3 + N_s$ Mixing

[Giunti, Zavanin, MPLA 31 (2015) 1650003]

$$\frac{\Delta m^2_{21} L}{4E} \ll \frac{\Delta m^2_{31} L}{4E} \ll 1$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} \simeq \delta_{\alpha\beta} - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2 \right) \sin^2 \Delta_{k1}$$
$$+ 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\Delta_{jk} = \frac{\Delta m^2_{jk} L}{4E} \qquad \eta_{\alpha\beta jk} = \arg\left[U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \right]$$

Survival Probabilities

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \sin^2 \Delta_{k1}$$
$$+ 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin \Delta_{j1} \sin \Delta_{k1} \cos \Delta_{jk}$$

Effective amplitude of $\overset{(-)}{\nu}_\alpha$ disappearance due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4 |U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4 |U_{\alpha k}|^2$$

$$|U_{\alpha k}|^2 \ll 1 \qquad (\alpha = e, \mu, \tau; \quad k = 4, \ldots, N)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2 \Delta_{k1}$$

Appearance Probabilities ($\alpha \neq \beta$)

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 |U_{\beta k}|^2 \sin^2 \Delta_{k1}$$
$$+ 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k}| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

Effective amplitude of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ transitions due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\beta} = 4 |U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\beta} \sin^2 \Delta_{k1}$$
$$+ 2 \sum_{k=4}^{N} \sum_{j=k+1}^{N} \sin 2\vartheta^{(k)}_{\alpha\beta} \sin 2\vartheta^{(j)}_{\alpha\beta} \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

$$\sin^2 2\vartheta^{(k)}_{\alpha\beta} = 4|U_{\alpha k}|^2|U_{\beta k}|^2$$

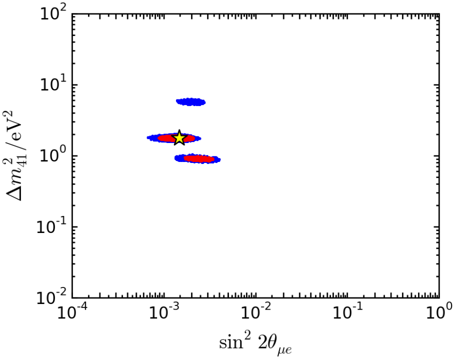
$$\boxed{\sin^2 2\vartheta^{(k)}_{\alpha\beta} \simeq \frac{1}{4} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2 2\vartheta^{(k)}_{\beta\beta}}$$

$$\left.\begin{array}{l} \sin^2 2\vartheta^{(k)}_{ee} \ll 1 \\ \sin^2 2\vartheta^{(k)}_{\mu\mu} \ll 1 \end{array}\right\} \quad \Rightarrow \quad \sin^2 2\vartheta^{(k)}_{e\mu} \quad \text{is quadratically suppressed}$$

on the other hand, observation of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ transitions due to Δm^2_{k1} imply that the corresponding $\overset{(-)}{\nu}_\alpha$ and $\overset{(-)}{\nu}_\beta$ disappearances must be observed

Collin, Arguelles, Conrad, Shaevitz

[NPB 908 (2016) 354]



Best Fit: $\Delta m^2_{41} = 1.75\,\text{eV}^2$

$|U_{e4}|^2 = 0.027 \qquad |U_{\mu 4}|^2 = 0.014$

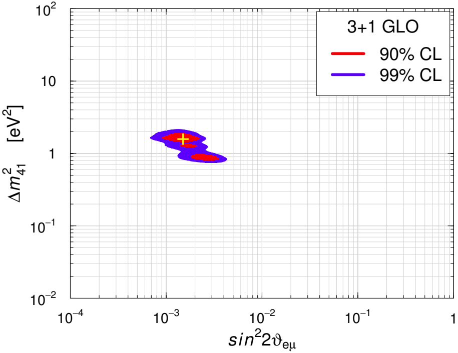
GoF $= 57\%$ ($\chi^2_{\text{min}}/\text{NDF} = 306.8/312$)

GoF$_{\text{null}} = 4.4\%$ ($\chi^2/\text{NDF} = 359.2/315$)

$\Delta\chi^2/\text{NDF} = 52.3/3$ ($\approx 6.7\sigma$)

Our Fit

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



Best Fit: $\Delta m^2_{41} = 1.6\,\text{eV}^2$

$|U_{e4}|^2 = 0.028 \qquad |U_{\mu 4}|^2 = 0.014$

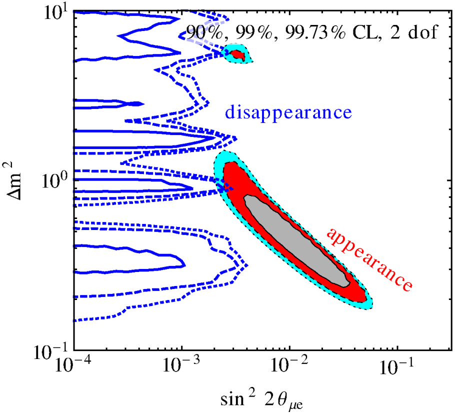
GoF $= 6\%$ ($\chi^2_{\text{min}}/\text{NDF} = 304.0/268$)

GoF$_{\text{null}} = 0.04\%$ ($\chi^2/\text{NDF} = 355.2/271$)

$\Delta\chi^2/\text{NDF} = 51.2/3$ ($\approx 6.6\sigma$)

Kopp, Machado, Maltoni, Schwetz

[JHEP 1305 (2013) 050]



Best Fit: $\Delta m^2_{41} = 0.93\,\text{eV}^2$

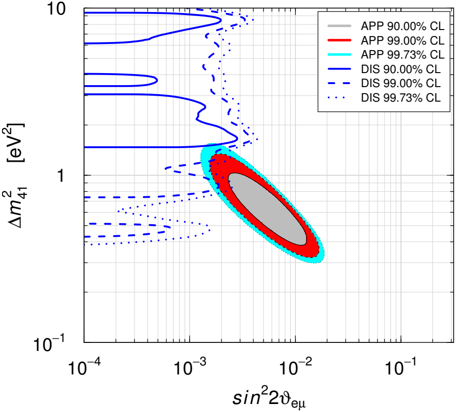
$|U_{e4}|^2 = 0.023$ $|U_{\mu 4}|^2 = 0.029$

$\text{GoF} = 19\%\,(\chi^2_{\text{min}}/\text{NDF} = 712/680)$

$\text{GoF}_{\text{PG}} = 0.01\%\,(\chi^2_{\text{PG}}/\text{NDF} = 18.0/2)$

Our Fit

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



Best Fit: $\Delta m^2_{41} = 1.6\,\text{eV}^2$

$|U_{e4}|^2 = 0.028$ $|U_{\mu 4}|^2 = 0.014$

$\text{GoF} = 6\%\,(\chi^2_{\text{min}}/\text{NDF} = 304.0/268)$

$\text{GoF}_{\text{PG}} = 0.06\%\,(\chi^2/\text{NDF} = 15.0/2)$

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- $\chi^2_{\rm min}$ has χ^2 distribution with Number of Degrees of Freedom

$$\mathrm{NDF} = N_{\rm D} - N_{\rm P}$$

$N_{\rm D}$ = Number of Data $\quad\quad$ $N_{\rm P}$ = Number of Fitted Parameters

