Neutrino Physics

Part I: Theory of Neutrino Masses and Mixing

Carlo Giunti

INFN, Torino, Italy

giunti@to.infn.it

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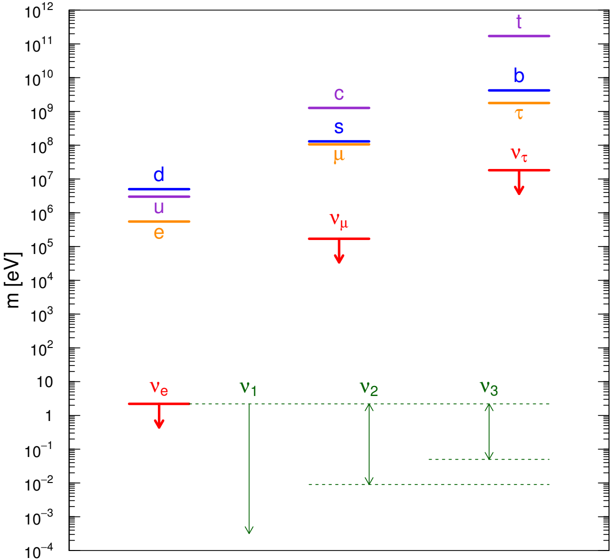
Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Sterile Neutrinos

Fermion Mass Spectrum



Dirac Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
  - Higgs Mechanism in SM
  - SM Extension: Dirac Neutrino Masses
  - Three-Generations Dirac Neutrino Masses
  - Mixing
  - CP Violation: Jarlskog Reparameterization Invariant
  - Lepton Numbers Violating Processes
- Majorana Neutrino Masses and Mixing
- Sterile Neutrinos

Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0 \qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$

- Dirac Lagrangian: $\mathscr{L}_{\text{D}}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$

- Chiral decomposition: $\nu_L \equiv P_L \nu\,, \qquad \nu_R \equiv P_R \nu\,, \qquad \nu = \nu_L + \nu_R$

  Left and Right-handed Projectors: $\quad P_L \equiv \dfrac{1-\gamma^5}{2}\,, \quad P_R \equiv \dfrac{1+\gamma^5}{2}$

$$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L + P_R = 1\,, \quad P_L P_R = P_R P_L = 0$$

$$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$$

- In SM only ν_L by assumption $\Longrightarrow$ no neutrino mass
  Note that all the other elementary fermion fields (charged leptons and quarks) have both left and right-handed components

- Oscillation experiments have shown that neutrinos are massive

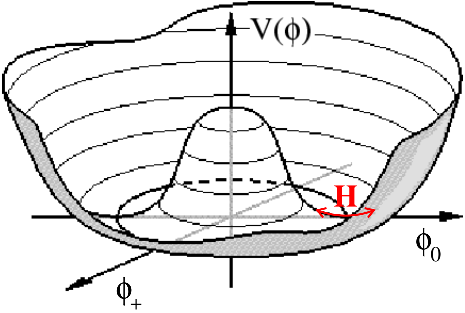
- Simplest and natural extension of the SM: consider also ν_R
  as for all the other elementary fermion fields

Higgs Mechanism in SM

- Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ $\qquad |\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$
- Higgs Lagrangian: $\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$
- Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$
- $\mu^2 < 0$ and $\lambda > 0$ $\implies$ $V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$

$$v \equiv \sqrt{-\frac{\mu^2}{\lambda}} = \left(\sqrt{2} G_F \right)^{-1/2} \simeq 246\,\text{GeV}$$

- Vacuum: V_{min} for $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- Spontaneous Symmetry Breaking: $SU(2)_L \times U(1)_Y \to U(1)_Q$



- Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \Longrightarrow |\Phi|^2 = \frac{v^2}{2} + vH + \frac{1}{2} H^2$

- $V = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \simeq 126\,\text{GeV}$$

$$-\mu^2 \simeq (89\,\text{GeV})^2 \qquad \lambda = -\frac{\mu^2}{v^2} \simeq 0.13$$

SM Extension: Dirac Neutrino Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -y^{\ell}\, \overline{L_L}\, \Phi\, \ell_R - y^{\nu}\, \overline{L_L}\, \widetilde{\Phi}\, \nu_R + \mathrm{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2\, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} = & -\frac{y^{\ell}}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & -\frac{y^{\nu}}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \mathrm{H.c.} \end{aligned}$$

$$\mathscr{L}_{H,\mathrm{L}} = -y^{\ell}\,\frac{v}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R - y^{\nu}\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R$$

$$-\frac{y^{\ell}}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R\,H - \frac{y^{\nu}}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R\,H + \mathrm{H.c.}$$

$$m_\ell = y^{\ell}\,\frac{v}{\sqrt{2}} \qquad\qquad m_\nu = y^{\nu}\,\frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_\ell}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_\nu}{v}$$

$$v = \left(\sqrt{2}\,G_{\mathrm{F}}\right)^{-1/2} = 246\,\mathrm{GeV}$$

PROBLEM: $y^{\nu} \lesssim 10^{-11} \ll y^{e} \sim 10^{-6}$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \text{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{\ell'_{\alpha L}} \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{\nu'_{\alpha L}} \, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \left[\overline{\boldsymbol{\ell}'_L} \, Y'^{\ell} \, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L} \, Y'^{\nu} \, \boldsymbol{\nu}'_R \right] + \mathrm{H.c.}$$

$$\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \boldsymbol{\ell}'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y'^{\ell} \equiv \begin{pmatrix} Y'^{\ell}_{ee} & Y'^{\ell}_{e\mu} & Y'^{\ell}_{e\tau} \\ Y'^{\ell}_{\mu e} & Y'^{\ell}_{\mu\mu} & Y'^{\ell}_{\mu\tau} \\ Y'^{\ell}_{\tau e} & Y'^{\ell}_{\tau\mu} & Y'^{\ell}_{\tau\tau} \end{pmatrix} \qquad Y'^{\nu} \equiv \begin{pmatrix} Y'^{\nu}_{ee} & Y'^{\nu}_{e\mu} & Y'^{\nu}_{e\tau} \\ Y'^{\nu}_{\mu e} & Y'^{\nu}_{\mu\mu} & Y'^{\nu}_{\mu\tau} \\ Y'^{\nu}_{\tau e} & Y'^{\nu}_{\tau\mu} & Y'^{\nu}_{\tau\tau} \end{pmatrix}$$

$$M'^{\ell} = \frac{v}{\sqrt{2}} \, Y'^{\ell} \qquad M'^{\nu} = \frac{v}{\sqrt{2}} \, Y'^{\nu}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}'_L}\,Y'^{\ell}\,\boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\,Y'^{\nu}\,\boldsymbol{\nu}'_R\right] + \mathrm{H.c.}$$

Diagonalization of Y'^{ℓ} and Y'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\,\boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\,\boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\,\boldsymbol{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\,\boldsymbol{n}_R$$

