Cosmic Neutrino

Part II: Cosmology

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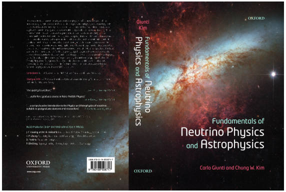
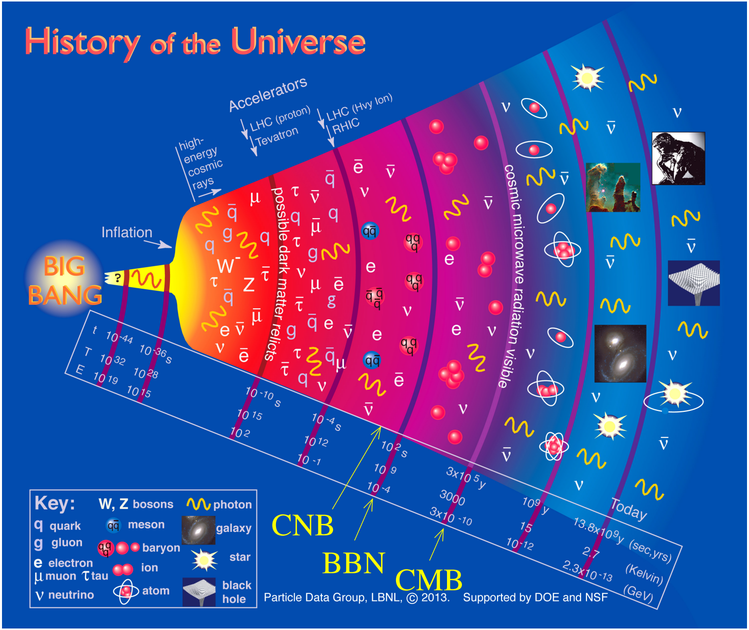


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages


History of the Universe
Accelerators
LHC (proton)
Tevatron
LHC (Hvy Ion)
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relics
cosmic microwave radiation visible
Today
Key:
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
W, Z bosons
meson
baryon
ion
atom
photon
galaxy
star
black hole
CNB
BBN
CMB
Particle Data Group, LBNL, © 2013. Supported by DOE and NSF

Basic Formalism

- Einstein equations of gravity: $\mathcal{R}^{\mu\nu} - \frac{1}{2}\,\mathcal{R}\,g^{\mu\nu} - \Lambda\,g^{\mu\nu} = 8\pi\,G_{\rm N}\,T^{\mu\nu}$
- Observations have shown that the Universe is spatially homogeneous and isotropic on large scales: $\gtrsim 100\,\text{Mpc}$.
- The Standard Cosmological Model assumes that there is a frame in which the total matter and radiation of the Universe can be described on large scales by a perfect fluid with the energy momentum tensor

$$T^{\mu\nu} = \text{diag}(\varrho, p, p, p)$$

- In this comoving frame, the geometry of space-time is described by the Friedmann–Robertson–Walker metric

$$d\tau^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1 - k\,r^2} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right]$$

- The rate of expansion is given by the Friedmann equation:

$$H^2 = \frac{8\pi G_{\rm N}}{3}\,\varrho - \frac{k}{R^2} \qquad\qquad H(t) \equiv \frac{\dot{R}(t)}{R(t)}$$

- Redshift: $z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\Delta\lambda}{\lambda} \implies \boxed{1 + z = \frac{R(t_0)}{R(t_e)}}$

- Radiation, matter and vacuum energy densities: $\varrho = \varrho_R + \varrho_M + \varrho_\Lambda$

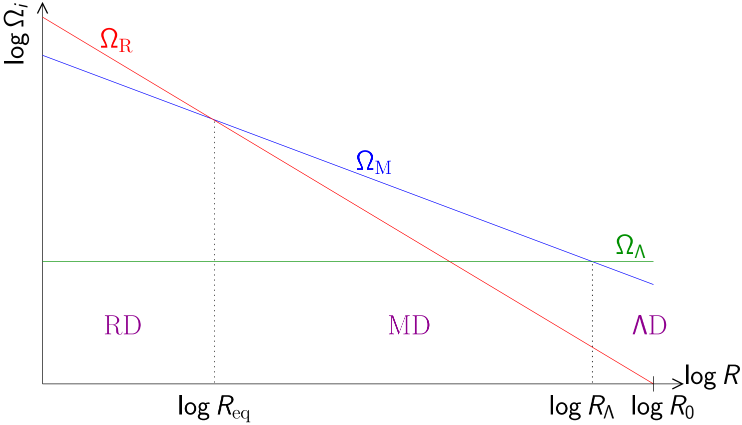
- Equation of state: $p_i = w_i\, \varrho_i$

Radiation:	$w_R = 1/3$	$\implies$	$\varrho_R \propto R^{-4} \propto (1+z)^4$
Matter:	$w_M = 0$	$\implies$	$\varrho_M \propto R^{-3} \propto (1+z)^3$
Vacuum Energy:	$w_\Lambda = -1$	$\implies$	$\varrho_\Lambda = \text{constant}$

- Flat Universe: $k = 0 \implies \varrho = \varrho_c \equiv \frac{3\,H^2}{8\pi\, G_N}$ critical density

$$\varrho_c^0 = \frac{3\,H_0^2}{8\pi\, G_N} = 10.54 h^2\, \text{keV}\, \text{cm}^{-3} \quad \text{with} \quad H_0 = 100\, h\, \text{km}\, \text{s}^{-1}\, \text{Mpc}^{-1}$$

- $\Omega_i \equiv \frac{\varrho_i}{\varrho_c} \implies \Omega_R + \Omega_M + \Omega_\Lambda = 1$ for a flat Universe



$$\Omega_\Lambda^0 \simeq 0.7 \qquad \Omega_\Lambda = \Omega_\Lambda^0$$

$$\Omega_M^0 \simeq 0.3 \qquad \Omega_M = \Omega_M^0 \, (R/R_0)^{-3} = \Omega_M^0 \, (1+z)^3$$

$$\Omega_R^0 \simeq 10^{-4} \qquad \Omega_R = \Omega_R^0 \, (R/R_0)^{-4} = \Omega_M^0 \, (1+z)^4$$

Friedmann equation for a flat Universe: $H^2 = \frac{8\pi}{3\,M_{\mathrm{P}}^2}\,\varrho$

$$\frac{H^2}{H_0^2} = \frac{\varrho}{\varrho^0} \quad \Longrightarrow \quad H^2 = H_0^2\,\frac{\varrho_\Lambda + \varrho_{\mathrm{M}} + \varrho_{\mathrm{R}}}{\varrho_{\mathrm{c}}^0}$$

$$\begin{aligned} \varrho_\Lambda &= \varrho_\Lambda^0 \\ \varrho_{\mathrm{M}} &= \varrho_{\mathrm{M}}^0 \left(\frac{R_0}{R}\right)^3 = \varrho_{\mathrm{M}}^0\,(1+z)^3 \\ \varrho_{\mathrm{R}} &= \varrho_{\mathrm{R}}^0 \left(\frac{R_0}{R}\right)^4 = \varrho_{\mathrm{R}}^0\,(1+z)^4 \end{aligned}$$

$$H^2 = H_0^2\,\frac{\varrho_\Lambda^0 + \varrho_{\mathrm{M}}^0\,(1+z)^3 + \varrho_{\mathrm{R}}^0\,(1+z)^4}{\varrho_{\mathrm{c}}^0}$$

$$H^2(z) = H_0^2\left[\Omega_\Lambda^0 + \Omega_{\mathrm{M}}^0\,(1+z)^3 + \Omega_{\mathrm{R}}^0\,(1+z)^4\right]$$

The expansion rate depends on H_0 and on Ω_Λ^0, Ω_{M}^0, Ω_{R}^0

Thermodynamics of the Early Universe

- Thermal equilibrium:

$$n_\chi = \frac{g_\chi}{(2\pi)^3} \int f_\chi(\vec{p})\, \mathrm{d}^3p$$

$$\varrho_\chi = \frac{g_\chi}{(2\pi)^3} \int E_\chi(\vec{p})\, f_\chi(\vec{p})\, \mathrm{d}^3p$$

$$p_\chi = \frac{g_\chi}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E_\chi(\vec{p})}\, f_\chi(\vec{p})\, \mathrm{d}^3p$$

- Statistical distribution: $f_\chi(\vec{p}) = \dfrac{1}{e^{(E_\chi(\vec{p})-\mu_\chi)/T_\chi} \pm 1}$

- Chemical potential:
  - $a + b \leftrightarrows c + d \quad \Longrightarrow \quad \mu_a + \mu_b = \mu_c + \mu_d$
  - $\mu_\gamma = 0$ and $\chi + \bar{\chi} \to \gamma\gamma \quad \Longrightarrow \quad \mu_\chi = -\mu_{\bar{\chi}}$
  - Conserved charge $\quad \Longrightarrow \quad \mu_\chi \neq 0$ if $n_\chi \neq n_{\bar{\chi}}$

- Relativistic limit: $T_\chi \gg m_\chi$ and $T_\chi \gg \mu_\chi \Longrightarrow f_\chi(\vec{p}) \simeq \dfrac{1}{e^{|\vec{p}|/T_\chi} \pm 1}$

$$n_\chi \simeq \begin{cases} \dfrac{\zeta(3)}{\pi^2} g_\chi T_\chi^3 & (\chi = \text{boson}) \\ \dfrac{3}{4} \dfrac{\zeta(3)}{\pi^2} g_\chi T_\chi^3 & (\chi = \text{fermion}), \end{cases}$$

$$\varrho_\chi \simeq \begin{cases} \dfrac{\pi^2}{30} g_\chi T_\chi^4 & (\chi = \text{boson}) \\ \dfrac{7}{8} \dfrac{\pi^2}{30} g_\chi T_\chi^4 & (\chi = \text{fermion}), \end{cases}$$

$$p_\chi \simeq \frac{1}{3} \varrho_\chi ,$$

- Average energy:

$$\langle E_\chi \rangle \simeq \langle |\vec{p}_\chi| \rangle \simeq \begin{cases} \dfrac{\pi^4}{30\,\zeta(3)} T_\chi \simeq 2.701\, T_\chi & (\chi = \text{boson}) \\ \dfrac{7\pi^4}{180\,\zeta(3)} T_\chi \simeq 3.151\, T_\chi & (\chi = \text{fermion}) \end{cases}$$

Radiation Temperature Scaling

$$\left.\begin{array}{l}\varrho_{\mathrm{R}} \propto R^{-4} \\ \varrho_{\mathrm{R}} \propto T^{4}\end{array}\right\} \implies \boxed{T \propto R^{-1}}$$

The Universe cools during expansion!