- $\langle \chi^2_{\rm min} \rangle = \mathrm{NDF}$ $\quad\quad$ $\mathrm{Var}(\chi^2_{\rm min}) = 2\mathrm{NDF}$
- $\mathrm{GoF} = \int_{\chi^2_{\rm min}}^{\infty} p_{\chi^2}(z, \mathrm{NDF})\, dz$ $\quad\quad$ $p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

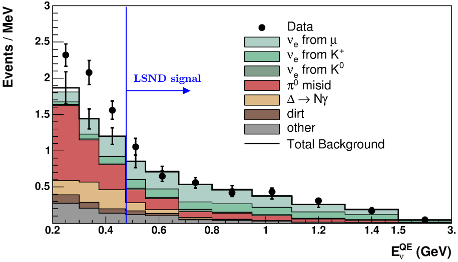
- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\rm PGoF} = (\chi^2_{\rm min})_{\rm A+B} - [(\chi^2_{\rm min})_{\rm A} + (\chi^2_{\rm min})_{\rm B}]$
- $\chi^2_{\rm PGoF}$ has χ^2 distribution with Number of Degrees of Freedom

$$\mathrm{NDF}_{\rm PGoF} = N_{\rm P}^{\rm A} + N_{\rm P}^{\rm B} - N_{\rm P}^{\rm A+B}$$

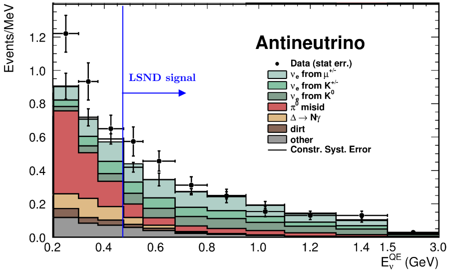
- $\mathrm{PGoF} = \int_{\chi^2_{\rm PGoF}}^{\infty} p_{\chi^2}(z, \mathrm{NDF}_{\rm PGoF})\, dz$

MiniBooNE Low-Energy Anomaly

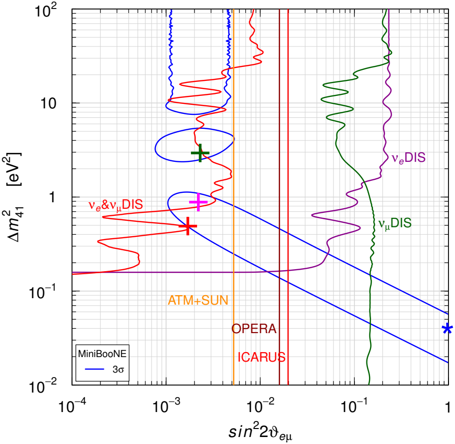
$\nu_\mu \to \nu_e$ [PRL 102 (2009) 101802]

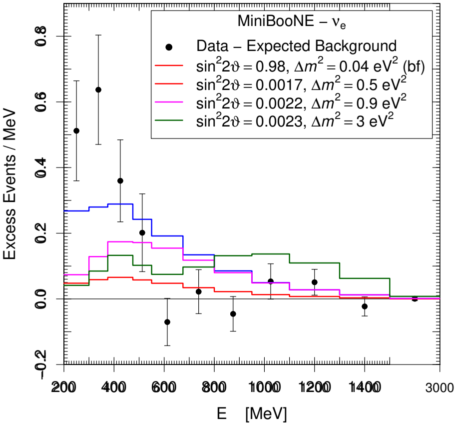


$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



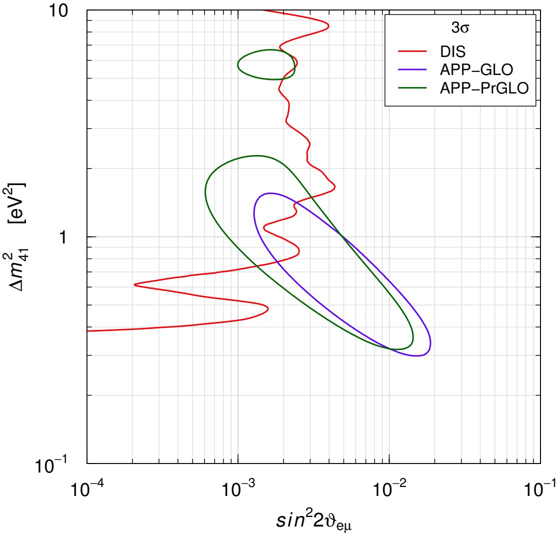
- Fit of MB Low-Energy Excess requires small Δm^2_{41} and large $\sin^2 2\vartheta_{e\mu}$, in contradiction with disappearance data
- MB low-energy excess is the main cause of bad APP-DIS $\text{GoF}_{\text{PG}} = 0.06\%$
- Multinucleon effects in neutrino energy reconstruction are not enough to solve the problem [Martini et al, PRD 85 (2012) 093012; PRD 87 (2013) 013009; PRD 93 (2016) 073008]
- Pragmatic Approach: discard the Low-Energy Excess because it is likely not due to oscillations [CG, Laveder, Li, Long, PRD 88 (2013) 073008]
- MicroBooNE is crucial for checking the MiniBooNE Low-Energy Anomaly and the consistency of different short-baseline data





No fit of low-energy excess for realistic $\sin^2 2\vartheta_{e\mu} \lesssim 3 \times 10^{-3}$

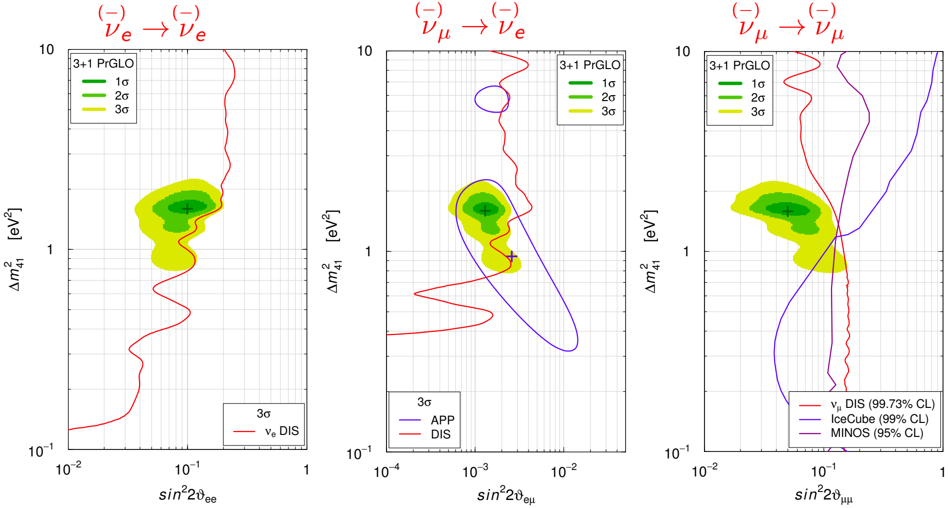
Global → Pragmatic



- APP-GLO: all MiniBooNE data
- APP-PrGLO: only MiniBooNE $E > 475\,\text{MeV}$ data (Pragmatic)