Important general remark: unitary transformations are allowed because they leave invariant the kinetic terms in the Lagrangian

$$\begin{aligned}\mathscr{L}_{\mathrm{kin}} &= \overline{\boldsymbol{\ell}'_L} i\partial\!\!\!/ \boldsymbol{\ell}'_L + \overline{\boldsymbol{\ell}'_R} i\partial\!\!\!/ \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L} i\partial\!\!\!/ \boldsymbol{\nu}'_L + \overline{\boldsymbol{\nu}'_R} i\partial\!\!\!/ \boldsymbol{\nu}'_R \\ &= \overline{\boldsymbol{\ell}_L} V^{\ell\dagger}_L i\partial\!\!\!/ V^{\ell}_L \boldsymbol{\ell}_L + \ldots \\ &= \overline{\boldsymbol{\ell}_L} i\partial\!\!\!/ \boldsymbol{\ell}_L + \overline{\boldsymbol{\ell}_R} i\partial\!\!\!/ \boldsymbol{\ell}_R + \overline{\boldsymbol{\nu}_L} i\partial\!\!\!/ \boldsymbol{\nu}_L + \overline{\boldsymbol{\nu}_R} i\partial\!\!\!/ \boldsymbol{\nu}_R\end{aligned}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}'_L}\, Y'^{\ell}\, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\, Y'^{\nu}\, \boldsymbol{\nu}'_R\right] + \text{H.c.}$$

Diagonalization of Y'^{ℓ} and Y'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \boldsymbol{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \boldsymbol{n}_R$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L}\, V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R \boldsymbol{\ell}_R + \overline{\boldsymbol{n}_L}\, V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R \boldsymbol{n}_R\right] + \text{H.c.}$$

$$V^{\ell\dagger}_L \; Y'^{\ell} \; V^{\ell}_R = Y^{\ell} \qquad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha}\, \delta_{\alpha\beta} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$V^{\nu\dagger}_L \; Y'^{\nu} \; V^{\nu}_R = Y^{\nu} \qquad Y^{\nu}_{kj} = y^{\nu}_{k}\, \delta_{kj} \qquad (k, j = 1, 2, 3)$$

Real and Positive y^{ℓ}_{α}, y^{ν}_{k}

$$\underset{9}{V^{\dagger}_L} \quad \underset{18}{Y'} \quad \underset{9}{V_R} \quad = \quad \underset{3}{Y}$$

Massive Chiral Lepton Fields

$V_L^{\ell\dagger}\,\boldsymbol{\ell}'_L = \boldsymbol{\ell}_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$V_R^{\ell\dagger}\,\boldsymbol{\ell}'_R = \boldsymbol{\ell}_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$V_L^{\nu\dagger}\,\boldsymbol{\nu}'_L = \boldsymbol{n}_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$V_R^{\nu\dagger}\,\boldsymbol{\nu}'_R = \boldsymbol{n}_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L}\,Y^{\ell}\,\boldsymbol{\ell}_R + \overline{\boldsymbol{n}_L}\,Y^{\nu}\,\boldsymbol{n}_R\right] + \text{H.c.}$$

$$= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y^{\ell}_{\alpha}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y^{\nu}_{k}\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \text{H.c.}$$

Massive Dirac Lepton Fields

$$\ell_\alpha \equiv \ell_{\alpha L} + \ell_{\alpha R} \qquad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \qquad (k = 1, 2, 3)$$

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell \, v}{\sqrt{2}} \, \overline{\ell_\alpha} \, \ell_\alpha - \sum_{k=1}^{3} \frac{y_k^\nu \, v}{\sqrt{2}} \, \overline{\nu_k} \, \nu_k \qquad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell}{\sqrt{2}} \, \overline{\ell_\alpha} \, \ell_\alpha \, H - \sum_{k=1}^{3} \frac{y_k^\nu}{\sqrt{2}} \, \overline{\nu_k} \, \nu_k \, H \qquad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^\ell v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \qquad\qquad m_k = \frac{y_k^\nu \, v}{\sqrt{2}} \qquad (k = 1, 2, 3)$$

Lepton-Higgs coupling $\propto$ Lepton Mass

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathsf{I}}^{(\mathsf{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \text{H.c.}$$

Weak Charged Current: $j_W^\rho = j_{W,\mathsf{L}}^\rho + j_{W,\mathsf{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\mathsf{L}}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\ell'_{\alpha L}} \gamma^\rho \nu'_{\alpha L} = 2 \overline{\boldsymbol{\ell}'_L} \gamma^\rho \boldsymbol{\nu}'_L$$

$$\boldsymbol{\ell}'_L = V_L^\ell \boldsymbol{\ell}_L \qquad \boldsymbol{\nu}'_L = V_L^\nu \boldsymbol{n}_L$$

$$j_{W,\mathsf{L}}^{\rho\dagger} = 2 \overline{\boldsymbol{\ell}_L} V_L^{\ell\dagger} \gamma^\rho V_L^\nu \boldsymbol{n}_L = 2 \overline{\boldsymbol{\ell}_L} \gamma^\rho V_L^{\ell\dagger} V_L^\nu \boldsymbol{n}_L = 2 \overline{\boldsymbol{\ell}_L} \gamma^\rho U \boldsymbol{n}_L$$

Mixing Matrix: $\boxed{U = V_L^{\ell\dagger} V_L^\nu}$

- Definition: Left-Handed Flavor Neutrino Fields

$$\boldsymbol{\nu}_L = U\,\boldsymbol{n}_L = V_L^{\ell\dagger}\,V_L^{\nu}\,\boldsymbol{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,\boldsymbol{\nu}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\ell_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L}$$

- Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton:

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2\left(\overline{e_L}\,\gamma^\rho\,\nu_{eL} + \overline{\mu_L}\,\gamma^\rho\,\nu_{\mu L} + \overline{\tau_L}\,\gamma^\rho\,\nu_{\tau L}\right)$$

- In practice left-handed flavor neutrino fields are useful for calculations in the SM approximation of massless neutrinos (interactions).

- If neutrino masses must be taken into account, it is necessary to use

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,U\,\boldsymbol{n}_L = 2\sum_{\alpha=e,\mu,\tau}\sum_{k=1}^{3}\overline{\ell_{\alpha L}}\,\gamma^\rho\,U_{\alpha k}\,\nu_{kL}$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Standard Model: Lepton numbers are conserved

- L_e, L_μ, L_τ are conserved in the Standard Model with massless neutrinos

- Dirac mass term:

$$\mathscr{L}^{\text{D}} = -\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m^{\text{D}}_{ee} & m^{\text{D}}_{e\mu} & m^{\text{D}}_{e\tau} \\ m^{\text{D}}_{\mu e} & m^{\text{D}}_{\mu\mu} & m^{\text{D}}_{\mu\tau} \\ m^{\text{D}}_{\tau e} & m^{\text{D}}_{\tau\mu} & m^{\text{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

- L is conserved: $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \implies |\Delta L| = 0$

Mixing Matrix

- $U = V_L^{\ell\dagger} V_L^{\nu} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$
- A unitary $N \times N$ matrix depends on N^2 independent real parameters:

$$N = 3 \quad \Longrightarrow \quad \begin{array}{ll} \dfrac{N(N-1)}{2} = 3 & \text{Mixing Angles} \\ \dfrac{N(N+1)}{2} = 6 & \text{Phases} \end{array}$$

- Not all phases are physical observables!
- Neutrino Lagrangian:

kinetic terms + mass terms + weak interactions

- Mixing is due to the diagonalization of the mass terms.
- The kinetic terms are invariant under unitary transformations of the fermion fields.
- What is the effect of mixing in weak interactions?