Neutrino Decoupling

- Neutrinos are in equilibrium in the early Universe through weak interactions:

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \qquad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N$$
$$\nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

- Interaction rate: $\Gamma_\nu = n_\nu \langle \sigma v \rangle \sim G_F^2 T^5$

$$n_\nu \sim T^3 \qquad \sigma \sim G_F^2 T^2 \qquad v \simeq 1$$

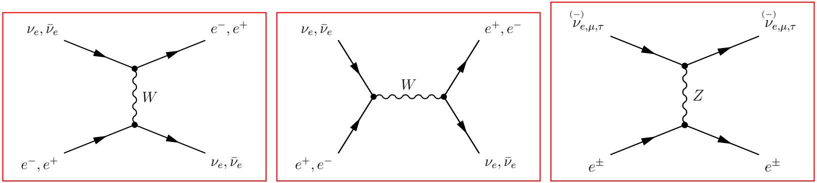
- In the radiation-dominated era: $H^2 \simeq \frac{8\pi}{3 M_P^2} \varrho_R$ with $\varrho_R = \frac{\pi^2}{30} g_* T^4$

$$H \simeq \frac{2\,\pi^{3/2}}{3\sqrt{5}\,M_P} \sqrt{g_*}\, T^2 \qquad g_* = \sum_{\substack{\chi=\text{relativistic}\\ \text{bosons}}} g_\chi + \frac{7}{8} \sum_{\substack{\chi=\text{relativistic}\\ \text{fermions}}} g_\chi$$

- Before ν decoupling: $g_* = g_*^{(\gamma)} + g_*^{(e^\pm)} + g_*^{(\nu)} = 2 + \frac{7}{8}4 + \frac{7}{8}6 = 10.75$

- Neutrino decoupling: $\Gamma_\nu \sim H \implies T^{\nu\text{-dec}} \sim \left(M_\mathrm{P}\, G_\mathrm{F}^2\right)^{-1/3} \sim 1\,\mathrm{MeV}$
- A more precise calculation takes into account that the dominant processes for $T \lesssim 100\,\mathrm{MeV}$ are

$$\nu\bar{\nu} \leftrightarrows e^+e^- \qquad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e$$



- Since the rates of these processes depend on neutrino energy $E \simeq p$, the decoupling temperature is not instantaneous and depends on p:

$$T^{\nu_e\text{-dec}}(p) \simeq 2.7 \left(\frac{p}{T}\right)^{-1/3} \qquad T^{\nu_{\mu,\tau}\text{-dec}}(p) \simeq 4.5 \left(\frac{p}{T}\right)^{-1/3}$$

- Taking into account that $\langle E \rangle \simeq 3T$, one obtains:

$$T^{\nu_e\text{-dec}} \simeq 1.9\,\mathrm{MeV} \qquad T^{\nu_{\mu,\tau}\text{-dec}} \simeq 3.1\,\mathrm{MeV}$$

- Hot relics: relativistic at decoupling $\Longrightarrow f_{\nu}^{\nu\text{-dec}}(\vec{p}) \simeq \dfrac{1}{e^{|\vec{p}|/T^{\nu\text{-dec}}}+1}$
- After decoupling: $f_{\nu}(\vec{p}) = f_{\nu}(\vec{p})|_{\nu\text{-dec}} = f_{\nu}^{\nu\text{-dec}}(\vec{p}_{\nu\text{-dec}})$
- Momentum scaling with expansion: $\vec{p} = \vec{p}_{\nu\text{-dec}} \left(\dfrac{R}{R_{\nu\text{-dec}}} \right)^{-1}$

$$f_{\nu}(\vec{p}) \simeq \left[\exp\left(\frac{|\vec{p}|\,(R/R_{\nu\text{-dec}})}{T^{\nu\text{-dec}}} \right) + 1 \right]^{-1} = \frac{1}{e^{|\vec{p}|/T_{\nu}}+1}$$

Effective temperature scales with expansion: $\boxed{T_{\nu} = T^{\nu\text{-dec}} \left(\dfrac{R}{R_{\nu\text{-dec}}} \right)^{-1}}$

Electron-Positron Annihilation

- After neutrino decoupling at $T \simeq 1\,\text{MeV}$ $e^{\pm}$ and γ are the only relativistic particles in thermal equilibrium.

- At $m_e/3 \simeq 0.2\,\text{MeV}$ electrons and positrons became nonrelativistic: out-of-equilibrium $e^- e^+ \to \gamma\gamma$ heat the photon distribution.

- During this phase the photon temperature does not scale as R^{-1}.

- Entropy density: $s = \dfrac{\varrho + p}{T} = \dfrac{2\pi^2}{45}\, g_s\, T_\gamma^3$

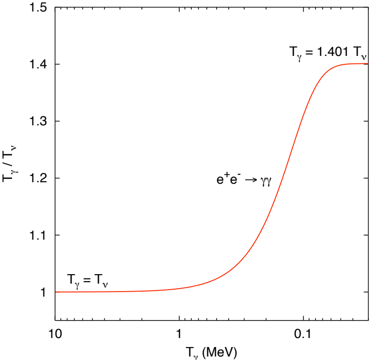
$$g_s = \sum_{\substack{\chi=\text{interacting}\\ \text{relativistic}\\ \text{bosons}}} g_\chi + \frac{7}{8} \sum_{\substack{\chi=\text{interacting}\\ \text{relativistic}\\ \text{fermions}}} g_\chi$$

- Entropy conservation: $s \propto R^{-3} \quad \Longrightarrow \quad T_\gamma \propto {g_s}^{-1/3}\, R^{-1}$

- Before and after $e^- e^+$ annihilation: $\dfrac{T_\nu^{\text{after}}}{T_\nu^{\text{before}}} = \left(\dfrac{R^{\text{after}}}{R^{\text{before}}}\right)^{-1}$

$$\frac{T_\gamma^{\text{after}}}{T_\gamma^{\text{before}}} = \left(\frac{g_s^{\text{after}}}{g_s^{\text{before}}}\right)^{-1/3} \left(\frac{R^{\text{after}}}{R^{\text{before}}}\right)^{-1} = \left(\frac{g_s^{\text{after}}}{g_s^{\text{before}}}\right)^{-1/3} \frac{T_\nu^{\text{after}}}{T_\nu^{\text{before}}}$$

- $T_\gamma^{\text{before}} = T_\nu^{\text{before}}$
- $g_s^{\text{before}} = g_s^{(\gamma)} + g_s^{(e^\pm)} = 2 + \frac{7}{8} 4 = \frac{11}{2}$
- $g_s^{\text{after}} = g_s^{(\gamma)} = 2$
- $T_\nu^{\text{after}} = \left(\frac{4}{11}\right)^{1/3} T_\gamma^{\text{after}} \simeq 0.7138\, T_\gamma^{\text{after}}$



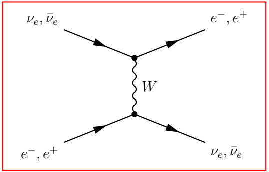
- $T_\nu^0 = \left(\frac{4}{11}\right)^{1/3} T_\gamma^0 = 1.945 \pm 0.001\,\text{K} = (1.676 \pm 0.001) \times 10^{-4}\,\text{eV}$

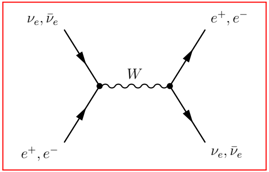
Effective Number of Relativistic Degrees Of Freedom

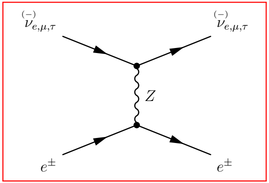
- Radiation density: $$\varrho_{\mathrm{R}} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right] \varrho_{\gamma}$$
- Three standard neutrinos: $N_{\mathrm{eff}}^{\nu} = 3.046$ Why $N_{\mathrm{eff}}^{\nu} > 3$?

