Massive Neutrinos in Cosmology

Part I: Theory and Phenomenology of Massive Neutrinos

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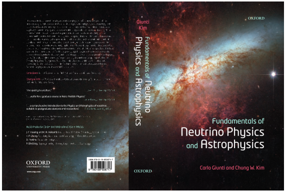
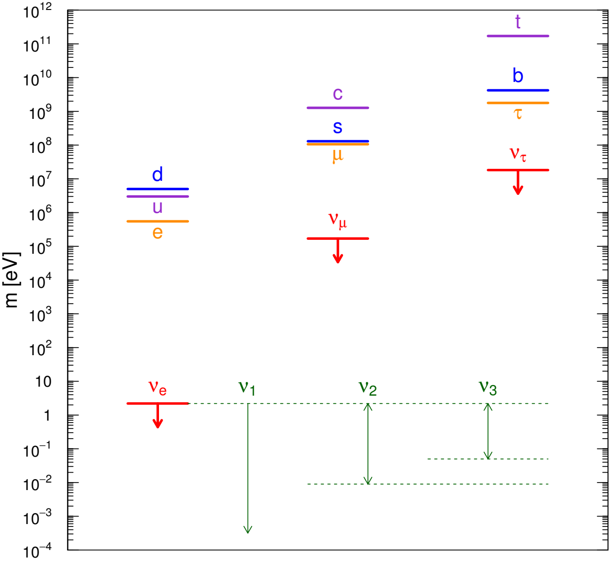


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Fermion Mass Spectrum



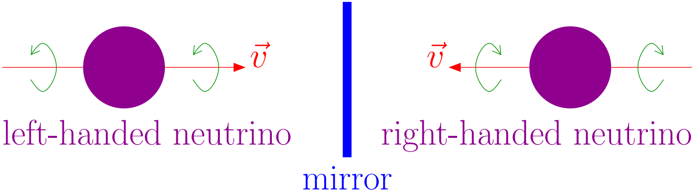
Standard Model

- Glashow (1961), Weinberg (1967) and Salam (1968) formulate the Standard Model of ElectroWeak Interactions (1979 Physics Nobel Prize) assuming that neutrinos are massless and left-handed

- Universal $V - A$ Weak Interactions

- Quantum Field Theory: $\nu_L \quad \Rightarrow \quad |\nu(h=-1)\rangle \quad$ and $\quad |\bar{\nu}(h=+1)\rangle$

- Parity is violated: $\nu_L \xrightarrow{P} \cancel{\nu_R} \qquad |\nu(h=-1)\rangle \xrightarrow{P} \cancel{|\nu(h=+1)\rangle}$



- Particle-Antiparticle symmetry (Charge Conjugation) is violated:

$\nu_L \xrightarrow{C} \cancel{(\nu^c)_L = (\nu_R)^c} \qquad |\nu(h=-1)\rangle \xrightarrow{C} \cancel{|\bar{\nu}(h=-1)\rangle}$

Standard Model: Massless Neutrinos

	1st Generation	2nd Generation	3rd Generation
Quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$ $\begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$ $\begin{matrix} t_R \\ b_R \end{matrix}$
Leptons	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ $\begin{matrix} \cancel{\nu_{eR}} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$ $\begin{matrix} \cancel{\nu_{\mu R}} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ $\begin{matrix} \cancel{\nu_{\tau R}} \\ \tau_R \end{matrix}$

- No ν_R $\Longrightarrow$ No Dirac mass Lagrangian $\mathcal{L}_{\text{D}} \sim m_{\text{D}} \overline{\nu_L} \nu_R$
- Majorana Neutrinos: $\nu = \nu^c$ $\Longrightarrow$ $\nu_R = (\nu^c)_R = \nu_L^c$

  Majorana mass Lagrangian: $\mathcal{L}_{\text{M}} \sim m_{\text{M}} \overline{\nu_L} \nu_L^c$

  forbidden by Standard Model $\text{SU}(2)_L \times \text{U}(1)_Y$ symmetry!
- In Standard Model neutrinos are massless!
- Experimentally allowed until 1998, when the Super-Kamiokande atmospheric neutrino experiment obtained a model-independent proof of

  Neutrino Oscillations

SM Extension: Massive Dirac Neutrinos

	1st Generation	2nd Generation	3rd Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$ $\begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$ $\begin{matrix} t_R \\ b_R \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ $\begin{matrix} \nu_{eR} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$ $\begin{matrix} \nu_{\mu R} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ $\begin{matrix} \nu_{\tau R} \\ \tau_R \end{matrix}$

- $\nu_R \implies$ Dirac mass Lagrangian $\mathcal{L}_{\mathrm{D}} \sim m_{\mathrm{D}} \overline{\nu_L} \nu_R$
- m_{D} is generated by the standard Higgs mechanism: $y \overline{L_L} \widetilde{\Phi} \nu_R \to y v \overline{\nu_L} \nu_R$
- Necessary assumption: lepton number conservation to forbid the Majorana mass terms

$$\mathcal{L}_{\mathrm{M}} \sim m_{\mathrm{M}} \overline{\nu_R} \nu_R^c \quad \text{singlet under SM symmetries!}$$

- Extremely small Yukawa couplings: $y \lesssim 10^{-11}$
- Not theoretically attractive.