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)

$$\ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau) \, , \qquad \nu_k \to e^{i\varphi_k} \, \nu_k \quad (k = 1, 2, 3)$$

- Performing this transformation, the Weak Charged Current becomes

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, e^{-i\varphi_\alpha} \, \gamma^{\rho} \, U_{\alpha k} \, e^{i\varphi_k} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \, \underbrace{e^{-i(\varphi_e - \varphi_1)}}_{1} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_\alpha - \varphi_e)}}_{2} \, \gamma^{\rho} \, U_{\alpha k} \, \underbrace{e^{i(\varphi_k - \varphi_1)}}_{2} \, \nu_{kL}$$

- There are 5 independent combinations of the phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the lepton fields leaves the Weak Charged Current invariant $\Longleftrightarrow$ conservation of Total Lepton Number.

- The mixing matrix contains 1 Physical Phase.

- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

  3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad$ $s_{ab} \equiv \sin\vartheta_{ab}$ $\quad$ $0 \le \vartheta_{ab} \le \frac{\pi}{2}$ $\quad$ $0 \le \delta_{13} < 2\pi$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23}e^{i\delta_{23}} \\ 0 & -s_{23}e^{-i\delta_{13}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12}e^{-i\delta'_{12}} & 0 \\ -s'_{12}e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

CP Violation: Jarlskog Reparameterization Invariant

- There is CPV if $U^* \neq U$.
- Simplest invariants: $|U_{\alpha k}|^2 = U_{\alpha k} U_{\alpha k}^* \,, \quad U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$
- Simplest CPV invariants: $\text{Im}\big[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}\big] = \pm J$

$$\text{Jarlskog invariant:} \quad J = \text{Im}\big[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}\big] = \text{Im} \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- In standard parameterization:

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos\vartheta_{13} \sin 2\vartheta_{13} \sin\delta_{13} \end{aligned}$$

- For CPV all mixing angles must be different from 0 and $\pi/2$!
- The Jarlskog invariant is useful for quantifying CPV in a parameterization-independent way.
- All measurable CPV effects depend on J.

GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- Neutral-Current Weak Interaction Lagrangian:

$$\mathscr{L}_{\text{I}}^{(\text{NC})} = -\frac{g}{2\cos\vartheta_{\text{W}}} j_Z^\rho Z_\rho \qquad j_Z^\rho = j_{Z,\text{L}}^\rho + j_{Z,\text{Q}}^\rho$$

- Leptonic Weak Neutral Current: $(g_L^\nu = \frac{1}{2},\, g_L^\ell = -\frac{1}{2} + \sin^2\vartheta_{\text{W}},\, g_R^\ell = \sin^2\vartheta_{\text{W}})$

$$j_{Z,\text{L}}^\rho = 2g_L^\nu\, \overline{\boldsymbol{\nu}'_L}\, \gamma^\rho\, \boldsymbol{\nu}'_L + 2g_L^\ell\, \overline{\boldsymbol{\ell}'_L}\, \gamma^\rho \boldsymbol{\ell}'_L + 2g_R^\ell\, \overline{\boldsymbol{\ell}'_R}\, \gamma^\rho \boldsymbol{\ell}'_R$$

- Invariant under mixing transformations with unitary V_L^ℓ, V_R^ℓ, V_L^ν:

$$\begin{aligned} j_{Z,\text{L}}^\rho &= 2g_L^\nu\, \overline{\boldsymbol{n}_L}\, V_L^{\nu\dagger}\, \gamma^\rho\, V_L^\nu\, \boldsymbol{n}_L + 2g_L^\ell\, \overline{\boldsymbol{\ell}_L}\, V_L^{\ell\dagger}\, \gamma^\rho\, V_L^\ell\, \boldsymbol{\ell}_L + 2g_R^\ell\, \overline{\boldsymbol{\ell}_R}\, V_R^{\ell\dagger}\, \gamma^\rho\, V_R^\ell\, \boldsymbol{\ell}_R \\ &= 2g_L^\nu\, \overline{\boldsymbol{n}_L}\, \gamma^\rho\, \boldsymbol{n}_L + 2g_L^\ell\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2g_R^\ell\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \end{aligned}$$

- Invariant also under the mixing transformation $\boldsymbol{\nu}_L = U\, \boldsymbol{n}_L$ which defines the flavor neutrino fields:

$$\begin{aligned} j_{Z,\text{L}}^\rho &= 2g_L^\nu\, \overline{\boldsymbol{\nu}_L}\, U\, \gamma^\rho\, U^\dagger\, \boldsymbol{\nu}_L + 2g_L^\ell\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2g_R^\ell\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \\ &= 2g_L^\nu\, \overline{\boldsymbol{\nu}_L}\, \gamma^\rho\, \boldsymbol{\nu}_L + 2g_L^\ell\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2g_R^\ell\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \end{aligned}$$

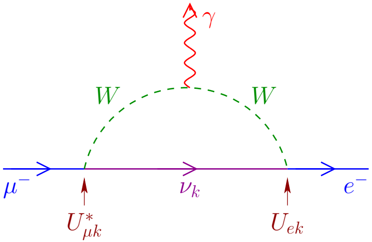
- Mixing has no effect in neutral-current weak interactions.