Pragmatic Global 3+1 Fit

Update of [Gariazzo, CG, Laveder, Li, Zavanin, JPG 43 (2016) 033001]



GoF = 24% PGoF = 7%

No Osc. disfavored at $\approx 6.2\sigma$

$\Delta\chi^2/\text{NDF} = 46.6/3$

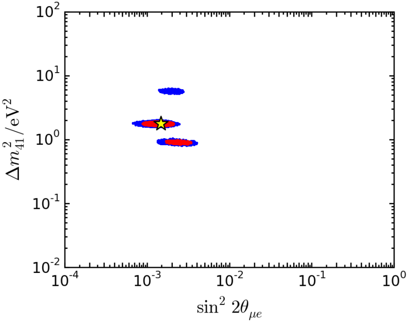
Not yet included:

- IceCube, arXiv:1605.01990
- MINOS, arXiv:1607.01176

SBL + IceCube

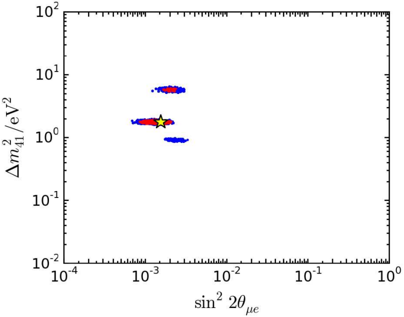
[Collin, Arguelles, Conrad, Shaevitz, arXiv:1607.00011]

SBL



→

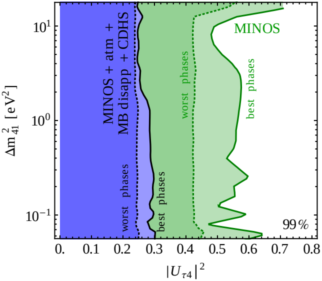
SBL + IceCube



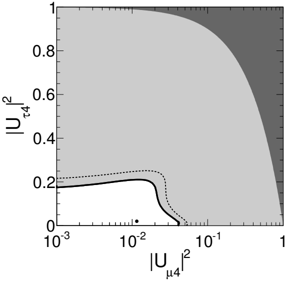
Red: 90% CL   Blue: 99% CL

3+1	Δm^2_{41}	$\lvert U_{e4}\rvert$	$\lvert U_{\mu4}\rvert$	$\lvert U_{\tau4}\rvert$	N_{bins}	χ^2_{min}	χ^2_{null}	$\Delta\chi^2$ (dof)
SBL	1.75	0.163	0.117	-	315	306.81	359.15	52.34 (3)
SBL+IC	1.75	0.164	0.119	0.00	524	518.59	568.84	50.26 (4)
IC	5.62	-	0.314	-	209	207.11	209.69	2.58 (2)

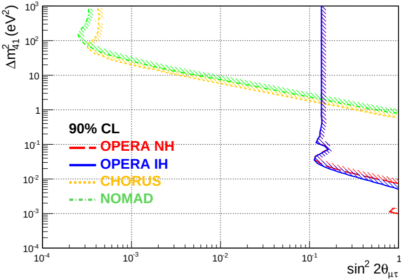
Bounds on $|U_{\tau 4}|$



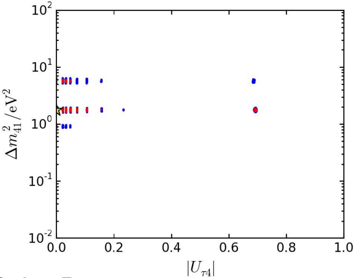
[Kopp et al, JHEP 1305 (2013) 050]



[Super-Kamiokande, PRD 91 (2015) 052019]



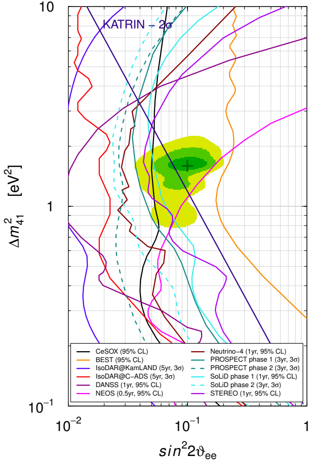
$\nu_\mu \to \nu_\tau$ [OPERA, JHEP 1506 (2015) 069]



IceCube Data [Collin et al, PRL 117 (2016) 221801]

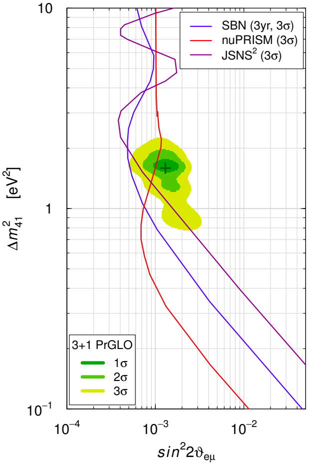
The Race for the Light Sterile

$\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$



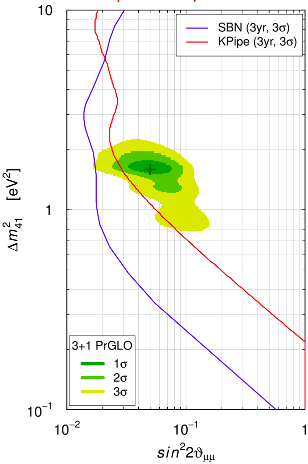
SBN (FNAL, USA)
[arXiv:1503.01520]
3 Liquid Argon TPCs
LAr1-ND $L \simeq 100$ m
MicroBooNE $L \simeq 470$ m
ICARUS T600 $L \simeq 600$ m

$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$



nuPRISM (J-PARC, Japan)
[Wilking@NNN2015]
$L \simeq 1$ km
50 m tall water Cherenkov detector
$1^\circ - 4^\circ$ off-axis

$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu$



KPipe (Japan) [arXiv:1510.06994]
KDAR: K Decay At Rest
$K^+ \to \mu^+ + \nu_\mu (E = 236 \text{ MeV})$
$L \simeq$ 30-150m
120 m long detector!