Beyond the SM: Massive Majorana Neutrinos

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu = \nu^c} \quad \longrightarrow \quad \cancel{L = -1}$$

$$\nu_L \quad \Longrightarrow \quad L = +1 \qquad\qquad \nu_L^c \quad \Longrightarrow \quad L = -1$$

$$\mathscr{L}^{\text{M}} = \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2} \left(\overline{\nu_L^c}\, \nu_L + \overline{\nu_L}\, \nu_L^c \right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta^-_{0\nu})$$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta^+_{0\nu})$$

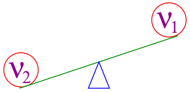
Seesaw Mechanism

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \mathrm{H.c.}$$

m_R^{M} can be arbitrarily large (not protected by SM symmetries)

$m_R^{\mathrm{M}} \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^{\mathrm{M}} \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \quad \Rightarrow \quad m_\ell \simeq \dfrac{(m^{\mathrm{D}})^2}{m_R^{\mathrm{M}}}, \qquad m_h \simeq m_R^{\mathrm{M}}$



seesaw mechanism

natural explanation of smallness
of light neutrino masses

massive neutrinos are Majorana!

3-GEN $\Rightarrow$ effective low-energy 3-ν mixing

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Neutrino Mixing

Left-handed Flavor Neutrinos produced in Weak Interactions

$$|\nu_e, -\rangle \qquad |\nu_\mu, -\rangle \qquad |\nu_\tau, -\rangle$$

$$\mathcal{H}_{\text{CC}} = \frac{g}{\sqrt{2}} W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right) + \text{H.c.}$$

Fields $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha, -\rangle = \sum_k U_{\alpha k}^* |\nu_k, -\rangle$ States

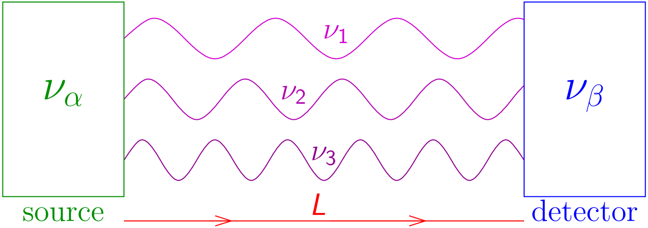
$$|\nu_1, -\rangle \qquad |\nu_2, -\rangle \qquad |\nu_3, -\rangle$$

Left-handed Massive Neutrinos propagate from Source to Detector

3×3 Unitary Mixing Matrix: $\quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

Neutrino Oscillations

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^* \, |\nu_1\rangle + U_{\alpha 2}^* \, |\nu_2\rangle + U_{\alpha 3}^* \, |\nu_3\rangle$$



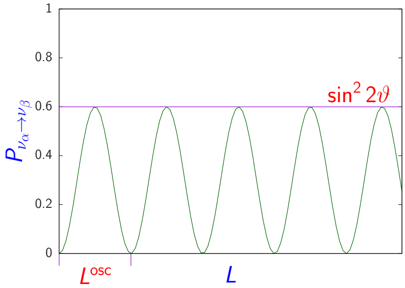
$$|\nu(t>0)\rangle = U_{\alpha 1}^* \, e^{-iE_1 t} \, |\nu_1\rangle + U_{\alpha 2}^* \, e^{-iE_2 t} \, |\nu_2\rangle + U_{\alpha 3}^* \, e^{-iE_3 t} \, |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

the oscillation probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \implies \boxed{L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}}$



Tiny neutrino masses lead to observable macroscopic oscillation distances!

$$\frac{L}{E} \lesssim \begin{cases} 10 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1}\,\text{eV}^2 \\ 10^3 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3}\,\text{eV}^2 \\ 10^4 \, \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2 \\ 10^{11} \, \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2 \end{cases}$$

Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

A Brief History of Neutrino Oscillations

- 1957: Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\Longrightarrow$ $\nu \leftrightarrows \bar{\nu}$
- In 1957 only one neutrino type $\nu = \nu_e$ was known! The possible existence of ν_μ was discussed by several authors. Maybe the first have been Sakata and Inoue in 1946 and Konopinski and Mahmoud in 1953. Maybe Pontecorvo did not know. He discussed the possibility to distinguish ν_μ from ν_e in 1959.
- 1962: Maki, Nakagava, Sakata proposed a model with ν_e and ν_μ and Neutrino Mixing:

  "weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrows \nu_\mu$"
- 1962: Lederman, Schwartz and Steinberger discover ν_μ
- 1967: Pontecorvo: intuitive $\nu_e \leftrightarrows \nu_\mu$ oscillations with maximal mixing. Applications to reactor and solar neutrinos ("prediction" of the solar neutrino problem).
- 1969: Gribov and Pontecorvo: $\nu_e - \nu_\mu$ mixing and oscillations. But no clear derivation of oscillations with a factor of 2 mistake in the phase (misprint?).