Lepton Numbers Violating Processes

Dirac mass term allows L_e, L_μ, L_τ violating processes

Example: $\mu^\pm \to e^\pm + \gamma\,, \qquad \mu^\pm \to e^\pm + e^+ + e^-$

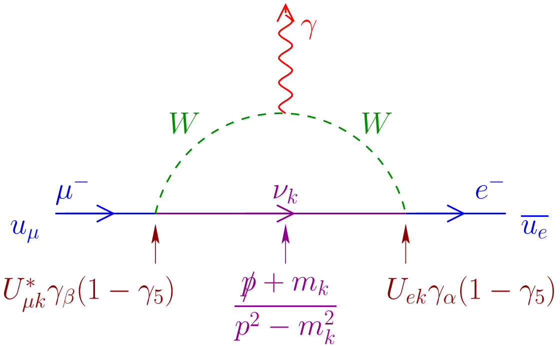
$$\boxed{\mu^- \to e^- + \gamma}$$



$$\sum_k U_{\mu k}^* U_{ek} = 0 \quad \Longrightarrow \quad \text{GIM suppression:} \quad A \propto \sum_k U_{\mu k}^* U_{ek} f(m_k)$$

$$\mathscr{L}_{\text{I}}^{(\text{CC})} = -\frac{g}{2\sqrt{2}} W^{\alpha} \left[\overline{\nu_e} \gamma_\alpha (1-\gamma_5) e + \overline{\nu_\mu} \gamma_\alpha (1-\gamma_5) \mu + \ldots \right]$$

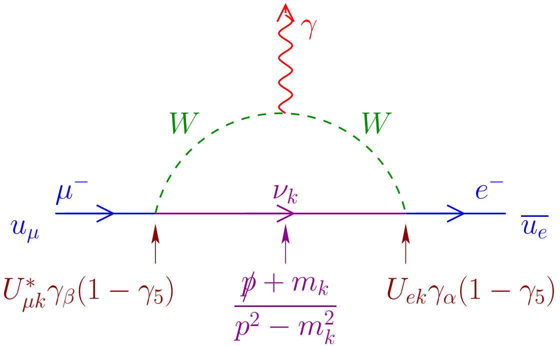
$$= -\frac{g}{2\sqrt{2}} W^{\alpha} \sum_k \left[\overline{\nu_k} U_{ek}^{*} \gamma_\alpha (1-\gamma_5) e + \overline{\nu_k} U_{\mu k}^{*} \gamma_\alpha (1-\gamma_5) \mu + \ldots \right]$$



$$A \propto \sum_k \overline{u_e} U_{ek} \gamma_\alpha (1-\gamma_5) \frac{\not{p} + m_k}{p^2 - m_k^2} U_{\mu k}^{*} \gamma_\beta (1-\gamma_5) u_\mu$$

$$\mathscr{L}_I^{(CC)} = -\frac{g}{2\sqrt{2}} W^\alpha \left[\overline{\nu_e}\gamma_\alpha (1-\gamma_5) e + \overline{\nu_\mu}\gamma_\alpha (1-\gamma_5) \mu + \ldots\right]$$

$$= -\frac{g}{2\sqrt{2}} W^\alpha \sum_k \left[\overline{\nu_k} U_{ek}^* \gamma_\alpha (1-\gamma_5) e + \overline{\nu_k} U_{\mu k}^* \gamma_\alpha (1-\gamma_5) \mu + \ldots\right]$$



$$A \propto \sum_k \overline{u_e} U_{ek} \gamma_\alpha (1-\gamma_5) \frac{\not{p} + \cancel{m_k}}{p^2 - m_k^2} (1+\gamma_5) \gamma_\beta U_{\mu k}^* u_\mu$$

$$\frac{1}{p^2 - m_k^2} = p^{-2}\left(1 - \frac{m_k^2}{p^2}\right)^{-1} \simeq p^{-2}\left(1 + \frac{m_k^2}{p^2}\right)$$

$$A \propto \sum_k U_{ek} U_{\mu k}^* \left(1 + \frac{m_k^2}{p^2}\right) = \sum_k U_{ek} U_{\mu k}^* \frac{m_k^2}{p^2} \to \sum_k U_{ek} U_{\mu k}^* \frac{m_k^2}{m_W^2}$$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi} \left| \sum_k U_{ek} U_{\mu k}^* \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$

[Petcov, SJNP 25 (1977) 340; Bilenky, Petcov, Pontecorvo, PLB 67 (1977) 309]

[Lee, Shrock, PRD 16 (1977) 1444]

Suppression factor: $\frac{m_k}{m_W} \lesssim 10^{-11}$ for $m_k \lesssim 1\,\text{eV}$

$(\text{BR})_{\text{the}} \lesssim 10^{-47}$ $(\text{BR})_{\text{exp}} \lesssim 10^{-11}$

Majorana Neutrino Masses and Mixing

Two-Component Theory of a Massless Neutrino

[Landau, NP 3 (1957) 127; Lee, Yang, PR 105 (1957) 1671; Salam, NC 5 (1957) 299]

- Dirac Equation: $(i\gamma^\mu \partial_\mu - m)\,\psi = 0$
- Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$i\gamma^\mu \partial_\mu \psi_L = m\,\psi_R$$
$$i\gamma^\mu \partial_\mu \psi_R = m\,\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^\mu \partial_\mu \psi_L = 0$$
$$i\gamma^\mu \partial_\mu \psi_R = 0$$

- A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor), which has only two independent components (half the number of degrees of freedom of a Dirac field, which has four independent components).

- Chiral representation of γ matrices:

$$\gamma^0 = \begin{pmatrix} 0 & -\mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix} \qquad P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}$$

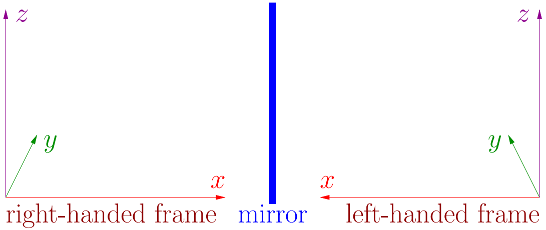
- Four-components Dirac spinor: $\psi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

- The Weyl spinors ψ_L and ψ_R have only two components:

$$\psi_L = P_L \psi = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \qquad \psi_R = P_R \psi = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to parity violation ($\psi_L \overset{P}{\rightleftharpoons} \psi_R$)

- Parity is the symmetry of space inversion (mirror transformation)



- Parity was considered to be an exact symmetry of nature

- 1956: Lee and Yang understand that Parity can be violated in Weak Interactions (1957 Physics Nobel Prize)

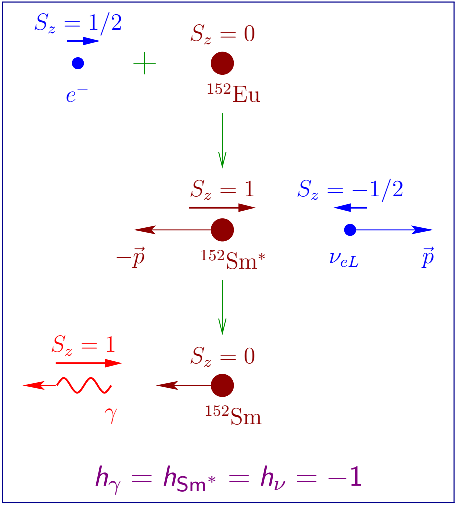
- 1957: Wu et al. discover Parity violation in β-decay of ^{60}Co