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\Delta_{kj} = \Delta m^2_{kj} L/4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\Delta_{41} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\Delta_{51} + 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin\Delta_{41} \sin\Delta_{51} \cos(\Delta_{54} \overset{(+)}{-} \eta)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2\Delta_{41} + |U_{\alpha 5}|^2 \sin^2\Delta_{51}) - 4|U_{\alpha 4}|^2|U_{\alpha 5}|^2 \sin^2\Delta_{54}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Archidiacono et al, PRD 86 (2012) 065028; Jacques, Krauss, Lunardini, PRD 87 (2013) 083515; Conrad et al, AHEP 2013 (2013) 163897; Archidiacono et al, PRD 87 (2013) 125034; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008; Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

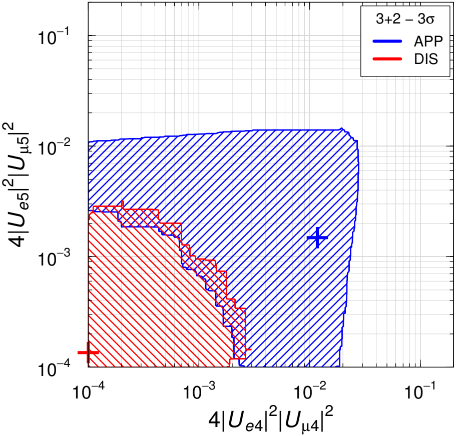
- Good: CP violation
- Bad: Two massive sterile neutrinos at the eV scale!

  4 more parameters: $\underbrace{\Delta m^2_{41}, |U_{e4}|^2, |U_{\mu 4}|^2}_{3+1}, \Delta m^2_{51}, |U_{e5}|^2, |U_{\mu 5}|^2, \eta$

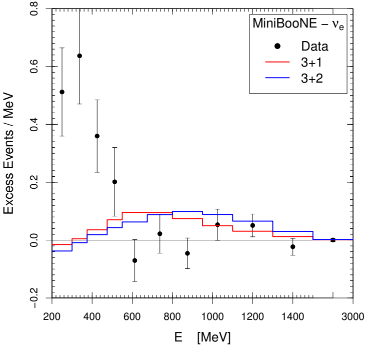
Global Fits	Our Fit		KMMS	
	3+1	3+2	3+1	3+2
GoF	6%	10%	19%	23%
PGoF	0.06%	0.3%	0.01%	0.003%

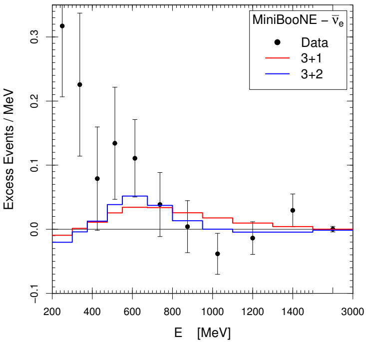
- Our Fit: Gariazzo, Giunti, Laveder, Li, Zavanin, JPG 43 (2016) 033001
- KMMS: Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050

APP-DIS 3+2 Tension:



3+2 cannot fit MiniBooNE Low-Energy Excess

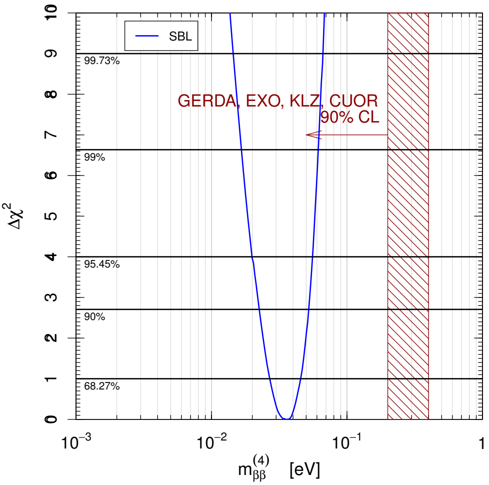




- Note difference between 3+2 ν_e and $\bar{\nu}_e$ histograms due to CP violation
- 3+2 can fit slightly better the small $\bar{\nu}_e$ excess at about 600 MeV
- 3+2 fit of low-energy excess as bad as 3+1
- Claims that 3+2 can fit low-energy excess do not take into account constraints from other data
- Conclusion: 3+2 is not needed

Neutrinoless Double-β Decay

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_{21}}\, m_2 + |U_{e3}|^2\, e^{i\alpha_{31}}\, m_3 + |U_{e4}|^2\, e^{i\alpha_{41}}\, m_4$$



Pragmatic 3+1 Fit

$$m_{\beta\beta}^{(k)} = |U_{ek}|^2 m_k$$

$$m_1 \ll m_4$$

$$\Downarrow$$

$$m_{\beta\beta}^{(4)} \simeq |U_{e4}|^2 \sqrt{\Delta m^2_{41}}$$

surprise:

possible cancellation with $m_{\beta\beta}^{(3\nu)}$

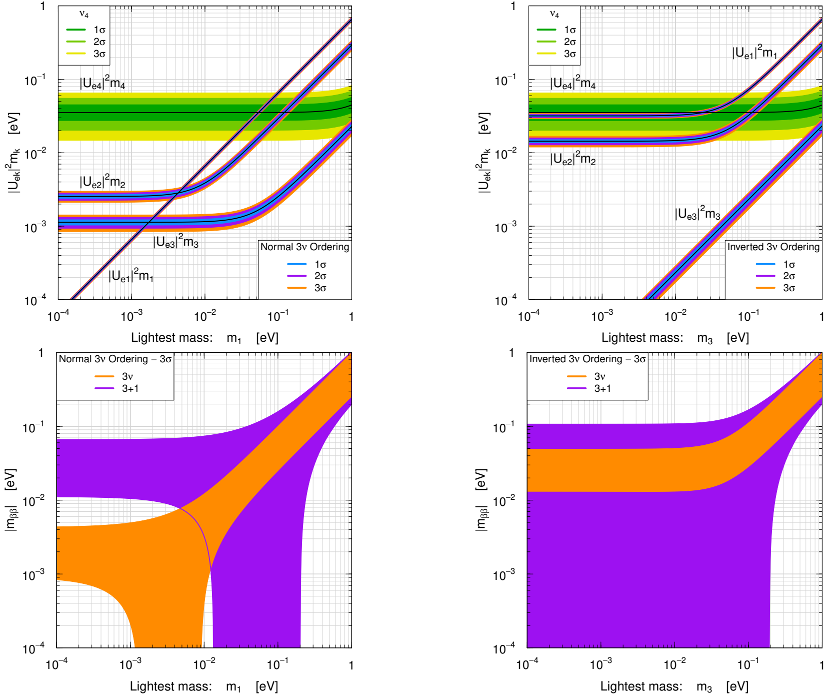
[Barry et al, JHEP 07 (2011) 091]

[Li, Liu, PLB 706 (2012) 406]