- 1975-76: Start of the "Modern Era" of Neutrino Oscillations with a general theory of neutrino mixing and a rigorous derivation of the oscillation probability by Eliezer and Swift, Fritzsch and Minkowski, and Bilenky and Pontecorvo. [Bilenky, Pontecorvo, Phys. Rep. (1978) 225]
- 1978: Wolfenstein discovers the effect on neutrino oscillations of the matter potential ("Matter Effect")
- 1985: Mikheev and Smirnov discover the resonant amplification of solar $\nu_e \to \nu_\mu$ oscillations due to the Matter Effect ("MSW Effect")
- 1998: the Super-Kamiokande experiment observed in a model-independent way the Vacuum Oscillations of atmospheric neutrinos ($\nu_\mu \to \nu_\tau$).
- 2002: the SNO experiment observed in a model-independent way the flavor transitions of solar neutrinos ($\nu_e \to \nu_\mu, \nu_\tau$), mainly due to adiabatic MSW transitions. [see: Smirnov, arXiv:1609.02386]
- 2015: Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (SNO) received the Physics Nobel Prize "for the discovery of neutrino oscillations, which shows that neutrinos have mass"

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \le \vartheta_{ab} \le \frac{\pi}{2}$ $\quad 0 \le \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS:
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent Δm^2_{kj}: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes $(\beta\beta_{0\nu})$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, Borexino; Super-Kamiokande; GALLEX/GNO, SAGE; Homestake, Kamiokande)

VLBL Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\to$

$$\begin{cases} \Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{S}} = \sin^2\vartheta_{12} \simeq 0.30 \end{cases}$$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Atmospheric $\nu_\mu \to \nu_\tau$	(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)	
LBL Accelerator ν_μ disappearance	(K2K, MINOS, T2K, NOνA)	$\to$ $\Delta m^2_{\text{A}} \simeq \lvert\Delta m^2_{31}\rvert \simeq 2.5 \times 10^{-3}\,\text{eV}^2$, $\sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23} \simeq 0.50$
LBL Accelerator $\nu_\mu \to \nu_\tau$	(OPERA)	

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

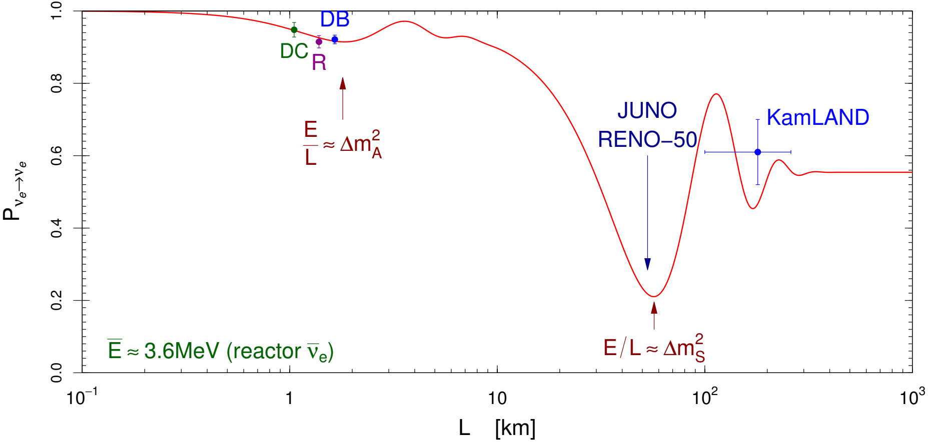
LBL Accelerator $\nu_\mu \to \nu_e$ (T2K, MINOS, NOνA)

LBL Reactor $\bar{\nu}_e$ disappearance (Daya Bay, RENO, Double Chooz)

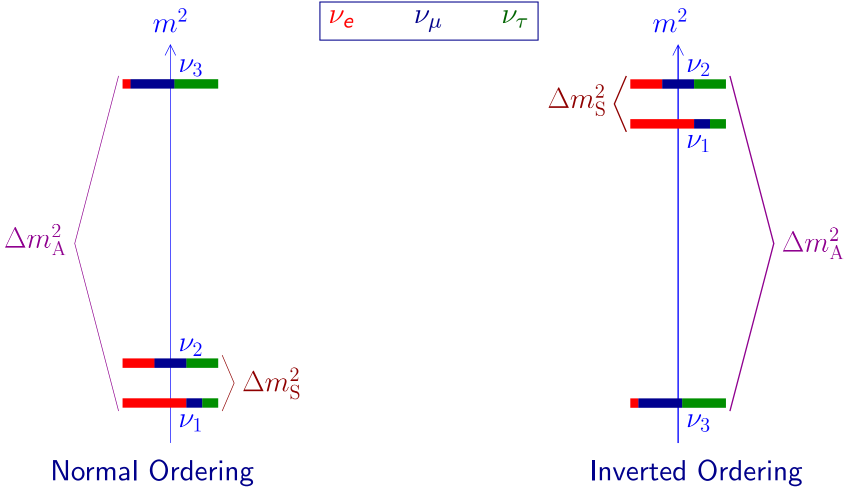
$\rightarrow$

$$\Delta m^2_{\text{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{13} \simeq 0.022$$


