Neutrino Physics

Part II: Phenomenology of Massive Neutrinos

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Neutrino Mixing

Left-handed Flavor Neutrinos produced in Weak Interactions

$$|\nu_e, -\rangle \qquad |\nu_\mu, -\rangle \qquad |\nu_\tau, -\rangle$$

$$\mathcal{H}_{\text{CC}} = \frac{g}{\sqrt{2}} W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right) + \text{H.c.}$$

Fields $\quad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha, -\rangle = \sum_k U_{\alpha k}^* |\nu_k, -\rangle \quad$ States

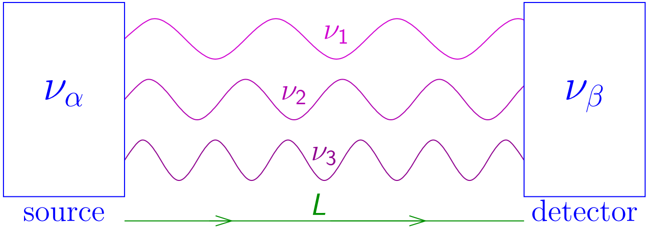
$$|\nu_1, -\rangle \qquad |\nu_2, -\rangle \qquad |\nu_3, -\rangle$$

Left-handed Massive Neutrinos propagate from Source to Detector

3×3 Unitary Mixing Matrix: $\quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

Neutrino Oscillations

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha 1}\,|\nu_1\rangle + U^*_{\alpha 2}\,|\nu_2\rangle + U^*_{\alpha 3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U^*_{\alpha 1}\,e^{-iE_1 t}\,|\nu_1\rangle + U^*_{\alpha 2}\,e^{-iE_2 t}\,|\nu_2\rangle + U^*_{\alpha 3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

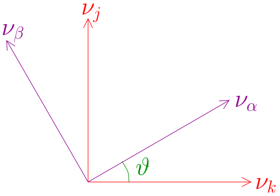
$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha\to\nu_\beta}(L) = |\langle\nu_\beta|\nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k}\,U^*_{\alpha k}\,U^*_{\beta j}\,U_{\alpha j}\,\exp\left(-i\frac{\Delta m^2_{kj} L}{2E}\right)$$

the oscillation probabilities depend on U and $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$

Effective Two-Neutrino Mixing Approximation

$$|\nu_\alpha\rangle = \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle$$
$$|\nu_\beta\rangle = -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle$$



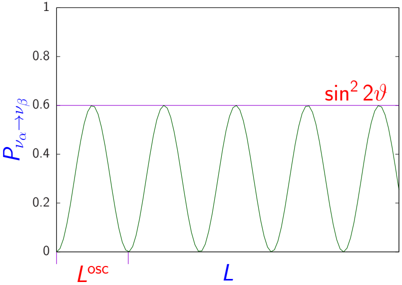
$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m_k^2 - m_j^2$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

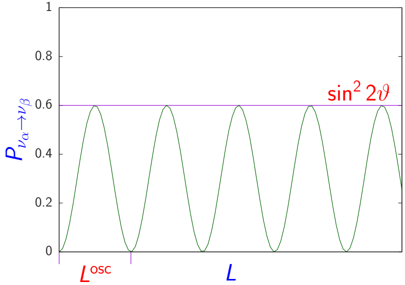
Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \implies L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$



- The effect of a tiny Δm^2 can be amplified by a large distance L.
- A tiny Δm^2 generates oscillations observable at macroscopic distances!
- Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

$$2\nu\text{-mixing:} \qquad P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]} \right)$$



$$\frac{L}{E} \gtrsim \begin{cases} 10 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1}\,\text{eV}^2 \\ 10^3 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3}\,\text{eV}^2 \\ 10^4 \, \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2 \\ 10^{11} \, \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2 \end{cases}$$

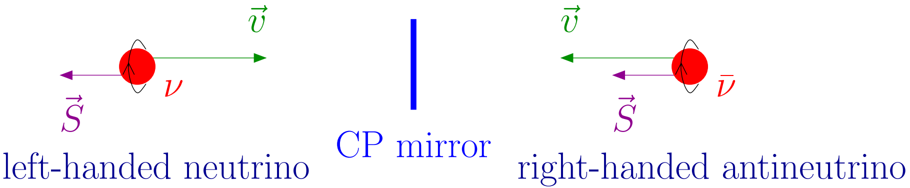
Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu_{\alpha L}^{\mathsf{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu_{\alpha L}}^T$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed



$$\text{Fields: } \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$$

$$\text{States: } |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\text{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\implies$ $A_{\alpha\beta}^{\text{CPT}} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $U \leftrightarrows U^*$ $\alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$A^{\text{CP}}_{\alpha\beta}(L, E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

Jarlskog rephasing invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

$$J \neq 0 \iff \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

$$
\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \leftarrow A^{\text{CP}}_{\alpha\beta} \\
&+ P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow -A^{\text{CPT}}_{\beta\alpha} = 0 \\
&+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{CP}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} \qquad \Longrightarrow \qquad \boxed{A^{\text{CP}}_{\alpha\beta} = -A^{\text{CP}}_{\beta\alpha}}
\end{aligned}
$$

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{T}} P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $A^{\text{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

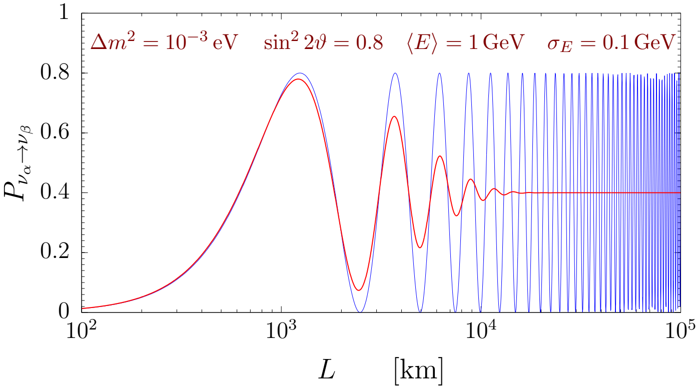
$$\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow A^{\text{T}}_{\alpha\beta} \\
&+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{T}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{T}}_{\alpha\beta} - A^{\text{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\text{T}}_{\alpha\beta} = A^{\text{CP}}_{\alpha\beta}}
\end{aligned}$$

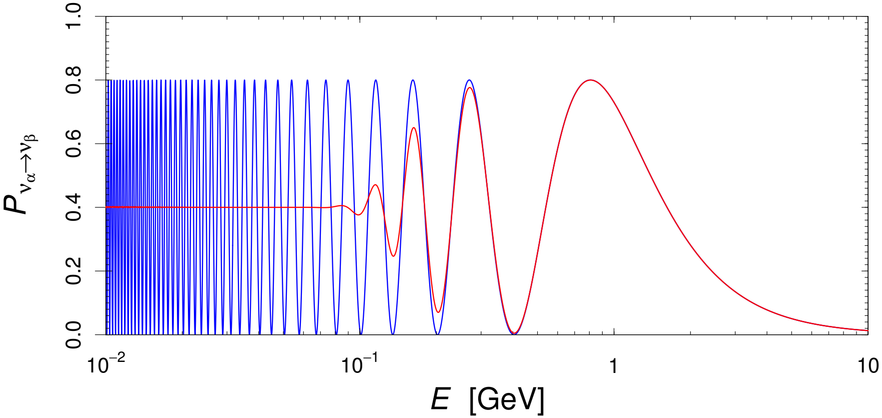
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

$$\Downarrow$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E)\rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\, \mathrm{d}E\right] \qquad (\alpha \neq \beta)$$





$\Delta m^2 = 10^{-3}\,\text{eV} \qquad \sin^2 2\vartheta = 0.8 \qquad L = 10^3\,\text{km} \qquad \sigma_E = 0.01\,\text{GeV}$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E)\, dE \right] \qquad (\alpha \neq \beta)$$

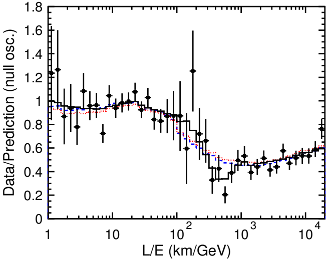
A Brief History of Neutrino Oscillations

- 1957: Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\implies$ $\nu \leftrightarrows \bar{\nu}$
- In 1957 only one neutrino type $\nu = \nu_e$ was known! The possible existence of ν_μ was discussed by several authors. Maybe the first have been Sakata and Inoue in 1946 and Konopinski and Mahmoud in 1953. Maybe Pontecorvo did not know. He discussed the possibility to distinguish ν_μ from ν_e in 1959.
- 1962: Maki, Nakagava, Sakata proposed a model with ν_e and ν_μ and Neutrino Mixing:

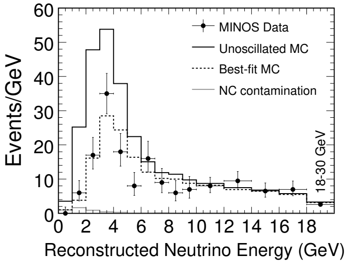
  "weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrows \nu_\mu$"
- 1962: Lederman, Schwartz and Steinberger discover ν_μ
- 1967: Pontecorvo: intuitive $\nu_e \leftrightarrows \nu_\mu$ oscillations with maximal mixing. Applications to reactor and solar neutrinos ("prediction" of the solar neutrino problem).
- 1969: Gribov and Pontecorvo: $\nu_e - \nu_\mu$ mixing and oscillations. But no clear derivation of oscillations with a factor of 2 mistake in the phase (misprint?).

- 1975-76: Start of the "Modern Era" of Neutrino Oscillations with a general theory of neutrino mixing and a rigorous derivation of the oscillation probability by Eliezer and Swift, Fritzsch and Minkowski, and Bilenky and Pontecorvo. [Bilenky, Pontecorvo, Phys. Rep. (1978) 225]
- 1978: Wolfenstein discovers the effect on neutrino oscillations of the matter potential ("Matter Effect")
- 1985: Mikheev and Smirnov discover the resonant amplification of solar $\nu_e \to \nu_\mu$ oscillations due to the Matter Effect ("MSW Effect")
- 1998: the Super-Kamiokande experiment observed in a model-independent way the Vacuum Oscillations of atmospheric neutrinos ($\nu_\mu \to \nu_\tau$).
- 2002: the SNO experiment observed in a model-independent way the flavor transitions of solar neutrinos ($\nu_e \to \nu_\mu, \nu_\tau$), mainly due to adiabatic MSW transitions. [see: Smirnov, arXiv:1609.02386]
- 2015: Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (SNO) received the Physics Nobel Prize "for the discovery of neutrino oscillations, which shows that neutrinos have mass".