[Mangano et al, NPB 729 (2005) 221] $\rightarrow N_{\mathrm{eff}}^{\nu} = 3.045$ [de Salas, Pastor, JCAP 1607 (2016) 051]

- Neutrino decoupling was not instantaneous at $T^{\nu\text{-dec}}$.
- Higher-energy neutrinos decoupled later and were not completely decoupled during $e^- e^+$ annihilation.
- This effect is different for $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_{\mu,\tau}$ because of the additional charged-current interactions of $\overset{(-)}{\nu}_e$:

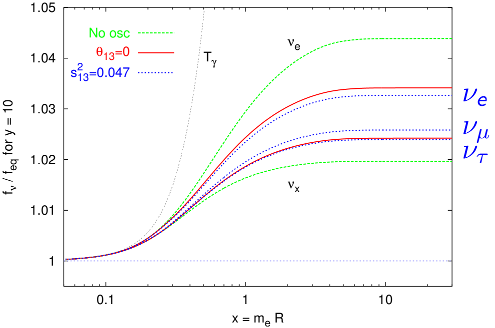






- Equilibrium distribution: $f_{\text{eq}}(\vec{p}) \simeq \dfrac{1}{e^{p/T}+1}$
- Nonthermal distortions: $f_{\nu_\alpha}(\vec{p},t) = f_{\text{eq}}(\vec{p})\,(1+\delta_{\nu_\alpha}(\vec{p},t))$
- Boltzmann equation:

$$\left(\frac{\partial}{\partial t} - Hp\,\frac{\partial}{\partial p}\right) f_{\nu_\alpha}(\vec{p},t) = C\left[f_{\nu_\alpha}; f_{\nu_\beta}, f_{e^\pm}\right]$$



$$R = T_\nu^{-1}$$

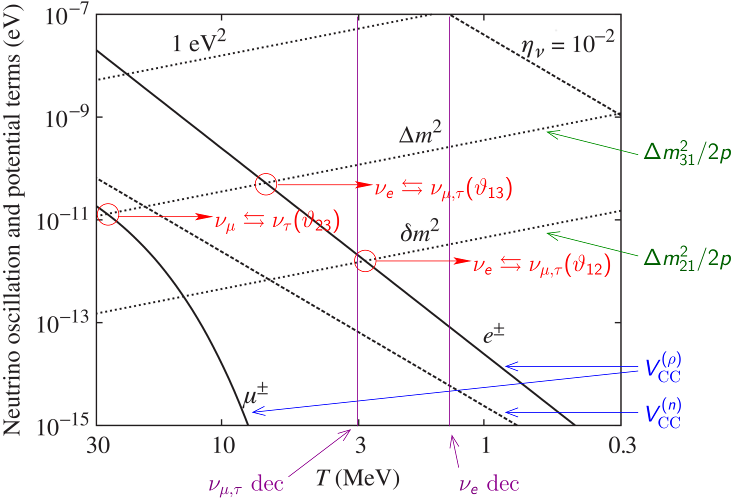
$$x = m_e R = m_e/T_\nu$$

$$y = pR = p/T_\nu$$

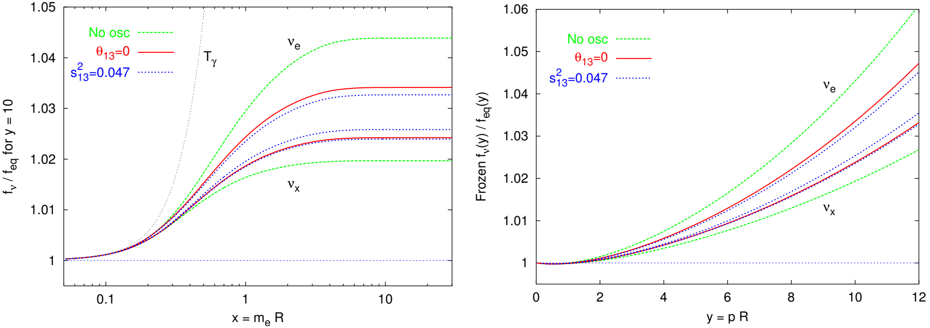
[Mangano et al, NPB 729 (2005) 221, arXiv:hep-ph/0506164]

- Neutrino oscillations mix the flavor distributions.

- Matter potential: $V_{\mathrm{CC}}^{(\ell)} = \sqrt{2} G_{\mathrm{F}} \left(n_{\ell^-} - n_{\ell^+} \right) - \dfrac{8\sqrt{2} G_{\mathrm{F}} p}{3 m_W^2} \left(\varrho_{\ell^-} + \varrho_{\ell^+} \right)$



[Lesgourgues, Mangano, Miele, Pastor, Neutrino Cosmology, Cambridge University Press, 2013]



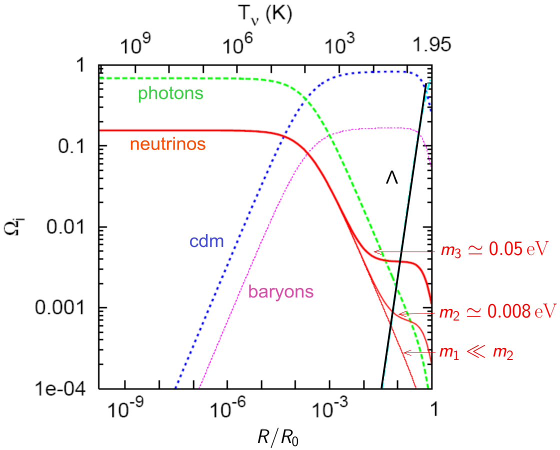
[Mangano et al, NPB 729 (2005) 221, arXiv:hep-ph/0506164]

$$R = T_\nu^{-1} \qquad x = m_e R = m_e / T_\nu \qquad y = pR = p/T_\nu$$

$$N_{\text{eff}} = 3 + \frac{\delta \varrho_{\nu_e}}{\varrho_\nu} + \frac{\delta \varrho_{\nu_\mu}}{\varrho_\nu} + \frac{\delta \varrho_{\nu_\tau}}{\varrho_\nu} = 3.046$$

$$f_{\nu_k} = \sum_{\alpha=e,\mu,\tau} |U_{\alpha k}|^2 f_{\nu_\alpha} \quad \Rightarrow \quad \begin{cases} f_{\nu_1} \simeq 0.7 f_{\nu_e} + 0.3 f_{\nu_{\mu,\tau}} \\ f_{\nu_2} \simeq 0.3 f_{\nu_e} + 0.7 f_{\nu_{\mu,\tau}} \\ f_{\nu_3} \simeq f_{\nu_{\mu,\tau}} \end{cases}$$

Energy Density



Rad: $\begin{cases} \varrho_c \simeq \varrho_R \propto R^{-4} \\ \dfrac{\varrho_M}{\varrho_c} \propto \dfrac{R^{-3}}{R^{-4}} \propto R \\ \dfrac{\varrho_\Lambda}{\varrho_c} \propto \dfrac{1}{R^{-4}} \propto R^4 \end{cases}$

Mat: $\begin{cases} \varrho_c \simeq \varrho_M \propto R^{-3} \\ \dfrac{\varrho_R}{\varrho_c} \propto \dfrac{R^{-4}}{R^{-3}} \propto R^{-1} \\ \dfrac{\varrho_\Lambda}{\varrho_c} \propto \dfrac{1}{R^{-3}} \propto R^3 \end{cases}$

Λ: $\begin{cases} \varrho_c \simeq \varrho_\Lambda = \text{const.} \\ \dfrac{\varrho_R}{\varrho_c} \propto R^{-4} \\ \dfrac{\varrho_M}{\varrho_c} \propto R^{-3} \end{cases}$

$$\Omega_i = \varrho_i / \varrho_c \qquad \varrho_c = \frac{3M_P^2}{8\pi} H^2$$

Nonrelativistic Transition

- After decoupling $T_\nu \propto R^{-1} \implies T_\nu = T_\nu^0 \left(\frac{R_0}{R}\right) = T_\nu^0 (1+z)$

- Nonrelativistic transition: $T_{\nu_i}^{\text{nr}} \simeq 3 m_i \Rightarrow z_{\nu_i}^{\text{nr}} \simeq \frac{m_i}{3\,T_\nu^0} \simeq 2.0 \times 10^3 \left(\frac{m_i}{\text{eV}}\right)$

  $m_3 \gtrsim 5 \times 10^{-2}\,\text{eV} \Rightarrow z_{\nu_3}^{\text{nr}} \gtrsim 100 \qquad m_2 \gtrsim 8 \times 10^{-3}\,\text{eV} \Rightarrow z_{\nu_2}^{\text{nr}} \gtrsim 16$

- After the nonrelativistic transition: $\varrho_{\nu_i} \simeq m_i\, n_{\nu_i}$

- $n_\nu^0 + n_{\bar\nu}^0 \simeq \frac{3}{2}\,\frac{\zeta(3)}{\pi^2}\,(T_\nu^0)^3 \simeq \frac{6}{11}\,\frac{\zeta(3)}{\pi^2}\,(T_\gamma^0)^3 = \frac{3}{11}\,n_\gamma^0 \simeq 112\,\text{cm}^{-3}$

- $\varrho_{\text{c}}^0 \equiv \frac{3\,H_0^2}{8\pi\,G_{\text{N}}} \simeq 10.54\,h^2\,\text{keV}\,\text{cm}^{-3} \Rightarrow \Omega_{\nu_i}^0 \simeq \frac{m_i(n_\nu^0 + n_{\bar\nu}^0)}{\varrho_{\text{c}}^0} \simeq \frac{m_i}{94.1\,h^2\,\text{eV}}$

- Nonthermal distortions $\implies$ $\boxed{\Omega_{\nu_i}^0 \simeq \frac{m_i}{93.1\,h^2\,\text{eV}}}$

  $\Omega_{\nu_3}^0 \gtrsim 5 \times 10^{-4}$

  $\Omega_{\nu_2}^0 \gtrsim 9 \times 10^{-5}$

- $\Omega^0_{\nu\text{-relativistic}} = \left(\frac{4}{11}\right)^{4/3} \Omega^0_\gamma \simeq 1.2 \times 10^{-5} \ll \Omega^0_{\nu_2} \gtrsim 9 \times 10^{-5}$

- Total contribution of neutrinos to the current energy density of the Universe: [Gershtein, Zeldovich, JETP Lett. 4 (1966) 120; Cowsik, McClelland, PRL 29 (1972) 669]

$$\boxed{\Omega^0_\nu \simeq \frac{\sum_i m_i}{93.1\, h^2\, \text{eV}}}$$

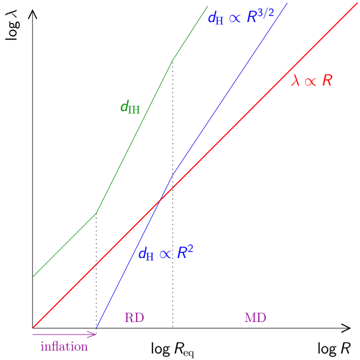
$$\left.\begin{array}{l} \Omega^0_\nu \leq \Omega^0_\text{M} \simeq 0.3 \\ h \simeq 0.7 \end{array}\right\} \implies \sum_i m_i \lesssim 14\,\text{eV}$$

- This bound is not competitive with the current kinematical laboratory limit:

$$m_i \lesssim m_\beta \lesssim 2\,\text{eV} \implies \sum_i m_i \lesssim 6\,\text{eV}$$

Matter-Radiation Equality

- Matter-radiation equality is important because subhorizon matter density fluctuations can grow only during the matter-dominated era.



