Massive Neutrinos in Cosmology

Part II: Cosmology

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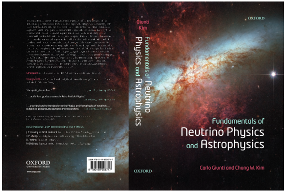


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Thermodynamics of the Early Universe

- Thermal equilibrium:

$$n_\chi = \frac{g_\chi}{(2\pi)^3} \int f_\chi(\vec{p}) \, \mathrm{d}^3 p$$

$$\rho_\chi = \frac{g_\chi}{(2\pi)^3} \int E_\chi(\vec{p}) \, f_\chi(\vec{p}) \, \mathrm{d}^3 p$$

$$p_\chi = \frac{g_\chi}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E_\chi(\vec{p})} \, f_\chi(\vec{p}) \, \mathrm{d}^3 p$$

- Statistical distribution: $f_\chi(\vec{p}) = \dfrac{1}{e^{(E_\chi(\vec{p}) - \mu_\chi)/T_\chi} \pm 1}$

- Chemical potential:
  - $a + b \leftrightarrows c + d \quad \Longrightarrow \quad \mu_a + \mu_b = \mu_c + \mu_d$
  - $\mu_\gamma = 0$ and $\chi + \bar{\chi} \to \gamma\gamma \quad \Longrightarrow \quad \mu_\chi = -\mu_{\bar{\chi}}$
  - Conserved charge $\Longrightarrow \quad \mu_\chi \neq 0$ if $n_\chi \neq n_{\bar{\chi}}$

- Relativistic limit: $T_\chi \gg m_\chi$ and $T_\chi \gg \mu_\chi \Longrightarrow f_\chi(\vec{p}) \simeq \dfrac{1}{e^{|\vec{p}|/T_\chi} \pm 1}$

$$n_\chi \simeq \begin{cases} \dfrac{\zeta(3)}{\pi^2}\, g_\chi\, T_\chi^3 & (\chi = \text{boson}) \\ \dfrac{3}{4}\,\dfrac{\zeta(3)}{\pi^2}\, g_\chi\, T_\chi^3 & (\chi = \text{fermion}), \end{cases}$$

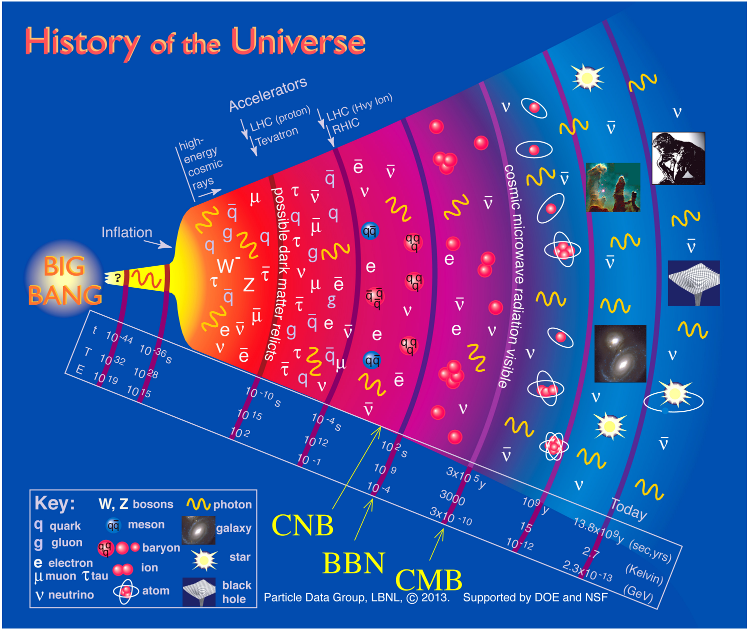
$$\rho_\chi \simeq \begin{cases} \dfrac{\pi^2}{30}\, g_\chi\, T_\chi^4 & (\chi = \text{boson}) \\ \dfrac{7}{8}\,\dfrac{\pi^2}{30}\, g_\chi\, T_\chi^4 & (\chi = \text{fermion}), \end{cases}$$

$$p_\chi \simeq \frac{1}{3}\,\rho_\chi\,,$$

- Average energy:

$$\langle E_\chi \rangle \simeq \langle |\vec{p}_\chi| \rangle \simeq \begin{cases} \dfrac{\pi^4}{30\,\zeta(3)}\, T_\chi \simeq 2.701\, T_\chi & (\chi = \text{boson}) \\ \dfrac{7\pi^4}{180\,\zeta(3)}\, T_\chi \simeq 3.151\, T_\chi & (\chi = \text{fermion}) \end{cases}$$

Neutrino Decoupling



- Neutrinos are in equilibrium in the early Universe through weak interactions:

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N$$
$$\nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

- Interaction rate: $\Gamma_\nu = n_\nu \langle \sigma v \rangle \sim G_F^2 T^5$

$$n_\nu \sim T^3 \qquad \sigma \sim G_F^2 T^2 \qquad v \simeq 1$$

- The rate of expansion is given by the Friedmann equation:

$$H^2 = \frac{8\pi}{3 M_P} \rho - \frac{k}{R^2} \qquad\qquad H(t) \equiv \frac{\dot{R}(t)}{R(t)}$$

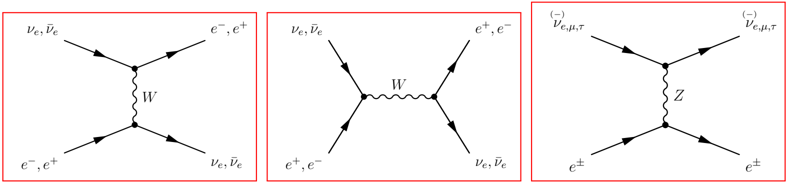
- In the radiation-dominated era: $H^2 \simeq \dfrac{8\pi}{3 M_P^2} \rho_{\rm rad}$ with $\rho_{\rm rad} = \dfrac{\pi^2}{30} g_* T^4$

$$H \simeq \frac{2\,\pi^{3/2}}{3\sqrt{5}\,M_P} \sqrt{g_*}\, T^2 \qquad\qquad g_* = \sum_{\substack{\chi=\text{relativistic}\\ \text{bosons}}} g_\chi + \frac{7}{8} \sum_{\substack{\chi=\text{relativistic}\\ \text{fermions}}} g_\chi$$

- Before ν decoupling: $g_* = g_*^{(\gamma)} + g_*^{(e^\pm)} + g_*^{(\nu)} = 2 + \dfrac{7}{8}\,4 + + \dfrac{7}{8}\,6 = 10.75$

- Neutrino decoupling: $\Gamma_\nu \sim H \implies T^{\nu\text{-dec}} \sim \left(M_\mathrm{P}\, G_\mathrm{F}^2\right)^{-1/3} \sim 1\,\mathrm{MeV}$
- A more precise calculation takes into account that the dominant processes for $T \lesssim 100\,\mathrm{MeV}$ are

$$\nu\bar{\nu} \leftrightarrows e^+ e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e$$



- Since the rates of these processes depend on neutrino energy $E \simeq p$, the decoupling temperature is not instantaneous and depends on p:

$$T^{\nu_e\text{-dec}}(p) \simeq 2.7 \left(\frac{p}{T}\right)^{-1/3} \qquad T^{\nu_{\mu,\tau}\text{-dec}}(p) \simeq 4.5 \left(\frac{p}{T}\right)^{-1/3}$$

- Taking into account that $\langle E \rangle \simeq 3T$, one obtains:

$$T^{\nu_e\text{-dec}} \simeq 1.9\,\mathrm{MeV} \qquad T^{\nu_{\mu,\tau}\text{-dec}} \simeq 3.1\,\mathrm{MeV}$$

- Hot relics: relativistic at decoupling $\Longrightarrow f_{\nu}^{\nu\text{-dec}}(\vec{p}) \simeq \dfrac{1}{e^{|\vec{p}|/T^{\nu\text{-dec}}} + 1}$

- FRW metric: $d\tau^2 = dt^2 - R^2(t)\left[\dfrac{dr^2}{1 - k\,r^2} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right]$

- Momentum scaling with expansion: $|\vec{p}| = |\vec{p}|_{\nu\text{-dec}}\left(\dfrac{R}{R_{\nu\text{-dec}}}\right)^{-1}$

$$f_{\nu}(\vec{p}) \simeq \left[\exp\left(\frac{|\vec{p}|\,(R/R_{\nu\text{-dec}})}{T^{\nu\text{-dec}}}\right) + 1\right]^{-1} = \frac{1}{e^{|\vec{p}|/T_{\nu}} + 1}$$

Effective temperature scales with expansion: $\boxed{T_{\nu} = T^{\nu\text{-dec}}\left(\dfrac{R}{R_{\nu\text{-dec}}}\right)^{-1}}$