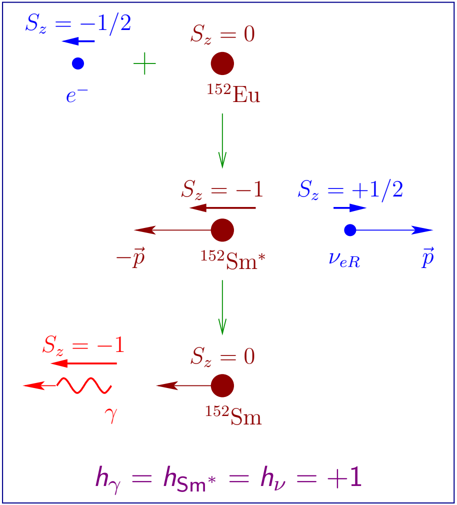
- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields $\Longrightarrow$ Two-component Theory of a Massless Neutrino (1957)

- 1958: Goldhaber, Grodzins and Sunyar measure the neutrino helicity with the electron capture process

$$
\begin{array}{l}
e^{-} + {}^{152}\text{Eu} \to {}^{152}\text{Sm}^{*} + \nu_{e} \\
\qquad\qquad\qquad\quad \downarrow \\
\qquad\qquad\qquad\quad {}^{152}\text{Sm}^{*} \to {}^{152}\text{Sm} + \gamma
\end{array}
$$

The neutrino helicity is the same as the measurable helicity of the photon when it is emitted in the same direction of the ${}^{152}\text{Sm}^{*}$ recoil.





$h_\gamma = -0.91 \pm 0.19 \Longrightarrow$ NEUTRINOS ARE LEFT-HANDED: ν_L

[Goldhaber, Grodzins and Sunyar, PR 109 (1958) 1015]

Quantization

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3\, 2E} \sum_{h=\pm1} \left[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + b^{(h)\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x} \right]$$

$$p^0 = E = \sqrt{\vec{p}^2 + m^2}$$

$$(\not{p} - m)\, u^{(h)}(p) = 0$$
$$(\not{p} + m)\, v^{(h)}(p) = 0$$

$$\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|}\, u^{(h)}(p) = h\, u^{(h)}(p)$$
$$\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|}\, v^{(h)}(p) = -h\, v^{(h)}(p)$$

$$\{a^{(h)}(p), a^{(h')\dagger}(p')\} = \{b^{(h)}(p), b^{(h')\dagger}(p')\} = (2\pi)^3\, 2E\, \delta^3(\vec{p} - \vec{p}\,')\, \delta_{hh'}$$
$$\{a^{(h)}(p), a^{(h')}(p')\} = \{a^{(h)\dagger}(p), a^{(h')\dagger}(p')\} = 0$$
$$\{b^{(h)}(p), b^{(h')}(p')\} = \{b^{(h)\dagger}(p), b^{(h')\dagger}(p')\} = 0$$
$$\{a^{(h)}(p), b^{(h')}(p')\} = \{a^{(h)\dagger}(p), b^{(h')\dagger}(p')\} = 0$$
$$\{a^{(h)}(p), b^{(h')\dagger}(p')\} = \{a^{(h)\dagger}(p), b^{(h')}(p')\} = 0$$

- Left-handed neutrino: $|\nu_L(p)\rangle = |\nu(p, h = -1)\rangle = a^{(-)\dagger}(p)|0\rangle$
- Right-handed neutrino: $|\nu_R(p)\rangle = |\nu(p, h = +1)\rangle = a^{(+)\dagger}(p)|0\rangle$

- Left-handed antineutrino: $|\bar{\nu}_L(p)\rangle = |\bar{\nu}(p, h = -1)\rangle = b^{(-)\dagger}(p)|0\rangle$
- Right-handed antineutrino: $|\bar{\nu}_R(p)\rangle = |\bar{\nu}(p, h = +1)\rangle = b^{(+)\dagger}(p)|0\rangle$

Helicity and Chirality

$$\nu_L(x) = \int \frac{d^3p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)}(p)\, u_L^{(h)}(p)\, e^{-ip\cdot x} + b^{(h)\dagger}(p)\, v_L^{(h)}(p)\, e^{ip\cdot x} \right]$$

$$u^{(h)\dagger}(p) u^{(h)}(p) = 2E \qquad u^{(h)\dagger}(p)\gamma^5 u^{(h)}(p) = 2h|\vec{p}|$$

$$v^{(h)\dagger}(p) v^{(h)}(p) = 2E \qquad v^{(h)\dagger}(p)\gamma^5 v^{(h)}(p) = -2h|\vec{p}|$$

$$u_L^{(h)\dagger}(p) u_L^{(h)}(p) = u^{(h)\dagger}(p) \left(\frac{1-\gamma^5}{2} \right) u^{(h)}(p) = E - h|\vec{p}|$$

$$u_L^{(-)\dagger}(p) u_L^{(-)}(p) = E + |\vec{p}| \simeq 2E - \frac{m^2}{2E}$$ left-handed neutrinos

$$u_L^{(+)\dagger}(p) u_L^{(+)}(p) = E - |\vec{p}| \simeq \frac{m^2}{2E}$$ suppressed right-handed neutrinos

$$v_L^{(h)\dagger}(p) v_L^{(h)}(p) = v^{(h)\dagger}(p) \left(\frac{1-\gamma^5}{2} \right) v^{(h)}(p) = E + h|\vec{p}|$$

$$v_L^{(-)\dagger}(p) v_L^{(-)}(p) = E - |\vec{p}| \simeq \frac{m^2}{2E}$$ right-handed antineutrinos

$$v_L^{(+)\dagger}(p) v_L^{(+)}(p) = E + |\vec{p}| \simeq 2E - \frac{m^2}{2E}$$ suppressed left-handed antineutrinos

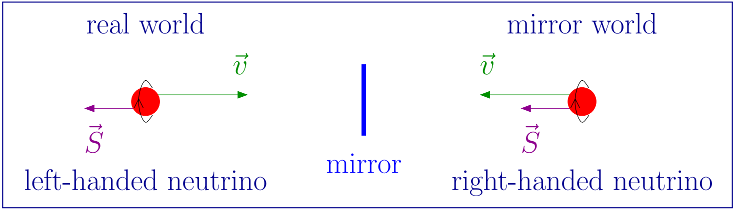
Standard Model

- Glashow (1961), Weinberg (1967) and Salam (1968) formulate the Standard Model of ElectroWeak Interactions (1979 Physics Nobel Prize) assuming that neutrinos are massless and left-handed

- Universal $V - A$ Weak Interactions

- Quantum Field Theory: $\nu_L \quad \Rightarrow \quad |\nu(h=-1)\rangle$ and $|\bar{\nu}(h=+1)\rangle$

- Parity is violated: $\nu_L \xrightarrow{\text{P}} \cancel{\nu_R}$ $\qquad |\nu(h=-1)\rangle \xrightarrow{\text{P}} \cancel{|\nu(h=+1)\rangle}$



- Particle-Antiparticle symmetry (Charge Conjugation) is violated:

$\nu_L \xrightarrow{\text{C}} \cancel{\nu_R}$ $\qquad |\nu(h=-1)\rangle \xrightarrow{\text{C}} \cancel{|\bar{\nu}(h=-1)\rangle}$

Majorana Equation

- Can a two-component spinor describe a massive fermion?