- Therefore structure formation starts at matter-radiation equality.
- Where neutrino still relativistic at matter-radiation equality?
- The answer to this question is important in order to determine the effect of neutrinos on structure formation.

- Redshift of matter-radiation equality:

$$\left.\begin{array}{l}\varrho_{\mathrm{M}} \propto R^{-3} \\ \varrho_{\mathrm{R}} \propto R^{-4}\end{array}\right\} \Rightarrow \frac{\varrho_{\mathrm{M}}}{\varrho_{\mathrm{R}}} = \frac{\varrho_{\mathrm{M}}^{0}}{\varrho_{\mathrm{R}}^{0}} \frac{R}{R_0} = \frac{\varrho_{\mathrm{M}}^{0}}{\varrho_{\mathrm{R}}^{0}} (1+z)^{-1} \Rightarrow 1 + z_{\mathrm{eq}} = \frac{\varrho_{\mathrm{M}}^{0}}{\varrho_{\mathrm{R}}^{0}} = \frac{\Omega_{\mathrm{M}}^{0}}{\Omega_{\mathrm{R}}^{0}}$$

- This relation assumes that the number of relativistic particles is not changed.
- If neutrinos were relativistic at matter-radiation equality:

$$1 + z_{\mathrm{eq}} = \frac{\Omega_{\mathrm{M}}^{0}}{\Omega_{\mathrm{R}}^{0}}(m_\nu = 0) = \frac{\Omega_{\mathrm{B}}^{0} + \Omega_{\mathrm{CDM}}^{0}}{\Omega_{\gamma}^{0} + \Omega_{\nu}^{0}(m_\nu = 0)}$$

$$\Omega_{\mathrm{R}}^{0}(m_\nu = 0) = \left[1 + 3\left(\frac{4}{11}\right)^{4/3}\right]\Omega_{\gamma}^{0} \simeq 4.4 \times 10^{-5}\, h^{-2}$$
$$\simeq 8.9 \times 10^{-5} \quad \text{for} \quad h \simeq 0.7$$

$$z_{\mathrm{eq}} \simeq 2.4 \times 10^{4} \left(\Omega_{\mathrm{B}}^{0} + \Omega_{\mathrm{CDM}}^{0}\right) h^{2} \simeq 3.5 \times 10^{3} \quad \text{for} \quad \Omega_{\mathrm{B}}^{0} + \Omega_{\mathrm{CDM}}^{0} \simeq 0.3$$

$$z_{\nu_i}^{\mathrm{nr}} \simeq 2.0 \times 10^{3} \left(\frac{m_i}{\mathrm{eV}}\right) < z_{\mathrm{eq}} \quad \text{for} \quad m_i \lesssim 1.75\,\mathrm{eV}$$

- From the current kinematical bound $m_i \lesssim 2\,\text{eV}$ it is likely that all the three standard massive neutrinos became nonrelativistic after matter-radiation equality.

- From t_{eq} to $t_{\nu_i}^{\text{nr}}$ neutrinos free stream.

- Subhorizon matter density fluctuations are suppressed by neutrino free streaming.

- Current physical free-streaming scale: $\lambda^0_{\nu_i\text{-fs}} \simeq z_{\nu_i\text{-nr}}\, d_{\text{H}}(z_{\nu_i\text{-nr}})$

- Matter-dominated era: $d_{\text{H}}(z) \simeq 2\, H_0^{-1}\, z^{-3/2}\, (\Omega^0_{\text{M}})^{-1/2}$

$$\lambda^0_{\nu_i\text{-fs}} \simeq 0.013 \left(\frac{m_i}{\text{eV}}\right)^{-1/2} (\Omega^0_{\text{M}})^{-1/2}\, h^{-1}\,\text{Mpc}$$

$$k^0_{\nu_i\text{-fs}} \simeq \frac{2\pi}{\lambda^0_{\nu_i\text{-fs}}} \simeq 0.047 \left(\frac{m_i}{\text{eV}}\right)^{1/2} \sqrt{\Omega^0_{\text{M}}}\, h\,\text{Mpc}^{-1}$$

Power Spectrum

- Density fluctuations: $\delta(t,\vec{x}) \equiv \dfrac{\varrho(t,\vec{x}) - \langle\varrho(t)\rangle}{\langle\varrho(t)\rangle} = \displaystyle\int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\delta(t,\vec{k})\,e^{i\vec{k}\cdot\vec{x}}$

- The Fourier transform transform differential equations into algebraic ones.

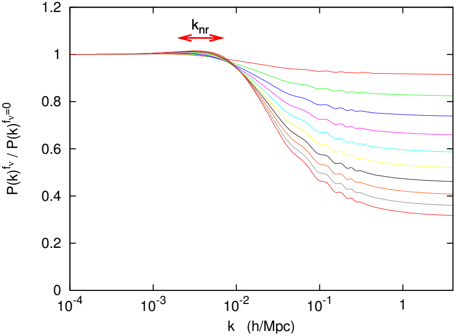
- In the linear theory, the algebraic equations for the amplitude of each fluctuation mode with wavenumber $\vec{k}$ are independent.

- The amplitude $\delta(t,\vec{k})$ of each fluctuation mode evolves in time independently of the others and can be conveniently studied separately.

- Power spectrum: $P(k,t) = \langle|\delta(t,\vec{k})|^2\rangle$

- The power spectrum is the variance of the distribution of fluctuations in Fourier space.

- Gaussian fluctuations are completely characterized by their variance, i.e. by the power spectrum.



[Lesgourgues, Pastor, Phys. Rept. 429 (2006) 307]

$$\omega_M^0 = \Omega_M^0 \, h^2 = 0.147$$

$$\Omega_\Lambda^0 = 0.70$$

$$m_1 \simeq m_2 \simeq m_3 \simeq \frac{1}{3} \sum_i m_i$$

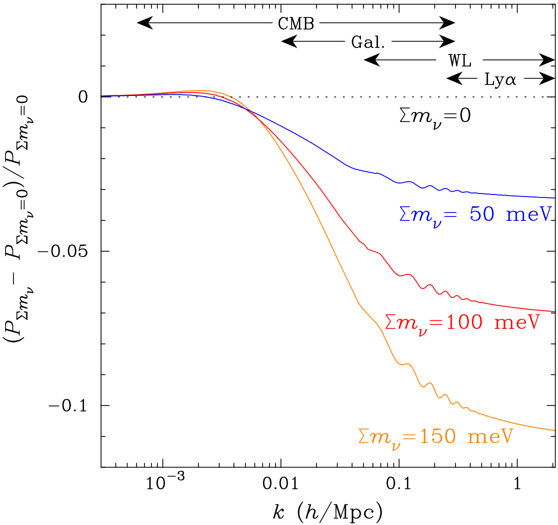
$$f_\nu \equiv \frac{\Omega_\nu^0}{\Omega_M^0} = 0.01, 0.02, \ldots, 0.10$$

$$\sum_i m_i = 0.046, 0.092, 0.138, 0.184, 0.230, 0.270, 0.322, 0.368, 0.414, 0.460 \, \text{eV}$$

$$\frac{\Delta P(k)}{P(k)} \simeq -8 \frac{\Omega_\nu^0}{\Omega_M^0} \simeq -0.8 \left(\frac{\sum_i m_i}{1 \, \text{eV}} \right) \left(\frac{0.1}{\Omega_M^0 \, h^2} \right)$$

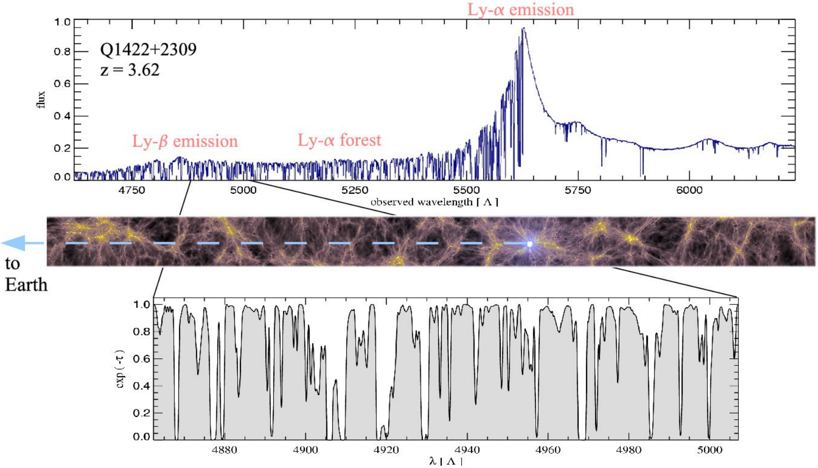
$$k \gtrsim k_{fs} \simeq 0.026 \sqrt{\frac{\sum_i m_i}{1 \, \text{eV}}} \sqrt{\Omega_M^0} \, h \, \text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]



[Abazajian et al, Astropart.Phys. 63 (2015) 66, arXiv:1309.5383.]