Electron-Positron Annihilation

- After neutrino decoupling at $T \simeq 1\,\text{MeV}$ $e^{\pm}$ and γ are the only relativistic particles in thermal equilibrium.
- At $m_e/3 \simeq 0.2\,\text{MeV}$ electrons and positrons became nonrelativistic: out-of-equilibrium $e^- e^+ \to \gamma\gamma$ heat the photon distribution.
- During this phase the photon temperature does not scate as R^{-1}.
- Entropy density: $s = \dfrac{\rho + p}{T} = \dfrac{2\pi^2}{45}\, g_s\, T_\gamma^3$

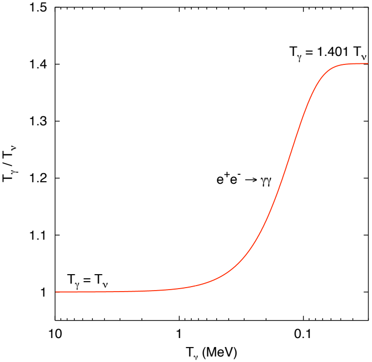
$$g_s = \sum_{\substack{\chi=\text{interacting}\\ \text{relativistic}\\ \text{bosons}}} g_\chi + \frac{7}{8} \sum_{\substack{\chi=\text{interacting}\\ \text{relativistic}\\ \text{fermions}}} g_\chi$$

- Entropy conservation: $s \propto R^{-3} \quad \Longrightarrow \quad T_\gamma \propto {g_s}^{-1/3}\, R^{-1}$

- Before and after $e^- e^+$ annihilation: $\dfrac{T_\nu^{\text{after}}}{T_\nu^{\text{before}}} = \left(\dfrac{R^{\text{after}}}{R^{\text{before}}} \right)^{-1}$

$$\frac{T_\gamma^{\text{after}}}{T_\gamma^{\text{before}}} = \left(\frac{g_s^{\text{after}}}{g_s^{\text{before}}} \right)^{-1/3} \left(\frac{R^{\text{after}}}{R^{\text{before}}} \right)^{-1} = \left(\frac{g_s^{\text{after}}}{g_s^{\text{before}}} \right)^{-1/3} \frac{T_\nu^{\text{after}}}{T_\nu^{\text{before}}}$$

- $T_\gamma^{\text{before}} = T_\nu^{\text{before}}$
- $g_s^{\text{before}} = g_s^{(\gamma)} + g_s^{(e^\pm)} = 2 + \dfrac{7}{8} 4$
- $g_s^{\text{after}} = g_s^{(\gamma)} = 2$
- $T_\nu^{\text{after}} = \left(\dfrac{4}{11} \right)^{1/3} T_\gamma^{\text{after}} \simeq 0.7138 \, T_\gamma^{\text{after}}$

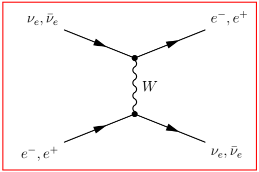


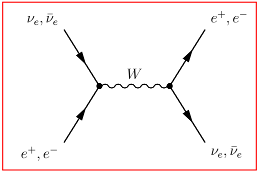
- $\boxed{T_\nu^0 = \left(\dfrac{4}{11} \right)^{1/3} T_\gamma^0 = 1.945 \pm 0.001 \, \text{K} = (1.676 \pm 0.001) \times 10^{-4} \, \text{eV}}$

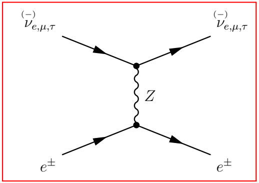
Effective Number of Relativistic Degrees Of Freedom

- Radiation density: $$\boxed{\rho_{\text{rad}} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \rho_\gamma}$$
- Three standard neutrinos: $N_{\text{eff}}^{\nu} = 3.046$ Why $N_{\text{eff}}^{\nu} > 3$?

  [Mangano et al, NPB 729 (2005) 221] $\rightarrow N_{\text{eff}}^{\nu} = 3.045$ [de Salas, Pastor, JCAP 1607 (2016) 051]
- Neutrino decoupling was not instantaneous at $T^{\nu\text{-dec}}$.
- Higher-energy neutrinos decoupled later and were not completely decoupled during $e^- e^+$ annihilation.
- This effect is different for $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_{\mu,\tau}$ because of the additional charged-current interactions of $\overset{(-)}{\nu}_e$:

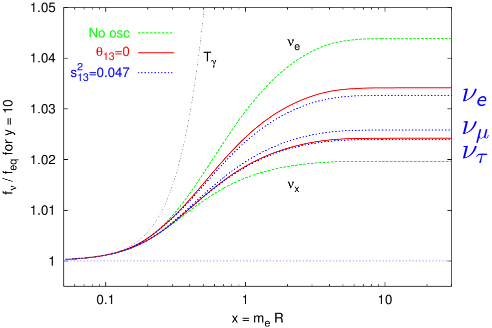






- Equilibrium distribution: $f_{\text{eq}}(\vec{p}) \simeq \dfrac{1}{e^{p/T}+1}$
- Nonthermal distortions: $f_{\nu_\alpha}(\vec{p},t) = f_{\text{eq}}(\vec{p})\,(1+\delta_{\nu_\alpha}(\vec{p},t))$
- Boltzmann equation:

$$\left(\frac{\partial}{\partial t} - Hp\,\frac{\partial}{\partial p}\right) f_{\nu_\alpha}(\vec{p},t) = C\left[f_{\nu_\alpha}; f_{\nu_\beta}, f_{e^\pm}\right]$$



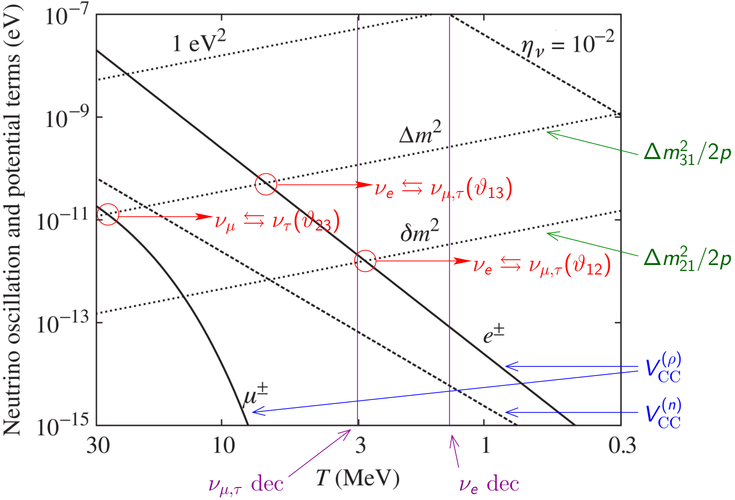
$R = T_\nu^{-1}$

$x = m_e R = m_e / T_\nu$

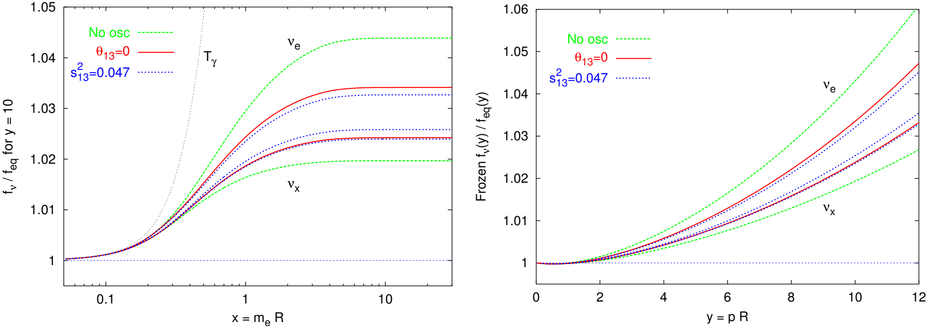
$y = pR = p / T_\nu$

[Mangano et al, NPB 729 (2005) 221, arXiv:hep-ph/0506164]

- Neutrino oscillations mix the flavor distributions.
- Matter potential: $V_{\mathrm{CC}}^{(\ell)} = \sqrt{2} G_{\mathrm{F}} \left(n_{\ell^-} - n_{\ell^+} \right) - \dfrac{8\sqrt{2} G_{\mathrm{F}} p}{3 m_W^2} \left(\rho_{\ell^-} + \rho_{\ell^+} \right)$



[Lesgourgues, Mangano, Miele, Pastor, Neutrino Cosmology, Cambridge University Press, 2013]