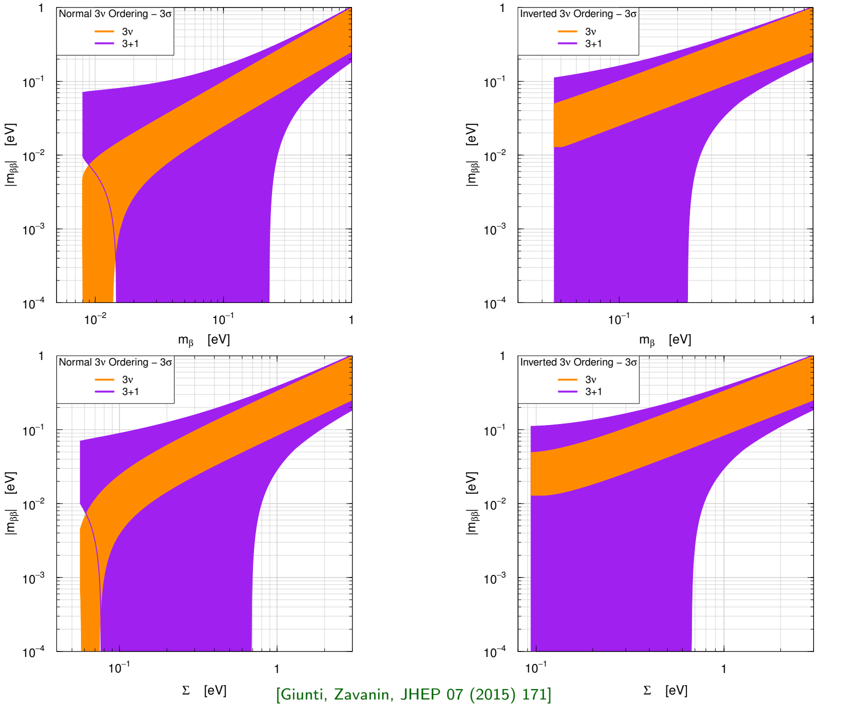
[Rodejohann, JPG 39 (2012) 124008]

[Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

[Giunti, Zavanin, JHEP 07 (2015) 171]



ν4
1σ
2σ
3σ
$|U_{e4}|^2 m_4$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
Normal 3ν Ordering
Inverted 3ν Ordering
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_1 [eV]
Lightest mass: m_3 [eV]
Normal 3ν Ordering – 3σ
Inverted 3ν Ordering – 3σ
3ν
3+1
$|m_{\beta\beta}|$ [eV]



[Giunti, Zavanin, JHEP 07 (2015) 171]

Effects of light sterile neutrinos should also be seen in:

- β Decay Experiments
  [Hannestad et al, JCAP 1102 (2011) 011, PRC 84 (2011) 045503; Formaggio, Barrett, PLB 706 (2011) 68; Esmaili, Peres, PRD 85 (2012) 117301; Gastaldo et al, JHEP 1606 (2016) 061]
- Neutrinoless Double-β Decay Experiments
  [Rodejohann et al, JHEP 1107 (2011) 091; Li, Liu, PLB 706 (2012) 406; Meroni et al, JHEP 1311 (2013) 146, PRD 90 (2014) 053002; Pascoli et al, PRD 90 (2014) 093005; CG, Zavanin, JHEP 1507 (2015) 171; Guzowski et al, PRD 92 (2015) 012002]
- Long-baseline Neutrino Oscillation Experiments
  [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, arXiv:1601.05995, arXiv:1603.03759, arXiv:1605.04299; Gandhi et al, JHEP 1511 (2015) 039; Pant et al, arXiv:1509.04096, Choubey, Pramanik, arXiv:1604.04731]
- Solar neutrinos
  [Dooling et al, PRD 61 (2000) 073011, Gonzalez-Garcia et al, PRD 62 (2000) 013005; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301; Li et al, PRD 80 (2009) 113007, PRD 87, 113004 (2013), JHEP 1308 (2013) 056; Kopp et al, JHEP 1305 (2013) 050]
- Atmospheric neutrinos
  [Goswami, PRD 55 (1997) 2931; Bilenky et al, PRD 60 (1999) 073007; Maltoni et al, NPB 643 (2002) 321, PRD 67 (2003) 013011; Choubey, JHEP 0712 (2007) 014; Razzaque, Smirnov, JHEP 1107 (2011) 084, PRD 85 (2012) 093010; Gandhi, Ghoshal, PRD 86 (2012) 037301; Barger et al, PRD 85 (2012) 011302; Esmaili et al, JCAP 1211 (2012) 041, JCAP 1307 (2013) 048, JHEP 1312 (2013) 014; Rajpoot et al, EPJC 74 (2014) 2936; Lindner et al, JHEP 1601 (2016) 124; Behera et al, arXiv:1605.08607]
- Supernova neutrinos
  [Caldwell, Fuller, Qian, PRD 61 (2000) 123005; Peres, Smirnov, NPB 599 (2001); Sorel, Conrad, PRD 66 (2002) 033009; Tamborra et al, JCAP 1201 (2012) 013; Wu et al, PRD 89 (2014) 061303; Esmaili et al, PRD 90 (2014) 033013]
- Cosmic neutrinos
  [Cirelli et al, NPB 708 (2005) 215; Donini, Yasuda, arXiv:0806.3029; Barry et al, PRD 83 (2011) 113012]
- Indirect dark matter detection [Esmaili, Peres, JCAP 1205 (2012) 002]
- Cosmology [see: Wong, ARNPS 61 (2011) 69; Archidiacono et al, AHEP 2013 (2013) 191047]

Effective 3+1 LBL Oscillation Probabilities

[de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, arXiv:1601.05995, arXiv:1603.03759, arXiv:1605.04299; Gandhi et al, JHEP 1511 (2015) 039]

$$|U_{e3}| \simeq \sin\vartheta_{13} \simeq 0.15 \sim \varepsilon \quad \Longrightarrow \quad \varepsilon^2 \sim 0.03$$

$$|U_{e4}| \simeq \sin\vartheta_{14} \simeq 0.17 \sim \varepsilon$$

$$|U_{\mu 4}| \simeq \sin\vartheta_{24} \simeq 0.11 \sim \varepsilon$$

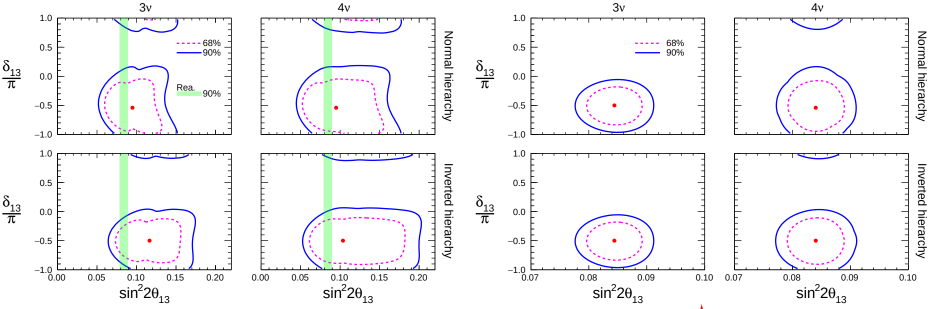
$$\alpha \equiv \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq \frac{7 \times 10^{-5}}{2.4 \times 10^{-3}} \simeq 0.031 \sim \varepsilon^2$$

At order ε^3: [Klop, Palazzo, PRD 91 (2015) 073017] $\Delta_{kj} \equiv \Delta m^2_{kj} L/4E$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq\ & 4\sin^2\vartheta_{13}\sin^2\vartheta_{23}\sin^2\Delta_{31} & \sim \varepsilon^2 \\ & +2\sin\vartheta_{13}\sin 2\vartheta_{12}\sin 2\vartheta_{23}(\alpha\Delta_{31})\sin\Delta_{31}\cos(\Delta_{32}+\delta_{13}) & \sim \varepsilon^3 \\ & +4\sin\vartheta_{13}\sin\vartheta_{14}\sin\vartheta_{24}\sin\vartheta_{23}\sin\Delta_{31}\sin(\Delta_{31}+\delta_{13}-\delta_{14}) & \sim \varepsilon^3 \end{aligned}$$