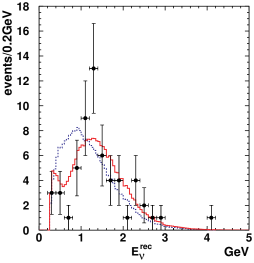
Observations of Neutrino Oscillations



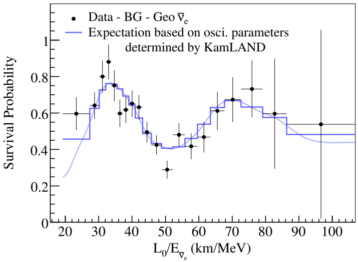
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



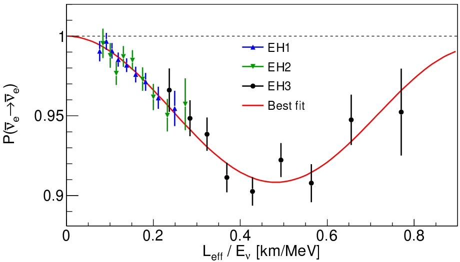
[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]



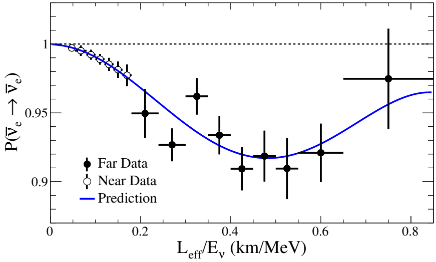
[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]



[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]



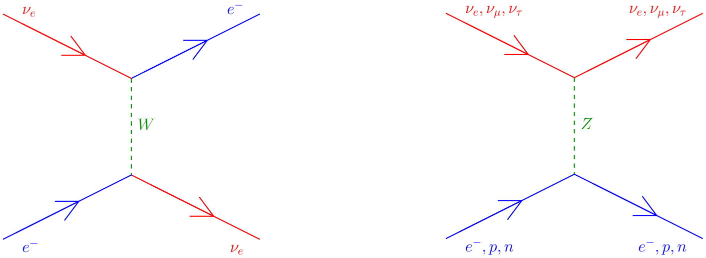
[Daya Bay, PRL, 112 (2014) 061801, arXiv:1310.6732]



[RENO, arXiv:1511.05849]

Effective Potentials in Matter

coherent interactions with medium: forward elastic CC and NC scattering



$$V_{\rm CC} = \sqrt{2} G_{\rm F} N_e \qquad V_{\rm NC}^{(e^-)} = -V_{\rm NC}^{(p)} \quad \Rightarrow \quad V_{\rm NC} = V_{\rm NC}^{(n)} = -\frac{\sqrt{2}}{2} G_{\rm F} N_n$$

$$V_e = V_{\rm CC} + V_{\rm NC} \qquad V_\mu = V_\tau = V_{\rm NC}$$

only $V_{\rm CC} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\rm CC} = -V_{\rm CC}$ $\quad \overline{V}_{\rm NC} = -V_{\rm NC}$

Evolution of Neutrino Flavors in Matter

- Flavor neutrino ν_α with momentum p: $|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* |\nu_k(p)\rangle$
- Evolution is determined by Hamiltonian
- Hamiltonian in vacuum: $\mathcal{H} = \mathcal{H}_0$

$$\mathcal{H}_0 |\nu_k(p)\rangle = E_k |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

- Hamiltonian in matter: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$ $\qquad \mathcal{H}_I |\nu_\alpha(p)\rangle = V_\alpha |\nu_\alpha(p)\rangle$
- Schrödinger evolution equation: $i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle$
- Initial condition: $|\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$
- For $t > 0$ the state $|\nu(p,t)\rangle$ is a superposition of all flavors:

$$|\nu(p,t)\rangle = \sum_\beta \varphi_\beta(p,t) |\nu_\beta(p)\rangle$$

- Transition probability: $P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2$

Neutrino Oscillations in Matter

$$i \frac{\mathrm{d}}{\mathrm{d}x} \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2EV_{\mathrm{CC}} = 2\sqrt{2}EG_{\mathrm{F}}N_e$$

effective mass-squared matrix in vacuum $\mathbb{M}^2_{\mathrm{VAC}} = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \mathbb{V} = \mathbb{M}^2_{\mathrm{MAT}}$ effective mass-squared matrix in matter

$\uparrow$ ($\mathbb{V}$): potential due to coherent forward elastic scattering

In Neutrino Oscillations Dirac = Majorana

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495; Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Evolution of Amplitudes: $i\,\frac{\mathrm{d}\psi_\alpha}{\mathrm{d}x} = \frac{1}{2E}\sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \psi_\beta$

difference: $\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})} D(\lambda) \end{cases}$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2 D \quad \Longrightarrow \quad DM^2 D^\dagger = M^2$$

$$U^{(\mathrm{M})} M^2 (U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})} D M^2 D^\dagger (U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})} M^2 (U^{(\mathrm{D})})^\dagger$$

Three-Neutrino Mixing Paradigm

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS:
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent Δm^2_{kj}: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes ($\beta\beta_{0\nu}$)

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Solar

$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, Borexino; Super-Kamiokande; GALLEX/GNO, SAGE; Homestake, Kamiokande)

VLBL Reactor

$\bar{\nu}_e$ disappearance

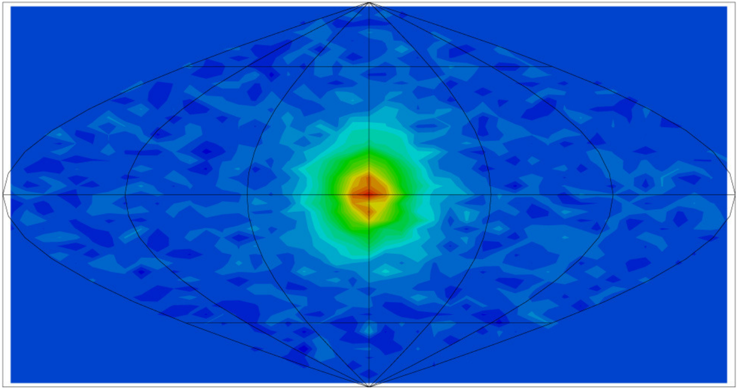
(KamLAND)

$\to$

$$\Delta m^2_{\rm S} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$$

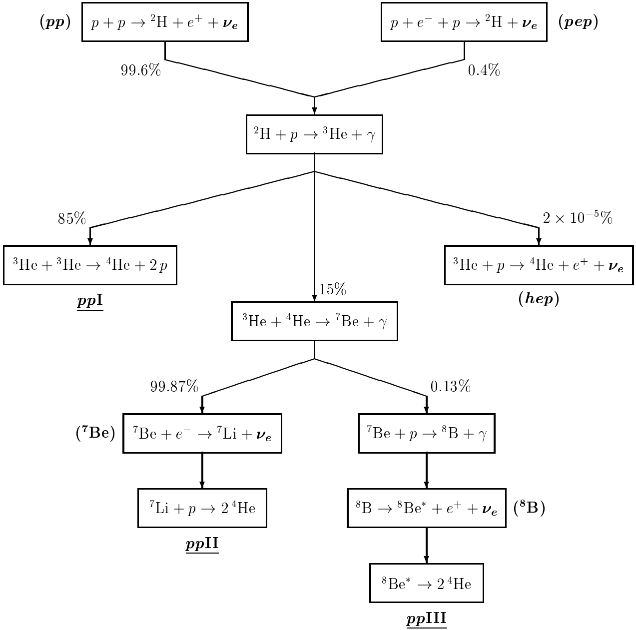
$$\sin^2\vartheta_{\rm S} = \sin^2\vartheta_{12} \simeq 0.30$$

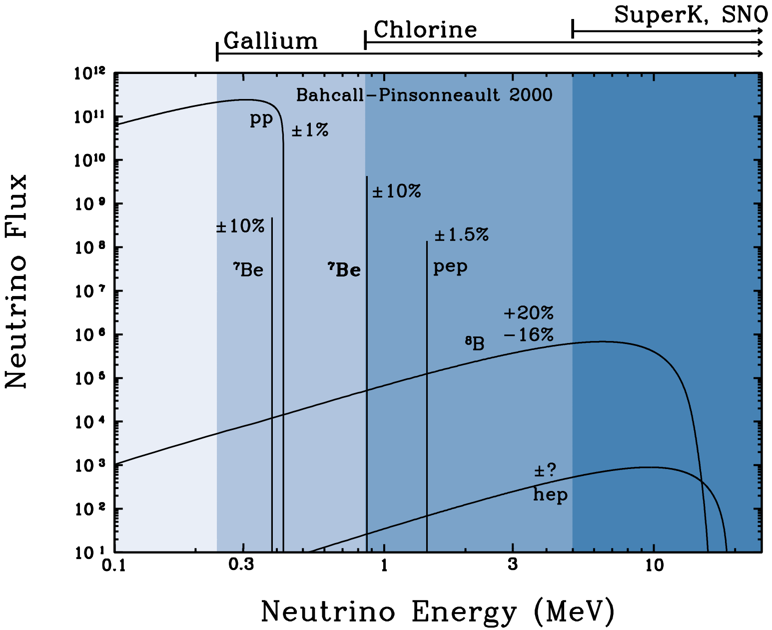
Solar Neutrinos



The sun observed through neutrinos by Super-Kamiokande

Standard Solar Model (SSM): *pp* chain





SuperK, SNO
Chlorine
Gallium
Bahcall−Pinsonneault 2000
pp
±1%
±10%
±10%
⁷Be
⁷Be
±1.5%
pep
+20%
−16%
⁸B
±?
hep
Neutrino Flux
Neutrino Energy (MeV)
0.1
0.3
1
3
10

Solar Neutrino Observations

- 1957: Bruno Pontecorvo suggests to observe solar neutrinos using a detector tank containing Clorine through the process

$$\nu_e + {}^{37}_{17}\text{Cl} \to {}^{37}_{18}\text{Ar} + e^-$$

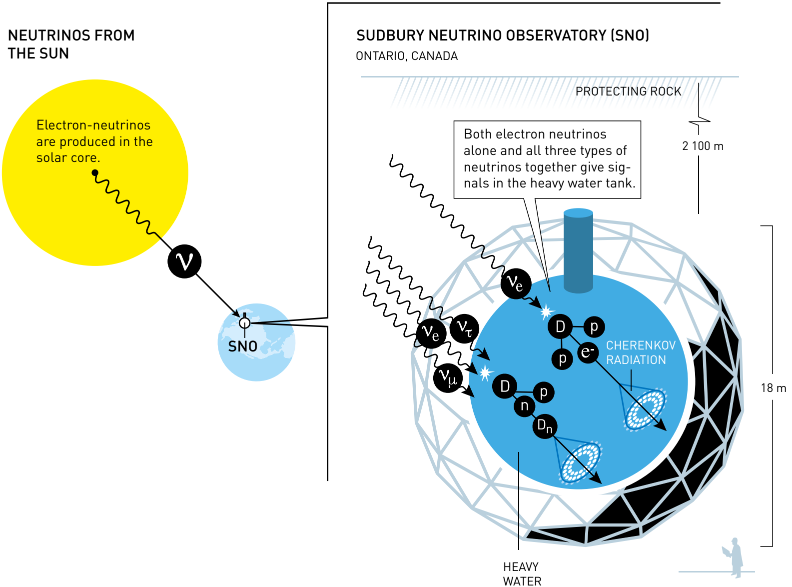
- 1964: John N. Bahcall calculates the cross sections and finds that it is enough to observe solar neutrinos.
- 1964: Raymond Davis proposes the Homestake experiment that is constructed in 1965–1967. It is based in the radiochemical counting of the ^{37}Ar produced by solar neutrinos in a tank with 615 tons of tetrachloroethylene (C_2Cl_4).
- 1970: Davis (2002 Physics Nobel Prize) and collaborators observe for the first time solar neutrinos counting ^{37}Ar atoms that are produced with a rate of about one every 2 days in the Homestake detector which contains about 2×10^{30} atoms!
- Solar neutrinos have been observed in the experiments Homestake (1970-1994), Kamiokande (1987-1995) SAGE (1990-2010), GALLEX/GNO (1991-2000), Super-Kamiokande (1996-2019), SNO (1999-2008), Borexino (2007-2019).