  Yes! (E. Majorana, 1937)

- Trick: ν_R and ν_L are not independent: $\boxed{\nu_R = \nu_L^c = \mathcal{C}\,\overline{\nu_L}^T}$

  charge-conjugation matrix: $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$

- The relation between ν_R and ν_L must satisfy two requirements:
  - It must have the correct chirality.

    This is satisfied, because ν_L^c is right-handed: $P_R\nu_L^c = \nu_L^c \quad P_L\nu_L^c = 0$
  - It must be compatible with the chiral Dirac equations

$$i\gamma^\mu\partial_\mu\nu_L = m\,\nu_R$$
$$i\gamma^\mu\partial_\mu\nu_R = m\,\nu_L$$

Check:

$$\begin{aligned} i\gamma^\mu\partial_\mu\nu_R &= i\gamma^\mu\partial_\mu\mathcal{C}\overline{\nu_L}^T = i\mathcal{C}\mathcal{C}^{-1}\gamma^\mu\mathcal{C}\partial_\mu\overline{\nu_L}^T = -i\mathcal{C}(\gamma^\mu)^T\partial_\mu\overline{\nu_L}^T \\ &= -i\mathcal{C}(\partial_\mu\overline{\nu_L}\gamma^\mu)^T = m\,\mathcal{C}\overline{\nu_R}^T = m\,\nu_L \qquad \text{OK} \end{aligned}$$

- $i\gamma^{\mu}\partial_{\mu}\nu_L = m\,\nu_R \quad \to \quad \boxed{i\gamma^{\mu}\partial_{\mu}\nu_L = m\,\nu_L^c}$ Majorana equation
- Majorana field: $\nu = \nu_L + \nu_R = \nu_L + \nu_L^c$

$$\boxed{\nu = \nu^c} \quad \text{Majorana condition}$$

- $\nu = \nu^c$ implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\nu}\gamma^{\mu}\nu = \overline{\nu^c}\gamma^{\mu}\nu^c = -\nu^T\mathcal{C}^{\dagger}\gamma^{\mu}\mathcal{C}\overline{\nu}^T = \overline{\nu}\mathcal{C}\gamma^{\mu\,T}\mathcal{C}^{\dagger}\nu = -\overline{\nu}\gamma^{\mu}\nu = 0$$

- Only two independent components: in the chiral representation

$$\nu = \begin{pmatrix} i\sigma^2\chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$$

- Majorana Lagrangian: $\mathscr{L}^{\mathrm{M}} = \frac{1}{2}\,\overline{\nu}\,(i\not{\partial} - m)\,\nu\big|_{\nu=\nu^c}$
- Quantized Dirac Neutrino Field:

$$\nu(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)}(p)\,u^{(h)}(p)\,e^{-ip\cdot x} + b^{(h)\dagger}(p)\,v^{(h)}(p)\,e^{ip\cdot x} \right]$$

- Quantized Majorana Neutrino Field: $b^{(h)}(p) = a^{(h)}(p)$

$$\nu(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)}(p)\,u^{(h)}(p)\,e^{-ip\cdot x} + a^{(h)\dagger}(p)\,v^{(h)}(p)\,e^{ip\cdot x} \right]$$

- A Majorana field has half the degrees of freedom of a Dirac field

Total Lepton Number

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu = \nu^c} \quad \longrightarrow \quad \cancel{L = -1}$$

$$\nu_L \quad \Longrightarrow \quad L = +1 \qquad\qquad \nu_L^c \quad \Longrightarrow \quad L = -1$$

$$\mathscr{L}^{\mathrm{M}} = \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta_{0\nu}^-)$$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta_{0\nu}^+)$$

No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T \, \mathcal{C}^\dagger \, \nu_L - \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed $Y = 2$ Higgs triplet ($I = 1$, $I_3 = -1$)
- Compare with Dirac Mass Term $\propto \overline{\nu_R} \nu_L$ with $I_3 = 1/2$ and $Y = -1$ balanced by $\phi_0 \to v$ with $I_3 = -1/2$ and $Y = +1$

Confusing Majorana Antineutrino Terminology

- A Majorana neutrino is the same as a Majorana antineutrino

- Neutrino interactions are described by the CC and NC Lagrangians

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_L}\, \gamma^{\mu}\, \ell_L\, W_{\mu} + \overline{\ell_L}\, \gamma^{\mu}\, \nu_L\, W_{\mu}^{\dagger} \right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\, \overline{\nu_L}\, \gamma^{\mu}\, \nu_L\, Z_{\mu}$$

- Dirac: ν_L { destroys left-handed neutrinos; creates right-handed antineutrinos

- Majorana: ν_L { destroys left-handed neutrinos; creates right-handed neutrinos

- Common implicit definitions:

  left-handed Majorana neutrino $\equiv$ neutrino
  right-handed Majorana neutrino $\equiv$ antineutrino

Effective Majorana Mass

- Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- Dimensionless action: $I = \int \mathrm{d}^4x \, \mathscr{L}(x) \Longrightarrow \mathscr{L}(x) \sim [E]^4$
- Kinetic terms: $\overline{\psi} i \partial\!\!\!/ \psi \sim [E]^4$, $(\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- Mass terms: $m \overline{\psi} \psi \sim [E]^4$, $m^2 \phi^\dagger \phi \sim [E]^4$
- CC weak interaction: $g \, \overline{\boldsymbol{\nu}_L} \gamma^\rho \boldsymbol{\ell}_L W_\rho \sim [E]^4$
- Yukawa couplings: $y \, \overline{L_L} \Phi \ell_R \sim [E]^4$
- Product of fields $\mathscr{O}_d$ with energy dimension $d \equiv$ dim-d operator
- $\mathscr{L}_{(\mathscr{O}_d)} = C_{(\mathscr{O}_d)} \mathscr{O}_d \Longrightarrow C_{(\mathscr{O}_d)} \sim [E]^{4-d}$
- $\mathscr{O}_{d>4}$ are not renormalizable

- SM Lagrangian includes all $\mathscr{O}_{d\leq4}$ invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- SM cannot be considered as the final theory of everything
- SM is an effective low-energy theory
- It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- It is plausible that at low-energy there are effective non-renormalizable $\mathscr{O}_{d>4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- All $\mathscr{O}_d$ must respect $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- $\mathscr{O}_{d>4}$ is suppressed by a coefficient $\mathcal{M}^{4-d}$, where $\mathcal{M}$ is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

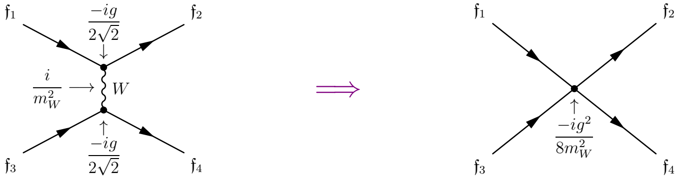
$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$