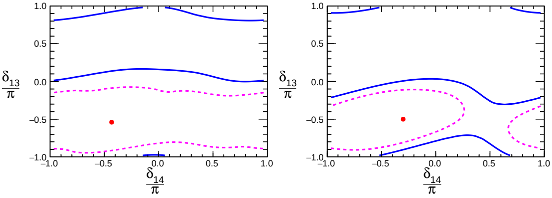
CP Violation in T2K and NOνA

[Capozzi, CG, Laveder, Palazzo, in preparation, with T2K and NOνA data presented at Neutrino 2016]



LBL + Reactors

Normal Ordering



Inverted Ordering

Inverted Ordering: Better agreement of LBL & Reactors for $\delta_{14} \approx -\pi/2$

Cosmology

- neutrinos in equilibrium in early Universe through weak interactions:

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N$$

$$\nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

- weak interactions freeze out $\Longrightarrow$ active $(\nu_e, \nu_\mu, \nu_\tau)$ neutrino decoupling

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H$$

$$T_{\nu\text{-dec}} \sim 1\,\text{MeV} \qquad t_{\nu\text{-dec}} \sim 1\,\text{s}$$

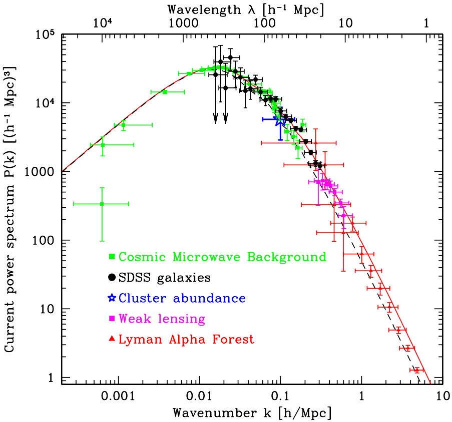
- relic neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$

- number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$

- density contribution: $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.1\,\text{eV}} \Rightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.1\,\text{eV}}}$
  $\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120; Cowsik, McClelland, PRL 29 (1972) 669]

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\rm M}^0 = 0.28\,,\ h = 0.72\,,\ \Omega_{\rm B}^0/\Omega_{\rm M}^0 = 0.16)$

Dashed Curve: $\sum_{k=1}^{3} m_k = 1\,\text{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\rm nr} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\text{eV}}}\,\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

[PRD 78 (2008) 033010, hep-ph/0805.2517]

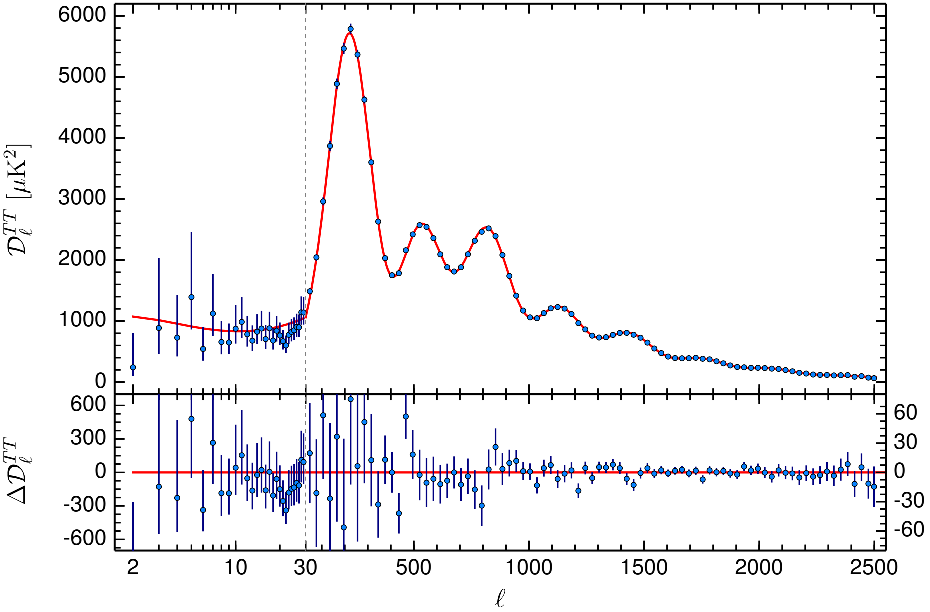
Flat ΛCDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

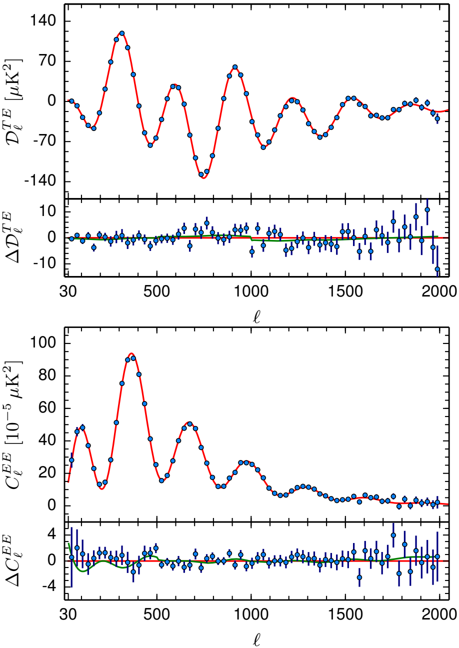
2σ (95% C.L.) constraints on the sum of ν masses Σ.

Planck

[arXiv:1502.01589]

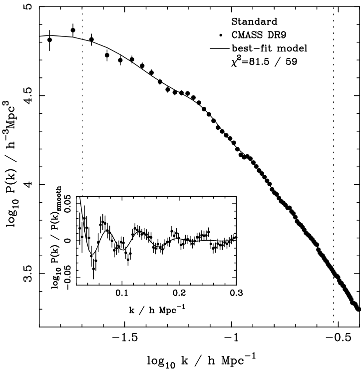


Planck Polarization Data



Planck Terminology

- TT denotes the Plank TT data (low-ℓ for $\ell < 30$ and high-ℓ for $\ell \geq 30$).
- lowP denotes the Planck polarization data at multipoles $\ell < 30$ (low-ℓ).
- TE denotes the Plank TE data at $\ell \geq 30$.
- EE denotes the Plank EE data at $\ell \geq 30$.
- Lensing denotes the Plank weak lensing data.
- BAO denotes the Baryon Acustic Oscillation data.