Lyman-alpha Forest

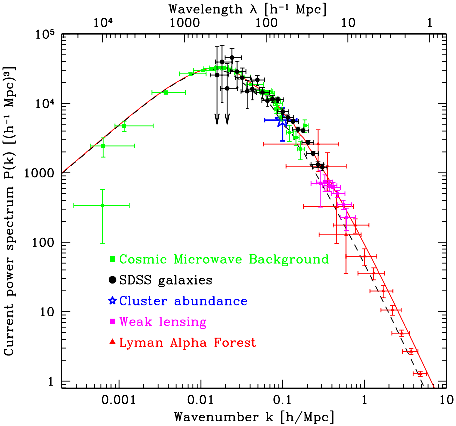


[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67\,\text{Å}$, $\lambda_\beta^0 = 1025.72\,\text{Å}$, $\lambda_\gamma^0 = 972.54\,\text{Å}$

Lyman-α forest: The region in which only Lyα photons can be absorbed:

$$[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$$



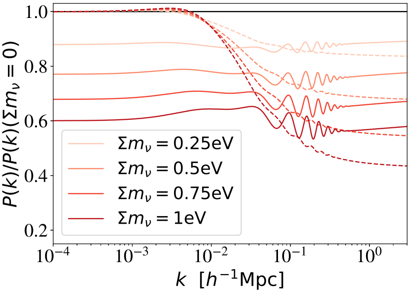
[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$$h = 0.72$$
$$\Omega_M^0 = 0.28$$
$$\Omega_B^0/\Omega_M^0 = 0.16$$

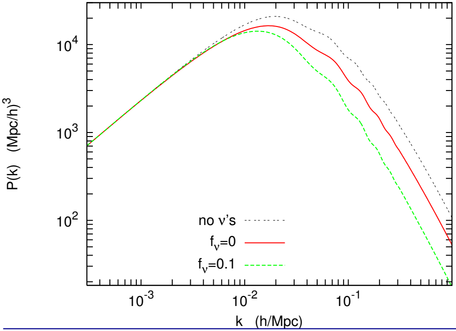
Dashed Curve: $\sum_{i=1}^{3} m_i = 1\,\text{eV}$

$$f_\nu \equiv \frac{\Omega_\nu^0}{\Omega_M^0} \simeq \frac{\sum_i m_i}{93.1\,h^2\,\text{eV}\,\Omega_M^0} \simeq 0.07$$



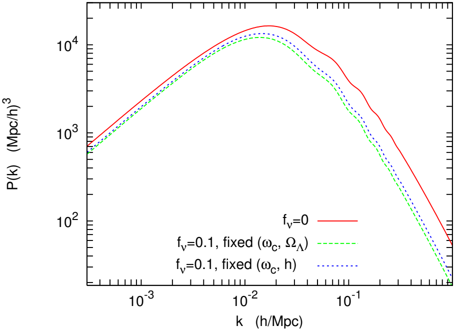
[Lesgourgues, Verde, Review of Particle Physics 2017]

- In the previous figures $\omega_M^0 = \Omega_M^0 h^2$ is fixed.
- $\Omega_B^0 + \Omega_{CDM}^0 = \Omega_M^0 - \Omega_\nu^0$
- $\Omega_\nu^0 \simeq \sum_i m_i \Big/ 93.1\, h^2\, \text{eV}$
- $z_{eq} \simeq 2.4 \times 10^4 \left(\Omega_B^0 + \Omega_{CDM}^0\right) h^2$
- z_{eq} decreases unless h is increased
- If z_{eq} is kept fixed by increasing h, there is also a suppression of the large-scale power spectrum.
- It is due to the decrease of time available for fluctuation growth as a consequence of the faster expansion.



- Fixed ω_B^0, ω_M^0, Ω_Λ^0
- $\omega_M^0 = \Omega_M^0 h^2 = \left(1 - \Omega_\Lambda^0\right) h^2$
- h fixed
- $z_\Lambda = (\Omega_\Lambda^0/\Omega_M^0)^{1/3}$ fixed
- $\omega_{CDM}^0 = \omega_M^0 - \omega_B^0 - \omega_\nu^0$
- ω_{CDM}^0, $\Omega_{CDM}^0 = \omega_{CDM}^0/h^2$ and z_{eq} decrease for $f_\nu > 0$

[Lesgourgues, Pastor, Phys. Rept. 429 (2006) 307]



- Fixed ω_B^0, ω_{CDM}^0 $\Rightarrow$ z_{eq} fixed
- $\omega_M^0 = \omega_B^0 + \omega_{CDM}^0 + \omega_\nu^0$ increases
- Fixed Ω_Λ^0 $\Rightarrow$ z_Λ fixed:

$$h = \sqrt{\frac{\omega_M^0}{1 - \Omega_\Lambda^0}} \text{ increases}$$

- Fixed h:

$$\Omega_\Lambda^0 = 1 - \omega_M^0 h^2 \text{ decreases}$$

Friedmann equation for a flat Universe: $H^2 = \frac{8\pi}{3\,M_{\mathrm{P}}^2}\,\varrho$

$$\frac{H^2}{H_0^2} = \frac{\varrho}{\varrho^0} \quad \Longrightarrow \quad H^2 = H_0^2\,\frac{\varrho_\Lambda + \varrho_{\mathrm{M}} + \varrho_{\mathrm{R}}}{\varrho_{\mathrm{c}}^0}$$

$$\varrho_\Lambda = \varrho_\Lambda^0$$

$$\varrho_{\mathrm{M}} = \varrho_{\mathrm{M}}^0 \left(\frac{R_0}{R}\right)^3 = \varrho_{\mathrm{M}}^0\,(1+z)^3$$

$$\varrho_{\mathrm{R}} = \varrho_{\mathrm{R}}^0 \left(\frac{R_0}{R}\right)^4 = \varrho_{\mathrm{R}}^0\,(1+z)^4$$

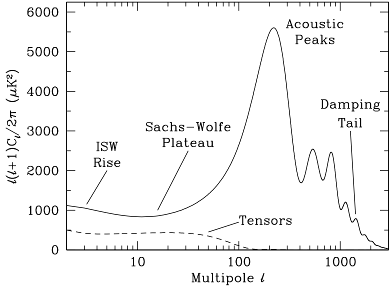
$$H^2 = H_0^2\,\frac{\varrho_\Lambda^0 + \varrho_{\mathrm{M}}^0\,(1+z)^3 + \varrho_{\mathrm{R}}^0\,(1+z)^4}{\varrho_{\mathrm{c}}^0}$$

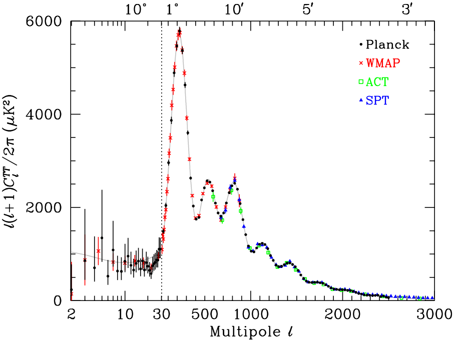
$$H^2 = H_0^2\left[\Omega_\Lambda^0 + \Omega_{\mathrm{M}}^0\,(1+z)^3 + \Omega_{\mathrm{R}}^0\,(1+z)^4\right]$$

Matter-dominated Universe: $H^2 \simeq H_0^2\left[1 - \Omega_{\mathrm{M}}^0 + \Omega_{\mathrm{M}}^0\,(1+z)^3\right]$

Increases with Ω_{M}^0 because $1+z>1$

Cosmic Microwave Background Radiation





- Temperature fluctuations:

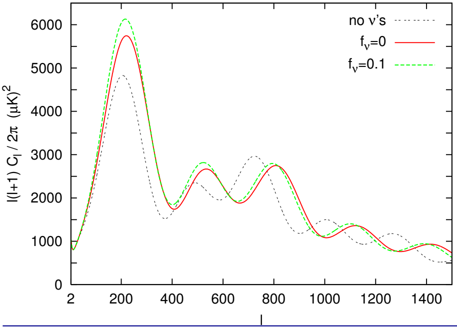
$$\frac{\Delta T_\gamma(\theta,\phi)}{T_\gamma} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}\, Y_\ell^m(\theta,\phi)$$

- Angular power spectrum:

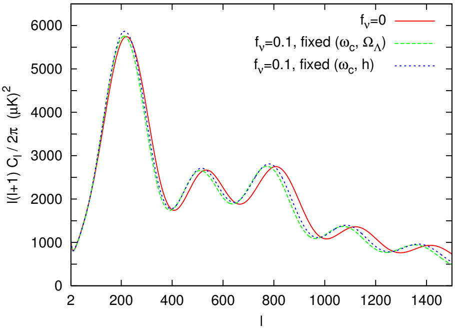
$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \langle |a_{\ell m}|^2 \rangle$$

- C_ℓ are the variances of the multipole moments $a_{\ell m}$.