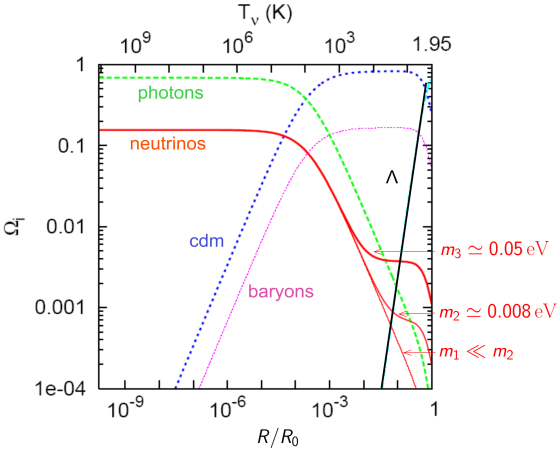
[Mangano et al, NPB 729 (2005) 221, arXiv:hep-ph/0506164]

$$R = T_\nu^{-1} \qquad x = m_e R = m_e / T_\nu \qquad y = pR = p/T_\nu$$

$$\boxed{N_{\text{eff}} = 3 + \frac{\delta\rho_{\nu_e}}{\rho_\nu} + \frac{\delta\rho_{\nu_\mu}}{\rho_\nu} + \frac{\delta\rho_{\nu_\tau}}{\rho_\nu} = 3.046}$$

$$f_{\nu_k} = \sum_{\alpha=e,\mu,\tau} |U_{\alpha k}|^2 f_{\nu_\alpha} \quad \Rightarrow \quad \left\{ \begin{array}{l} f_{\nu_1} \simeq 0.7 f_{\nu_e} + 0.3 f_{\nu_{\mu,\tau}} \\ f_{\nu_2} \simeq 0.3 f_{\nu_e} + 0.7 f_{\nu_{\mu,\tau}} \\ f_{\nu_3} \simeq f_{\nu_{\mu,\tau}} \end{array} \right.$$

Energy Density



Rad: $\begin{cases} \rho_c \simeq \rho_R \propto R^{-4} \\ \dfrac{\rho_M}{\rho_c} \propto \dfrac{R^{-3}}{R^{-4}} \propto R \\ \dfrac{\rho_\Lambda}{\rho_c} \propto \dfrac{1}{R^{-4}} \propto R^4 \end{cases}$

Mat: $\begin{cases} \rho_c \simeq \rho_M \propto R^{-3} \\ \dfrac{\rho_R}{\rho_c} \propto \dfrac{R^{-4}}{R^{-3}} \propto R^{-1} \\ \dfrac{\rho_\Lambda}{\rho_c} \propto \dfrac{1}{R^{-3}} \propto R^3 \end{cases}$

Λ: $\begin{cases} \rho_c \simeq \rho_\Lambda = \text{const.} \\ \dfrac{\rho_R}{\rho_c} \propto R^{-4} \\ \dfrac{\rho_M}{\rho_c} \propto R^{-3} \end{cases}$

$$\Omega_i = \rho_i / \rho_c \qquad \rho_c = \frac{3 M_P}{8\pi} H^2$$

Nonrelativistic Transition

- After decouling $T_\nu \propto R^{-1} \quad \Longrightarrow \quad T_\nu = T_\nu^0 \left(\frac{R_0}{R}\right) = T_\nu^0 \, (1+z)$

- Nonrelativistic transition: $T_{\nu_i}^{\text{nr}} \simeq 3 m_i \Rightarrow z_{\nu_i}^{\text{nr}} \simeq \frac{m_i}{3\, T_\nu^0} \simeq 2.0 \times 10^3 \left(\frac{m_i}{\text{eV}}\right)$

  $m_3 \gtrsim 5 \times 10^{-2}\,\text{eV} \Rightarrow z_{\nu_3}^{\text{nr}} \gtrsim 100 \qquad m_2 \gtrsim 8 \times 10^{-3}\,\text{eV} \Rightarrow z_{\nu_2}^{\text{nr}} \gtrsim 16$

- After the nonrelativistic transition: $\rho_{\nu_i} \simeq m_i n_{\nu_i}$

- $n_\nu^0 + n_{\bar\nu}^0 \simeq \frac{3}{2} \frac{\zeta(3)}{\pi^2} (T_\nu^0)^3 \simeq \frac{6}{11} \frac{\zeta(3)}{\pi^2} (T_\gamma^0)^3 = \frac{3}{11} n_\gamma^0 \simeq 112\,\text{cm}^{-3}$

- $\rho_c^0 \equiv \frac{3\, H_0^2}{8\pi\, G_{\text{N}}} \simeq 10.54\, h^2\,\text{keV}\,\text{cm}^{-3} \Rightarrow \Omega_{\nu_i}^0 \simeq \frac{m_i (n_\nu^0 + n_{\bar\nu}^0)}{\rho_c^0} \simeq \frac{m_i}{94.1\, h^2\,\text{eV}}$

- Nonthermal distortions $\Longrightarrow \boxed{\Omega_{\nu_i}^0 \simeq \frac{m_i}{93.1\, h^2\,\text{eV}}}$

  $\Omega_{\nu_3}^0 \gtrsim 5 \times 10^{-4}$

  $\Omega_{\nu_2}^0 \gtrsim 9 \times 10^{-5}$

- $\Omega^0_{\nu\text{-relativistic}} = \left(\frac{4}{11}\right)^{4/3} \Omega^0_\gamma \simeq 1.2 \times 10^{-5} \ll \Omega^0_{\nu_2} \gtrsim 9 \times 10^{-5}$
- Total contribution of neutrinos to the current energy density of the Universe: [Gershtein, Zeldovich, JETP Lett. 4 (1966) 120; Cowsik, McClelland, PRL 29 (1972) 669]

$$\boxed{\Omega^0_\nu \simeq \frac{\sum_i m_i}{93.1\, h^2\, \mathrm{eV}}}$$

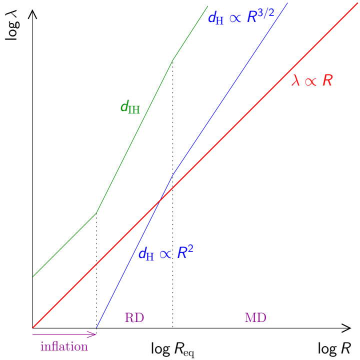
$$\left.\begin{array}{l} \Omega^0_\nu \leq \Omega^0_\mathrm{M} \simeq 0.3 \\ h \simeq 0.7 \end{array}\right\} \implies \sum_i m_i \lesssim 14\,\mathrm{eV}$$

- This bound is not competitive with the current kinematical laboratory limit:

$$m_i \lesssim m_\beta \lesssim 2\,\mathrm{eV} \implies \sum_i m_i \lesssim 6\,\mathrm{eV}$$

Matter-Radiation Equality

- Matter-radiation equality is important because subhorizon matter density fluctuations can grow only during the matter-dominated era.



- Therefore structure formation starts at matter-radiation equality.
- Where neutrino still relativistic at matter-radiation equality?
- The answer to this question is important in order to determine the effect of neutrinos on structure formation.

- Redshift of matter-radiation equality:

$$\left.\begin{array}{l}\rho_{\text{M}} \propto R^{-3} \\ \rho_{\text{R}} \propto R^{-4}\end{array}\right\} \Rightarrow \frac{\rho_{\text{M}}}{\rho_{\text{R}}} = \frac{\rho_{\text{M}}^0}{\rho_{\text{R}}^0}\frac{R}{R_0} = \frac{\rho_{\text{M}}^0}{\rho_{\text{R}}^0}(1+z)^{-1} \Rightarrow 1 + z_{\text{eq}} = \frac{\rho_{\text{M}}^0}{\rho_{\text{R}}^0} = \frac{\Omega_{\text{M}}^0}{\Omega_{\text{R}}^0}$$

- This relation assumes that the number of relativistic particles is not changed.
- If neutrinos were relativistic at matter-radiation equality:

$$1 + z_{\text{eq}} = \frac{\Omega_{\text{M}}^0}{\Omega_{\text{R}}^0}(m_\nu = 0) = \frac{\Omega_{\text{B}}^0 + \Omega_{\text{CDM}}^0}{\Omega_\gamma^0 + \Omega_\nu^0(m_\nu = 0)}$$

$$\begin{aligned}\Omega_{\text{R}}^0(m_\nu = 0) &= \left[1 + 3\left(\frac{4}{11}\right)^{4/3}\right]\Omega_\gamma^0 \simeq 4.4 \times 10^{-5}\, h^{-2} \\ &\simeq 8.9 \times 10^{-5} \quad \text{for} \quad h \simeq 0.7\end{aligned}$$

$$z_{\text{eq}} \simeq 2.4 \times 10^4 \left(\Omega_{\text{B}}^0 + \Omega_{\text{CDM}}^0\right) h^2 \simeq 3.5 \times 10^3 \quad \text{for} \quad \Omega_{\text{B}}^0 + \Omega_{\text{CDM}}^0 \simeq 0.3$$

$$z_{\nu_i}^{\text{nr}} \simeq 2.0 \times 10^3 \left(\frac{m_i}{\text{eV}}\right) < z_{\text{eq}} \quad \text{for} \quad m_i \lesssim 1.75\,\text{eV}$$