1.0
0.8
0.6
0.4
0.2
0.0
$P_{\nu_e \to \nu_e}$
DB
DC
R
$\frac{E}{L} \approx \Delta m^2_A$
JUNO
RENO–50
KamLAND
$E/L \approx \Delta m^2_S$
$\overline{E} \approx 3.6$MeV (reactor $\overline{\nu}_e$)
10^{-1}
1
10
10^2
10^3
L [km]

Mass Ordering



$\Delta m^2_{31} > \Delta m^2_{32} > 0$ $\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

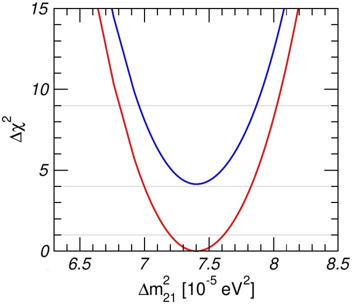
Towards Precision Neutrino Physics

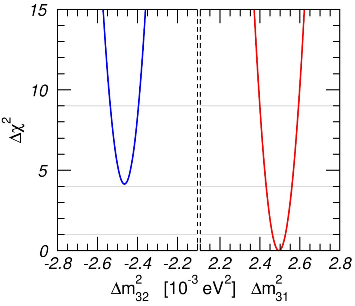
[NuFIT 3.2 (2018), www.nu-fit.org; T. Schwetz @ CERN Neutrino Platform Week, 1 Feb 2018]

[See also: Capozzi et al., Phys.Rev. D95 (2017) 096014, arXiv:1703.04471; de Salas et al., arXiv:1708.01186]

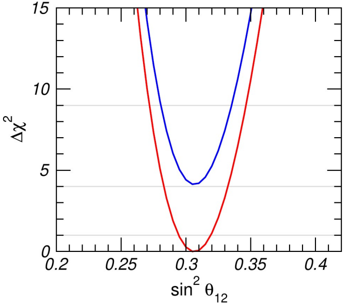
SOL: $\Delta m^2_{21} = 7.40^{+0.21}_{-0.20} \times 10^{-5}\,\text{eV}^2$ precision $\simeq 2.8\%$

ATM:
- NO : $\Delta m^2_{31} = 2.494^{+0.033}_{-0.031} \times 10^{-3}\,\text{eV}^2$ precision $\simeq 1.3\%$
- IO : $\Delta m^2_{32} = -2.465^{+0.032}_{-0.031} \times 10^{-3}\,\text{eV}^2$ precision $\simeq 1.3\%$





Normal Ordering is preferred by $\Delta\chi^2 = 4.1$



Solar

$\nu_e \to \nu_\mu, \nu_\tau$

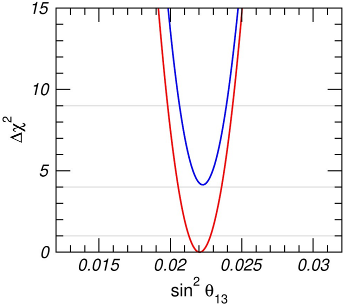
(SNO, Borexino, Super-Kamiokande, GALLEX/GNO, SAGE, Homestake, Kamiokande)

VLBL Reactor

$\bar{\nu}_e$ disappearance

(KamLAND)

$\sin^2\vartheta_{12} = 0.307^{+0.013}_{-0.012}$ precision $\simeq 4.2\%$



LBL Accelerator

$\nu_\mu \to \nu_e$

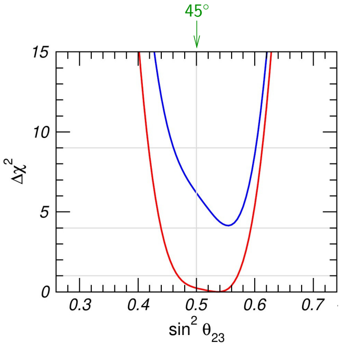
(T2K, MINOS, NOνA)

LBL Reactor

$\bar{\nu}_e$ disappearance

(Daya Bay, RENO, Double Chooz)

$$\sin^2\vartheta_{13} = \begin{cases} 0.02206 \pm 0.00075 \quad \text{(NO)} \\ \text{precision} \simeq 3.4\% \\ 0.02227 \pm 0.00074 \quad \text{(IO)} \\ \text{precision} \simeq 3.3\% \end{cases}$$



Atmospheric

$\nu_\mu \to \nu_\tau$

LBL Accelerator

ν_μ disappearance

LBL Accelerator

$\nu_\mu \to \nu_\tau$

(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)