- Analogy with Fermi effective low-energy theory of weak interactions:

$$\mathscr{L}_{\text{eff}}^{(\text{CC})} \propto G_{\text{F}}\,(\overline{\nu_{eL}}\gamma^{\rho}e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots$$

$$\mathscr{O}_6 \to (\overline{\nu_{eL}}\gamma^{\rho}e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots \qquad\qquad \frac{g_6}{\mathcal{M}^2} \to \frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

$$G_{\mu\nu}^{(W)}(p) = i\,\frac{-g_{\mu\nu} + \frac{p_\mu\,p_\nu}{m_W^2}}{p^2 - m_W^2} \xrightarrow{p^2 \ll m_W^2} i\,\frac{g_{\mu\nu}}{m_W^2}$$



- $\mathcal{M}^{4-d}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM

- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality

- $\mathcal{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)

- $\mathcal{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)

- Majorana neutrino masses provide the most accessible low-energy window on new physics beyond the SM.

- Indeed, the existence of neutrino masses is the first and so far the only well established phenomenon beyond the SM.

- The only $SU(2)_L \times U(1)_Y$ invariant dim-5 Lagrangian term that can be constructed with SM fields:

$$\mathscr{L}_5 = -\frac{g_5}{\mathcal{M}} \left[\left(\overline{L_L} \, \widetilde{\Phi} \right) \left(\widetilde{\Phi}^T \, L_L^c \right) + \left(\overline{L_L^c} \, \widetilde{\Phi}^* \right) \left(\widetilde{\Phi}^\dagger \, L_L \right) \right]$$

- Electroweak Symmetry Breaking:

$$\widetilde{\Phi} = i\sigma_2 \, \Phi^* \quad \xrightarrow[\text{Breaking}]{\text{EW Symmetry}} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

- $$\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{EW Symmetry}} \mathscr{L}_{\text{mass}}^{\text{M}} = -\frac{1}{2} \frac{g_5 \, v^2}{\mathcal{M}} \left(\overline{\nu_L} \, \nu_L^c + \overline{\nu_L^c} \, \nu_L \right)$$

- Majorana neutrino mass: $$\boxed{m = \frac{g_5 \, v^2}{\mathcal{M}}}$$

- General Seesaw Mechanism: $m \propto \frac{v^2}{\mathcal{M}} = v \, \frac{v}{\mathcal{M}}$

  natural explanation of the strong suppression of neutrino masses with respect to the electroweak scale

- Example: $\mathcal{M} \sim 10^{15}\,\text{GeV}$ (GUT scale)

$$v \sim 10^2\,\text{GeV} \quad \Longrightarrow \quad \frac{v}{\mathcal{M}} \sim 10^{-13} \quad \Longrightarrow \quad m \sim 10^{-2}\,\text{eV}$$

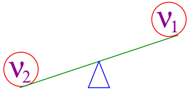
Type-I Seesaw Mechanism

$$\mathcal{L}^{\text{D+M}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m^{\text{D}} \\ m^{\text{D}} & m_R^{\text{M}} \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

m_R^{M} can be arbitrarily large (not protected by SM symmetries)

$m_R^{\text{M}} \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^{\text{M}} \gg m^{\text{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\text{D}} \\ m^{\text{D}} & m_R^{\text{M}} \end{pmatrix} \Longrightarrow m_{\text{light}} \simeq \dfrac{(m^{\text{D}})^2}{m_R^{\text{M}}}, \; m_{\text{heavy}} \simeq m_R^{\text{M}}$



seesaw mechanism

natural explanation of smallness
of light neutrino masses

massive neutrinos are Majorana!

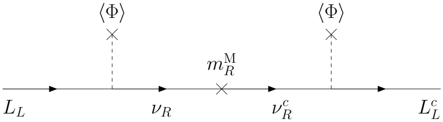
3-GEN $\Rightarrow$ effective low-energy 3-ν mixing

[Minkowski, PLB 67 (1977) 42]

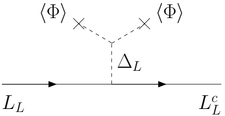
[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Three Seesaw Types

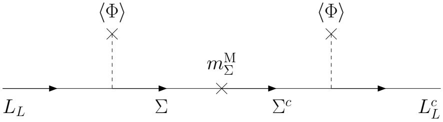
- Since combining the two doublets L_L and Φ one can form singlets and triplets, there are three types of Seesaw types that can be generated at the tree level.
- Type-I Seesaw: intermediate fermion singlets ν_R



- Type-II Seesaw: coupling with boson triplets Δ_L



- Type-III Seesaw: intermediate fermion triplets Σ



Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\mathcal{L}_\Phi = -y_d \left(\overline{L_L}\, \Phi\, \nu_R + \overline{\nu_R}\, \Phi^\dagger\, L_L \right) \xrightarrow[\langle\Phi\rangle\neq 0]{} -m_{\text{D}} \left(\overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right)$$

$$\mathcal{L}_\eta = -y_s \left(\eta\, \overline{\nu_R^c}\, \nu_R + \eta^\dagger\, \overline{\nu_R}\, \nu_R^c \right) \xrightarrow[\langle\eta\rangle\neq 0]{} -\frac{1}{2}\, m_R \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$\eta = 2^{-1/2} \left(\langle\eta\rangle + \rho + i\,\chi \right) \qquad \mathcal{L}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\underset{\text{scale of } L \text{ violation}}{m_R} \gg \underset{\text{EW scale}}{m_{\text{D}}} \implies \text{Type-I Seesaw:} \quad \boxed{m_1 \simeq \frac{m_{\text{D}}^2}{m_R}}$$

ρ = massive scalar, χ = Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{i y_s}{\sqrt{2}}\,\chi \left[\overline{\nu_2}\gamma^5\nu_2 - \frac{m_{\text{D}}}{m_R} \left(\overline{\nu_2}\gamma^5\nu_1 + \overline{\nu_1}\gamma^5\nu_2 \right) + \left(\frac{m_{\text{D}}}{m_R} \right)^2 \overline{\nu_1}\gamma^5\nu_1 \right]$$

Mixing of Three Majorana Neutrinos

- $\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$

$$\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^T_L \, \mathcal{C}^\dagger \, M^L \, \boldsymbol{\nu}'_L + \text{H.c.}$$

$$= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^T_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} + \text{H.c.}$$

- In general, the matrix M^L is a complex symmetric matrix

$$\sum_{\alpha,\beta} \nu'^T_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} = \sum_{\alpha,\beta} \left(\nu'^T_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\beta L} \right)^T$$

$$= - \sum_{\alpha,\beta} \nu'^T_{\beta L} \, M^L_{\alpha\beta} \, (\mathcal{C}^\dagger)^T \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu'^T_{\beta L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu'_{\alpha L}$$

$$= \sum_{\alpha,\beta} \nu'^T_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\beta\alpha} \, \nu'_{\beta L}$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \quad \Longleftrightarrow \quad M^L = {M^L}^T$$