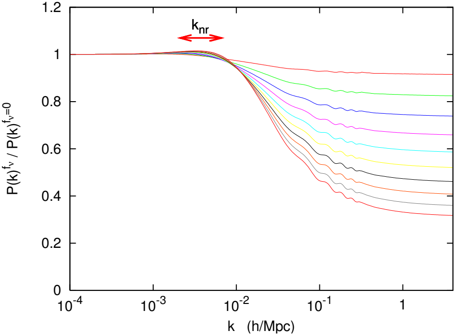
- From the current kinematical bound $m_i \lesssim 2\,\text{eV}$ it is likely that all the three standard massive neutrinos became nonrelativistic after matter-radiation equality.
- From t_{eq} to $t_{\nu_i}^{\text{nr}}$ neutrinos free stream.
- Subhorizon matter density fluctuations are suppressed by neutrino free streaming.
- Current physical free-streaming scale: $\lambda^0_{\nu_i\text{-fs}} \simeq z_{\nu_i\text{-nr}}\, d_{\text{H}}(z_{\nu_i\text{-nr}})$
- Matter-dominated era: $d_{\text{H}}(z) \simeq 2\, H_0^{-1}\, z^{-3/2}\, (\Omega^0_{\text{M}})^{-1/2}$

$$\lambda^0_{\nu_i\text{-fs}} \simeq 0.013 \left(\frac{m_i}{\text{eV}}\right)^{-1/2} (\Omega^0_{\text{M}})^{-1/2}\, h^{-1}\,\text{Mpc}$$

$$k^0_{\nu_i\text{-fs}} \simeq \frac{2\pi}{\lambda^0_{\nu_i\text{-fs}}} \simeq 0.047 \left(\frac{m_i}{\text{eV}}\right)^{1/2} \sqrt{\Omega^0_{\text{M}}}\, h\,\text{Mpc}^{-1}$$

Power Spectrum

- Density fluctuations: $\delta(t,\vec{x}) \equiv \dfrac{\rho(t,\vec{x}) - \langle \rho(t) \rangle}{\langle \rho(t) \rangle} = \displaystyle\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta(t,\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$
- The Fourier transform transform differential equations into algebraic ones.
- In the linear theory, the algebraic equations for the amplitude of each fluctuation mode with wavenumber $\vec{k}$ are independent.
- The amplitude $\delta(t,\vec{k})$ of each fluctuation mode evolves in time independently of the others and can be conveniently studied separately.
- Power spectrum: $P(k,t) = \langle |\delta(t,\vec{k})|^2 \rangle$
- The power spectrum is the variance of the distribution of fluctuations in Fourier space.
- Gaussian fluctuations are completely characterized by their variance, i.e. by the power spectrum.



[Lesgourgues, Pastor, Phys. Rept. 429 (2006) 307]

$$\omega_{\mathrm{M}}^0 = \Omega_{\mathrm{M}}^0 \, h^2 = 0.147$$

$$\Omega_\Lambda^0 = 0.70$$

$$m_1 \simeq m_2 \simeq m_3 \simeq \frac{1}{3} \sum_i m_i$$

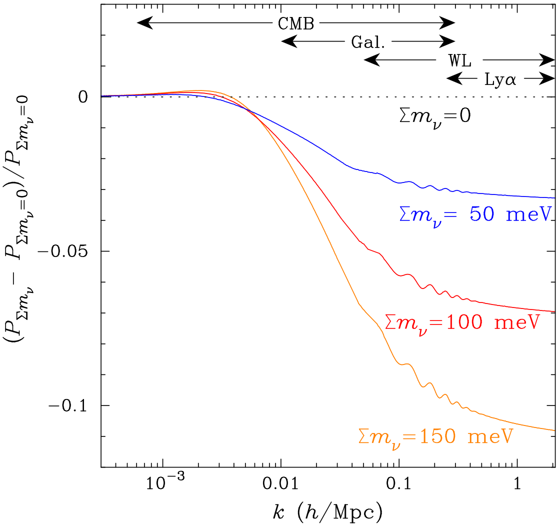
$$f_\nu \equiv \frac{\Omega_\nu^0}{\Omega_{\mathrm{M}}^0} = 0.01, 0.02, \ldots, 0.10$$

$$\sum_i m_i = 0.046, 0.092, 0.138, 0.184, 0.230, 0.270, 0.322, 0.368, 0.414, 0.460 \, \mathrm{eV}$$

$$\frac{\Delta P(k)}{P(k)} \simeq -8 \, \frac{\Omega_\nu^0}{\Omega_{\mathrm{M}}^0} \simeq -0.8 \left(\frac{\sum_i m_i}{1 \, \mathrm{eV}} \right) \left(\frac{0.1}{\Omega_{\mathrm{M}}^0 \, h^2} \right)$$

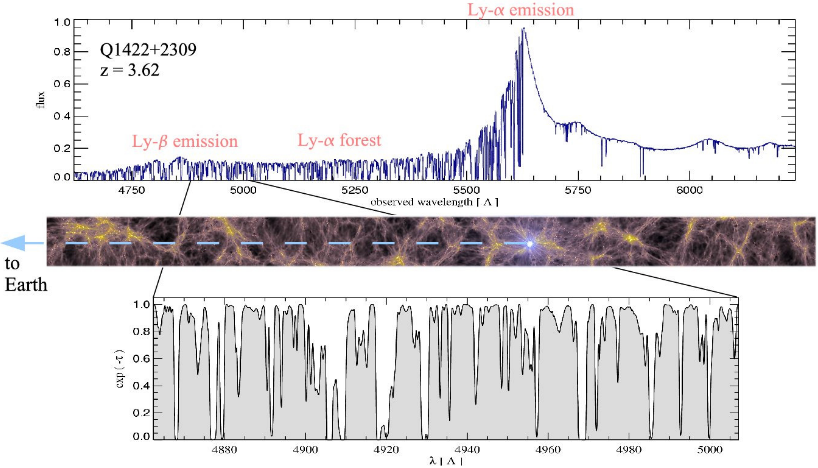
$$k \gtrsim k_{\mathrm{fs}} \simeq 0.026 \sqrt{\frac{\sum_i m_i}{1 \, \mathrm{eV}}} \sqrt{\Omega_{\mathrm{M}}^0} \, h \, \mathrm{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]



[Abazajian et al, Astropart.Phys. 63 (2015) 66, arXiv:1309.5383.]

Lyman-alpha Forest

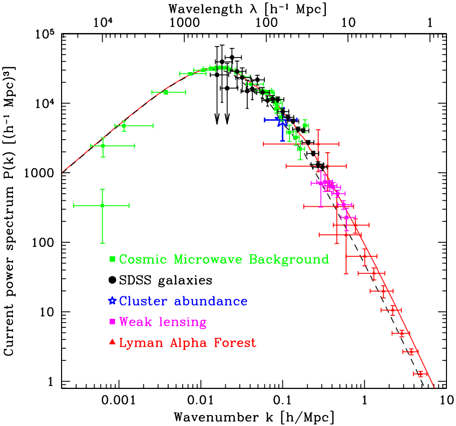


[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67\,\text{Å}$, $\lambda_\beta^0 = 1025.72\,\text{Å}$, $\lambda_\gamma^0 = 972.54\,\text{Å}$

Lyman-α forest: The region in which only Lyα photons can be absorbed:

$$[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$$



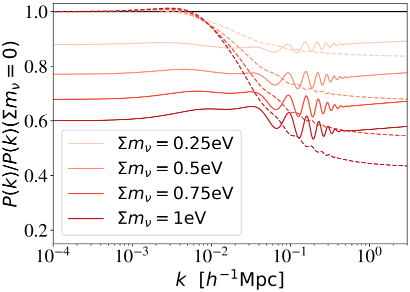
[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$$h = 0.72$$
$$\Omega_{\text{M}}^{0} = 0.28$$
$$\Omega_{\text{B}}^{0}/\Omega_{\text{M}}^{0} = 0.16$$

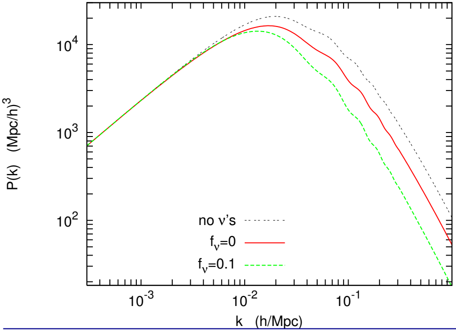
Dashed Curve: $\sum_{i=1}^{3} m_i = 1\,\text{eV}$

$$f_\nu \equiv \frac{\Omega_\nu^0}{\Omega_{\text{M}}^0} \simeq \frac{\sum_i m_i}{93.1\,h^2\,\text{eV}\,\Omega_{\text{M}}^0} \simeq 0.07$$



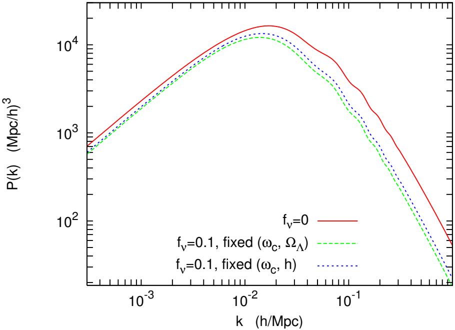
[Lesgourgues, Verde, Review of Particle Physics 2017]

- In the previous figures $\omega_{\text{M}}^0 = \Omega_{\text{M}}^0 \, h^2$ is fixed.
- $\Omega_{\text{B}}^0 + \Omega_{\text{CDM}}^0 = \Omega_{\text{M}}^0 - \Omega_\nu^0$
- $\Omega_\nu^0 \simeq \sum_i m_i \Big/ 93.1 \, h^2 \, \text{eV}$
- $z_{\text{eq}} \simeq 2.4 \times 10^4 \left(\Omega_{\text{B}}^0 + \Omega_{\text{CDM}}^0\right) h^2$
- z_{eq} decreases unless h is increased
- If z_{eq} is kept fixed by increasing h, there is also a suppression of the large-scale power spectrum.
- It is due to the decrease of time available for fluctuation growth as a consequence of the faster expansion.