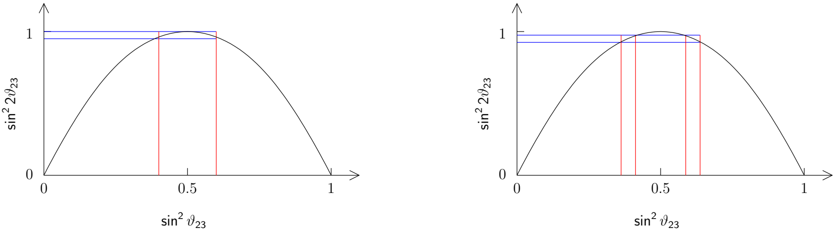
(K2K, MINOS, T2K, NOνA)

(OPERA)

$$\sin^2\vartheta_{23} = \begin{cases} 0.538^{+0.033}_{-0.069} \quad \text{(NO)} \quad \text{precision} \simeq 13\% \\ \quad \text{Maximal Mixing allowed at} < 1\sigma \\ \\ 0.554^{+0.023}_{-0.033} \quad \text{(IO)} \quad \text{precision} \simeq 6\% \\ \quad \text{Second octant “favored” by } \Delta\chi^2 \simeq 2 \end{cases}$$

Difficulty of measuring precisely ϑ_{23}

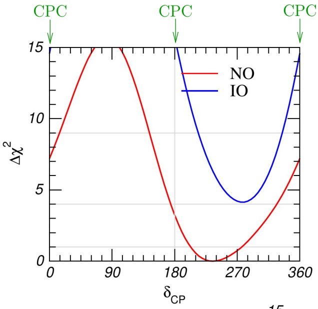
$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \sin^2 2\vartheta_{23} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \qquad \sin^2 2\vartheta_{23} = 4 \sin^2\vartheta_{23} \left(1 - \sin^2\vartheta_{23}\right)$$



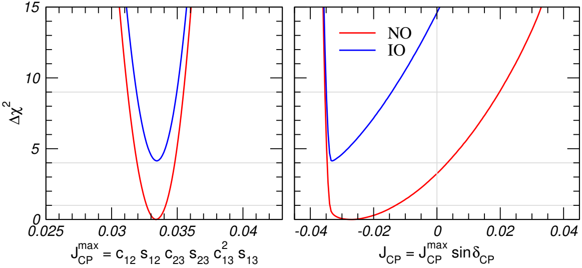
The octant degeneracy is resolved by small ϑ_{13} effects:

$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \left[\sin^2 2\vartheta_{23} \cos^2\vartheta_{13} + \sin^4\vartheta_{23} \sin^2 2\vartheta_{13}\right] \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$



$$\frac{\delta_{13}}{\pi} = \begin{cases} 1.3^{+0.24}_{-0.17} \quad \text{(NO)} \quad \text{precision} \simeq 18\% \\ \text{CP Conservation allowed at} < 2\sigma \\ 1.54^{+0.14}_{-0.16} \quad \text{(IO)} \quad \text{precision} \simeq 10\% \\ \text{CP Violation favored at } 3\sigma \end{cases}$$



$$J_{CP}^{\max} = 0.033 \pm 0.0007$$

J_{CP} can be 10^3 larger than $J_{CP}^{\text{quarks}} = (3.04^{+0.21}_{-0.20}) \times 10^{-5}$

Towards a precise determination of the mixing matrix

well determined

totally unknown

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

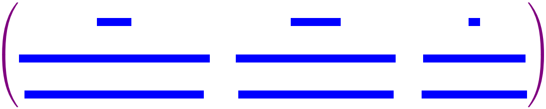
large uncertainty due to ϑ_{23} and δ_{13}

medium uncertainty due to ϑ_{23}

NuFIT 3.2 (2018)

$$|U|_{3\sigma} = \begin{pmatrix} 0.799 \to 0.844 & 0.516 \to 0.582 & 0.141 \to 0.156 \\ 0.242 \to 0.494 & 0.467 \to 0.678 & 0.639 \to 0.774 \\ 0.284 \to 0.521 & 0.490 \to 0.695 & 0.615 \to 0.754 \end{pmatrix}$$

$|U|_{3\sigma} =$



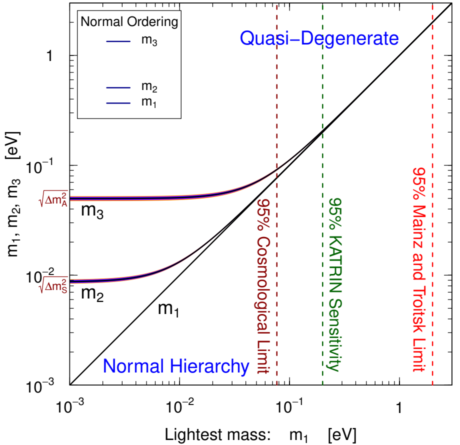
only the mass composition of ν_e is well determined