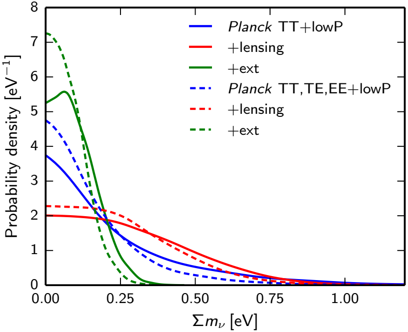


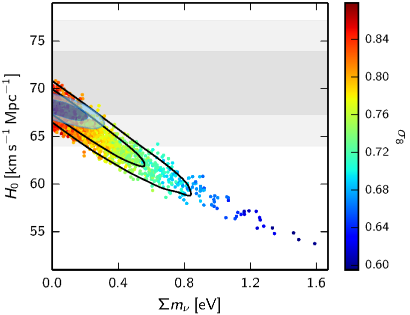
Baryon Oscillation Spectroscopic Survey (BOSS)
part of the Sloan Digital Sky Survey III (SDSS-III)
Data Release 9 (DR9) CMASS sample
[arXiv:1203.6594]

Limits on the Sum of Standard Light Neutrino Masses

[Planck, arXiv:1502.01589]

Cosmological data set	Σ (at 95% C.L.)
Plank TT + lowP	< 0.72 eV
Plank TT + lowP + BAO	< 0.21 eV
Plank TT,TE,EE + lowP	< 0.49 eV
Plank TT,TE,EE + lowP + BAO	< 0.17 eV
Plank TT + lowP + lensing	< 0.68 eV
Plank TT,TE,EE + lowP + lensing	< 0.59 eV
Plank TT + lowP + lensing + BAO + H_0	< 0.23 eV





Sterile Neutrinos in Cosmology

- sterile neutrinos can be produced by $\nu_{e,\mu,\tau} \to \nu_s$ oscillations before active neutrino decoupling $(t_{\nu\text{-dec}} \sim 1\,\text{s})$
- energy density of radiation before matter-radiation equality:

$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right]\rho_\gamma \qquad (t < t_{\text{eq}} \sim 6 \times 10^4\,\text{y})$$

$$N_{\text{eff}}^{\text{SM}} = 3.046 \qquad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$$

- sterile neutrino contribution:

$$\rho_s = (T_s/T_\nu)^4 \rho_\nu \implies \Delta N_{\text{eff}} = (T_s/T_\nu)^4$$

- sterile neutrino $\nu_s \simeq \nu_4$ with mass $m_s = m_4 \simeq \sqrt{\Delta m_{41}^2} \sim 1\,\text{eV}$ becomes non-relativistic at $T_\nu \sim m_s/3$, that is at $t_{\nu_s\text{-nr}} \sim 2.0 \times 10^5\,\text{y}$, before recombination at $t_{\text{rec}} \sim 3.8 \times 10^5\,\text{y}$
- current energy density of sterile neutrinos:

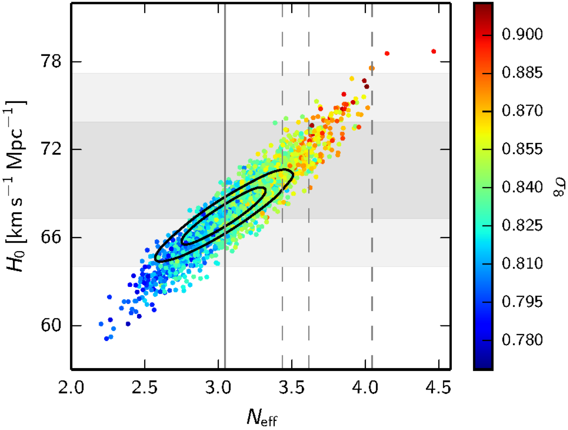
$$\Omega_s = \frac{n_s m_s}{\rho_c} \simeq \frac{1}{h^2}\frac{(T_s/T_\nu)^3 m_s}{94.1\,\text{eV}} = \frac{1}{h^2}\frac{\Delta N_{\text{eff}}^{3/4} m_s}{94.1\,\text{eV}} = \frac{1}{h^2}\frac{m_s^{\text{eff}}}{94.1\,\text{eV}}$$

$$m_s^{\text{eff}} = \Delta N_{\text{eff}}^{3/4} m_s = (T_s/T_\nu)^3 m_s$$

Limits on Dark Radiation

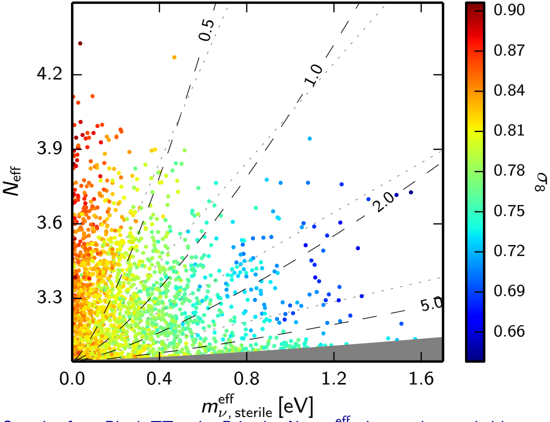
[Planck, arXiv:1502.01589]

Cosmological data set	$N_{\rm eff}$
Plank TT + lowP	3.13 ± 0.32
Plank TT + lowP + BAO	3.15 ± 0.23
Plank TT,TE,EE + lowP	2.99 ± 0.20
Plank TT,TE,EE + lowP + BAO	3.04 ± 0.18



Limits on Massive Sterile Neutrinos

$N_{\rm eff} < 3.7 \qquad m_s^{\rm eff} < 0.52 \qquad$ (95%; Plank TT + lowP + lensing + BAO)



Samples from Plank TT + lowP in the $N_{\rm eff}$–$m_s^{\rm eff}$ plane, colour-coded by σ_8, in models with one massive sterile neutrino family, with effective mass $m_s^{\rm eff}$, and the three active neutrinos as in the base ΛCDM model. The physical mass of the sterile neutrino in the thermal scenario, $m_s^{\rm thermal}$, is constant along the grey dashed lines, with the indicated mass in eV; the grey region shows the region excluded by our prior $m_s^{\rm thermal} < 10\,$eV, which excludes most of the area where the neutrinos behave nearly like dark matter. The physical mass in the Dodelson-Widrow scenario, $m_s^{\rm DW}$, is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

[arXiv:1502.01589]

- $m_s^{\rm eff} \equiv 94.1 \Omega_s h^2\,$eV
- Thermally distributed:

$$f_s(E) = \frac{1}{e^{E/T_s} + 1}$$

$$m_s^{\rm eff} = \left(\frac{T_s}{T_\nu}\right)^3 m_4 = (\Delta N_{\rm eff})^{3/4} m_4$$