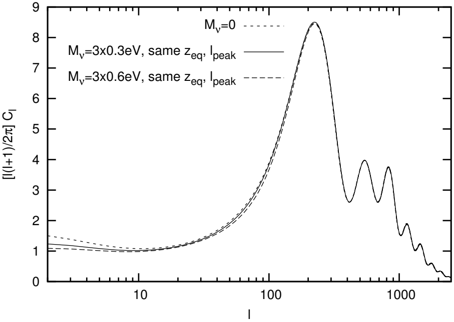
- Gaussian fluctuations are completely characterized by the variances C_ℓ.



- Fixed ω_B^0, ω_M^0, Ω_Λ^0



- Fixed ω_B^0, ω_{CDM}^0 $\Rightarrow$ z_{eq} fixed



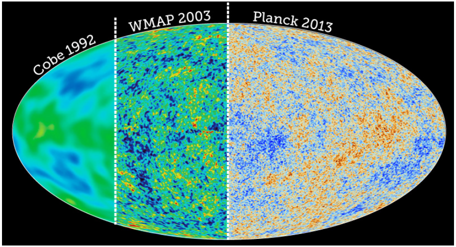
[Lesgourgues, Pastor, New J. Phys. 16 (2014) 065002]

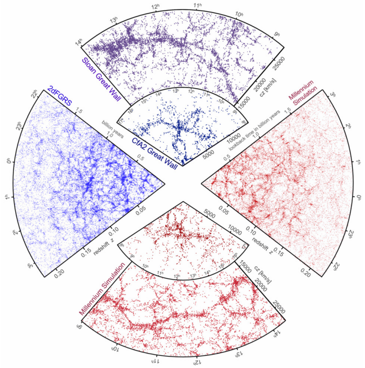
- ΛMDM: Λ Mixed Dark Matter model, where Mixed refers to the inclusion of some HDM component.
- Flat ΛCDM parameters:

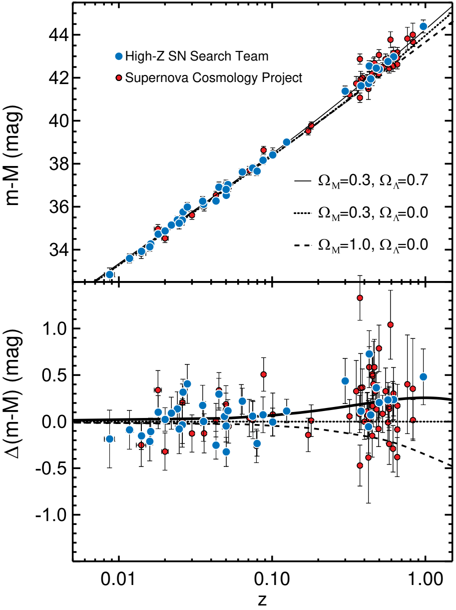
$$\omega_{\mathrm{B}}^{0},\ \omega_{\mathrm{M}}^{0},\ \Omega_{\Lambda}^{0},\ A_s,\ n_s,\ \tau$$

- Some of the parameters of the ΛMDM model have been varied together with $M_\nu = \sum_i m_i$ in order to keep fixed the redshift of equality and the angular diameter distance to last scattering.
- We conclude that the CMB alone is not a very powerful tool for constraining sub-eV neutrino masses, and should be used in combination with homogeneous cosmology constraints and/or measurements of the LSS power spectrum, for instance from galaxy clustering, galaxy lensing or CMB lensing.

Cosmological Bound on Neutrino Masses







WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

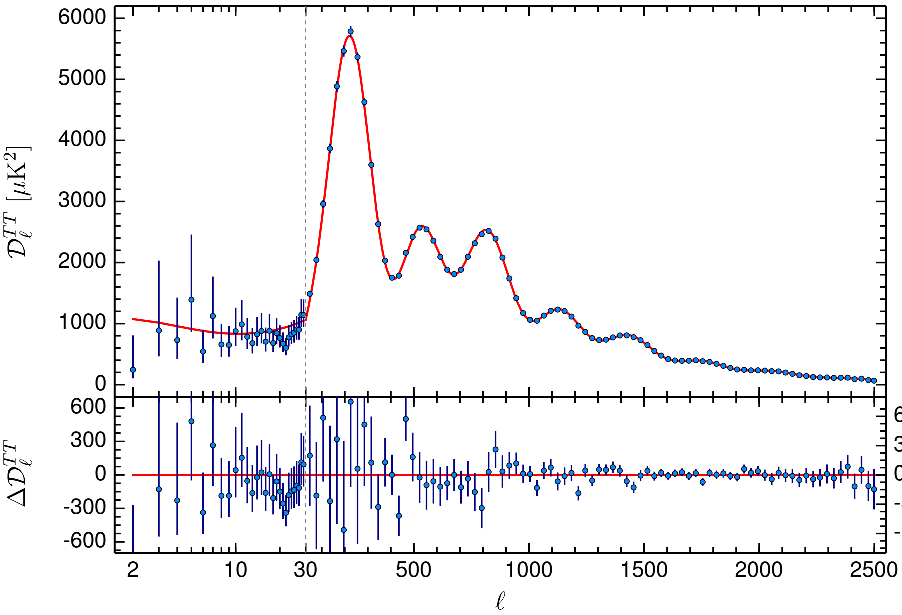
[PRD 78 (2008) 033010, hep-ph/0805.2517]

Flat ΛCDM

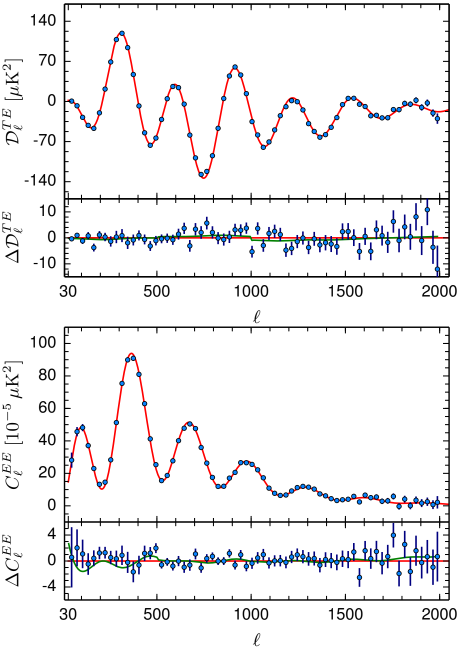
Case	Cosmological data set	$\sum_i m_i$ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

Planck

[arXiv:1502.01589]

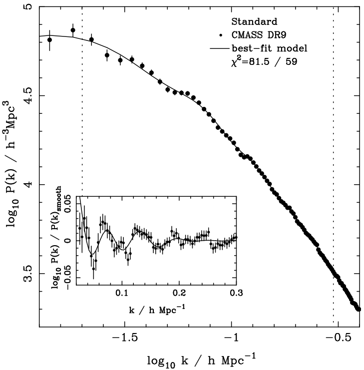


Planck Polarization Data

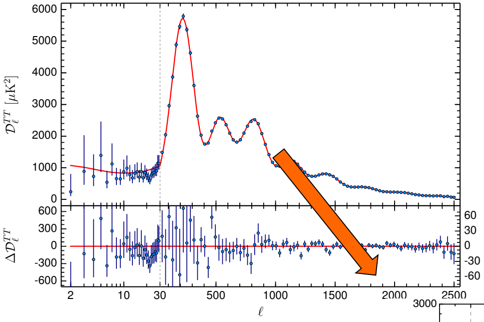


Planck Terminology

- TT denotes the Plank TT data (low-ℓ for $\ell < 30$ and high-ℓ for $\ell \geq 30$).
- lowP denotes the Planck polarization data at multipoles $\ell < 30$ (low-ℓ).
- TE denotes the Plank TE data at $\ell \geq 30$.
- EE denotes the Plank EE data at $\ell \geq 30$.
- Lensing denotes the Plank weak lensing data.
- BAO denotes the Baryon Acoustic Oscillation data.



Baryon Oscillation Spectroscopic Survey (BOSS)
part of the Sloan Digital Sky Survey III (SDSS-III)
Data Release 9 (DR9) CMASS sample
[arXiv:1203.6594]

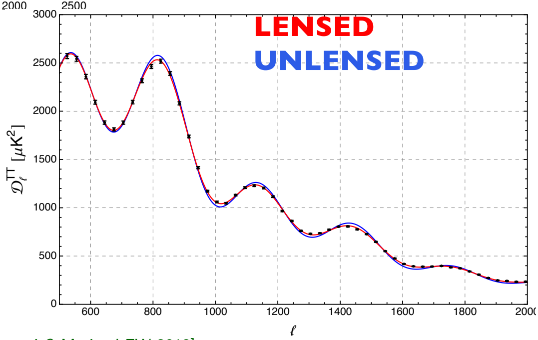


Lensing smooths the peaks of the CMB power spectrum…

… and introduces nongaussianities in the map (nonzero 4-point c.f.)