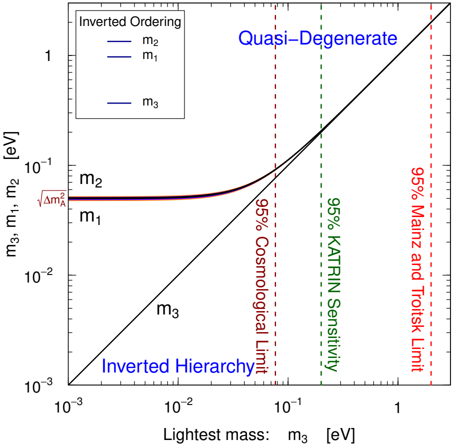
Why it is important to measure accurately the mixing parameters?

- They are fundamental parameters.
- They lead to selection in huge model space. Examples:
  - Deviation from Tribimaximal Mixing $U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$
  - Violation of μ-τ symmetry ($|U_{\mu k}| = |U_{\tau k}|$)
- They have phenomenological usefulness (e.g. to determine the initial flavor composition of astrophysical neutrinos).
- CP:
  - CP conservation would need an explanation (a new symmetry?).
  - CP violation may be linked to the CP violation in the sector of heavy neutrinos which generate the matter-antimatter asymmetry in the Universe through leptogenesis (CP-violating decay of heavy neutrinos).

Absolute Scale of Neutrino Masses

Mass Hierarchy or Degeneracy?





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}\,\text{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

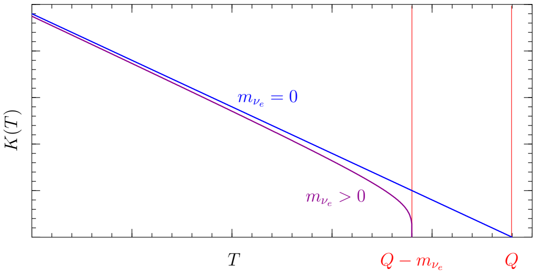
Tritium Beta-Decay

$$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

Kurie function: $K(T) = \left[(Q - T) \sqrt{(Q-T)^2 - m_{\nu_e}^2} \right]^{1/2}$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$



$m_{\nu_e} < 2.2\,\text{eV}$ (95% C.L.)

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

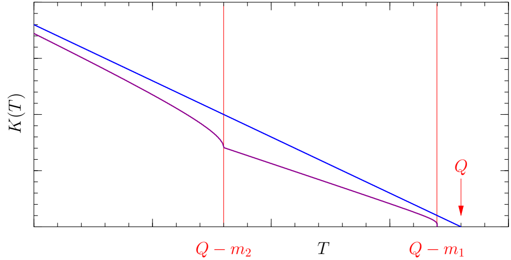
future: KATRIN

[www.katrin.kit.edu]

start data taking 2016?

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses $(m_k \ll Q-T)$

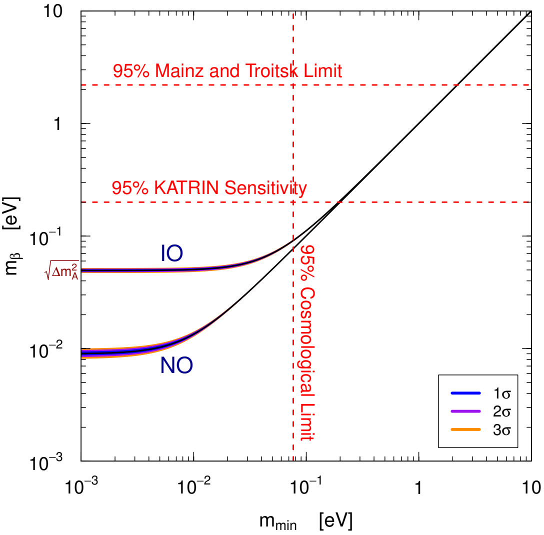
effective mass: $$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

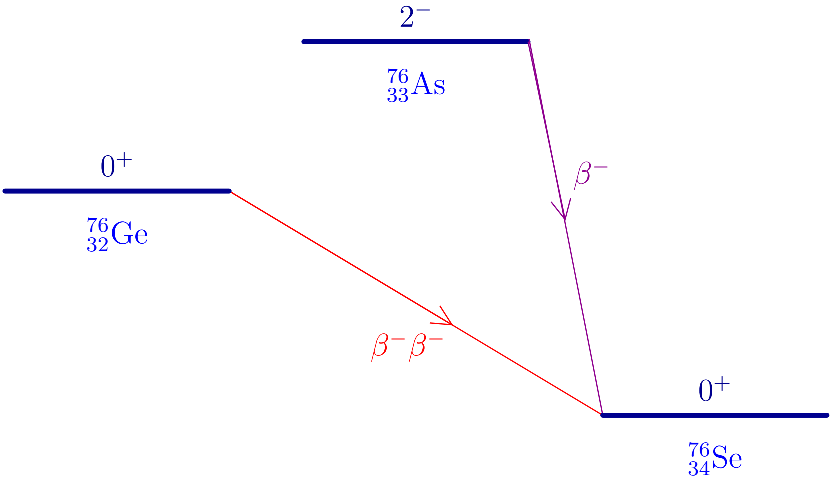
Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