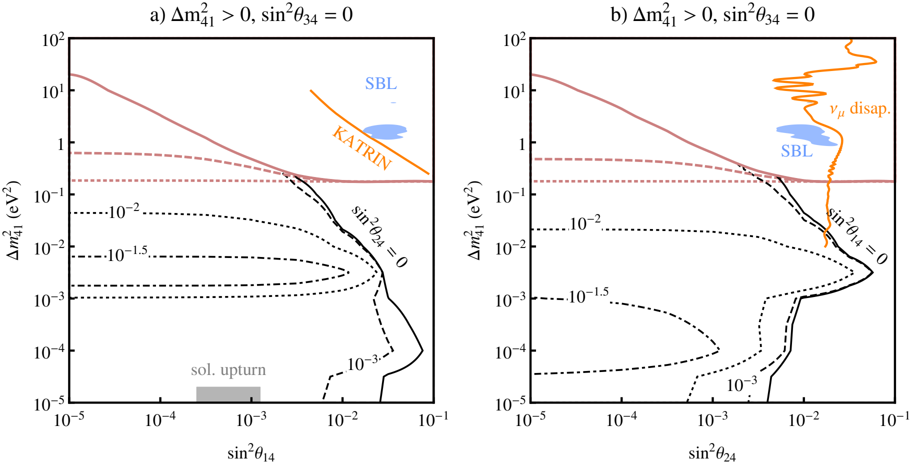
- Dodelson-Widrow:

$$f_s(E) = \frac{\chi}{e^{E/T_\nu} + 1}$$

$$m_s^{\rm eff} = \chi_s m_4$$

Standard Cosmological Scenario Mixing Bounds

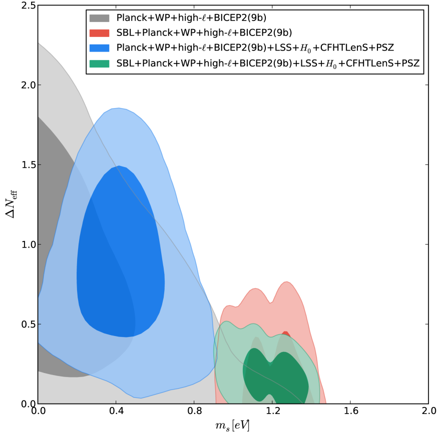
[Mirizzi, Mangano, Saviano, Borriello, Giunti, Miele, Pisanti, PLB 726 (2013) 8, arXiv:1303.5368]

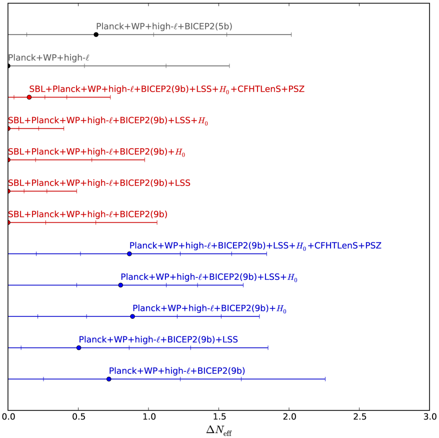


Non-standard mechanism for partial thermalization of ν_s is needed

Large primordial neutrino asymmetry?

[Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009; Saviano, Mirizzi, Pisanti, Serpico, Mangano, Miele, PRD 87 (2013) 073006]





[Archidiacono, Fornengo, Gariazzo, Giunti, Hannestad, Laveder, arXiv:1404.1794]

See also:
- [Gariazzo, Giunti, Laveder, JCAP 1504 (2015) 023]
- [Bergstrom, Gonzalez-Garcia, Niro, Salvado, JHEP 1410 (2014) 104]

Without oscillation data:
- [Giusarma, Di Valentino, Lattanzi, Melchiorri, Mena, arXiv:1403.4852]
- [Zhang, Li, Zhang, arXiv:1403.7028]
- [Dvorkin, Wyman, Rudd, Hu, arXiv:1403.8049]
- [Zhang, Li, Zhang, arXiv:1404.3598]

Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1\,\text{eV}$

Sterile neutrinos are thermalized $(\Delta N_{\text{eff}} = 1)$ by active-sterile oscillations before neutrino decoupling [Dolgov, Villante, NPB 679 (2004) 261]

Proposed mechanisms to avoid the tension:

- Large lepton asymmetry [Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009; Saviano et al., PRD 87 (2013) 073006; Hannestad, Hansen, Tram, JCAP 1304 (2013) 032]
- Interactions in the sterile sector [Hannestad, Hansen, Tram, PRL 112 (2014) 031802; Dasgupta, Kopp et al, PRL 112 (2014) 031803, JCAP 1510 (2015) 011; Bringmann, Hasenkamp, Kersten, JCAP 1407 (2014) 042; Ko, Tang, PLB 739 (2014) 62; Archidiacono, Hannestad et al, PRD 91 (2015) 065021, PRD 93 (2016) 045004, JCAP 1608 (2016) 067; Mirizzi, Mangano, Pisanti, Saviano, PRD 90 (2014) 113009, PRD 91 (2015) 025019; Tang, PLB 750 (2015) 201; Cherry, Friedland, Shoemaker, arXiv:1411.1071]
- A larger cosmic expansion rate at the time of sterile neutrino production [Rehagen, Gelmini JCAP 1406 (2014) 044]
- MeV dark matter annihilation [Ho, Scherrer, PRD 87 (2013) 065016]
- Invisible decay [Gariazzo, Giunti, Laveder, arXiv:1404.6160]
- Free primordial power spectrum of scalar fluctuations (Inflationary Freedom) [Gariazzo, Giunti, Laveder, JCAP 1504 (2015) 023]

Conclusions

Robust 3ν-Mixing Paradigm

$$\nu_e \to \nu_\mu, \nu_\tau \quad \text{with} \quad \Delta m^2_{\text{SOL}} \simeq 7.5 \times 10^{-5}\,\text{eV}^2$$

$$\nu_\mu \to \nu_\tau \quad \text{with} \quad \Delta m^2_{\text{ATM}} \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{12} \simeq 0.3 \qquad \sin^2\vartheta_{23} \simeq 0.5 \qquad \sin^2\vartheta_{13} \simeq 0.02$$

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow m_1, m_2, m_3 \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

Experiments: Measure mass ordering and CP violation.

Find absolute mass scale and Majorana or Dirac.

Find if sterile neutrinos exist.

Conclusions on Light Sterile Neutrinos

- Short-Baseline ν_e and $\bar{\nu}_e$ Disappearance:
  - Experimental data agree on Reactor $\bar{\nu}_e$ and Gallium ν_e disappearance.
  - Problem: total rates may have unknown systematic uncertainties.
  - Many promising projects to test unambiguously short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources.
  - Because of 5 MeV bump we know that the calculated spectrum must be corrected: oscillations must be observed as a function of distance!
  - Independent tests through effect of m_4 in β-decay and $\beta\beta_{0\nu}$-decay.
- Short-Baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ LSND Signal:
  - Not seen by other SBL $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ experiments.
  - Experiments with near detector are needed to check LSND signal!
  - Promising Fermilab program aimed at a conclusive solution of the mystery: a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-T600), all Liquid Argon Time Projection Chambers.
- Pragmatic 3+1 Fit is fine: moderate APP-DIS tension.
- 3+2 is not needed: same APP-DIS tension and no exp. CP violation.
- Cosmology:
  - Tension between $\Delta N_{\text{eff}} = 1$ and $m_s \approx 1\,\text{eV}$.
  - Cosmological and oscillation data may be reconciled by a non-standard cosmological mechanism.