Neutrino free streaming damps matter perturbations and *reduces* lensing

The effect is proportional to ν energy density

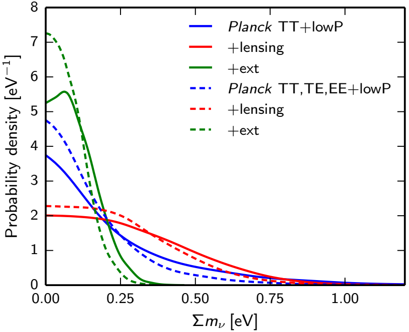


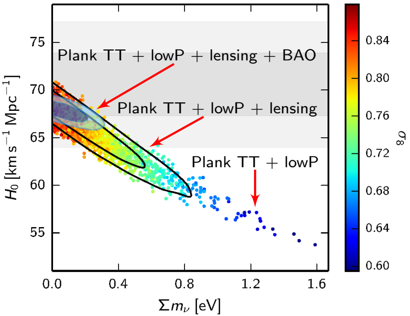
[M. Lattanzi @ Moriond EW 2018]

Planck Limits on $\sum m_\nu$

[Planck, A&A 594 (2016) A13, arXiv:1502.01589]

Cosmological data set	$\sum m_\nu$ (95% C.L.)
Plank TT + lowP	< 0.72 eV
Plank TT + lowP + BAO	< 0.21 eV
Plank TT,TE,EE + lowP	< 0.49 eV
Plank TT,TE,EE + lowP + BAO	< 0.17 eV
Plank TT + lowP + lensing	< 0.68 eV
Plank TT,TE,EE + lowP + lensing	< 0.59 eV
Plank TT + lowP + lensing + BAO + JLA + H_0	< 0.23 eV

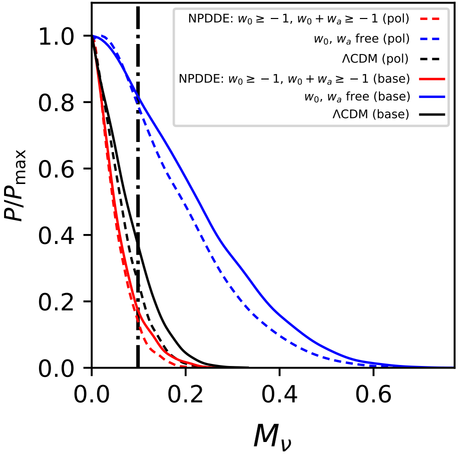




	Model	95% CL (eV)	Ref.
CMB alone			
Pl15[TT+lowP]	ΛCDM+$\sum m_\nu$	< 0.72	[29]
Pl15[TT+lowP]	ΛCDM+$\sum m_\nu$+N_{eff}	< 0.73	[35]
Pl16[TT+SimLow]	ΛCDM+$\sum m_\nu$	< 0.59	[32]
CMB + probes of background evolution			
Pl15[TT+lowP] + BAO	ΛCDM+$\sum m_\nu$	< 0.21	[29]
Pl15[TT+lowP] + JLA	ΛCDM+$\sum m_\nu$	< 0.33	[35]
Pl15[TT+lowP] + BAO	ΛCDM+$\sum m_\nu$+N_{eff}	< 0.27	[35]
CMB + probes of background evolution + LSS			
Pl15[TT+lowP+lensing]	ΛCDM+$\sum m_\nu$	< 0.68	[29]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$	< 0.25	[35]
Pl15[TT+lowP] + $P(k)_{\text{DR12}}$	ΛCDM+$\sum m_\nu$	< 0.30	[50]
Pl15[TT,TE,EE+lowP] + BAO+ $P(k)_{\text{WZ}}$	ΛCDM+$\sum m_\nu$	< 0.14	[52]
Pl15[TT,TE,EE+lowP] + BAO+ $P(k)_{\text{DR7}}$	ΛCDM+$\sum m_\nu$	< 0.13	[52]
Pl15[TT+lowP+lensing] + Lyα	ΛCDM+$\sum m_\nu$	< 0.12	[48]
Pl16[TT+SimLow+lensing] + BAO	ΛCDM+$\sum m_\nu$	< 0.17	[48]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$+Ω_k	< 0.37	[35]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$+w	< 0.37	[35]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$+N_{eff}	< 0.32	[29]
Pl15[TT,TE,EE+lowP+lensing]	ΛCDM+$\sum m_\nu$+5-params.	< 0.66	[34]

[Lesgourgues, Verde, Review of Particle Physics 2017]

- The neutrino mass bound can be loosened in extended cosmological models.
- For example with a varying Dark Energy equation of state.



[Vagnozzi et al, arXiv:1801.08553]

$$p_{DDE} = w_{DDE}\, \varrho_{DDE}$$

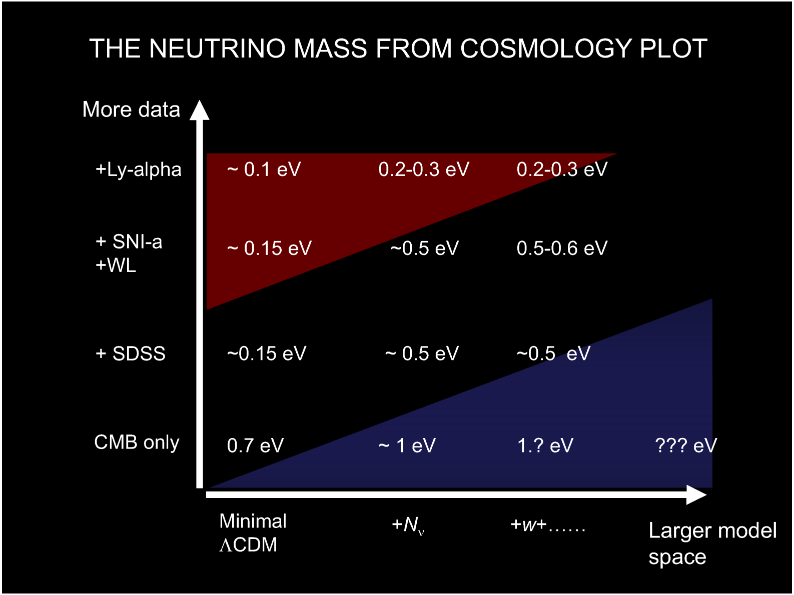
DDE: Dynamical Dark Energy

$$w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$$

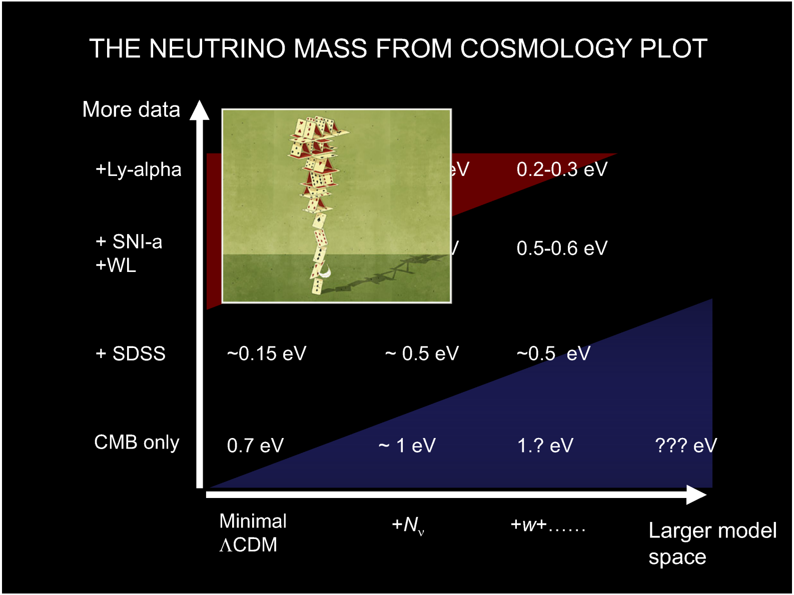
NPDDE: Non-Phantom DDE

$$w_{DE}(z) \geq -1$$

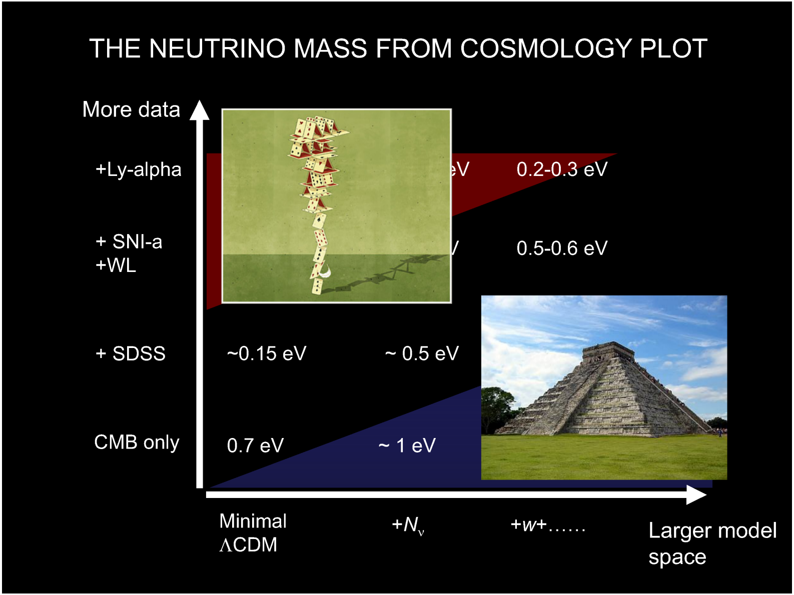
$$w_0 \geq -1 \qquad w_0 + w_a \geq -1$$



[S. Hannestad, 2018]

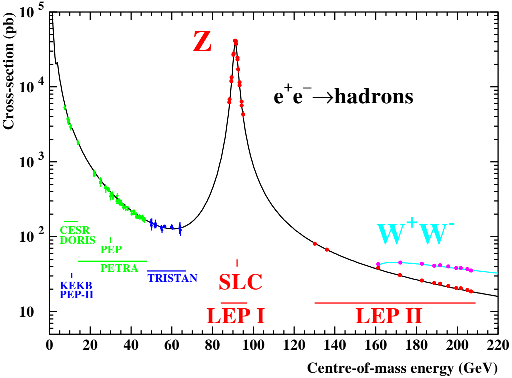


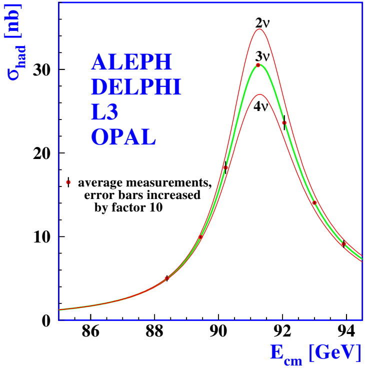
[S. Hannestad, 2018]