- Fixed ω_B^0, ω_M^0, Ω_Λ^0
- $\omega_M^0 = \Omega_M^0 h^2 = \left(1 - \Omega_\Lambda^0\right) h^2$
- h fixed
- $z_\Lambda = (\Omega_\Lambda^0 / \Omega_M^0)^{1/3}$ fixed
- $\omega_{CDM}^0 = \omega_M^0 - \omega_B^0 - \omega_\nu^0$
- ω_{CDM}^0, $\Omega_{CDM}^0 = \omega_{CDM}^0 / h^2$ and z_{eq} decrease for $f_\nu > 0$

[Lesgourgues, Pastor, Phys. Rept. 429 (2006) 307]



- Fixed ω_B^0, $\omega_{CDM}^0 \Rightarrow z_{eq}$ fixed
- $\omega_M^0 = \omega_B^0 + \omega_{CDM}^0 + \omega_\nu^0$ increases
- Fixed $\Omega_\Lambda^0 \Rightarrow z_\Lambda$ fixed:

$$h = \sqrt{\frac{\omega_M^0}{1 - \Omega_\Lambda^0}} \text{ increases}$$

- Fixed h:

$$\Omega_\Lambda^0 = 1 - \omega_M^0 h^2 \text{ decreases}$$

Friedmann equation for a flat Universe: $H^2 = \frac{8\pi}{3\,M_P}\,\rho$

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho^0} \quad \Longrightarrow \quad H^2 = H_0^2\,\frac{\rho_\Lambda + \rho_M + \rho_R}{\rho_c^0}$$

$$\boxed{\begin{aligned} \rho_\Lambda &= \rho_\Lambda^0 \\ \rho_M &= \rho_M^0 \left(\frac{R_0}{R}\right)^3 = \rho_M^0\,(1+z)^3 \\ \rho_R &= \rho_R^0 \left(\frac{R_0}{R}\right)^4 = \rho_R^0\,(1+z)^4 \end{aligned}}$$

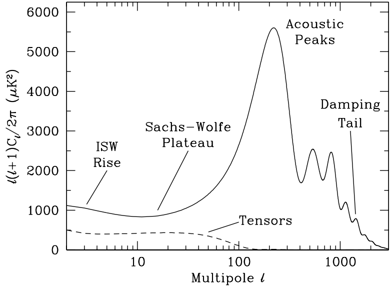
$$H^2 = H_0^2\,\frac{\rho_\Lambda^0 + \rho_M^0\,(1+z)^3 + \rho_R^0\,(1+z)^4}{\rho_c^0}$$

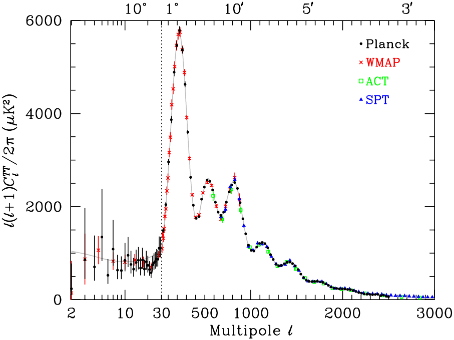
$$\boxed{H^2 = H_0^2 \left[\Omega_\Lambda^0 + \Omega_M^0\,(1+z)^3 + \Omega_R^0\,(1+z)^4\right]}$$

Matter-dominated Universe: $H^2 \simeq H_0^2 \left[1 - \Omega_M^0 + \Omega_M^0\,(1+z)^3\right]$

Increases with Ω_M^0 because $1 + z > 1$

Cosmic Microwave Background Radiation





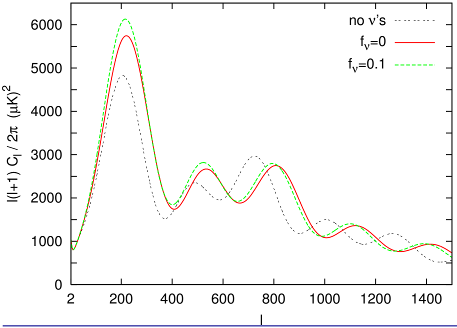
- Temperature fluctuations:

$$\frac{\Delta T_\gamma(\theta,\phi)}{T_\gamma} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} \, Y_\ell^m(\theta,\phi)$$

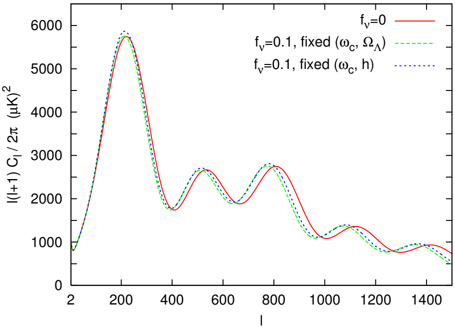
- Angular power spectrum:

$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \langle |a_{\ell m}|^2 \rangle$$

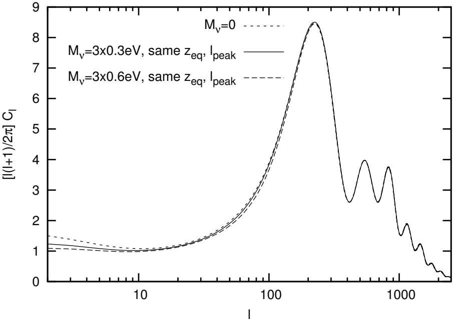
- C_ℓ are the variances of the multipole moments $a_{\ell m}$.
- Gaussian fluctuations are completely characterized by the variances C_ℓ.



- Fixed ω_B^0, ω_M^0, Ω_Λ^0



- Fixed ω_B^0, ω_{CDM}^0 $\Rightarrow$ z_{eq} fixed



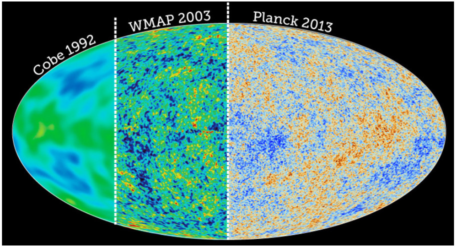
[Lesgourgues, Pastor, New J. Phys. 16 (2014) 065002]

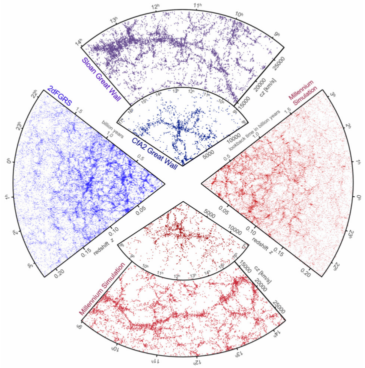
- ΛMDM: Λ Mixed Dark Matter model, where Mixed refers to the inclusion of some HDM component.
- Flat ΛCDM parameters:

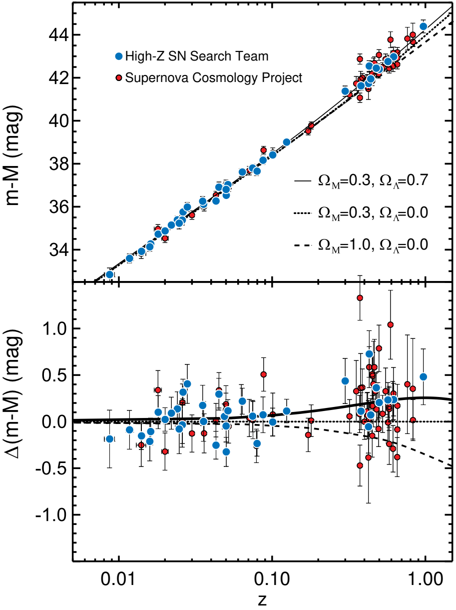
$$\omega_{\mathrm{B}}^{0},\ \omega_{\mathrm{M}}^{0},\ \Omega_{\Lambda}^{0},\ A_s,\ n_s,\ \tau$$

- Some of the parameters of the ΛMDM model have been varied together with $M_\nu = \sum_i m_i$ in order to keep fixed the redshift of equality and the angular diameter distance to last scattering.
- We conclude that the CMB alone is not a very powerful tool for constraining sub-eV neutrino masses, and should be used in combination with homogeneous cosmology constraints and/or measurements of the LSS power spectrum, for instance from galaxy clustering, galaxy lensing or CMB lensing.