The solar neutrino problem

- 1968: Bruno Pontecorvo suggests that part of solar ν_e's can disappear into ν_μ (or ν_τ) due to oscillations.
- 1970: Discovery of the solar neutrino problem in the Homestake experiment that counts about 0.5 ^{37}Ar atoms per day with a SSM prediction of about 1.5 ^{37}Ar atoms per day.
- All the other solar neutrino experiments observed a suppression of the solar ν_e signal.
- From 1970 to 2002 experts debated on the possible solutions of the solar neutrino problem.
- The two solutions that were considered more likely are:
  - There is a mistake in the SSM prediction of the solar ν_e flux.
  - Part of the solar ν_e's disappear into ν_μ (or ν_τ) due to oscillations as suggested by Pontecorvo.

The SNO Experiment

1 kton of D_2O, Cherenkov detector, 2100 m underground



- Observed SNO rates relative to the SSM predictions:

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02 \qquad \frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.02 \pm 0.13$$

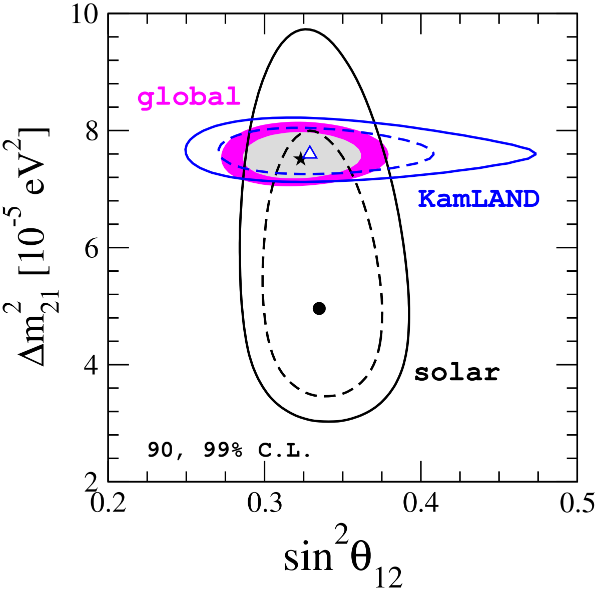
- The CC measurements confirms the solar neutrino problem: ν_e disappear.

- The NC measurement shows that the total flux of ν_e, ν_μ, ν_τ in agreement with the SSM prediction.

- The only possible explanation of the two measurements is that solar ν_e's transform into ν_μ and/or ν_τ. (A. McDonald: 2015 Physics Nobel Prize)

- The simplest and most plausible mechanism are neutrino oscillations.

- The oscillations of solar neutrinos have been confirmed in 2002 by the KamLAND very-long-baseline reactor neutrino experiment.



[M. Tortola @ Neutrino 2018]

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2, IceCube, ANTARES)

LBL Accelerator
ν_μ disappearance

(K2K, MINOS, T2K, NOνA)

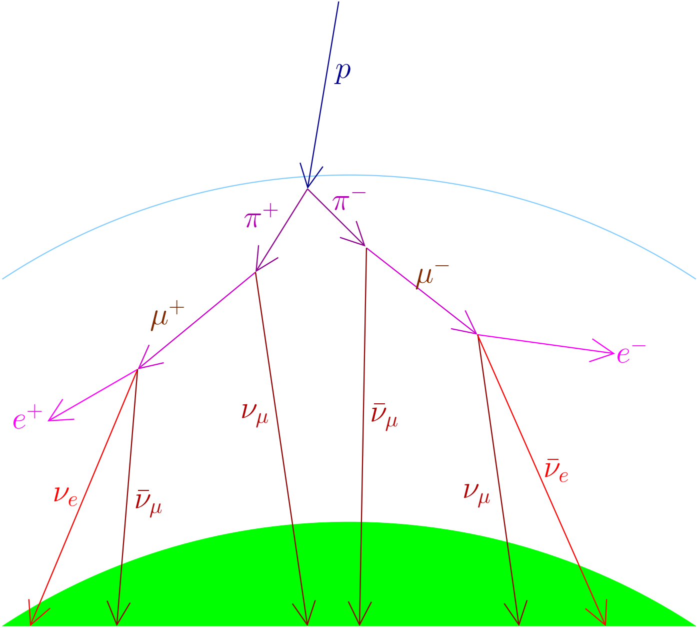
LBL Accelerator
$\nu_\mu \to \nu_\tau$

(OPERA)

$\to$

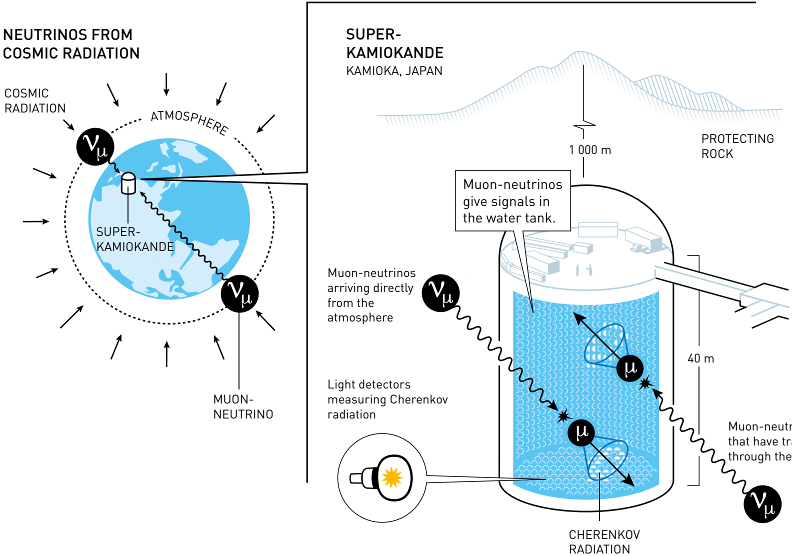
$$\begin{cases} \Delta m^2_{\mathrm{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2 \\ \sin^2\vartheta_{\mathrm{A}} = \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

Atmosferic Neutrinos

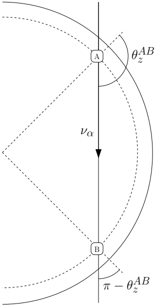


The Super-Kamiokande Experiment

50 ktons of water, Cherenkov detector, 1000 m underground



The Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi-\theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(B)}_{\nu_\alpha}(\theta_z) = \phi^{(B)}_{\nu_\alpha}(\pi-\theta_z)$$

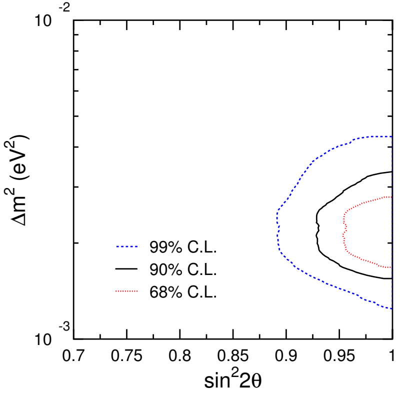
$$A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}}\right) = -0.296 \pm 0.048 \pm 0.01$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

(T. Kajita: 2015 Physics Nobel Prize)

Fit of Super-Kamiokande Atmospheric Data



$\nu_\mu \to \nu_\tau$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20$events/yr
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$N_{\nu_\tau}^{\text{the}} = 78 \pm 26$ @ $\Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$

$$N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}$$

$N_{\nu_\tau} > 0$ @ 2.4σ

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

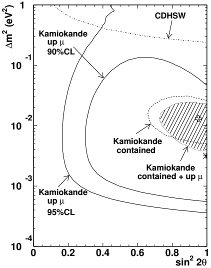
Check: OPERA ($\nu_\mu \to \nu_\tau$)

CERN to Gran Sasso (CNGS)

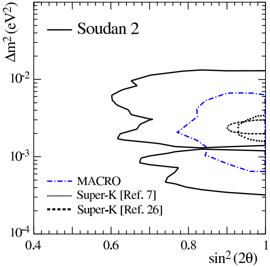
$L \simeq 732\,\text{km}$ $\langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

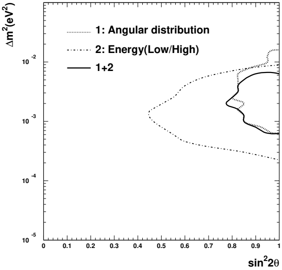
Kamiokande, Soudan-2, MACRO and MINOS



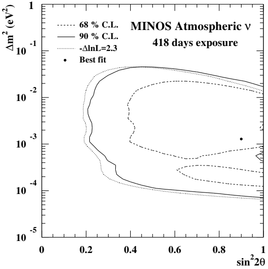
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



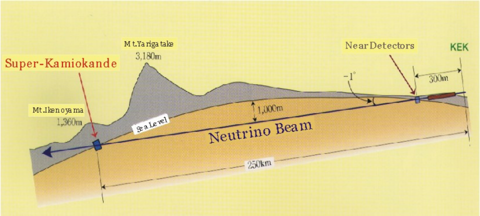
[MACRO, hep-ex/0304037]



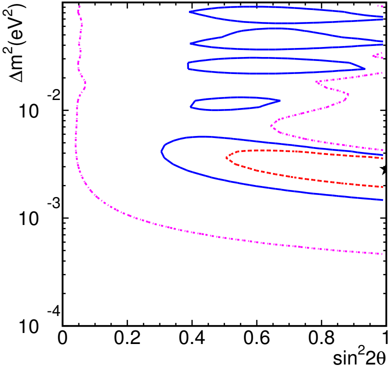
[MINOS, hep-ex/0512036]