- Quasi-Degenerate:
  $$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$
- Inverted Hierarchy:
  $$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$$
- Normal Hierarchy:
  $$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$$
  $$\simeq 2\times 10^{-5} + 6\times 10^{-5}\,\text{eV}^2$$
- If $m_\beta \lesssim 4\times 10^{-2}\,\text{eV}$
  $\Downarrow$
  Normal Spectrum

Neutrinoless Double-Beta Decay



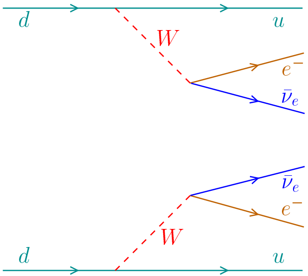
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

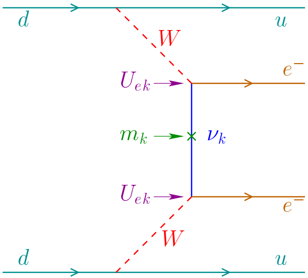


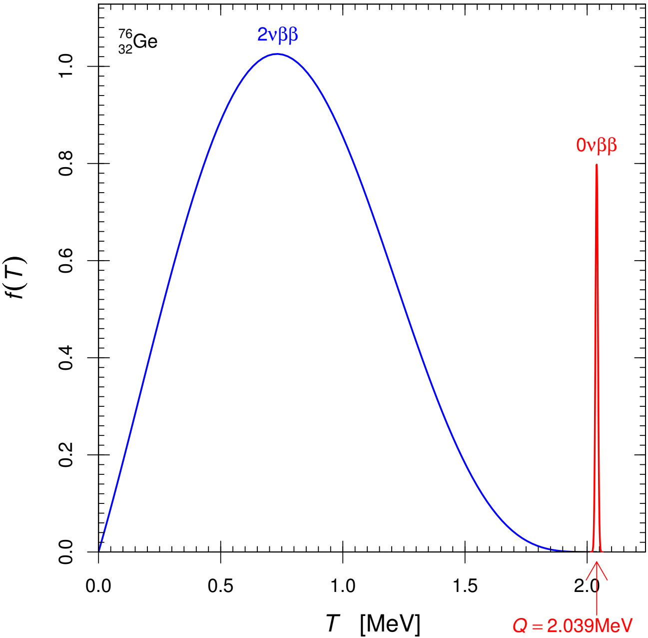
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$





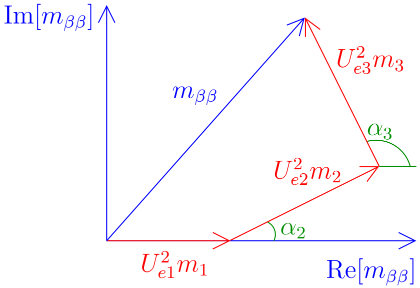
$^{76}_{32}$Ge
2νββ
0νββ
f(T)
T [MeV]
Q = 2.039MeV
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.5
2.0

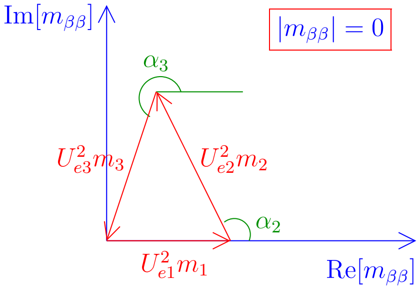
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} ⇒ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

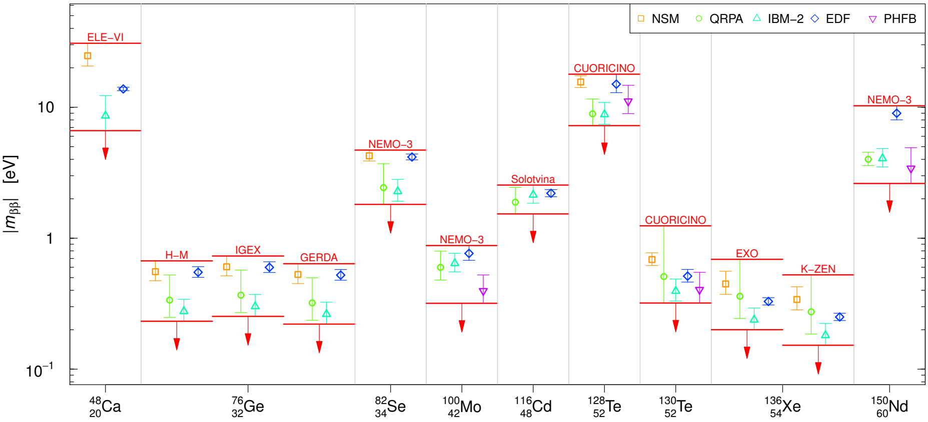
$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$