- $\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^{T}_{L} \, \mathcal{C}^{\dagger} \, M^{L} \, \boldsymbol{\nu}'_{L} + \text{H.c.}$

- $\boldsymbol{\nu}'_{L} = V^{\nu}_{L} \, \boldsymbol{n}_{L} \qquad \Longrightarrow \qquad \mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^{T}_{L} \, (V^{\nu}_{L})^{T} \, \mathcal{C}^{\dagger} \, M^{L} \, V^{\nu}_{L} \, \boldsymbol{\nu}'_{L} + \text{H.c.}$

- $(V^{\nu}_{L})^{T} \, M^{L} \, V^{\nu}_{L} = M \,, \qquad M_{kj} = m_{k} \, \delta_{kj} \qquad (k, j = 1, 2, 3)$

- Left-handed chiral fields with definite mass: $\boldsymbol{n}_{L} = V^{\nu\dagger}_{L} \, \boldsymbol{\nu}'_{L} = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}^{\text{M}}_{\text{mass}} &= \frac{1}{2} \left(\boldsymbol{n}^{T}_{L} \, \mathcal{C}^{\dagger} \, M \, \boldsymbol{n}_{L} - \overline{\boldsymbol{n}_{L}} \, M \, \mathcal{C} \, \boldsymbol{n}^{T}_{L} \right) \\ &= \frac{1}{2} \sum_{k=1}^{3} m_{k} \left(\nu^{T}_{kL} \, \mathcal{C}^{\dagger} \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu^{T}_{kL} \right) \end{aligned}$$

- Majorana fields of massive neutrinos: $\nu_{k} = \nu_{kL} + \nu^{c}_{kL}$ $\qquad \boxed{\nu^{c}_{k} = \nu_{k}}$

- $\boldsymbol{n} = \begin{pmatrix} \nu_{1} \\ \nu_{2} \\ \nu_{3} \end{pmatrix} \Longrightarrow \mathscr{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^{3} \overline{\nu_{k}} \, (i\partial\!\!\!/ - m_{k}) \, \nu_{k} \big|_{\nu_{k} = \nu^{c}_{k}}$

Mixing Matrix

- Leptonic Weak Charged Current:

$$j^{\rho\dagger}_{W,L} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,U\,\boldsymbol{n}_L \qquad \text{with} \qquad U = V^{\ell\dagger}_L\,V^\nu_L$$

- As in the Dirac case, we define the left-handed flavor neutrino fields as

$$\boldsymbol{\nu}_L = U\,\boldsymbol{n}_L = V^{\ell\dagger}_L\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- In this way, as in the Dirac case, the Leptonic Weak Charged Current has the SM form

$$j^{\rho\dagger}_{W,L} = 2\,\overline{\boldsymbol{\ell}_L}\,\gamma^\rho\,\boldsymbol{\nu}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\ell_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- Majorana Mass Term $\mathscr{L}^{\mathrm{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k \, \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + \text{H.c.}$ is not invariant under the global U(1) gauge transformations

$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \quad (k = 1, 2, 3)$$

- For eliminating some of the 6 phases of the unitary mixing matrix we can use only the global phase transformations (3 arbitrary phases)

$$\ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$

- Performing the transformation $\ell_{\alpha} \to e^{i\varphi_{\alpha}} \, \ell_{\alpha}$ we obtain

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, e^{-i\varphi_{\alpha}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} = 2 \underbrace{e^{-i\varphi_{e}}}_{1} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_{\alpha}-\varphi_{e})}}_{2} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$$

- We can eliminate 3 phases of the mixing matrix: one overall phase and two phases which can be factorized on the left.

- In the Dirac case we could eliminate also two phases which can be factorized on the right.

- In the Majorana case there are two additional physical Majorana phases which can be factorized on the right of the mixing matrix:

$$\boxed{U = U^{\mathrm{D}}\, D^{\mathrm{M}}} \qquad D^{\mathrm{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- U^{D} is a Dirac mixing matrix, with one Dirac phase

- Standard parameterization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- $D^{\mathrm{M}} = \mathrm{diag}\left(e^{i\lambda_1}\,,\, e^{i\lambda_2}\,,\, e^{i\lambda_3}\right)$, but only two Majorana phases are physical

- All measurable quantities depend only on the differences of the Majorana phases because $e^{i(\lambda_k - \lambda_j)}$ remains constant under the allowed phase transformation

$$\ell_\alpha \to e^{i\varphi} \ell_\alpha \quad \Longrightarrow \quad e^{i\lambda_k} \to e^{i(\lambda_k - \varphi)}$$

- Our convention: $\lambda_1 = 0 \Longrightarrow D^{\mathrm{M}} = \mathrm{diag}\left(1\,,\, e^{i\lambda_2}\,,\, e^{i\lambda_3}\right)$

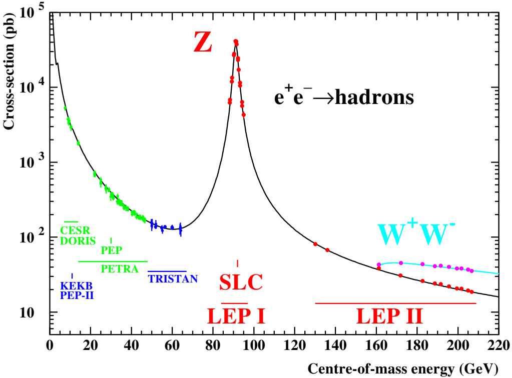
- CP is conserved if all the elements of each column of the mixing matrix are either real or purely imaginary:

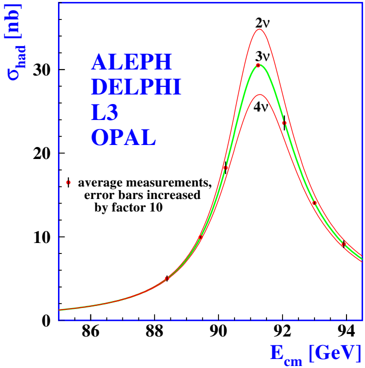
$$\delta_{13} = 0 \text{ or } \pi \quad \text{and} \quad \lambda_k = 0 \text{ or } \pi/2 \text{ or } \pi \text{ or } 3\pi/2$$

Sterile Neutrinos

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Sterile Neutrinos

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

$$e^+e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots$$

Sterile Neutrinos

- Sterile means no standard model interactions

  [Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

- Obviously no electromagnetic interactions as normal active neutrinos
- Thus sterile means no standard weak interactions
- But sterile neutrinos are not absolutely sterile:
  - Gravitational Interactions
  - New non-standard interactions of the physics beyond the Standard Model which generates the masses of sterile neutrinos
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos
  - Indirect evidence through combined fit of data
- Powerful window on new physics beyond the Standard Model