K2K

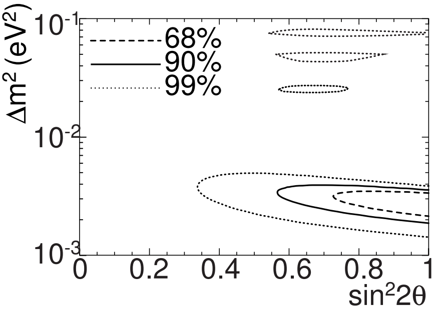
confirmation of atmospheric allowed region (June 2002)



KEK to Kamioka
(Super-Kamiokande)
250 km
$\nu_\mu \to \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]

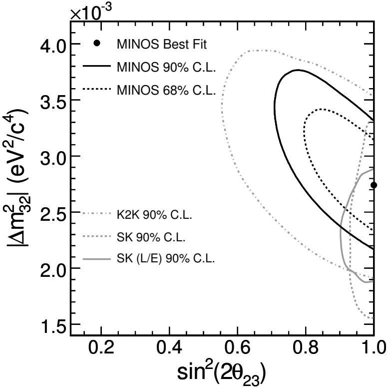


[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

MINOS

May 2005 – Feb 2006

http://www-numi.fnal.gov/



Near Detector: 1 km

$$\nu_\mu \to \nu_\mu$$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\vartheta > 0.87 \;@\; 68\%\,CL$$

[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]

OPERA

PRL **115,** 121802 (2015) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2015

Discovery of τ Neutrino Appearance in the CNGS Neutrino Beam with the OPERA Experiment

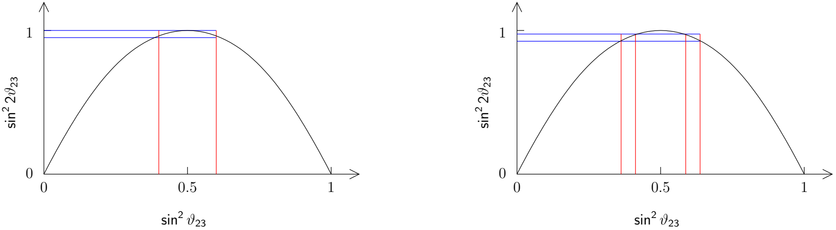
The OPERA experiment was designed to search for $\nu_\mu \to \nu_\tau$ oscillations in appearance mode, i.e., by detecting the τ leptons produced in charged current ν_τ interactions. The experiment took data from 2008 to 2012 in the CERN Neutrinos to Gran Sasso beam. The observation of the $\nu_\mu \to \nu_\tau$ appearance, achieved with four candidate events in a subsample of the data, was previously reported. In this Letter, a fifth ν_τ candidate event, found in an enlarged data sample, is described. Together with a further reduction of the expected background, the candidate events detected so far allow us to assess the discovery of $\nu_\mu \to \nu_\tau$ oscillations in appearance mode with a significance larger than 5σ.

	Expected background					
Channel	Charm	Had. reinterac.	Large μ scat.	Total	Expected signal	Observed
$\tau \to 1h$	0.017 ± 0.003	0.022 ± 0.006		0.04 ± 0.01	0.52 ± 0.10	3
$\tau \to 3h$	0.17 ± 0.03	0.003 ± 0.001		0.17 ± 0.03	0.73 ± 0.14	1
$\tau \to \mu$	0.004 ± 0.001		0.0002 ± 0.0001	0.004 ± 0.001	0.61 ± 0.12	1
$\tau \to e$	0.03 ± 0.01			0.03 ± 0.01	0.78 ± 0.16	0
Total	0.22 ± 0.04	0.02 ± 0.01	0.0002 ± 0.0001	0.25 ± 0.05	2.64 ± 0.53	5

Difficulty of measuring precisely ϑ_{23}

$$P^{\text{LBL}}_{\nu_\mu \to \nu_\mu} \simeq 1 - \sin^2 2\vartheta_{23} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\sin^2 2\vartheta_{23} = 4 \sin^2\vartheta_{23} \left(1 - \sin^2\vartheta_{23}\right)$$



The octant degeneracy is resolved by small ϑ_{13} effects:

$$P^{\text{LBL}}_{\nu_\mu \to \nu_\mu} \simeq 1 - \left[\sin^2 2\vartheta_{23} \cos^2 \vartheta_{13} + \sin^4 \vartheta_{23} \sin^2 2\vartheta_{13}\right] \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$P^{\text{LBL}}_{\nu_\mu \to \nu_e} \simeq \sin^2 \vartheta_{23} \sin^2 2\vartheta_{13} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

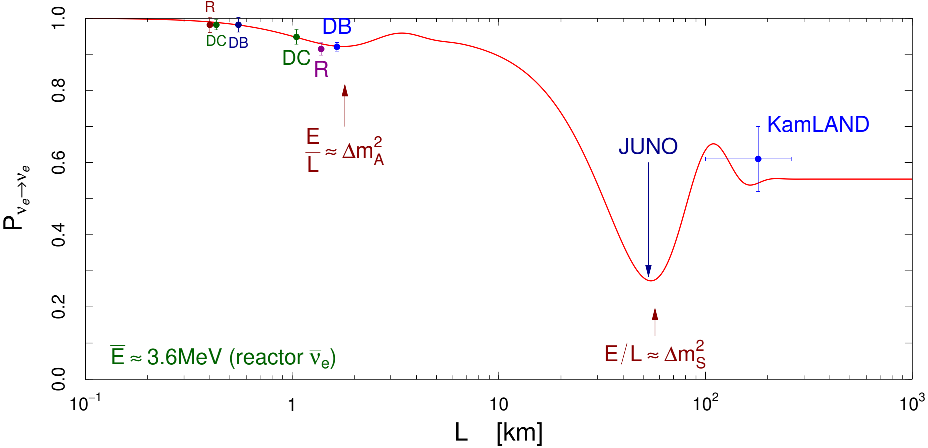
LBL Accelerator
$\nu_\mu \to \nu_e$ (T2K, MINOS, NOνA)

LBL Reactor
$\bar{\nu}_e$ disappearance (Daya Bay, RENO, Double Chooz)

$\rightarrow$

$$\Delta m^2_{\text{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{13} \simeq 0.022$$



$P_{\nu_e \to \nu_e}$
1.0
0.8
0.6
0.4
0.2
0.0
R
DC
DB
DB
DC
R
$\frac{E}{L} \approx \Delta m^2_A$
JUNO
KamLAND
$E/L \approx \Delta m^2_S$
$\overline{E} \approx 3.6\text{MeV}$ (reactor $\overline{\nu}_e$)
10^{-1}
1
10
10^2
10^3
L [km]

Towards a precise determination of neutrino mixing

well determined

totally unknown

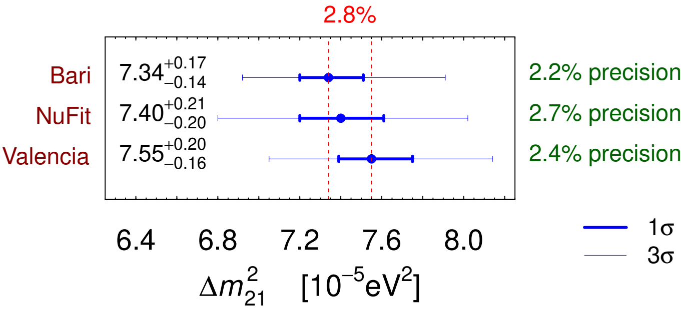
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

large uncertainty due to ϑ_{23} and δ_{13}

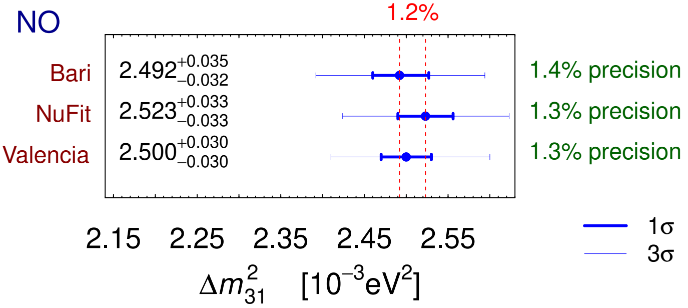
medium uncertainty due to ϑ_{23}

$|U|_{3\sigma} =$ 

only the mass composition of ν_e is well determined

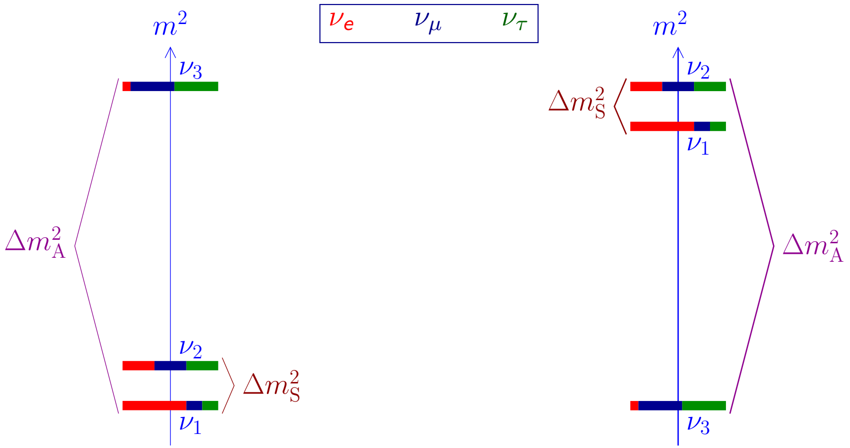


2.8%
Bari
$7.34^{+0.17}_{-0.14}$
2.2% precision
NuFit
$7.40^{+0.21}_{-0.20}$
2.7% precision
Valencia
$7.55^{+0.20}_{-0.16}$
2.4% precision
6.4
6.8
7.2
7.6
8.0
1σ
3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$



NO
1.2%
Bari
$2.492^{+0.035}_{-0.032}$
1.4% precision
NuFit
$2.523^{+0.033}_{-0.033}$
1.3% precision
Valencia
$2.500^{+0.030}_{-0.030}$
1.3% precision
2.15
2.25
2.35
2.45
2.55
1σ
3σ
Δm^2_{31} $[10^{-3}\text{eV}^2]$

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), ...
- CP violation ? $\delta_{13} \approx 3\pi/2$?
  - T2K (Japan), NOνA (USA), DUNE (USA), HyperK (Japan), ...
- Mass Ordering ?
  - JUNO (China), PINGU (Antarctica), ORCA (EU), INO (India), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