2015 90% C.L. Experimental Bounds

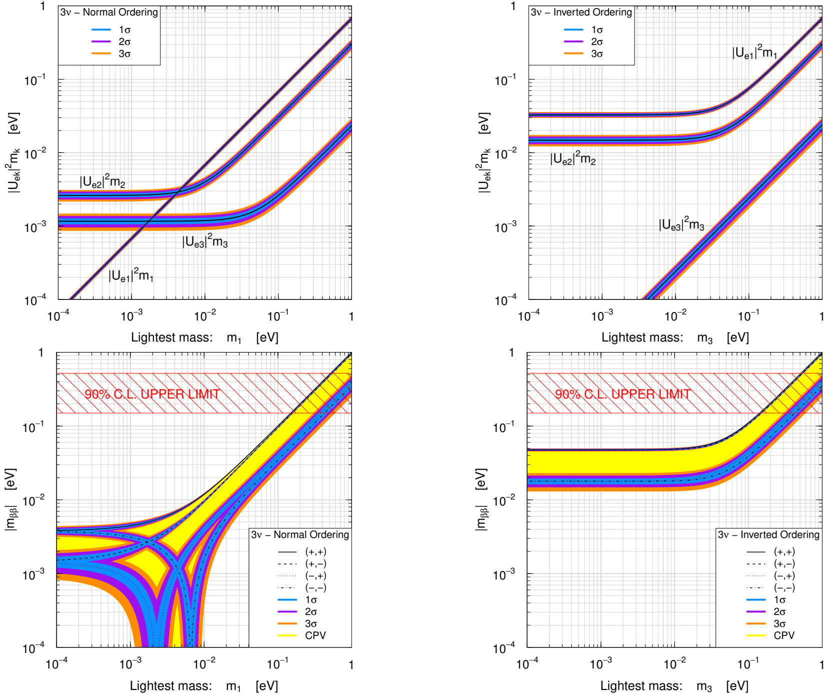
$\beta\beta^-$ decay	experiment	$T_{1/2}^{0\nu}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	GERDA	$> 2.1 \times 10^{25}$	$< 0.22 - 0.64$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORICINO	$> 2.8 \times 10^{24}$	$< 0.32 - 1.2$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.2 - 0.69$
	KamLAND-Zen	$> 1.9 \times 10^{25}$	$< 0.15 - 0.52$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



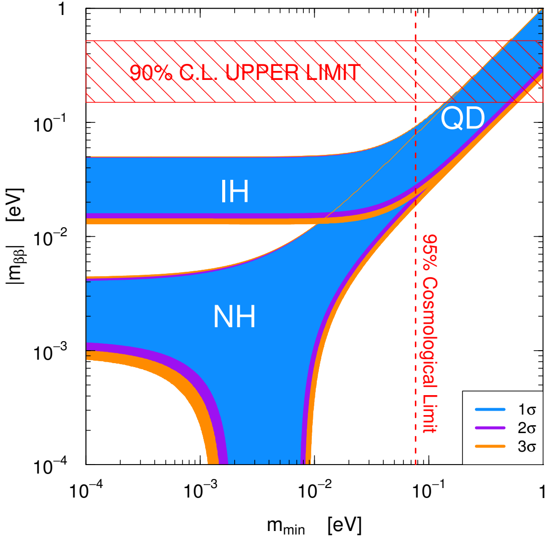
[Bilenky, CG, IJMPA 30 (2015) 0001]

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$


3ν – Normal Ordering
1σ
2σ
3σ
|U_e2|^2 m_2
|U_e3|^2 m_3
|U_e1|^2 m_1
|U_ek|^2 m_k [eV]
Lightest mass: m_1 [eV]
3ν – Inverted Ordering
|U_e1|^2 m_1
|U_e2|^2 m_2
|U_e3|^2 m_3
Lightest mass: m_3 [eV]
90% C.L. UPPER LIMIT
|m_ββ| [eV]
(+,+)
(+,−)
(−,+)
(−,−)
CPV

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12}\sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13}\sqrt{\Delta m^2_{\text{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\ \text{eV}$$

$$|m_{\beta\beta}| \lesssim 10^{-2}\ \text{eV} \Longrightarrow \text{Normal Spectrum}$$

Summary

Robust 3ν-Mixing Paradigm

$$\nu_e \to \nu_\mu, \nu_\tau \quad \text{with} \quad \Delta m^2_{\text{S}} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$$

$$\nu_\mu \to \nu_\tau \quad \text{with} \quad \Delta m^2_{\text{A}} \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{12} \simeq 0.3 \qquad \sin^2\vartheta_{23} \simeq 0.5 \qquad \sin^2\vartheta_{13} \simeq 0.02$$

β and $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_1, m_2, m_3 \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

Experiments: Measure mass ordering and CP violation.

Find absolute mass scale and Majorana or Dirac.