[S. Hannestad, 2018]

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$N_{\nu_{\text{active}}}^{\text{LEP}} = 2.9840 \pm 0.0082$$

$$e^+ e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots$$

Sterile Neutrinos

- Sterile means no standard model interactions

  [Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

- Obviously no electromagnetic interactions as normal active neutrinos

- Thus sterile means no standard weak interactions

- But sterile neutrinos are not absolutely sterile:

  - Gravitational Interactions

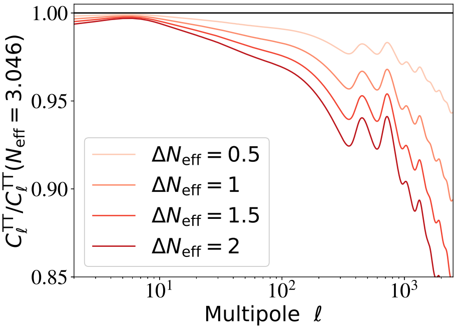
  - New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos

- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)

- Observables:
  - Disappearance of active neutrinos (neutral current deficit) $\leftarrow$ CEνNS
  - Indirect evidence through combined fit of data (current indication)

- Powerful window on new physics beyond the Standard Model

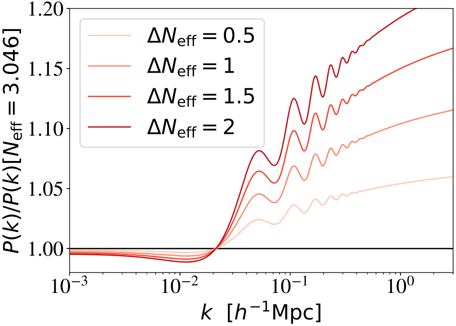
Dark Radiation



- Photons feel gravitational forces from a denser neutrino component.
- Decreases the acoustic peaks because the distribution of free-streaming neutrinos is smoother that that of the photons.

- $\varrho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\varrho_\gamma$
- $\Delta N_{\rm eff} = N_{\rm eff} - 3.046$
- Fixed $z_{\rm eq}$, z_Λ, ω_B^0
- $z_{\rm eq} \simeq \frac{\Omega_M^0 h^2}{\omega_\gamma^0 (1 + 0.227 N_{\rm eff})}$
- $z_\Lambda \simeq \left(\frac{\Omega_\Lambda^0}{\Omega_M^0}\right)^{1/3} \simeq \left(\frac{1-\Omega_M^0}{\Omega_M^0}\right)^{1/3}$
- Therefore fixed Ω_M^0
- $\omega_B^0 = \Omega_B^0 h^2$
- It can be done by increasing h^2 and decreasing Ω_B^0 with an increase of $\Omega_{CDM}^0 = \Omega_M^0 - \Omega_B^0$

Dark Radiation



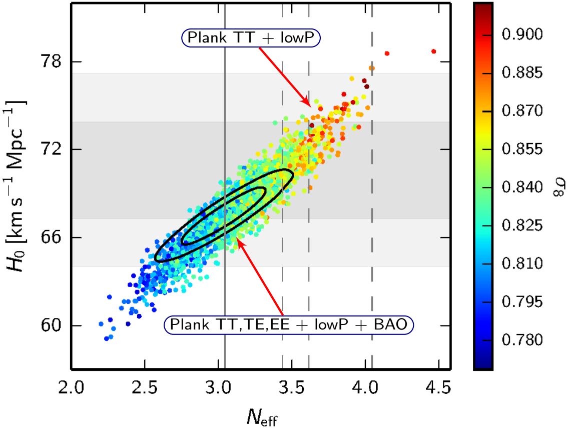
- Increased fluctuations due to increased $\Omega^0_{\rm CDM}$.
- Decreased BAO due to decreased $\Omega^0_{\rm B}$.

- $\varrho_{\rm R} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\rm eff} \right] \varrho_\gamma$
- $\Delta N_{\rm eff} = N_{\rm eff} - 3.046$
- Fixed $z_{\rm eq}$, z_Λ, $\omega^0_{\rm B}$
- $z_{\rm eq} \simeq \dfrac{\Omega^0_{\rm M} h^2}{\omega^0_\gamma (1 + 0.227 N_{\rm eff})}$
- $z_\Lambda \simeq \left(\dfrac{\Omega^0_\Lambda}{\Omega^0_{\rm M}} \right)^{1/3} \simeq \left(\dfrac{1 - \Omega^0_{\rm M}}{\Omega^0_{\rm M}} \right)^{1/3}$
- Therefore fixed $\Omega^0_{\rm M}$
- $\omega^0_{\rm B} = \Omega^0_{\rm B} h^2$
- It can be done by increasing h^2 and decreasing $\Omega^0_{\rm B}$ with an increase of $\Omega^0_{\rm CDM} = \Omega^0_{\rm M} - \Omega^0_{\rm B}$

Planck Limits on Dark Radiation

[Planck, A&A 594 (2016) A13, arXiv:1502.01589]

Cosmological data set	N_{eff}
Plank TT + lowP	3.13 ± 0.32
Plank TT + lowP + BAO	3.15 ± 0.23
Plank TT,TE,EE + lowP	2.99 ± 0.20
Plank TT,TE,EE + lowP + BAO	3.04 ± 0.18



Massive Sterile Neutrinos

- sterile neutrinos can be produced by $\nu_{e,\mu,\tau} \to \nu_s$ oscillations before active neutrino decoupling ($t_{\nu\text{-dec}} \sim 1\,\text{s}$)
- energy density of radiation before matter-radiation equality:

$$\varrho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \varrho_\gamma \qquad (t < t_{\text{eq}} \sim 6 \times 10^4\,\text{y})$$

$$N_{\text{eff}}^{\text{SM}} = 3.046 \qquad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$$

- sterile neutrino contribution:

$$\varrho_s = (T_s/T_\nu)^4 \varrho_\nu \quad \Longrightarrow \quad \Delta N_{\text{eff}} = (T_s/T_\nu)^4$$

- sterile neutrino $\nu_s \simeq \nu_4$ with mass $m_s = m_4 \sim 1\,\text{eV}$ becomes non-relativistic at $T_\nu \sim m_s/3$, that is at $t_{\nu_s\text{-nr}} \sim 2.0 \times 10^5\,\text{y}$, before recombination at $t_{\text{rec}} \sim 3.8 \times 10^5\,\text{y}$
- current energy density of sterile neutrinos:

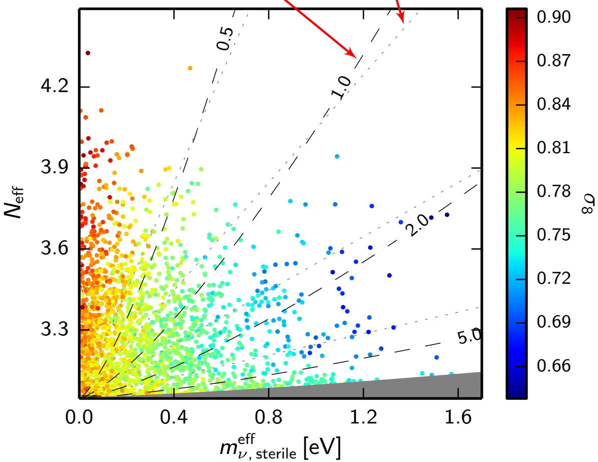
$$\Omega_s = \frac{n_s m_s}{\varrho_c} \simeq \frac{(T_s/T_\nu)^3 m_s}{93.1\,h^2\,\text{eV}} = \frac{\Delta N_{\text{eff}}^{3/4} m_s}{93.1\,h^2\,\text{eV}} = \frac{m_s^{\text{eff}}}{93.1\,h^2\,\text{eV}}$$

$$m_s^{\text{eff}} = \Delta N_{\text{eff}}^{3/4} m_s = (T_s/T_\nu)^3 m_s$$

Limits on Massive Sterile Neutrinos

$N_{\text{eff}} < 3.7$ $m_s^{\text{eff}} < 0.52$ (95%; Plank TT + lowP + lensing + BAO)

Constant m_s: Thermal and DW



- $m_s^{\text{eff}} \equiv 93.1 \Omega_s h^2$ eV
- Thermally distributed:

$$f_s(E) = \frac{1}{e^{E/T_s} + 1}$$

$$m_s^{\text{eff}} = \left(\frac{T_s}{T_\nu}\right)^3 m_s = (\Delta N_{\text{eff}})^{3/4} m_s$$

- Dodelson-Widrow:

$$f_s(E) = \frac{\chi_s}{e^{E/T_\nu} + 1}$$

$$m_s^{\text{eff}} = \chi_s \, m_s = \Delta N_{\text{eff}} \, m_s$$

Conclusions

- Normal light neutrinos are Hot Dark Matter.
- Their effects on cosmological observables depend on their masses.
- Cosmological data give information on neutrino physics, but it is model-dependent.
- Neutrino physics may contribute to solve tensions in the Cosmological data.
- Light sterile neutrinos are allowed only if their thermalization is suppressed.
- Heavy sterile neutrinos with mass of the order of keV can contribute to the Dark Matter (not discussed).