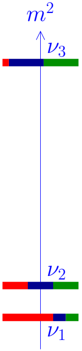
Determination of Mass Ordering

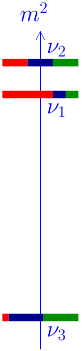
1. Matter Effects: Atmospheric (PINGU, ORCA), Long-Baseline, Supernova Experiments
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \dfrac{\Delta m^2_{31} \cos 2\vartheta_{13}}{2E} \Leftrightarrow \Delta m^2_{31} > 0$ NO
   - $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\dfrac{\Delta m^2_{31} \cos 2\vartheta_{13}}{2E} \Leftrightarrow \Delta m^2_{31} < 0$ IO
2. Phase Difference: Reactor $\bar{\nu}_e \to \bar{\nu}_e$ (JUNO)

Normal Ordering

$|\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$

$|\Delta m^2_{31}| > |\Delta m^2_{32}|$

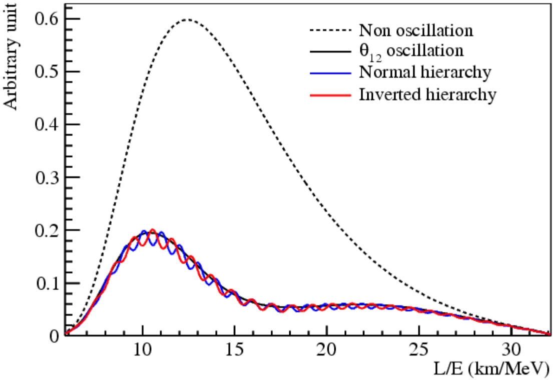




Inverted Ordering

$|\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$

$|\Delta m^2_{31}| < |\Delta m^2_{32}|$



Neutrino Physics with JUNO, arXiv:1507.05613

$$P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} = 1 - \cos^4\vartheta_{13}\sin^2 2\vartheta_{12}\sin^2\left(\Delta m^2_{21}L/4E\right)$$
$$- \cos^2\vartheta_{12}\sin^2 2\vartheta_{13}\sin^2\left(\Delta m^2_{31}L/4E\right)$$
$$- \sin^2\vartheta_{12}\sin^2 2\vartheta_{13}\sin^2\left(\Delta m^2_{32}L/4E\right)$$

[Petcov, Piai, PLB 533 (2002) 94; Choubey, Petcov, Piai, PRD 68 (2003) 113006; Learned, Dye, Pakvasa, Svoboda, PRD 78 (2008) 071302; Zhan, Wang, Cao, Wen, PRD 78 (2008) 111103, PRD 79 (2009) 073007]

CP Violation?

$$A_{\alpha\beta}^{\mathrm{CP}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

$$= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin\delta_{13}$$

Necessary conditions for observation of CP violation:

- Sensitivity to all mixing angles, including small ϑ_{13}
- Sensitivity to oscillations due to Δm_{21}^2 and Δm_{31}^2

LBL $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad A = \frac{2EV}{\Delta m^2_{31}} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad \Delta m^2_{21}/\Delta m^2_{31} \ll 1$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq\ & \sin^2 \underset{\vartheta_{13}\downarrow}{2\vartheta_{13}} \sin^2 \underset{\vartheta_{23}\ \text{octant}\downarrow}{\vartheta_{23}} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ & + \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \underset{\uparrow\ \text{CPV}}{\delta_{13}}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ & + \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2 \sin^2 2\vartheta_{12} \cos^2\vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

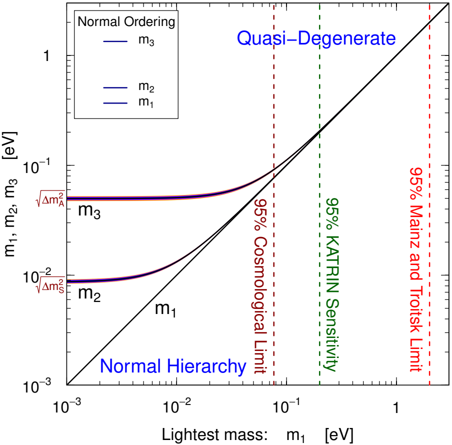
NO: $\Delta m^2_{31} > 0$ IO: $\Delta m^2_{31} < 0$

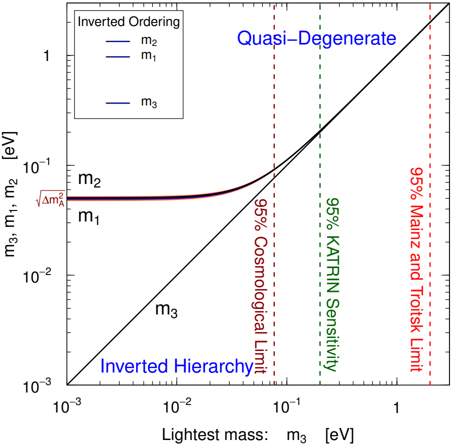
For antineutrinos: $\delta_{13} \to -\delta_{13}$ (CPV) and $A \to -A$ (Matter Effect)

[see: Mezzetto, Schwetz, JPG 37 (2010) 103001]

Absolute Scale of Neutrino Masses

Mass Hierarchy or Degeneracy?





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{S}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{A}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{A}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{A}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_{\text{A}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

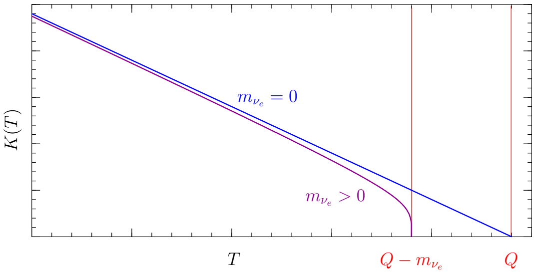
Tritium Beta-Decay

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

Kurie function: $K(T) = \left[(Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2} \right]^{1/2}$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$



$m_{\nu_e} < 2.2\,\mathrm{eV}$ (95% C.L.)

Mainz & Troitsk

[Weinheimer, hep-ex/0210050]

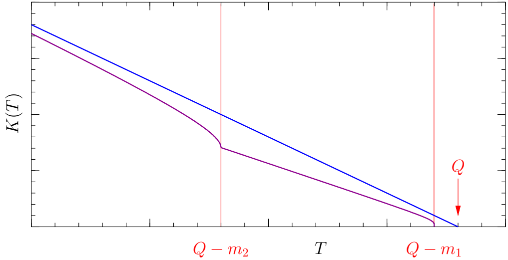
future: KATRIN

[www.katrin.kit.edu]

started data taking 2018

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters $\left(\sum_k |U_{ek}|^2 = 1 \right)$

if experiment is not sensitive to masses $(m_k \ll Q - T)$

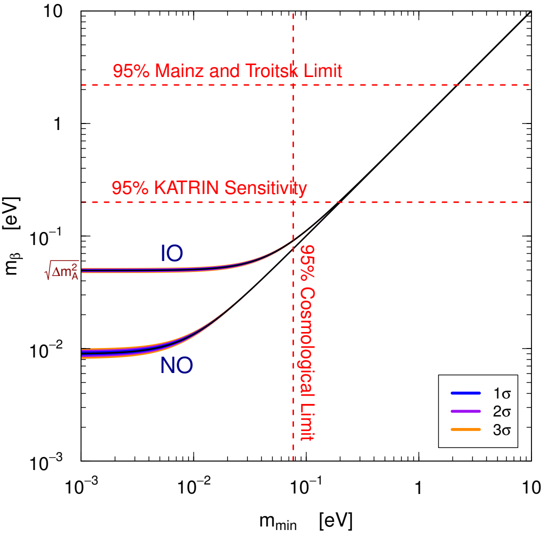
effective mass: $m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



- Quasi-Degenerate:

$$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- Inverted Hierarchy:

$$m_\beta^2 \simeq (1 - s_{13}^2)\Delta m_A^2 \simeq \Delta m_A^2$$

- Normal Hierarchy:

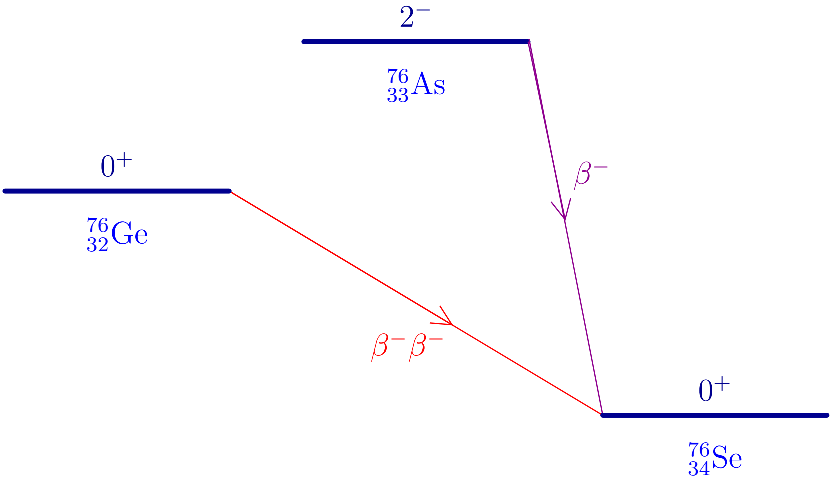
$$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$$
$$\simeq 2 \times 10^{-5} + 6 \times 10^{-5}\, \mathrm{eV}^2$$

- If $m_\beta \lesssim 4 \times 10^{-2}\, \mathrm{eV}$

$\Downarrow$

Normal Spectrum

Neutrinoless Double-Beta Decay



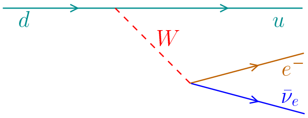
Effective Majorana Neutrino Mass: $$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

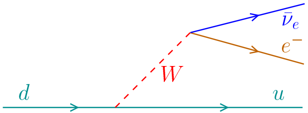
Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model



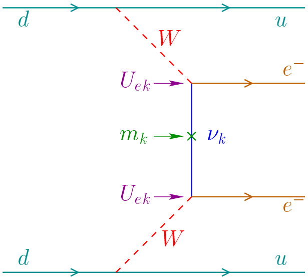


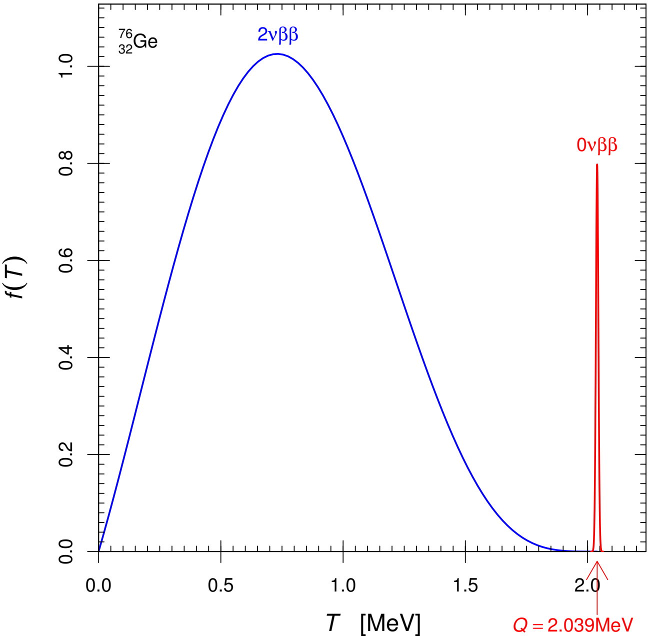
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$