Cosmological Bound on Neutrino Masses







WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

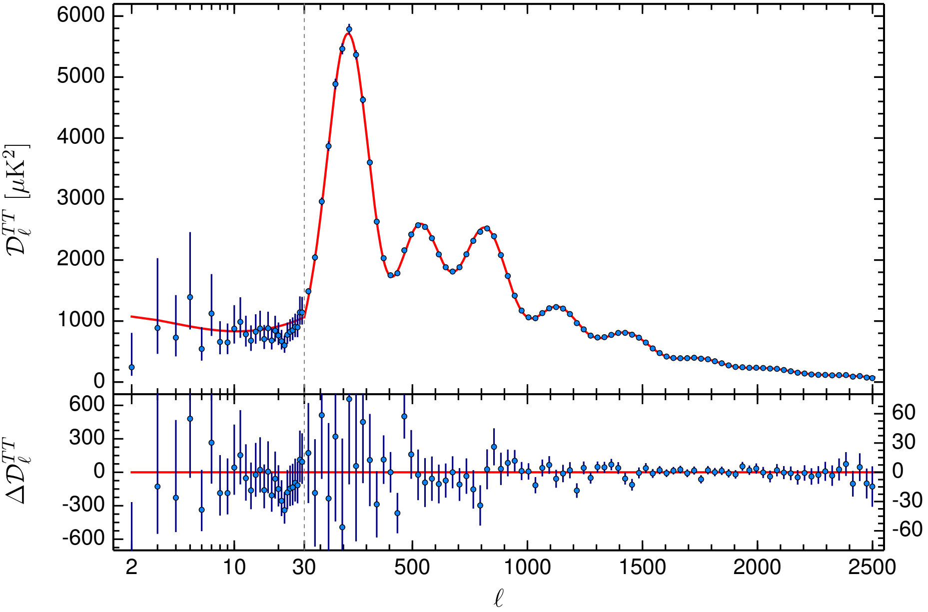
[PRD 78 (2008) 033010, hep-ph/0805.2517]

Flat ΛCDM

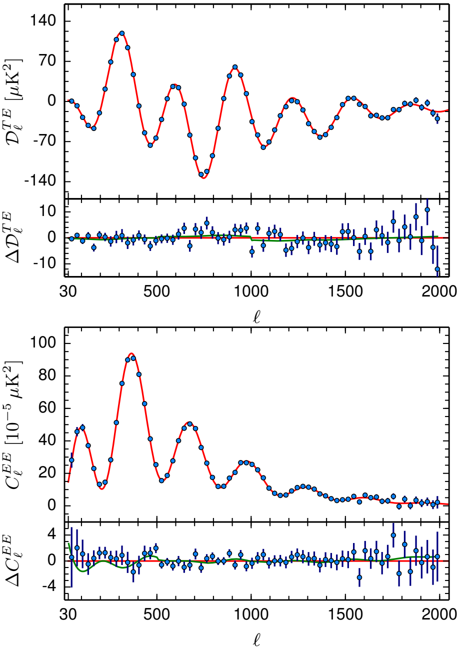
Case	Cosmological data set	$\sum_i m_i$ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

Planck

[arXiv:1502.01589]

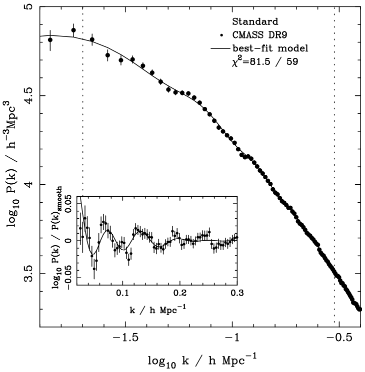


Planck Polarization Data

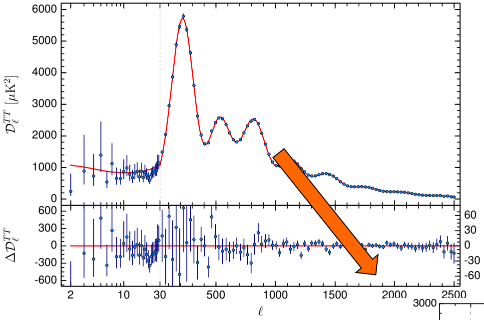


Planck Terminology

- TT denotes the Plank TT data (low-ℓ for $\ell < 30$ and high-ℓ for $\ell \geq 30$).
- lowP denotes the Planck polarization data at multipoles $\ell < 30$ (low-ℓ).
- TE denotes the Plank TE data at $\ell \geq 30$.
- EE denotes the Plank EE data at $\ell \geq 30$.
- Lensing denotes the Plank weak lensing data.
- BAO denotes the Baryon Acustic Oscillation data.

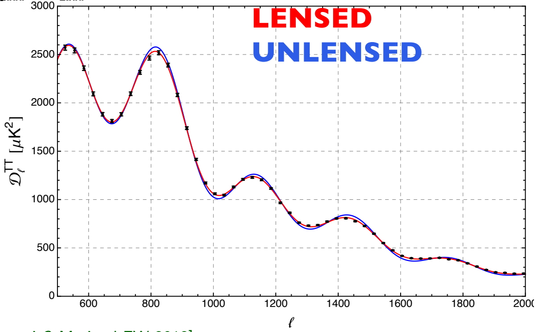


Baryon Oscillation Spectroscopic Survey (BOSS)
part of the Sloan Digital Sky Survey III (SDSS-III)
Data Release 9 (DR9) CMASS sample
[arXiv:1203.6594]



Lensing smooths the peaks of the CMB power spectrum…

… and introduces nongaussianities in the map (nonzero 4-point c.f.)



Neutrino free streaming damps matter perturbations and *reduces* lensing

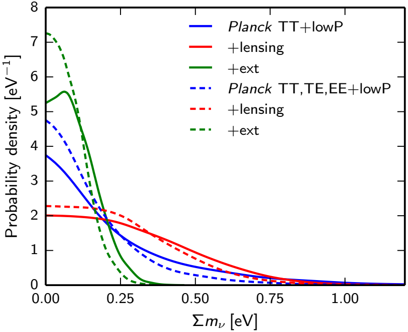
The effect is proportional to ν energy density

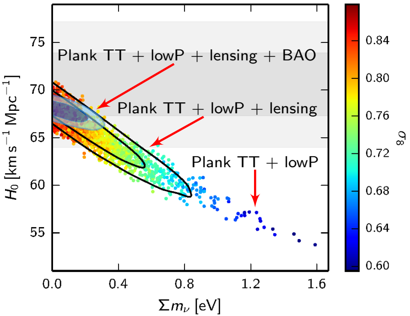
[M. Lattanzi @ Moriond EW 2018]

Planck Limits on $\sum m_\nu$

[Planck, A&A 594 (2016) A13, arXiv:1502.01589]

Cosmological data set	$\sum m_\nu$ (95% C.L.)
Plank TT + lowP	< 0.72 eV
Plank TT + lowP + BAO	< 0.21 eV
Plank TT,TE,EE + lowP	< 0.49 eV
Plank TT,TE,EE + lowP + BAO	< 0.17 eV
Plank TT + lowP + lensing	< 0.68 eV
Plank TT,TE,EE + lowP + lensing	< 0.59 eV
Plank TT + lowP + lensing + BAO + JLA + H_0	< 0.23 eV