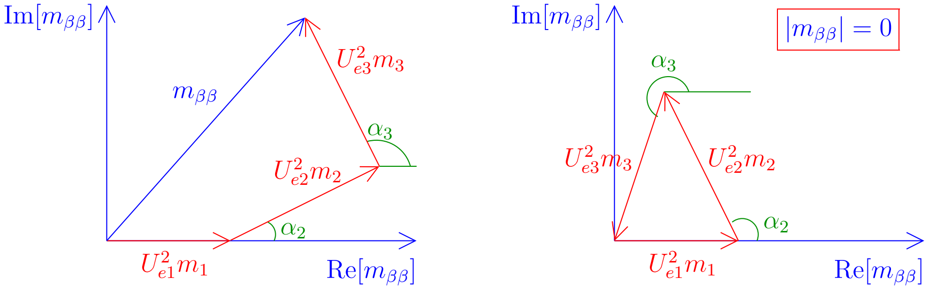
$^{76}_{32}$Ge
2νββ
0νββ
f(T)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
T [MeV]
Q = 2.039MeV

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} $\Rightarrow$ possible cancellations

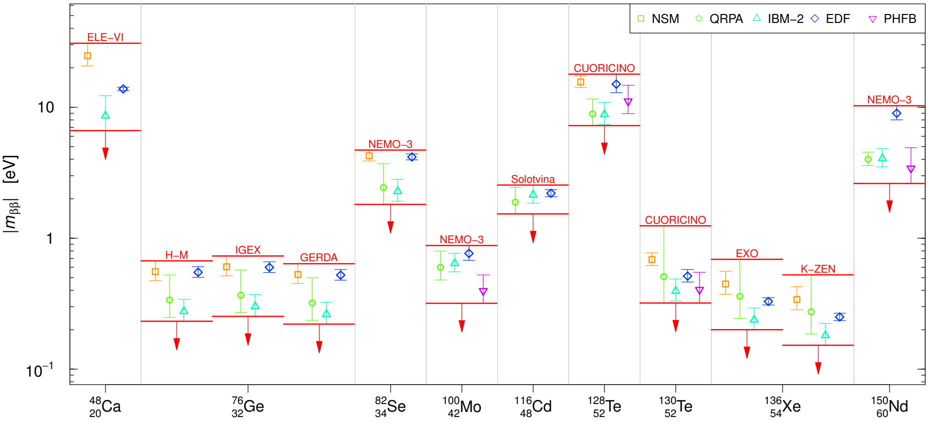
$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$



90% C.L. Experimental Bounds

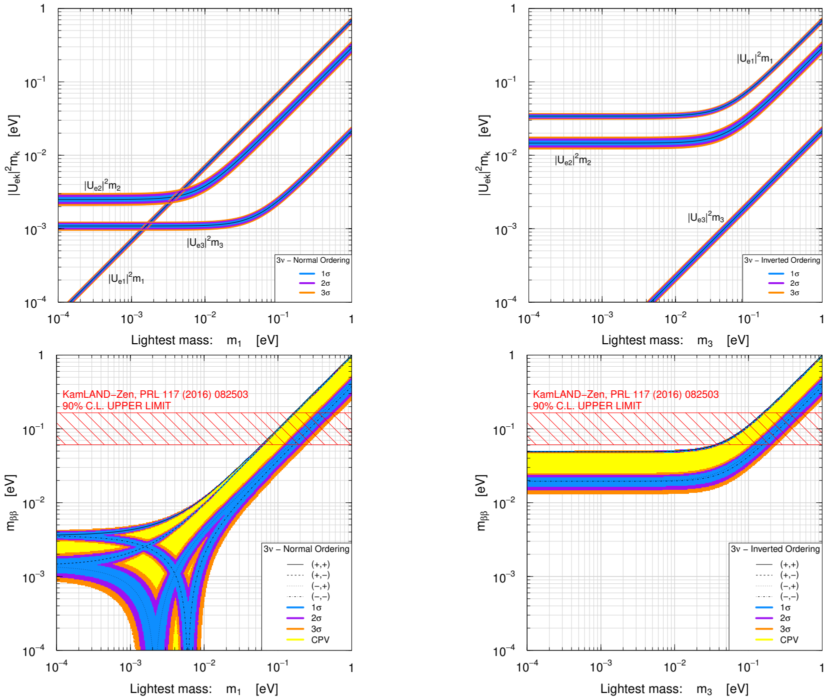
$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	Majorana	$> 4.8 \times 10^{25}$	$< 0.20 - 0.43$
	GERDA	$> 8.0 \times 10^{25}$	$< 0.12 - 0.26$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORE	$> 1.5 \times 10^{25}$	$< 0.11 - 0.52$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.17 - 0.49$
	KamLAND-Zen	$> 1.1 \times 10^{26}$	$< 0.06 - 0.16$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



[Bilenky, CG, IJMPA 30 (2015) 0001]

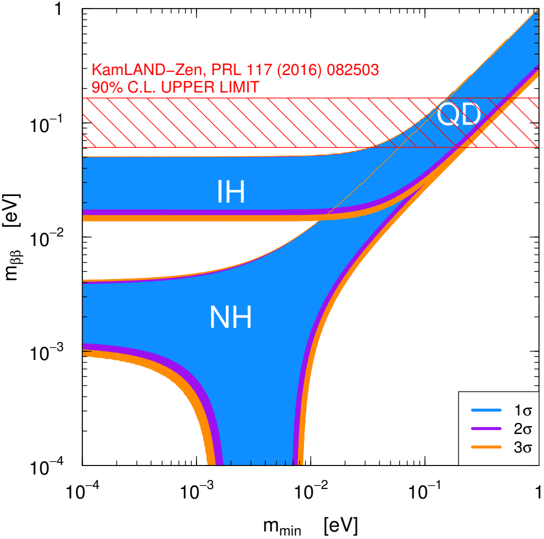
Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$



$|U_{ek}|^2 m_k$ [eV]
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
3ν – Normal Ordering
1σ
2σ
3σ
Lightest mass: m_1 [eV]
3ν – Inverted Ordering
Lightest mass: m_3 [eV]
KamLAND–Zen, PRL 117 (2016) 082503
90% C.L. UPPER LIMIT
$m_{\beta\beta}$ [eV]
(+,+)
(+,−)
(−,+)
(−,−)
CPV

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\text{A}}}|$$
$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\,\text{eV}$$

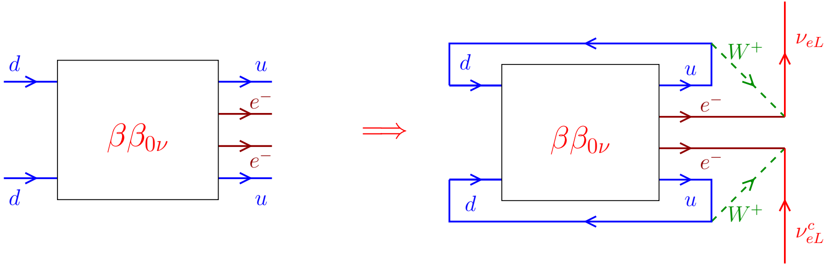
- If $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV}$

$$\Downarrow$$

Normal Spectrum

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

- $|m_{\beta\beta}|$ can vanish because of unfortunate cancellations among the ν_1, ν_2, ν_3 contributions or because neutrinos are Dirac particles.
- However, $\beta\beta_{0\nu}$ decay can be generated by another mechanism beyond the Standard Model.
- In this case, a Majorana mass for ν_e is generated by radiative corrections:



[Schechter, Valle, PRD 25 (1982) 2951; Takasugi, PLB 149 (1984) 372]

- Majorana Mass Term: $\mathcal{L}^{\mathrm{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right)$
- Very small four-loop diagram contribution: $m_{ee} \sim 10^{-24}\,\mathrm{eV}$

[Duerr, Lindner, Merle, JHEP 06 (2011) 091 (arXiv:1105.0901)]

- In any case finding $\beta\beta_{0\nu}$ decay is important for
  - Finding total Lepton number violation ($\Delta L = \pm 2$).
  - Establishing the Majorana (or pseudo-Dirac) nature of neutrinos.
- On the other hand, even if $\beta\beta_{0\nu}$ decay is not found, it is not possible to prove experimentally that neutrinos are Dirac particles, because
  - A Dirac neutrino is equivalent to 2 Majorana neutrinos with the same mass.
  - It is impossible to prove experimentally that the mass splitting is exactly zero.

Summary of Three-Neutrino Mixing

Robust 3ν-Mixing Paradigm

$$\Delta m^2_{\text{S}} \simeq 7.4 \times 10^{-5}\,\text{eV}^2 \qquad \Delta m^2_{\text{A}} \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{12} \simeq 0.3 \qquad \sin^2\vartheta_{23} \simeq 0.5 \qquad \sin^2\vartheta_{13} \simeq 0.02$$

β and $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_1, m_2, m_3 \lesssim 1\,\text{eV}$

To Do

Theory: Why lepton mixing $\neq$ quark mixing?
(Due to Majorana nature of ν's?)
Why $0 < \sin^2\vartheta_{13} \ll \sin^2\vartheta_{12} < \sin^2\vartheta_{23} \simeq 0.5$?

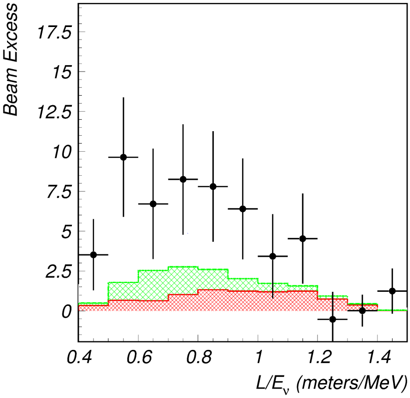
Experiments: Measure mass ordering and CP violation.
Find absolute mass scale and Majorana or Dirac.