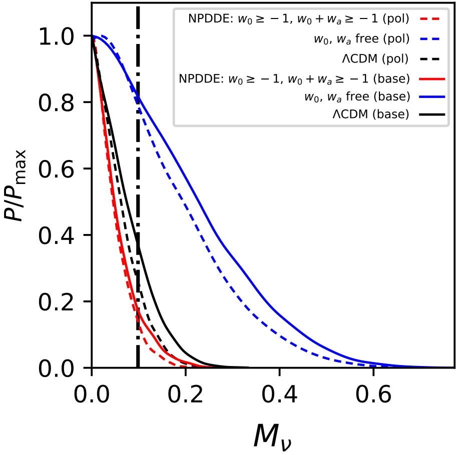




	Model	95% CL (eV)	Ref.
CMB alone			
Pl15[TT+lowP]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.72	[29]
Pl15[TT+lowP]	$\Lambda\text{CDM}+\sum m_\nu+N_{\text{eff}}$	< 0.73	[35]
Pl16[TT+SimLow]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.59	[32]
CMB + probes of background evolution			
Pl15[TT+lowP] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	< 0.21	[29]
Pl15[TT+lowP] + JLA	$\Lambda\text{CDM}+\sum m_\nu$	< 0.33	[35]
Pl15[TT+lowP] + BAO	$\Lambda\text{CDM}+\sum m_\nu+N_{\text{eff}}$	< 0.27	[35]
CMB + probes of background evolution + LSS			
Pl15[TT+lowP+lensing]	$\Lambda\text{CDM}+\sum m_\nu$	< 0.68	[29]
Pl15[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	< 0.25	[35]
Pl15[TT+lowP] + $\text{P(k)}_{\text{DR12}}$	$\Lambda\text{CDM}+\sum m_\nu$	< 0.30	[50]
Pl15[TT,TE,EE+lowP] + BAO+ P(k)_{WZ}	$\Lambda\text{CDM}+\sum m_\nu$	< 0.14	[52]
Pl15[TT,TE,EE+lowP] + BAO+ P(k)_{DR7}	$\Lambda\text{CDM}+\sum m_\nu$	< 0.13	[52]
Pl15[TT+lowP+lensing] + Lyα	$\Lambda\text{CDM}+\sum m_\nu$	< 0.12	[48]
Pl16[TT+SimLow+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	< 0.17	[48]
Pl15[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu+\Omega_k$	< 0.37	[35]
Pl15[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu+w$	< 0.37	[35]
Pl15[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu+N_{\text{eff}}$	< 0.32	[29]
Pl15[TT,TE,EE+lowP+lensing]	$\Lambda\text{CDM}+\sum m_\nu$+5-params.	< 0.66	[34]

[Lesgourgues, Verde, Review of Particle Physics 2017]

- The neutrino mass bound can be loosened in extended cosmological models.
- For example with a varying Dark Energy equation of state.



[Vagnozzi et al, arXiv:1801.08553]

$$p_{\mathrm{DDE}} = w_{\mathrm{DDE}}\, \rho_{\mathrm{DDE}}$$

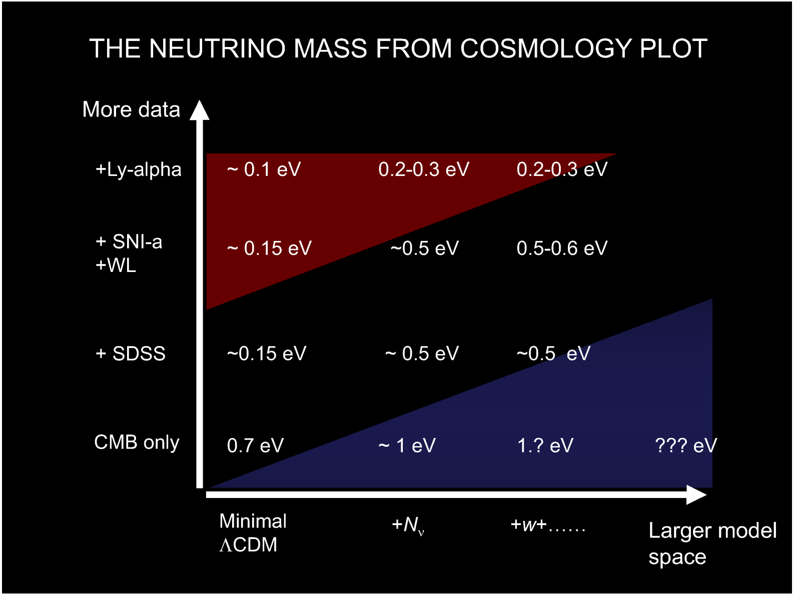
DDE: Dynamical Dark Energy

$$w_{\mathrm{DE}}(z) = w_0 + w_{\mathrm{a}} \frac{z}{1+z}$$

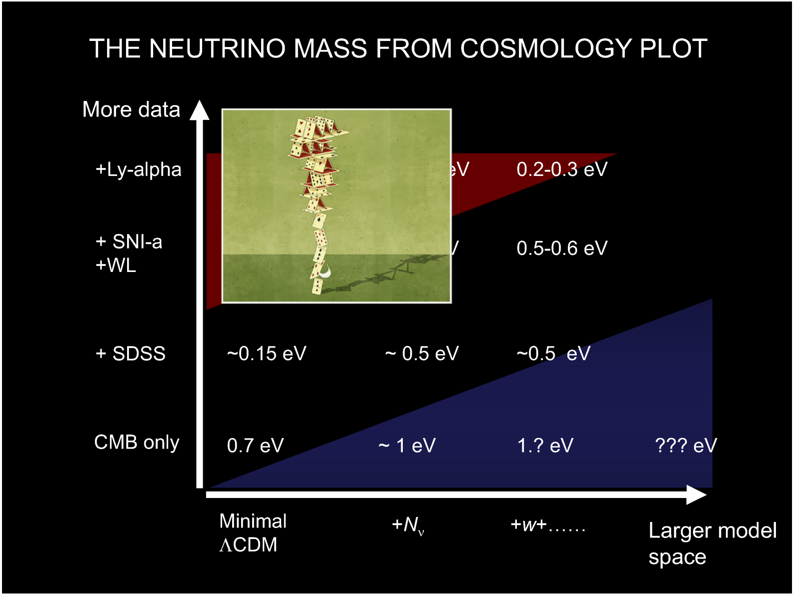
NPDDE: Non-Phantom DDE

$$w_{\mathrm{DE}}(z) \geq -1$$

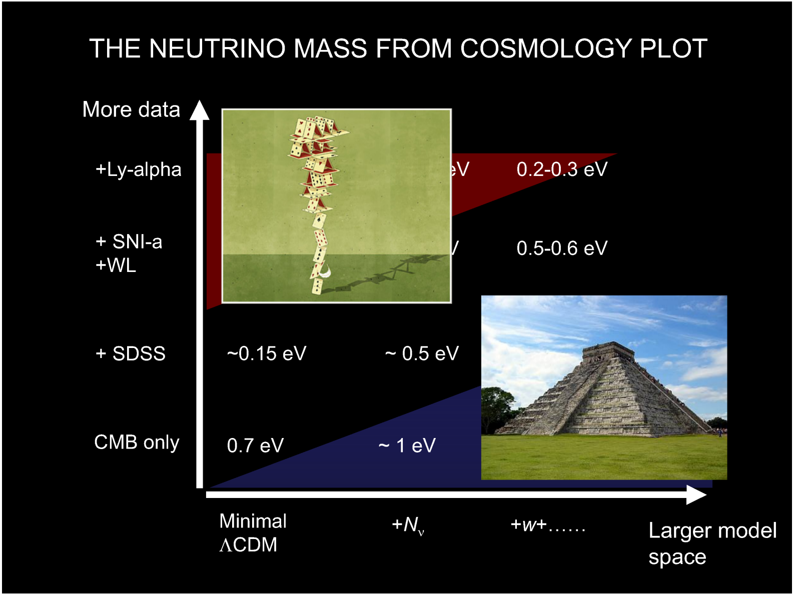
$$w_0 \geq -1 \qquad w_0 + w_{\mathrm{a}} \geq -1$$



[S. Hannestad, 2018]

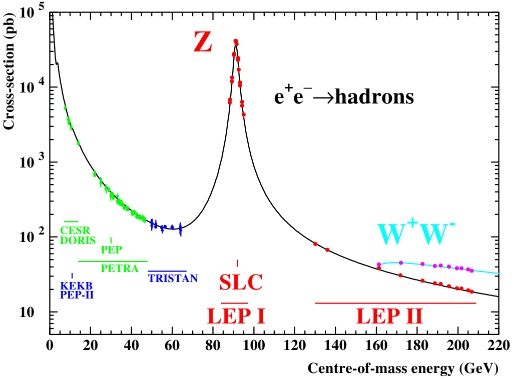


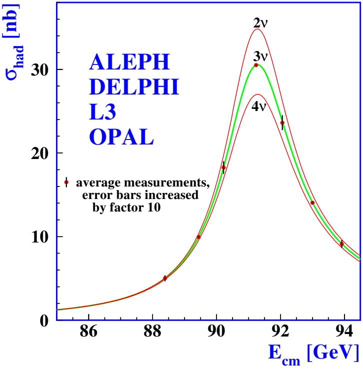
[S. Hannestad, 2018]



[S. Hannestad, 2018]

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_{\nu_{\text{active}}}^{\text{LEP}} = 2.9840 \pm 0.0082}$$

$$e^+e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \; \nu_\mu \; \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
		ACTIVE			STERILE	

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots}$$

Sterile Neutrinos

- Sterile means no standard model interactions

  [Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

- Obviously no electromagnetic interactions as normal active neutrinos

- Thus sterile means no standard weak interactions

- But sterile neutrinos are not absolutely sterile:

  - Gravitational Interactions

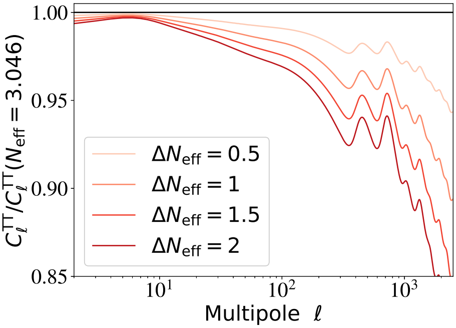
  - New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos

- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)

- Observables:
  - Disappearance of active neutrinos (neutral current deficit) $\leftarrow$ CEνNS
  - Indirect evidence through combined fit of data (current indication)

- Powerful window on new physics beyond the Standard Model

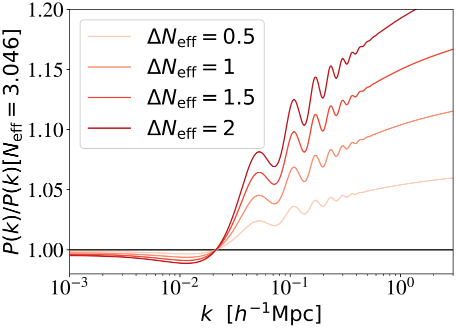
Dark Radiation



- Photons feel gravitational forces from a denser neutrino component.
- Decreases the acoustic peaks because the distribution of free-streaming neutrinos is smoother that that of the photons.

- $\rho_{\rm rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\rho_\gamma$
- $\Delta N_{\rm eff} = N_{\rm eff} - 3.046$
- Fixed $z_{\rm eq}$, z_Λ, $\omega_{\rm B}^0$
- $z_{\rm eq} \simeq \dfrac{\Omega_{\rm M}^0 h^2}{\omega_\gamma^0 (1 + 0.227 N_{\rm eff})}$
- $z_\Lambda \simeq \left(\dfrac{\Omega_\Lambda^0}{\Omega_{\rm M}^0}\right)^{1/3} \simeq \left(\dfrac{1-\Omega_{\rm M}^0}{\Omega_{\rm M}^0}\right)^{1/3}$
- Therefore fixed $\Omega_{\rm M}^0$
- $\omega_{\rm B}^0 = \Omega_{\rm B}^0 h^2$
- It can be done by increasing h^2 and decreasing $\Omega_{\rm B}^0$ with an increase of $\Omega_{\rm CDM}^0 = \Omega_{\rm M}^0 - \Omega_{\rm B}^0$

Dark Radiation



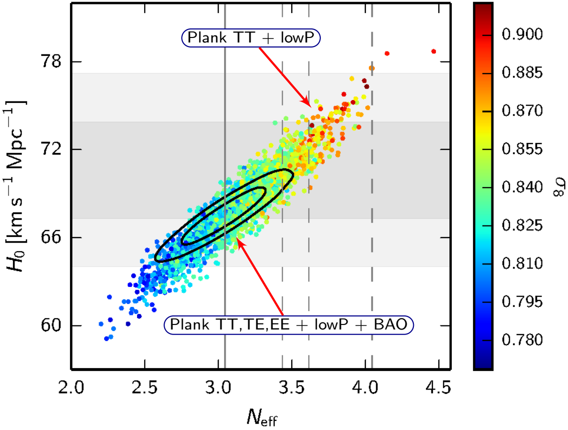
- Increased fluctuations due to increased Ω^0_{CDM}.
- Decreased BAO due to decreased Ω^0_{B}.

- $\rho_{\mathrm{rad}} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right] \rho_\gamma$
- $\Delta N_{\mathrm{eff}} = N_{\mathrm{eff}} - 3.046$
- Fixed z_{eq}, z_Λ, ω^0_{B}
- $z_{\mathrm{eq}} \simeq \frac{\Omega^0_{\mathrm{M}} h^2}{\omega^0_\gamma (1 + 0.227 N_{\mathrm{eff}})}$
- $z_\Lambda \simeq \left(\frac{\Omega^0_\Lambda}{\Omega^0_{\mathrm{M}}}\right)^{1/3} \simeq \left(\frac{1 - \Omega^0_{\mathrm{M}}}{\Omega^0_{\mathrm{M}}}\right)^{1/3}$
- Therefore fixed Ω^0_{M}
- $\omega^0_{\mathrm{B}} = \Omega^0_{\mathrm{B}} h^2$
- It can be done by increasing h^2 and decreasing Ω^0_{B} with an increase of $\Omega^0_{\mathrm{CDM}} = \Omega^0_{\mathrm{M}} - \Omega^0_{\mathrm{B}}$

Planck Limits on Dark Radiation

[Planck, A&A 594 (2016) A13, arXiv:1502.01589]

Cosmological data set	N_{eff}
Plank TT + lowP	3.13 ± 0.32
Plank TT + lowP + BAO	3.15 ± 0.23
Plank TT,TE,EE + lowP	2.99 ± 0.20
Plank TT,TE,EE + lowP + BAO	3.04 ± 0.18



Massive Sterile Neutrinos

- sterile neutrinos can be produced by $\nu_{e,\mu,\tau} \to \nu_s$ oscillations before active neutrino decoupling $(t_{\nu\text{-dec}} \sim 1\,\text{s})$
- energy density of radiation before matter-radiation equality:

$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right]\rho_\gamma \qquad\qquad (t < t_{\text{eq}} \sim 6 \times 10^4\,\text{y})$$

$$N_{\text{eff}}^{\text{SM}} = 3.046 \qquad\qquad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$$

- sterile neutrino contribution:

$$\rho_s = (T_s/T_\nu)^4 \rho_\nu \quad \Longrightarrow \quad \Delta N_{\text{eff}} = (T_s/T_\nu)^4$$

- sterile neutrino $\nu_s \simeq \nu_4$ with mass $m_s = m_4 \sim 1\,\text{eV}$ becomes non-relativistic at $T_\nu \sim m_s/3$, that is at $t_{\nu_s\text{-nr}} \sim 2.0 \times 10^5\,\text{y}$, before recombination at $t_{\text{rec}} \sim 3.8 \times 10^5\,\text{y}$
- current energy density of sterile neutrinos:

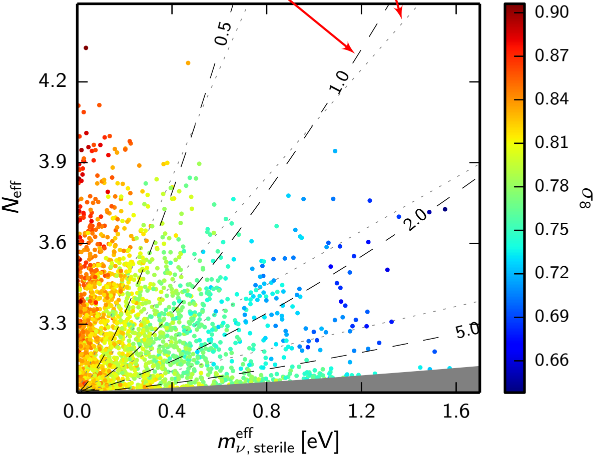
$$\Omega_s = \frac{n_s m_s}{\rho_c} \simeq \frac{(T_s/T_\nu)^3 m_s}{93.1\,h^2\,\text{eV}} = \frac{\Delta N_{\text{eff}}^{3/4} m_s}{93.1\,h^2\,\text{eV}} = \frac{m_s^{\text{eff}}}{93.1\,h^2\,\text{eV}}$$

$$m_s^{\text{eff}} = \Delta N_{\text{eff}}^{3/4} m_s = (T_s/T_\nu)^3 m_s$$

Limits on Massive Sterile Neutrinos

$N_{\rm eff} < 3.7 \qquad m_s^{\rm eff} < 0.52 \qquad$ (95%; Plank TT + lowP + lensing + BAO)

Constant m_s: Thermal and DW



- $m_s^{\rm eff} \equiv 93.1 \Omega_s h^2$ eV
- Thermally distributed:

$$f_s(E) = \frac{1}{e^{E/T_s}+1}$$

$$m_s^{\rm eff} = \left(\frac{T_s}{T_\nu}\right)^3 m_s = (\Delta N_{\rm eff})^{3/4} m_s$$

- Dodelson-Widrow:

$$f_s(E) = \frac{\chi_s}{e^{E/T_\nu}+1}$$

$$m_s^{\rm eff} = \chi_s\, m_s = \Delta N_{\rm eff}\, m_s$$

Conclusions

- Normal light neutrinos are Hot Dark Matter.
- Their effects on cosmological observables depend on their masses.
- Cosmological data give information on neutrino physics, but it is model-dependent.
- Neutrino physics may contribute to solve tensions in the Cosmological data.
- Light sterile neutrinos are allowed only if their thermalization is suppressed.
- Heavy sterile neutrinos with mass of the order of keV can contribute to the Dark Matter (not discussed).