Short-Baseline Neutrino Oscillation Anomalies

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e \qquad 20\,\text{MeV} \leq E \leq 52.8\,\text{MeV}$



$\Delta m^2_{\text{SBL}} \gtrsim 0.1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$

- Well-known and pure source of $\bar{\nu}_\mu$

$\underset{800\,\text{MeV}}{p} + \text{target} \to \pi^+ \xrightarrow{\text{at rest}} \mu^+ + \nu_\mu$

$\mu^+ \xrightarrow[\text{at rest}]{} e^+ + \nu_e + \bar{\nu}_\mu$

$L \simeq 30\,\text{m}$

$\bar{\nu}_e + p \to n + e^+$

Well-known detection process of $\bar{\nu}_e$

- $\approx 3.8\sigma$ excess
- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

[PRD 65 (2002) 112001]

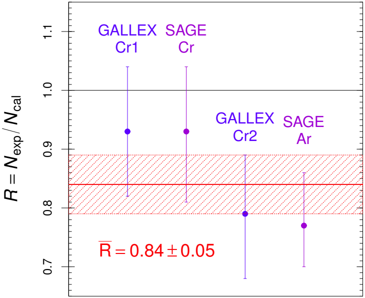
Gallium Anomaly

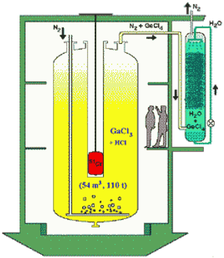
Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

$E \simeq 0.75\,\text{MeV}$ $\quad E \simeq 0.81\,\text{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$





$\approx 2.9\sigma$ deficit

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$ $\quad \langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

$\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807; Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344, MPLA 22 (2007) 2499, PRD 78 (2008) 073009, PRC 83 (2011) 065504]

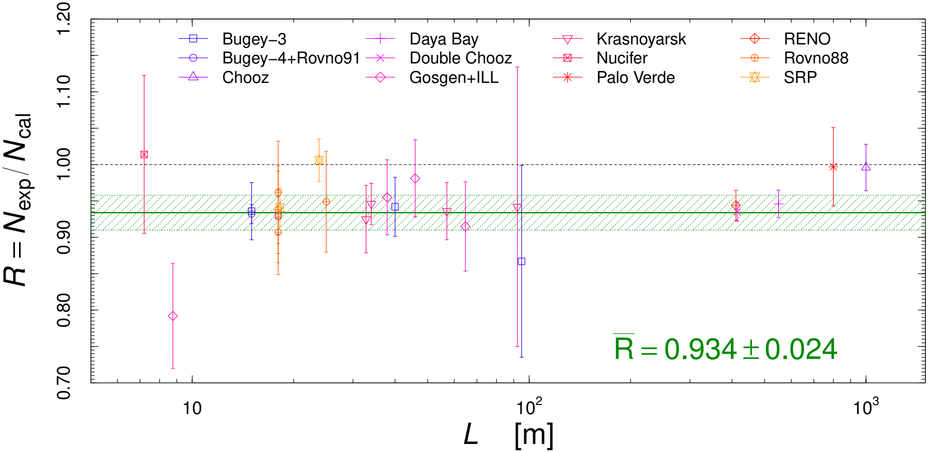
- ${}^{3}\text{He} + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + {}^{3}\text{H}$ cross section measurement [Frekers et al., PLB 706 (2011) 134]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes: Huber-Mueller (HM)

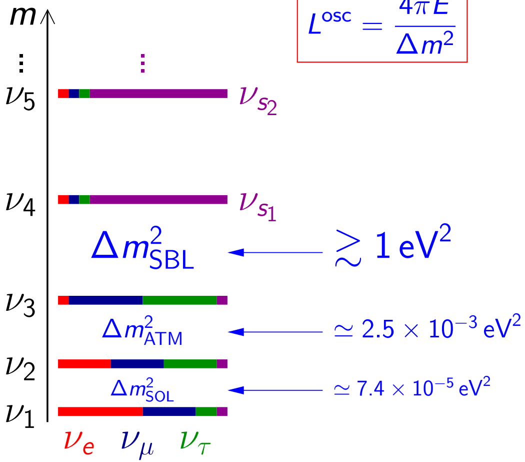
[Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]

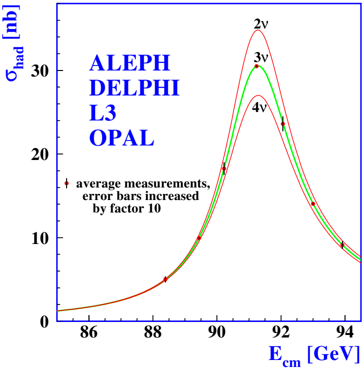


$\approx 2.8\sigma$ deficit

Beyond Three-Neutrino Mixing: Sterile Neutrinos

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$





$$N^{\text{LEP}}_{\nu_{\text{active}}} = 2.9840 \pm 0.0082$$

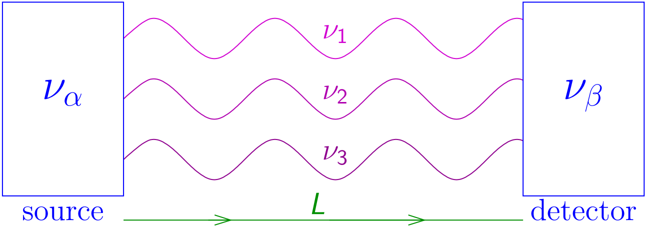
Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

Short-Baseline Neutrino Oscillations

Three-Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle$$



$$|\nu_{\text{detector}}\rangle \simeq U_{\alpha 1}\,e^{-iEL}\,|\nu_1\rangle + U_{\alpha 2}\,e^{-iEL}\,|\nu_2\rangle + U_{\alpha 3}\,e^{-iEL}\,|\nu_3\rangle = e^{-iEL}|\nu_\alpha\rangle$$

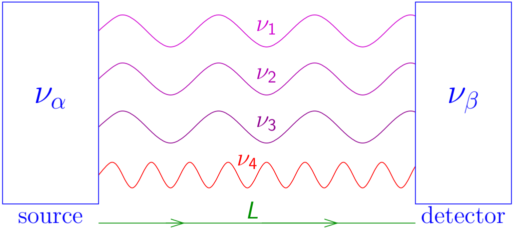
$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \simeq |e^{-iEL}\langle \nu_\beta | \nu_\alpha \rangle|^2 = \delta_{\alpha\beta}$$

No Observable Short-Baseline Neutrino Oscillations!

Short-Baseline Neutrino Oscillations

3+1 Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle + U_{\alpha 4}\,|\nu_4\rangle$$



$$|\nu_{\text{detector}}\rangle \simeq e^{-iEL}\left(U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle\right) + U_{\alpha 4}\,e^{-iE_4 L}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \neq \delta_{\alpha\beta}$$

Observable Short-Baseline Neutrino Oscillations!

The oscillation probabilities depend on U and
$\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$

Effective 3+1 SBL Oscillation Probabilities

Appearance ($\alpha \neq \beta$)

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- $\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$
- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, PRD 87, 113004 (2013) 113004]

3+1: Appearance vs Disappearance

- SBL Oscillation parameters: Δm^2_{41} $|U_{e4}|^2$ $|U_{\mu4}|^2$ $(|U_{\tau4}|^2)$
- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

quadratically suppressed for small $|U_{e4}|^2$ and $|U_{\mu4}|^2$

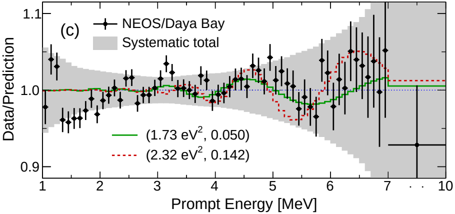
$\Downarrow$

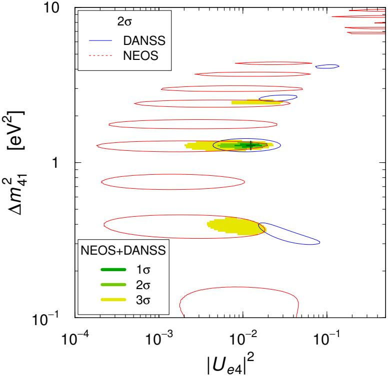
Appearance-Disappearance Tension

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

Reactor Spectral Ratios

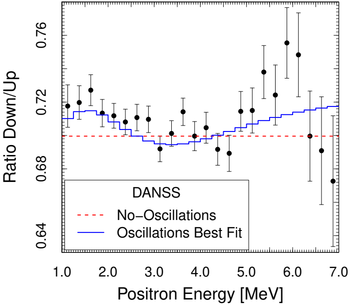
NEOS [PRL 118 (2017) 121802, arXiv:1610.05134]





DANSS

[PLB 787 (2018) 56, arXiv:1804.04046]



MODEL INDEPENDENT!

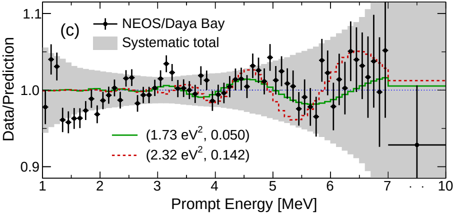
$\sim 3.5\sigma$

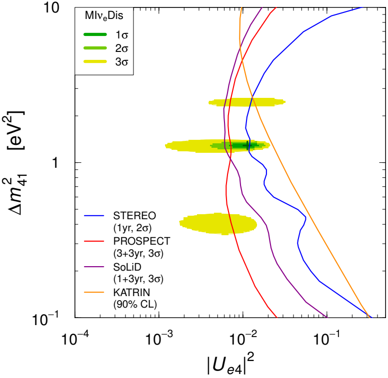
[Gariazzo, CG, Laveder, Li, PLB 782 (2018) 13, arXiv:1801.06467]

[See also: Dentler et al, JHEP 1808 (2018) 010, arXiv:1803.10661]

Reactor Spectral Ratios

NEOS [PRL 118 (2017) 121802, arXiv:1610.05134]





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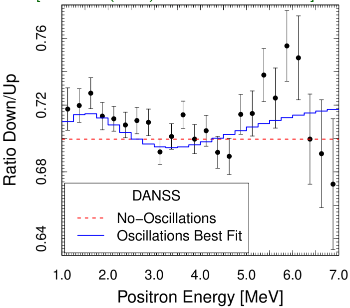

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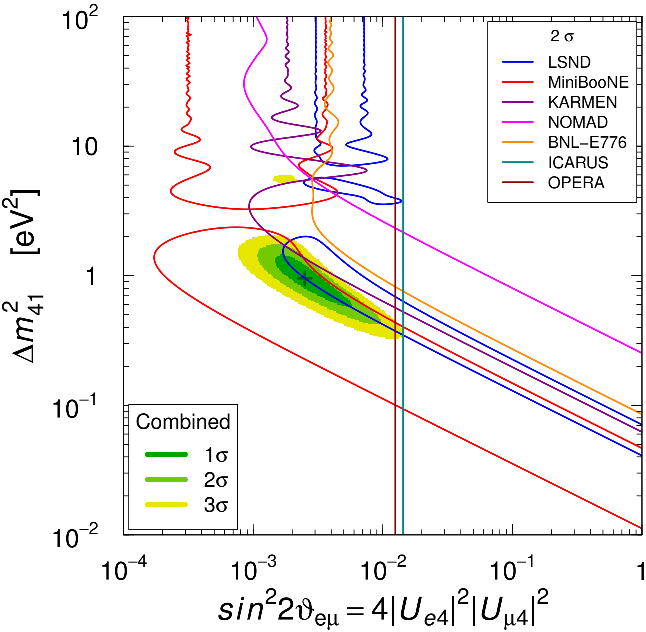
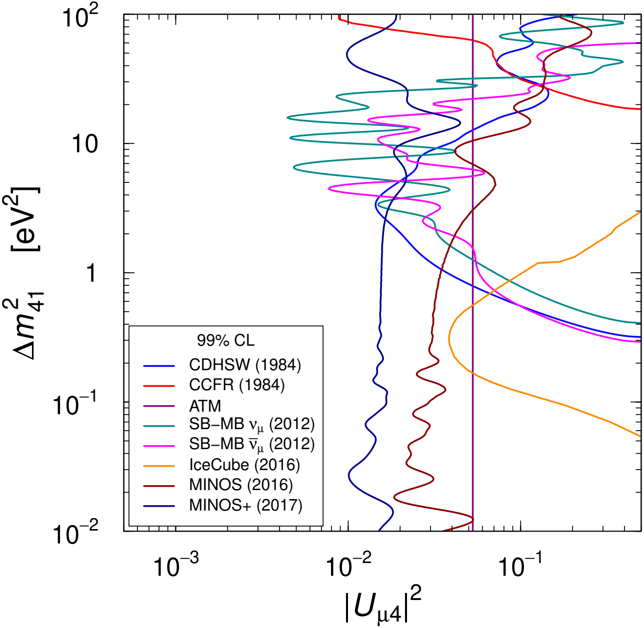
$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ Appearance



ν_μ and $\bar{\nu}_\mu$ Disappearance

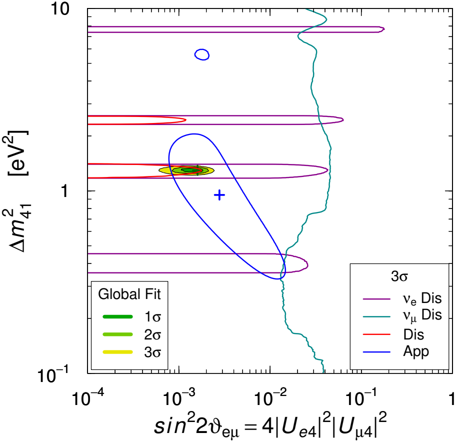


3+1 Appearance-Disappearance Tension

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$

$\nu_\mu \to \nu_e$ APP
$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- Global Fit without MINOS+
  $\chi^2_{\text{PG}}/\text{NDF}_{\text{PG}} = 7.8/2 \Rightarrow \text{GoF}_{\text{PG}} = 2\%$
- Similar tension in
  $3+2, \quad 3+3, \quad \ldots, \quad 3+N_s$

[CG, Zavanin, MPLA 31 (2015) 1650003]

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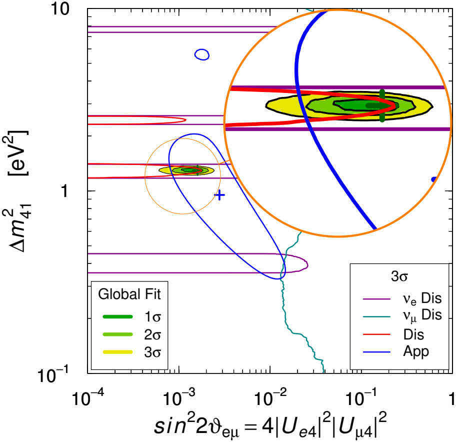

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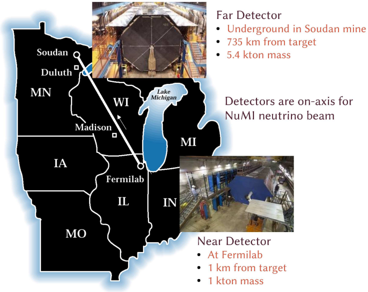
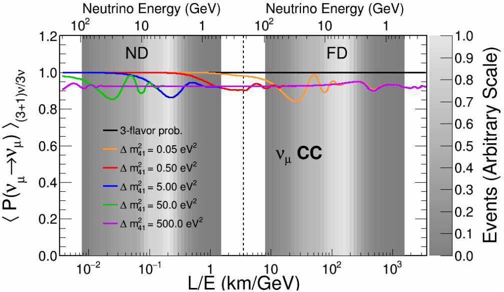
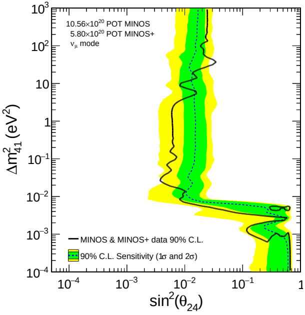
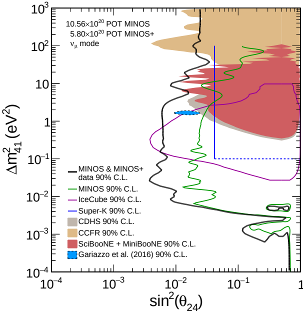
New Bound from MINOS+



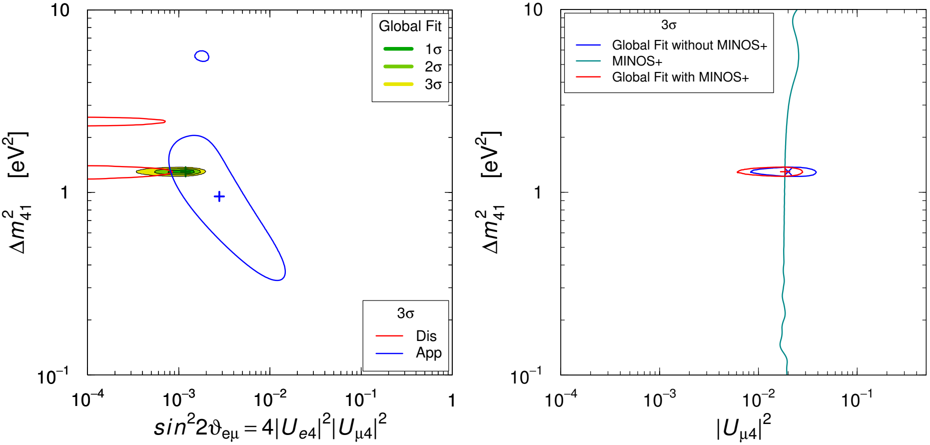
[arXiv:1710.06488]







Effects of MINOS+



- $\chi^2_{PG}/\text{NDF}_{PG} = 18.3/2 \Rightarrow \text{GoF}_{PG} = 0.01\%$ $\longleftarrow$ Intolerable tension!
- The MINOS+ bound (if correct) disfavors the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal.

[See also Dentler, Hernandez-Cabezudo, Kopp, Machado, Maltoni, Martinez-Soler, Schwetz, arXiv:1803.10661]

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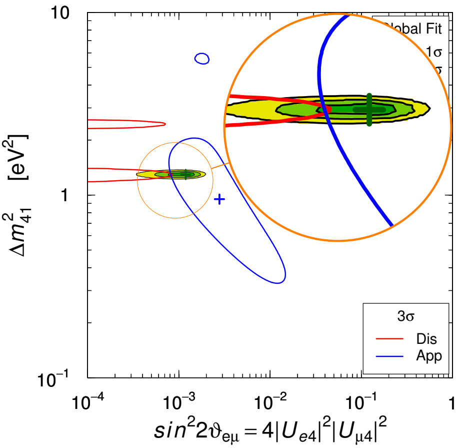

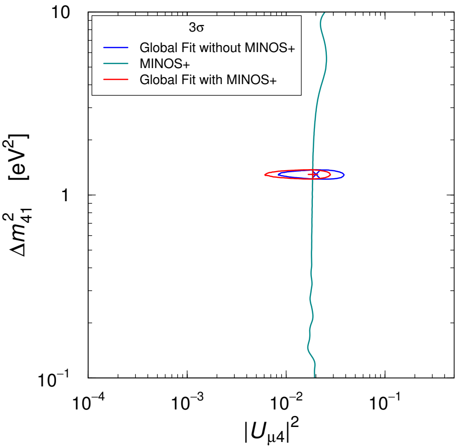

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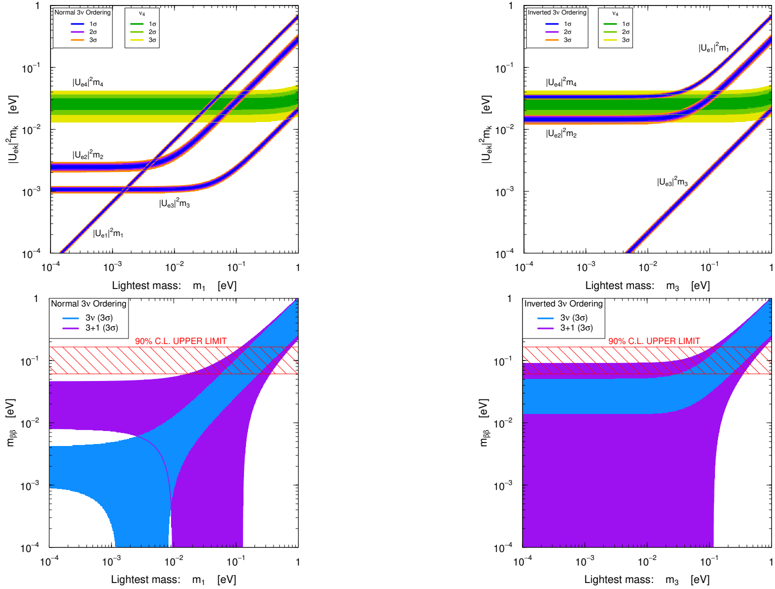
Neutrinoless Double-Beta Decay

$$m_{\beta\beta} = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3 + |U_{e4}|^2 e^{i\alpha_{41}} m_4 \right|$$



Conclusions

- Exciting model-independent indication of light sterile neutrinos at the eV scale from the NEOS and DANSS experiments $\Longrightarrow$ New Physics beyond the Standard Model?

- Agreement with the Reactor and Gallium Anomalies $\Longrightarrow$ Needed revision of the ^{235}U calculation and small decrease of the GALLEX and SAGE efficiencies.

- Can be checked in the near future by the reactor experiments PROSPECT, SoLid, STEREO.

- Independent tests through effect of m_4 in β-decay (KATRIN), EC (ECHo, HOLMES) and $\beta\beta_{0\nu}$-decay.

- The MINOS+ bound (if correct) disfavors the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal.