Light Sterile Neutrinos – Theory

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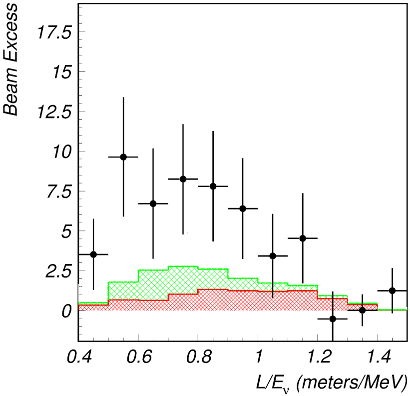
Indications of SBL Oscillations Beyond 3ν

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad$ $20\,\text{MeV} \leq E \leq 52.8\,\text{MeV}$



$\Delta m^2_{\text{SBL}} \gtrsim 0.1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$

- Well-known and pure source of $\bar{\nu}_\mu$

p (800 MeV) $+ \text{target} \to \pi^+ \xrightarrow{\text{at rest}} \mu^+ + \nu_\mu$

$\mu^+ \xrightarrow[\text{at rest}]{} e^+ + \nu_e + \bar{\nu}_\mu$

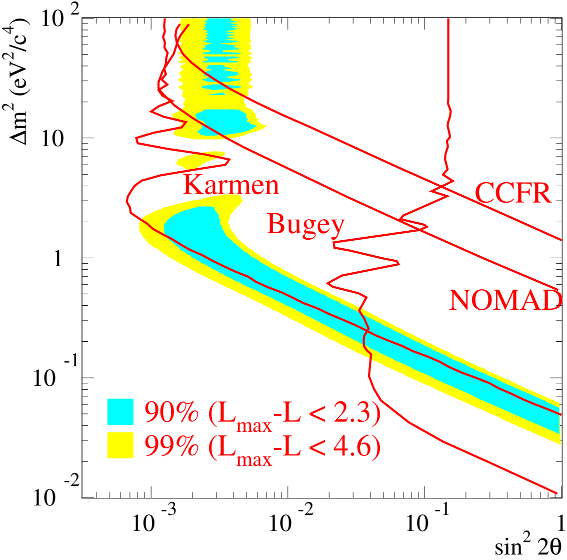
$L \simeq 30\,\text{m}$

$\bar{\nu}_e + p \to n + e^+$

Well-known detection process of $\bar{\nu}_e$

- $\approx 3.8\sigma$ excess
- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

[PRD 65 (2002) 112001]



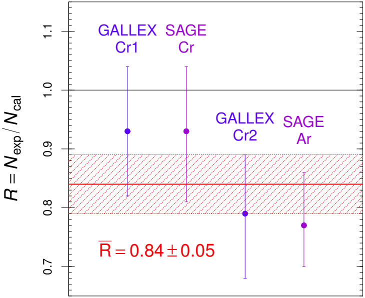
$$\Delta m^2_{\text{SBL}} \gtrsim 3 \times 10^{-2}\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}} \simeq 2.5 \times 10^{-3}\,\text{eV}^2 \gg \Delta m^2_{\text{SOL}}$$

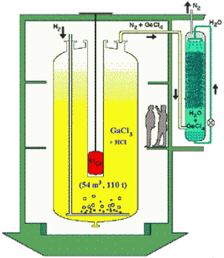
Gallium Anomaly

Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad E \simeq 0.75\,\text{MeV}$ $\qquad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$ $\quad E \simeq 0.81\,\text{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$





$\approx 2.9\sigma$ deficit

$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m} \quad \langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

$$\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$$

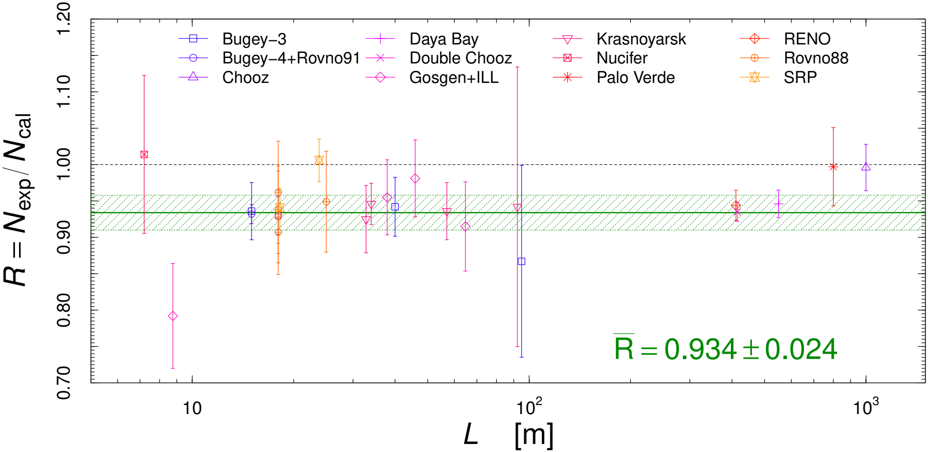
[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807; Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344, MPLA 22 (2007) 2499, PRD 78 (2008) 073009, PRC 83 (2011) 065504]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes: Huber-Mueller (HM)

[Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]



$\approx 2.8\sigma$ deficit

Standard Three Neutrino Mixing

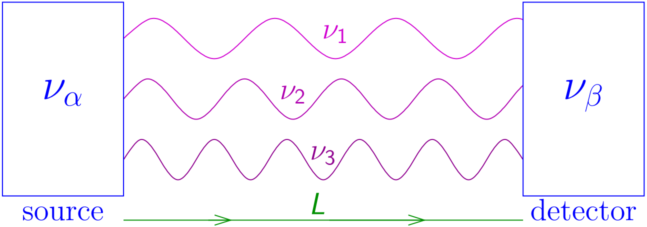
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- Neutrino Mixing: a Flavor Neutrino is a superposition of Massive Neutrinos

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{e2}^* & U_{e3}^* \\ U_{\mu1}^* & U_{\mu2}^* & U_{\mu3}^* \\ U_{\tau1}^* & U_{\tau2}^* & U_{\tau3}^* \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

- U is the 3×3 unitary Neutrino Mixing Matrix

Neutrino Oscillations

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^{*}\,|\nu_1\rangle + U_{\alpha 2}^{*}\,|\nu_2\rangle + U_{\alpha 3}^{*}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{\alpha 1}^{*}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{\alpha 2}^{*}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{\alpha 3}^{*}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

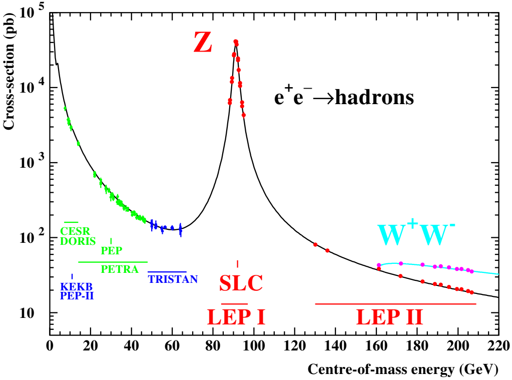
$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

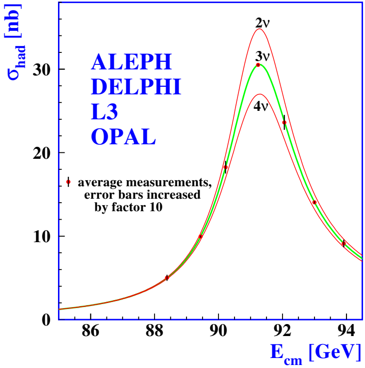
$$P_{\nu_\alpha\to\nu_\beta}(L) = |\langle\nu_\beta|\nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k}\,U_{\alpha k}^{*}\,U_{\beta j}^{*}\,U_{\alpha j}\,\exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

The oscillation probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

- In the standard framework of three-neutrino mixing there are two independent Δm^2's:
  - $\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$
  - $\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$
- Atmospheric and solar neutrino oscillations are detectable at the distances
  - $L^{\text{osc}}_{\text{ATM}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{ATM}}} \approx 1\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
  - $L^{\text{osc}}_{\text{SOL}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{SOL}}} \approx 50\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
- The atmospheric and solar neutrino oscillations cannot explain flavor neutrino transitions at shorter distances.

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

$$e^+ e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$$

3 light active flavor neutrinos

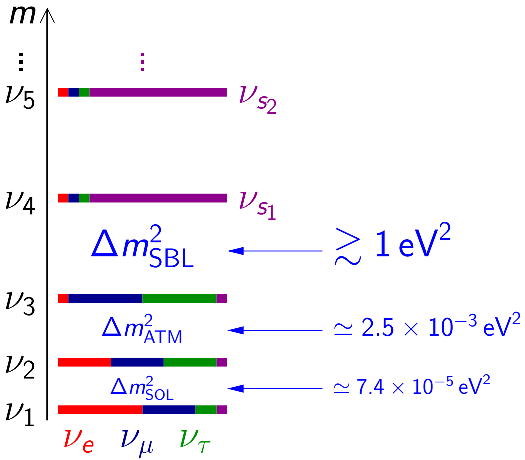
mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots}$$

Beyond Three-Neutrino Mixing: Sterile Neutrinos



$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$

Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

Sterile Neutrinos from Physics Beyond the SM

- Neutrinos are special in the Standard Model: the only neutral fermions
- Active left-handed neutrinos can mix with non-SM singlet fermions often called right-handed neutrinos
- Light left-handed anti-ν_R are light sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions

[Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into light sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Short-baseline anomalies + 3ν-mixing:

$$\Delta m^2_{21} \ll |\Delta m^2_{31}| \ll |\Delta m^2_{41}| \leq \ldots$$

ν_1	ν_2	ν_3	ν_4	$\ldots$
ν_e	ν_μ	ν_τ	ν_{s_1}	$\ldots$

- Here I consider sterile neutrinos with mass scale $\sim 1\,\text{eV}$ in light of short-baseline anomalies.

- Other possibilities (not incompatible):

  - Very light sterile neutrinos with mass scale $\ll 1\,\text{eV}$: important for solar neutrino phenomenology

    [de Holanda, Smirnov, PRD 69 (2004) 113002; PRD 83 (2011) 113011]

    [Das, Pulido, Picariello, PRD 79 (2009) 073010]

    Recent Daya Bay constraints for $10^{-3} \lesssim \Delta m^2 \lesssim 10^{-1}\,\text{eV}^2$ [PRL 113 (2014) 141802]

  - Heavy sterile neutrinos with mass scale $\gg 1\,\text{eV}$: could be Warm Dark Matter

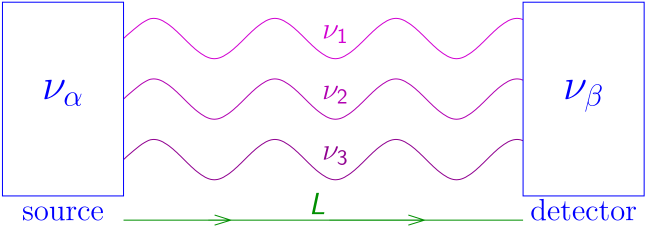
    [Asaka, Blanchet, Shaposhnikov, PLB 631 (2005) 151; Asaka, Shaposhnikov, PLB 620 (2005) 17; Asaka, Shaposhnikov, Kusenko, PLB 638 (2006) 401; Asaka, Laine, Shaposhnikov, JHEP 0606 (2006) 053, JHEP 0701 (2007) 091]

    [Reviews: Kusenko, Phys. Rept. 481 (2009) 1; Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191; Boyarsky, Iakubovskyi, Ruchayskiy, Phys. Dark Univ. 1 (2012) 136; Drewes, IJMPE, 22 (2013) 1330019]

Short-Baseline Neutrino Oscillations?

Three-Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle$$



$$\begin{aligned} |\nu_{\text{detector}}\rangle &\simeq U_{\alpha 1}^* e^{-iEL} |\nu_1\rangle + U_{\alpha 2}^* e^{-iEL} |\nu_2\rangle + U_{\alpha 3}^* e^{-iEL} |\nu_3\rangle \\ &= e^{-iEL} \left(U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle \right) = e^{-iEL} |\nu_\alpha\rangle \end{aligned}$$

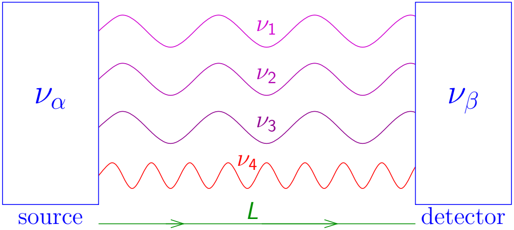
$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \simeq |e^{-iEL} \langle \nu_\beta | \nu_\alpha \rangle|^2 = \delta_{\alpha\beta}$$

No Short-Baseline Neutrino Oscillations!

Short-Baseline Neutrino Oscillations?

3+1 Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^{*} |\nu_1\rangle + U_{\alpha 2}^{*} |\nu_2\rangle + U_{\alpha 3}^{*} |\nu_3\rangle + U_{\alpha 4}^{*} |\nu_4\rangle$$



$$|\nu_{\text{detector}}\rangle \simeq e^{-iEL} \left(U_{\alpha 1}^{*} |\nu_1\rangle + U_{\alpha 2}^{*} |\nu_2\rangle + U_{\alpha 3}^{*} |\nu_3\rangle \right) + U_{\alpha 4}^{*} e^{-iE_4 L} |\nu_3\rangle \not\propto |\nu_\alpha\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \neq \delta_{\alpha\beta}$$

Short-Baseline Neutrino Oscillations!

The oscillation probabilities depend on U and

$$\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$$

- Some authors that probably did not think about the quantum mechanics of neutrino oscillations present $\nu_\mu \to \nu_e$ short-baseline transitions due to sterile neutrinos as

$$\nu_\mu \to \nu_s \to \nu_e$$

- This is wrong!

THERE IS NO INTERMEDIATE ν_s !

Two possible interpretations of $\nu_\mu \to \nu_s \to \nu_e$:

1. There is a transition from ν_μ to ν_s, and then to ν_e: wrong!
   Because the intermediate determination of the neutrino flavor interrupts the quantum evolution.
   Moreover, ν_s is not detectable!

2. There is an intermediate linear combination of massive neutrinos that corresponds to $|\nu_s\rangle$: wrong!
   This is possible only with the mixing $(|a|^2 + |b|^2 + |c|^2 = 1)$

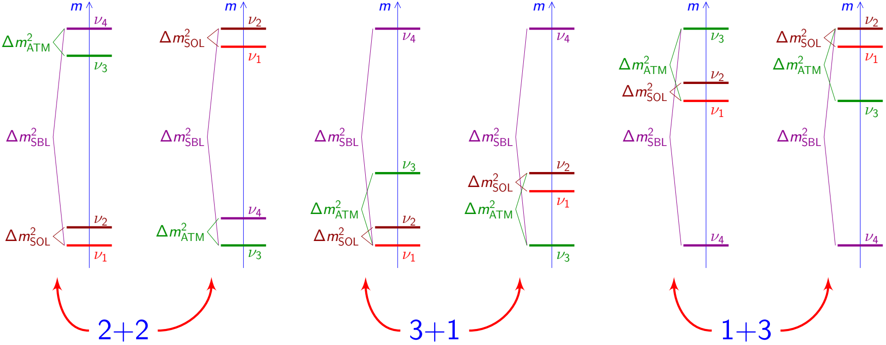
$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \\ |\nu_s\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cdots & \cdots & \cdots & 0 \\ a & b & c & 1 \\ \cdots & \cdots & \cdots & 0 \\ -a & -b & -c & 1 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \end{pmatrix}$$

$$|\nu(L)\rangle = \frac{e^{-iEL}}{\sqrt{2}} \left[a\,|\nu_1\rangle + b\,|\nu_2\rangle + c\,|\nu_3\rangle + e^{-i(E_4-E)L}\,|\nu_4\rangle \right]$$

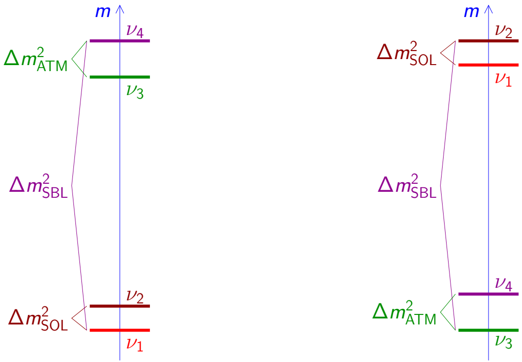
$$|\nu(L)\rangle = |\nu_\mu\rangle \quad \text{for} \quad L = 0 \quad \text{and} \quad |\nu(L)\rangle \propto |\nu_s\rangle \quad \text{for} \quad e^{-i(E_4-E)L} = -1$$

but in this case there are no SBL $\nu_\mu \to \nu_e$ transitions!

Four-Neutrino Schemes: 2+2, 3+1 and 1+3



2+2 Four-Neutrino Schemes

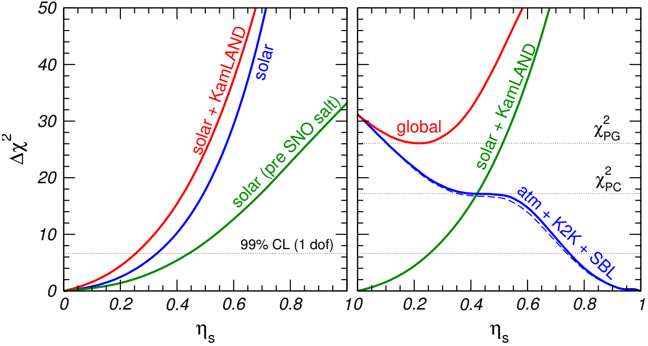


- After LSND (1995) 2+2 was preferred to 3+1, because of the 3+1 appearance-disappearance tension

  [Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

- This is not a perturbation of 3-ν Mixing $\Longrightarrow$ Large active–sterile oscillations for solar or atmospheric neutrinos!

2+2 Schemes are Strongly Disfavored



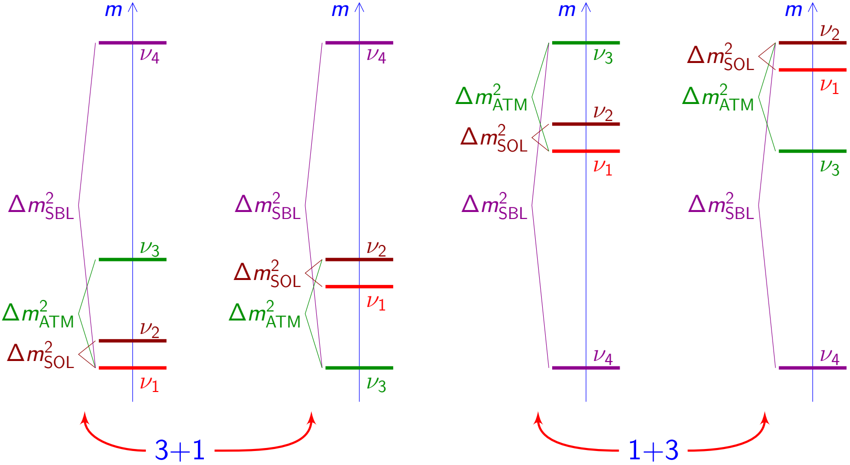
Solar: Matter Effects + SNO NC   Atmospheric: Matter Effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 = 1 - |U_{s3}|^2 + |U_{s4}|^2$$

$$99\% \text{ CL: } \begin{cases} \eta_s < 0.25 & \text{(Solar + KamLAND)} \\ \eta_s > 0.75 & \text{(Atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122]

3+1 and 1+3 Four-Neutrino Schemes



- Perturbation of 3-ν Mixing: $|U_{e4}|^2, |U_{\mu 4}|^2, |U_{\tau 4}|^2 \ll 1 \quad |U_{s4}|^2 \simeq 1$
- 1+3 schemes are disfavored by cosmology (ΛCDM):

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV}$$

[Planck, Astron. Astrophys. 594 (2016) A13 (arXiv:1502.01589)]

Effective 3+1 SBL Oscillation Probabilities

$$|\nu_\alpha\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} |\nu_k\rangle \quad \xrightarrow{t} \quad |\nu_\alpha(t)\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} e^{-iE_k t} |\nu_k\rangle$$

$$A_{\nu_\alpha \to \nu_\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \qquad (\langle \nu_\beta | \nu_k \rangle = U_{\beta k})$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 \\ &= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2 \\ &= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2 \end{aligned}$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2$$

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} \quad \Longrightarrow \quad E_k - E_1 \simeq \frac{\Delta m_{k1}^2}{2p}$$

$$E = p \qquad\qquad t \simeq L$$

$$P_{\nu_\alpha \to \nu_\beta} \simeq \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} \exp\left(-i \frac{\Delta m_{21}^2 L}{2E} \right) + U_{\alpha 3}^* U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m_{41}^2 L}{2E} \right) \right|^2$$

SBL $\Longrightarrow$ $\frac{\Delta m_{21}^2 L}{2E} \ll 1$ $\quad$ $\frac{\Delta m_{31}^2 L}{2E} \ll 1$

$$P_{\nu_\alpha \to \nu_\beta}^{\text{SBL}} \simeq \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m_{41}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^* U_{\beta 4}$$

$$P^{\rm SBL}_{\nu_\alpha\to\nu_\beta} \simeq \left|\delta_{\alpha\beta} - U^*_{\alpha4} U_{\beta4} \left[1 - \exp\left(-i\frac{\Delta m^2_{41} L}{2E}\right)\right]\right|^2$$

$$= \delta_{\alpha\beta} + |U_{\alpha4}|^2 |U_{\beta4}|^2 \left(2 - 2\cos\frac{\Delta m^2_{41} L}{2E}\right)$$

$$- 2\delta_{\alpha\beta} |U_{\alpha4}|^2 \left(1 - \cos\frac{\Delta m^2_{41} L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 2|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right) \left(1 - \cos\frac{\Delta m^2_{41} L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 4|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right) \sin^2 \frac{\Delta m^2_{41} L}{4E}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P^{\rm SBL}_{\nu_\alpha\to\nu_\beta} \simeq 4|U_{\alpha4}|^2 |U_{\beta4}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P^{\rm SBL}_{\nu_\alpha\to\nu_\alpha} \simeq 1 - 4|U_{\alpha4}|^2 \left(1 - |U_{\alpha4}|^2\right) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Appearance $(\alpha \neq \beta)$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\alpha}} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- $\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$
- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, PRD 87, 113004 (2013) 113004]

Common Parameterization of 4 × 4 Mixing Matrix

$$U = \left[W^{34} R^{24} W^{14} R^{23} W^{13} R^{12}\right] \mathrm{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}, e^{i\lambda_{41}}\right)$$

$$= \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ \cdots & \cdots & \cdots & c_{14}s_{24} \\ \cdots & \cdots & \cdots & c_{14}c_{24}s_{34}e^{-i\delta_{34}} \\ \cdots & \cdots & \cdots & c_{14}c_{24}c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 & 0 \\ 0 & 0 & e^{i\lambda_{31}} & 0 \\ 0 & 0 & 0 & e^{i\lambda_{41}} \end{pmatrix}$$

$$|U_{e4}|^2 = \sin^2\vartheta_{14} \quad \Rightarrow \quad \sin^2 2\vartheta_{ee} = 4|U_{e4}|^2\left(1 - |U_{e4}|^2\right) = \sin^2 2\vartheta_{14}$$

$$|U_{\mu4}|^2 = \cos^2\vartheta_{14}\sin^2\vartheta_{24} \simeq \sin^2\vartheta_{24} \Rightarrow \sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2\left(1 - |U_{\mu4}|^2\right) \simeq \sin^2 2\vartheta_{24}$$

3+1: Appearance vs Disappearance

- SBL Oscillation parameters: Δm^2_{41} $|U_{e4}|^2$ $|U_{\mu4}|^2$ $(|U_{\tau4}|^2)$
- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$

quadratically suppressed for small $|U_{e4}|^2$ and $|U_{\mu4}|^2$

$\Downarrow$

Appearance-Disappearance Tension

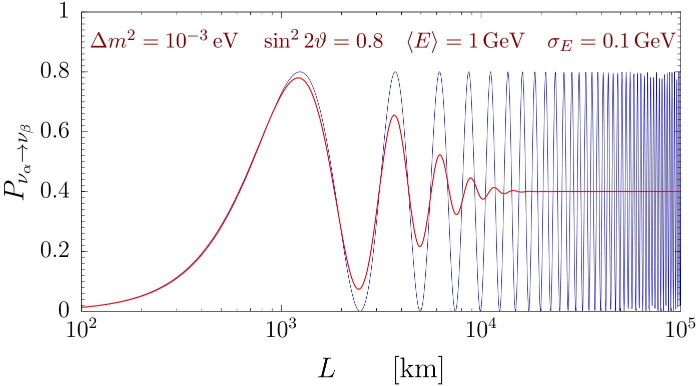
[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

Average over Energy Resolution of the Detector

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\sin^2 2\vartheta\left[1-\cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

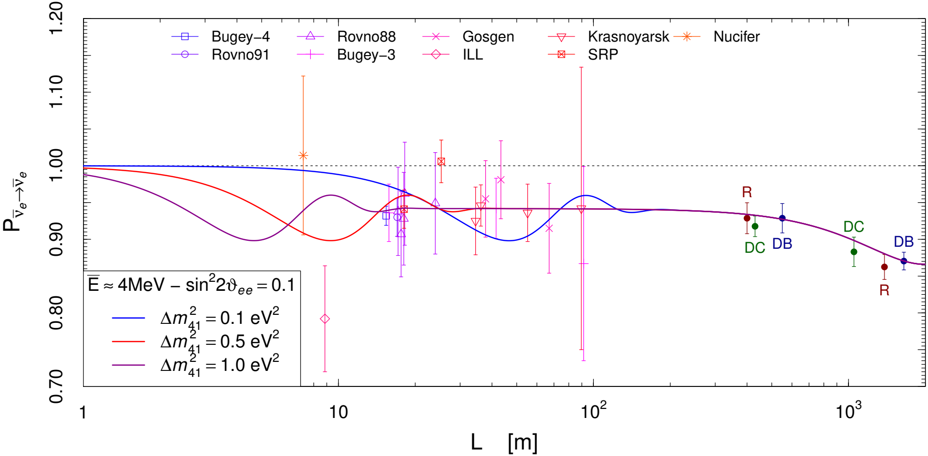
$$\Downarrow$$

$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,dE\right] \qquad (\alpha\neq\beta)$$



ν_e and $\bar{\nu}_e$ Disappearance

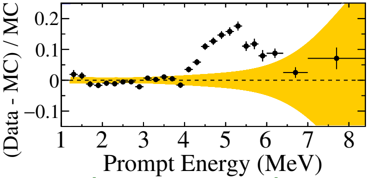
Short-Baseline Reactor Neutrino Oscillations



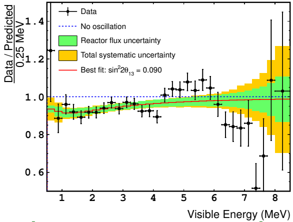
$$\Delta m^2_{\rm SBL} \gtrsim 0.5\,{\rm eV}^2 \gg \Delta m^2_{\rm ATM}$$

- SBL oscillations are averaged at the Daya Bay, RENO, and Double Chooz near detectors $\Longrightarrow$ no spectral distortion

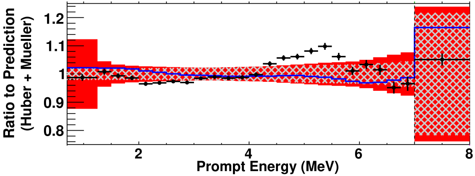
Reactor Antineutrino 5 MeV Bump



[RENO, arXiv:1511.05849]



[Double Chooz, arXiv:1406.7763]



[Daya Bay, arXiv:1508.04233]

- Cannot be explained by neutrino oscillations (SBL oscillations are averaged in RENO, DC, DB).
- It is likely due to a theoretical miscalculation of the spectrum.
- Heretic solution: detector energy nonlinearity. [Mention et al, PLB 773 (2017) 307]

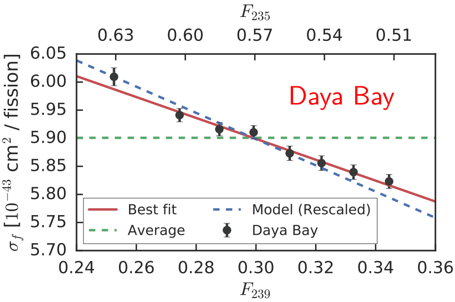
- $\sim 3\%$ effect on total flux, but if it is an excess it increases the anomaly!
- No post-bump complete calculation of the neutrino fluxes.
- Nominal Huber-Mueller flux calculation uncertainty: $\sim 2.7\%$.
- Post-bump estimate of the flux uncertainty due to unknown forbidden decays: $\sim 5\%$.

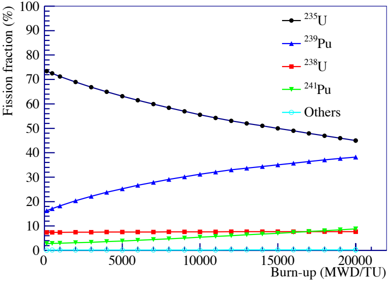
[Hayes and Vogel, ARNPS 66 (2016) 219]

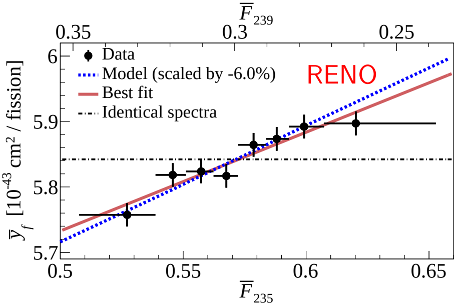
Reactor Fuel Evolution

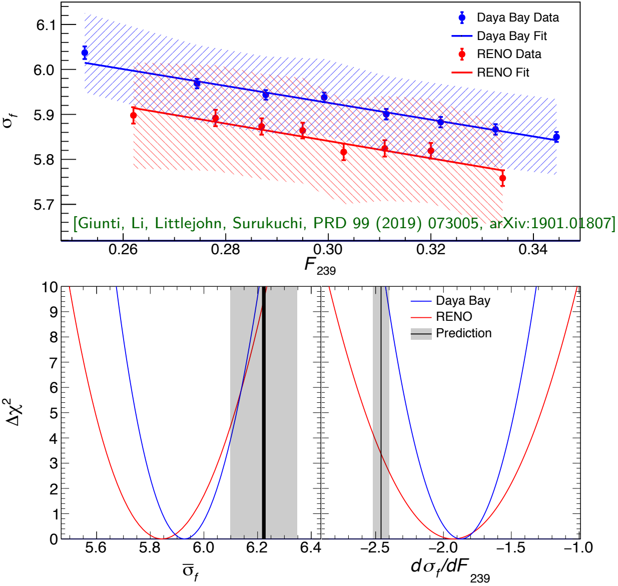
- Reactor $\bar{\nu}_e$ flux produced by the β decays of the fission products of
  $^{235}U \quad ^{238}U \quad ^{239}Pu \quad ^{241}Pu$
- Effective fission fractions:
  $F_{235} \quad F_{238} \quad F_{239} \quad F_{241}$
- Cross section per fission (IBD yield):

$$\sigma_f = \sum_{k=235,238,239,241} F_k \, \sigma_{f,k}$$

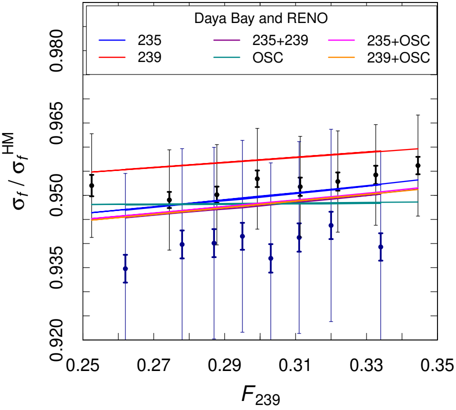


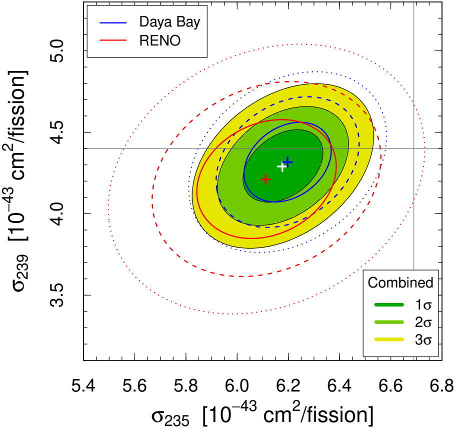






$$\sigma_f(F_{239}) = \bar{\sigma}_f + \frac{d\sigma_f}{dF_{239}} \left(F_{239} - \overline{F}_{239}\right)$$





235: $\quad r_{235} = 0.985 \pm 0.015$

$\chi^2/\text{NDF} = 9.0/15 \quad \text{GoF} = 88\%$

OSC: $\quad P_{\bar{\nu}_e \to \bar{\nu}_e} = 0.939 \pm 0.024$

$\chi^2/\text{NDF} = 16.3/15 \quad \text{GoF} = 37\%$

$$235{+}239: \begin{cases} r_{235} = 0.923 \pm 0.015 \\ r_{239} = 0.975 \pm 0.032 \end{cases}$$

$\chi^2/\text{NDF} = 8.7/14 \quad \text{GoF} = 85\%$

$$235{+}\text{OSC}: \begin{cases} r_{235} = 0.938 \pm 0.029 \\ P_{\bar{\nu}_e \to \bar{\nu}_e} = 0.986 \pm 0.022 \end{cases}$$

$\chi^2/\text{NDF} = 8.8/14 \quad \text{GoF} = 85\%$

[Giunti, Li, Littlejohn, Surukuchi, arXiv:1901.01807]

- Daya Bay and RENO favor a suppression of the ^{235}U flux (235) over oscillations (OSC).

- However, a practically equally good fit is obtained with the hybrid model 235+OSC.

- Moreover, the addition of other reactor data favors oscillations or, better, ^{235}U and/or ^{239}U flux suppression plus oscillations.

[Giunti, Ji, Laveder, Li, Littlejohn, JHEP 1710 (2017) 143, arXiv:1708.01133]

- Even if there are short-baseline neutrino oscillations, it is likely that the reactor antineutrino flux calculations must be corrected (most likely the ^{235}U flux) to fit:

  1. The 5 MeV bump
  2. The fuel evolution data

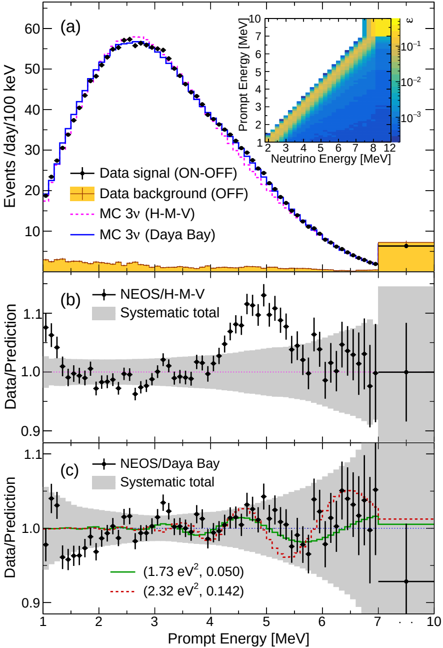
- The search for short-baseline neutrino oscillations needs

model-independent information

$\Uparrow$

ratios of spectra at different distances

NEOS



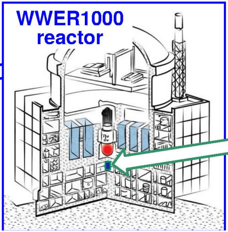
[PRL 118 (2017) 121802, arXiv:1610.05134]

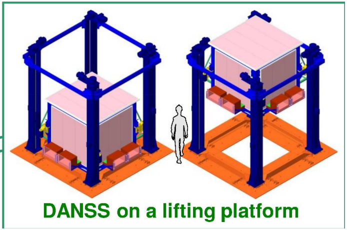
- Hanbit Nuclear Power Complex in Yeong-gwang, Korea.
- Thermal power of 2.8 GW.
- Detector: a ton of Gd-loaded liquid scintillator in a gallery approximately 24 m from the reactor core.
- The measured antineutrino event rate is 1976 per day with a signal to background ratio of about 22.

DANSS

[PLB 787 (2018) 56, arXiv:1804.04046]

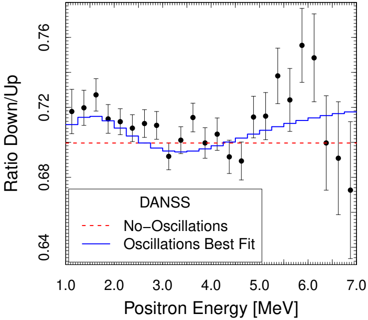
Detector of reactor AntiNeutrino based on Solid Scintillator





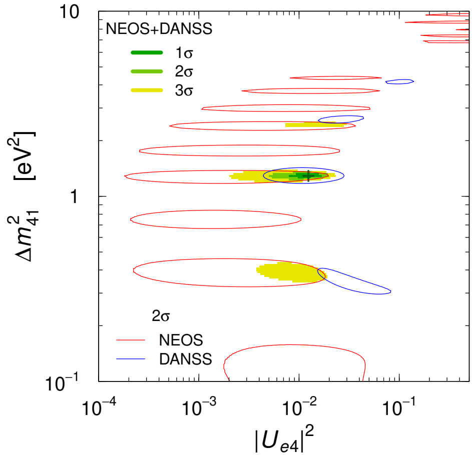
- Installed on a movable platform under a 3 GW reactor.
- Large neutrino flux.
- Reactor shielding of cosmic rays.
- Variable source-detector distance with the same detector!

$$\text{Down} = 12.7\,\text{m}$$
$$\text{Up} = 10.7\,\text{m}$$



Model-Independent $\bar{\nu}_e$ SBL Oscillations

[Gariazzo, Giunti, Laveder, Li, PLB 782 (2018) 13, arXiv:1801.06467]



$\sim 3.5\sigma$

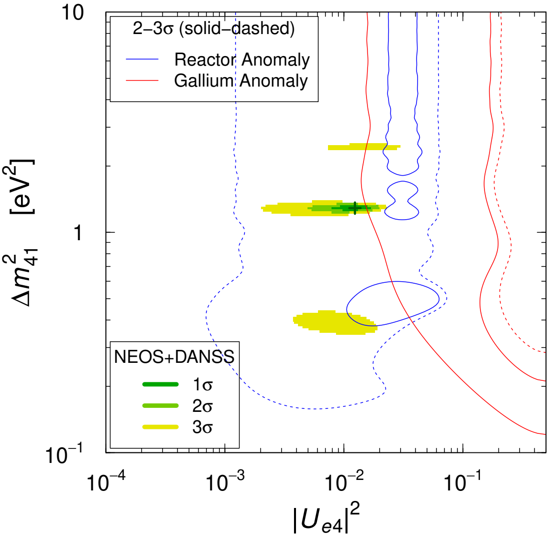
$\Delta m^2_{41} = 1.29 \pm 0.03$

$|U_{e4}|^2 = 0.012 \pm 0.003$

$|U_{e3}|^2 = 0.022 \pm 0.001$

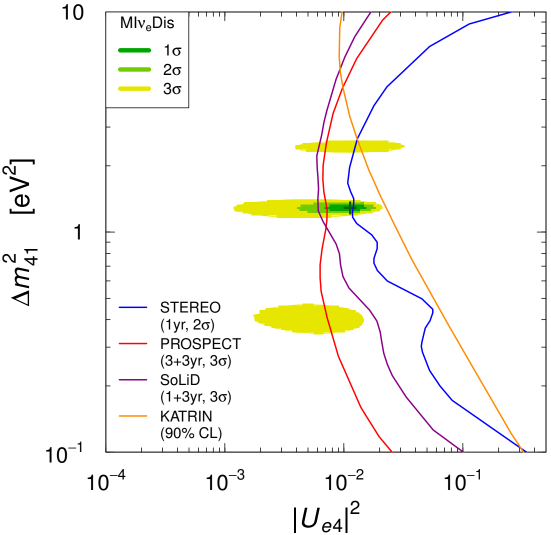
[See also Dentler, Hernandez-Cabezudo, Kopp, Machado, Maltoni, Martinez-Soler, Schwetz, JHEP 1808 (2018) 010, arXiv:1803.10661]

Comparison with the Reactor and Gallium Anomalies



- 3σ agreement.
- 2σ tension.
- Small overestimate of the reactor fluxes.
- Small overestimate of the GALLEX and SAGE efficiencies.

Global Model-Independent ν_e and $\bar{\nu}_e$ Disappearance



- NEOS and DANSS.
- Reactor rates with free ^{235}U and ^{239}Pu fluxes: r_{235} and r_{239}.
- Gallium data with free GALLEX and SAGE efficiencies: η_G and η_S.
- New reactor experiments: PROSPECT, STEREO, Neutrino-4, SoLiD
- Kinematic ν_4 mass measurement: KATRIN

[See also Dentler, Hernandez-Cabezudo, Kopp, Machado, Maltoni, Martinez-Soler, Schwetz, arXiv:1803.10661]

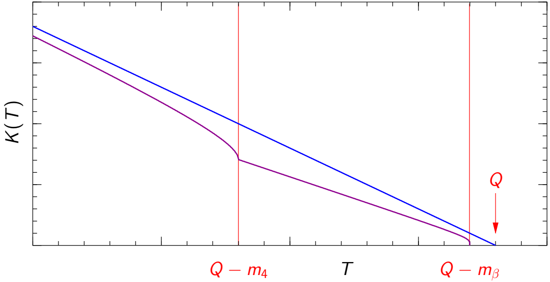
Tritium Beta-Decay: $^3\text{H} \to {^3\text{He}} + e^- + \bar{\nu}_e$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

$$\frac{K^2(T)}{Q-T} = \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2}\; \theta(Q - T - m_k)$$

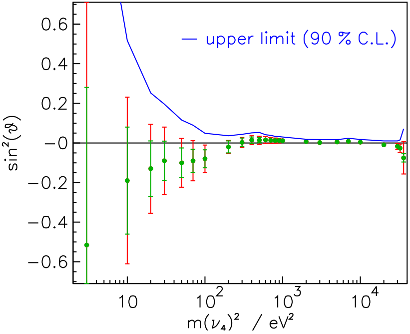
$$m_4 \gg m_{1,2,3} \Rightarrow \simeq \left(1 - |U_{e4}|^2\right) \sqrt{(Q-T)^2 - m_\beta^2}\; \theta(Q - T - m_\beta) + |U_{e4}|^2 \sqrt{(Q-T)^2 - m_4^2}\; \theta(Q - T - m_4)$$



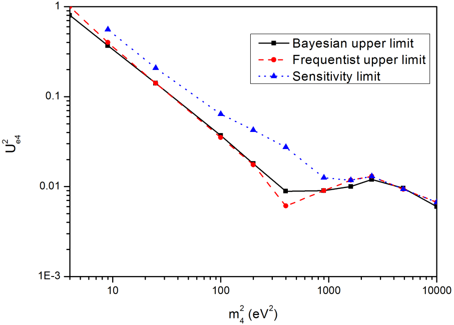
$$m_\beta^2 = \sum_{k=1}^{3} |U_{ek}|^2 m_k^2$$

Mainz and Troitsk Limit on $\Delta m^2_{41} \simeq m^2_4$

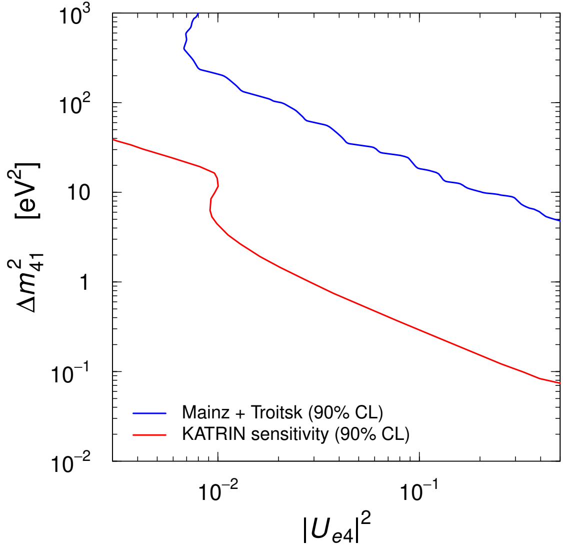
$$m_4 \gg m_{1,2,3} \quad \Longrightarrow \quad \Delta m^2_{41} \equiv m^2_4 - m^2_1 \simeq m^2_4$$



[Kraus, Singer, Valerius, Weinheimer, EPJC 73 (2013) 2323]

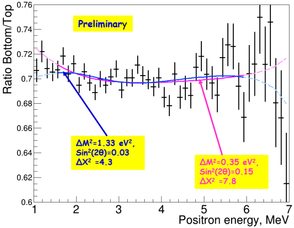


[Belesev et al, JPG 41 (2014) 015001]


10^3
10^2
10
1
10^{-1}
10^{-2}
Δm^2_{41} [eV^2]
10^{-2}
10^{-1}
$|U_{e4}|^2$
Mainz + Troitsk (90% CL)
KATRIN sensitivity (90% CL)

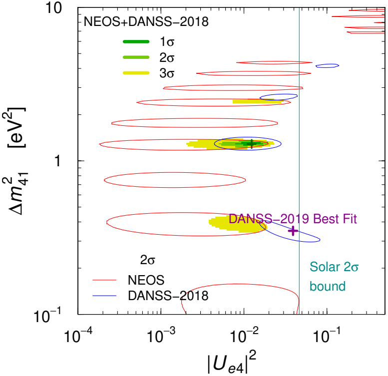
New DANSS results @ EPS-HEP 2019

Ratio of positron energy spectra at down and up detector positions (Full data set)



- The best 4ν point (ΔM^2=0.35eV^2, $Sin^2(2\theta)$=0.15, ΔX^2 =7.8) has CL of 1.8σ.
- Best point in old data (ΔM^2=1.33 eV^2) is also shown

[Danilov @ EPS-HEP 2019]



- The DANSS-2019 best fit has too large mixing.
- The agreement between NEOS and DANSS has diminished.
- Reactor indications in favor of SBL oscillations seem to be fading away.
- We wait independent checks of PROSPECT, STEREO and SoLiD.

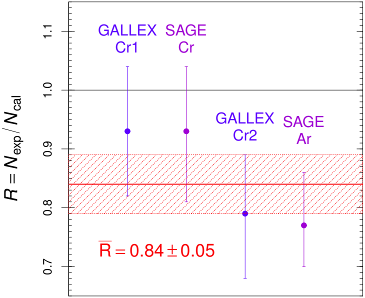
Gallium Anomaly

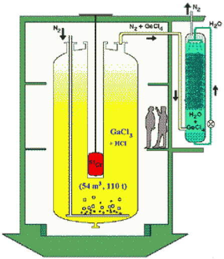
Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

$E \simeq 0.75\,\text{MeV}$ $\quad$ $E \simeq 0.81\,\text{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$





$\approx 2.9\sigma$ deficit

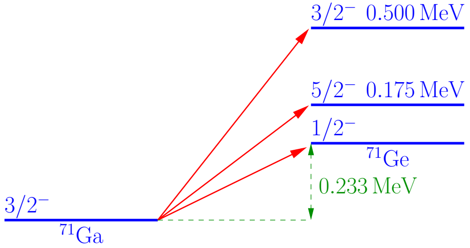
$\langle L \rangle_{\text{GALLEX}} = 1.9\,\text{m}$ $\quad$ $\langle L \rangle_{\text{SAGE}} = 0.6\,\text{m}$

$\Delta m^2_{\text{SBL}} \gtrsim 1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807;
Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344,
MPLA 22 (2007) 2499, PRD 78 (2008) 073009,
PRC 83 (2011) 065504]

- Deficit could be due to overestimate of
$$\sigma(\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-)$$

- Calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\text{G.S.}}$ from $T_{1/2}({}^{71}\text{Ge}) = 11.43 \pm 0.03\,\text{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\text{G.S.}}({}^{51}\text{Cr}) = 55.3 \times 10^{-46}\,\text{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\text{Cr}) = \sigma_{\text{G.S.}}({}^{51}\text{Cr}) \left(1 + 0.669\,\frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}} + 0.220\,\frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}}\right)$

- Contribution of excited states only 5%!

		$\frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}}$	$\frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}}$
Krofcheck et al. PRL 55 (1985) 1051	${}^{71}\text{Ga}(p,n){}^{71}\text{Ge}$	< 0.057	0.126 ± 0.023
Haxton PLB 431 (1998) 110	Shell Model + Exp.	0.19 ± 0.18	
Frekers et al. PLB 706 (2011) 134	${}^{71}\text{Ga}({}^{3}\text{He},{}^{3}\text{H}){}^{71}\text{Ge}$	0.040 ± 0.031	0.207 ± 0.016

- The ${}^{71}\text{Ga}({}^{3}\text{He},{}^{3}\text{H}){}^{71}\text{Ge}$ data confirm the contribution of the two excited states.
- Haxton: for BGT_{175} "the calculation predicts destructive interference between the (p,n) spin and spin-tensor matrix elements"

$$\langle f||O_{(p,n)}||i\rangle = \langle f||O_{\mathrm{GT}}||i\rangle + \delta\langle f||O_{L=2}||i\rangle \qquad \delta \approx 0.097$$

Transition	$\langle f\|O_{GT}\|i\rangle$	$\langle f\|O_{L=2}\|i\rangle$
$3/2^- \to 1/2^-$ (0 keV)	-0.451	0.348
$3/2^- \to 5/2^-$ (175 keV)	0.082	-2.23
$3/2^- \to 3/2^-$ (500 keV)	0.056	0.104

The Gallium Anomaly Revisited

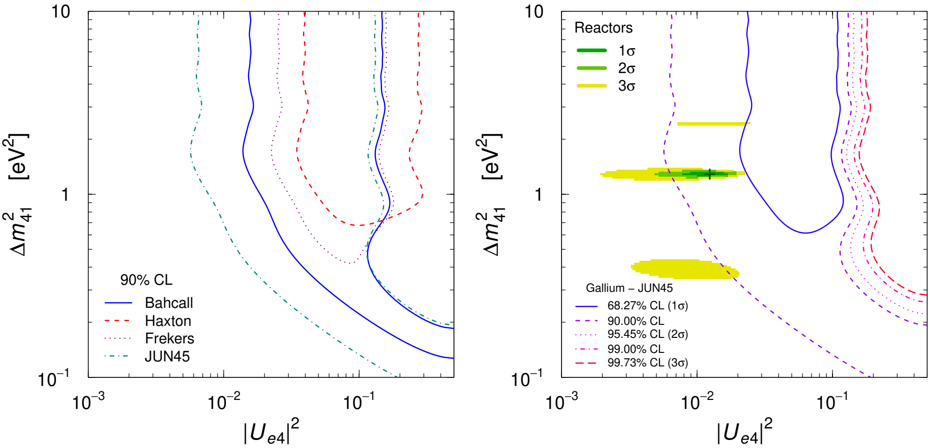
[Kostensalo, Suhonen, Giunti, Srivastava, PLB 795 (2019) 542, arXiv:1906.10980]

- New JUN45 shell-model calculation of the cross section of

$$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$$

Transition	$\langle f \| O_{\mathrm{GT}} \| i \rangle$	$\langle f \| O_{L=2} \| i \rangle$	$\mathrm{BGT}_{\beta}^{\mathrm{SM}}$	$\mathrm{BGT}_{(\mathrm{p,n})}^{\mathrm{SM}}$
$3/2^-_{\mathrm{g.s.}} \to 1/2^-_{\mathrm{g.s.}}$	-0.795	0.465	0.158	0.141
$3/2^-_{\mathrm{g.s.}} \to 5/2^-$ (175 keV)	0.144	-1.902	0.0052	0.0004
$3/2^-_{\mathrm{g.s.}} \to 3/2^-$ (500 keV)	0.100	0.0482	0.0025	0.0027

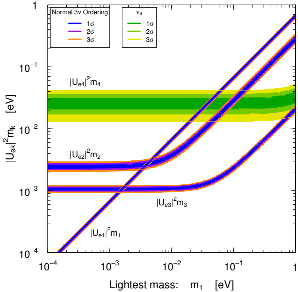
		$\frac{BGT_{175}}{BGT_{G.S.}}$	$\frac{BGT_{500}}{BGT_{G.S.}}$
Krofcheck et al. (1985)	$^{71}Ga(p,n)^{71}Ge$	< 0.057	0.126 ± 0.023
Haxton (1998)	Shell Model + Exp.	0.19 ± 0.18	
Frekers et al. (2011)	$^{71}Ga(^{3}He,^{3}H)^{71}Ge$	0.040 ± 0.031	0.207 ± 0.016
JUN45 (2019)	Shell Model	0.033 ± 0.017	0.016 ± 0.008

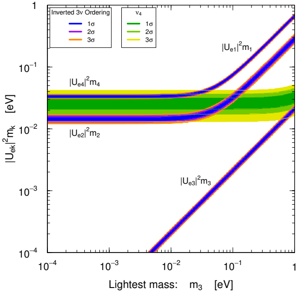


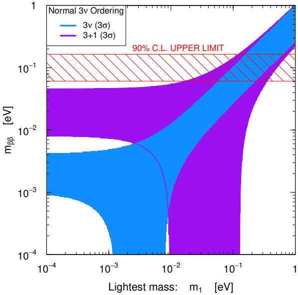
- With the new JUN45 shell-model calculation the statistical significance of the gallium anomaly is reduced from 3.0σ to 2.3σ.
- The Gallium data are more compatible with the indication of SBL oscillations obtained from the reactor neutrino NEOS and 2018 DANSS data, or with the absence of SBL oscillations.

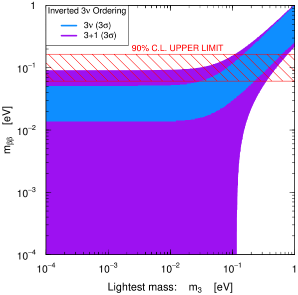
Neutrinoless Double-Beta Decay

$$m_{\beta\beta} = \left| |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3 + |U_{e4}|^2 \, e^{i\alpha_{41}} \, m_4 \right|$$

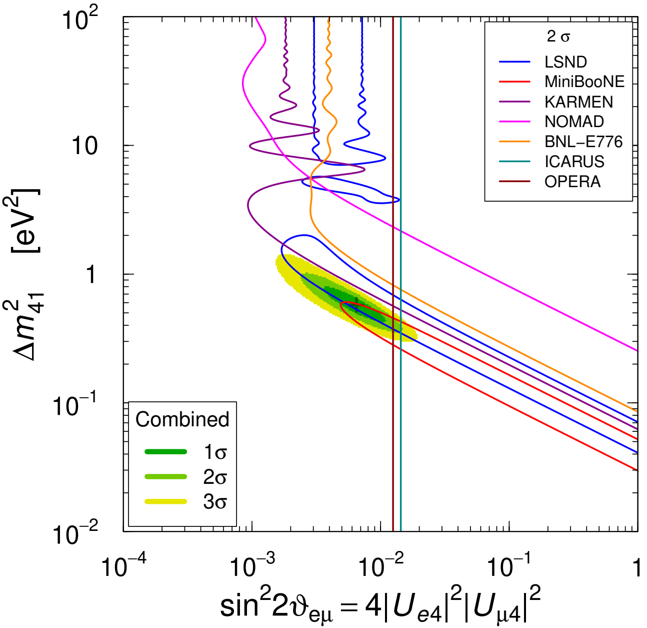






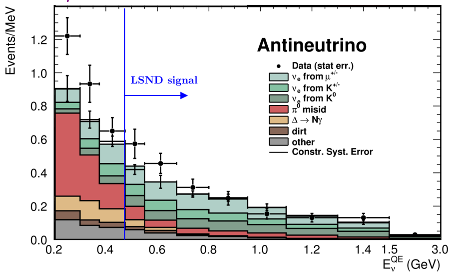


$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ Appearance

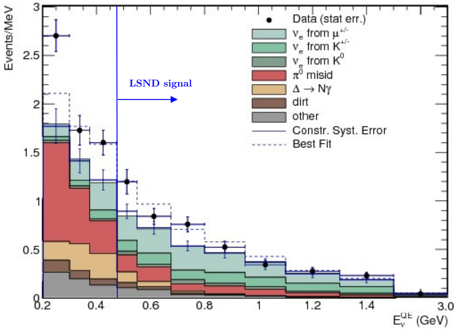


MiniBooNE

$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



$\nu_\mu \to \nu_e$ [PRL 121 (2018) 221801]



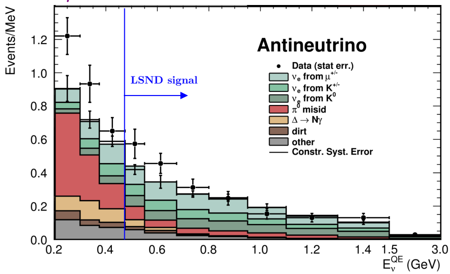
- Purpose: check the LSND signal
- Different $L \simeq 541\,\text{m}$
- Different $200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$
- Similar $L/E \iff$ oscillations
- No money, no Near Detector
- LSND signal expected for

  $E \gtrsim 475\,\text{MeV}$
- New low-energy anomaly for

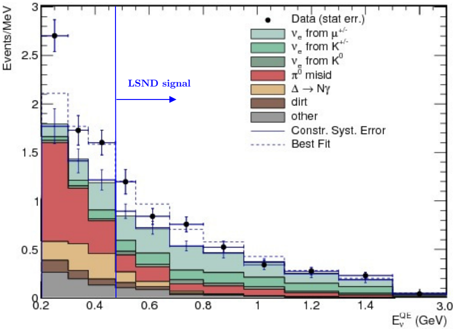
  $E < 475\,\text{MeV}$

MiniBooNE

$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]

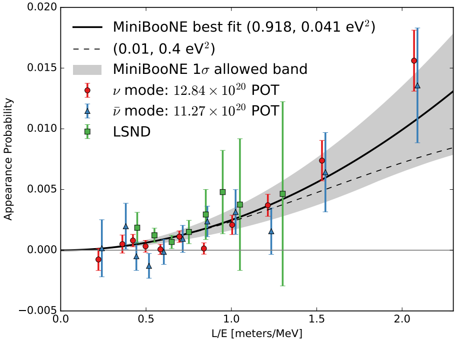


$\nu_\mu \to \nu_e$ [PRL 121 (2018) 221801]

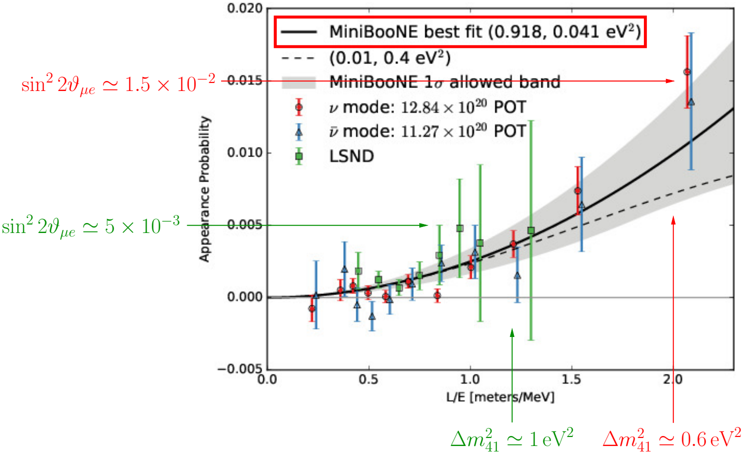


- LSND: excess for $\dfrac{L}{E} \lesssim 1.2\,\dfrac{m}{\text{MeV}}$
- MiniBooNE: the LSND excess should be at

$$E \gtrsim \frac{541\,\text{m}}{1.2\,\text{m}}\,\text{MeV} \simeq 451\,\text{MeV}$$



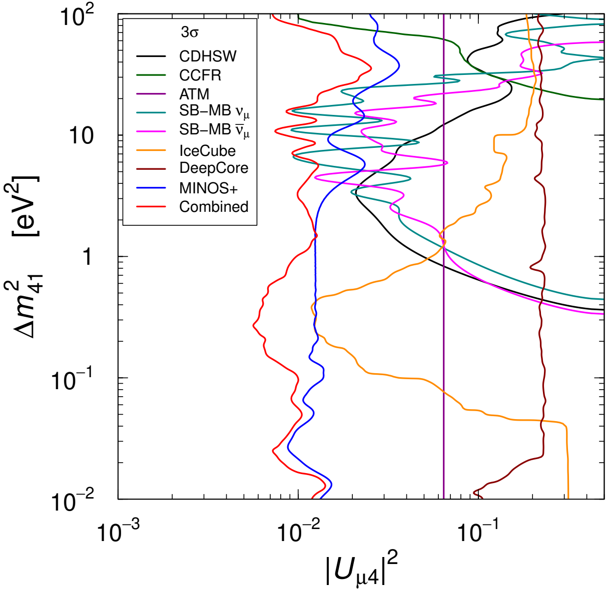
[MiniBooNE, PRL 121 (2018) 221801]



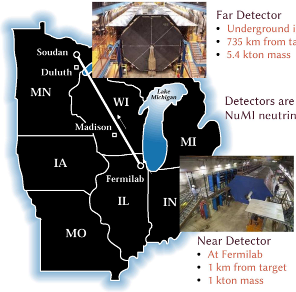
$$P_{\nu_\mu \to \nu_e} = \sin^2 2\vartheta_{\mu e} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \implies P^{\max}_{\nu_\mu \to \nu_e} = \sin^2 2\vartheta_{\mu e}$$

for $\dfrac{\Delta m^2_{41} L}{4E} = \dfrac{\pi}{2} \implies \dfrac{L\,[\text{m}]}{E\,[\text{MeV}]} \simeq \dfrac{1.2}{\Delta m^2_{41}\,[\text{eV}^2]}$

ν_μ and $\bar{\nu}_\mu$ Disappearance



MINOS+

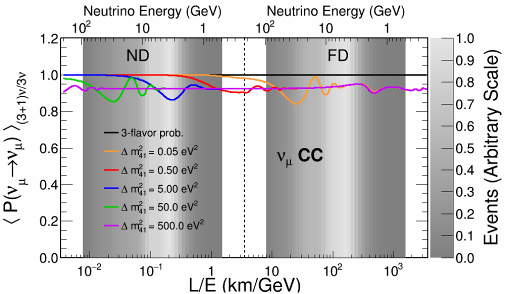


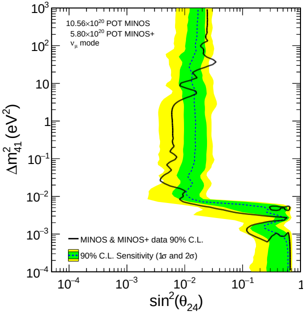
Far Detector
- Underground in Soudan mine
- 735 km from target
- 5.4 kton mass

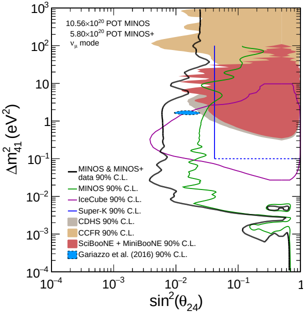
Detectors are on-axis for NuMI neutrino beam

Near Detector
- At Fermilab
- 1 km from target
- 1 kton mass

[PRL 122 (2019) 091803, arXiv:1710.06488]





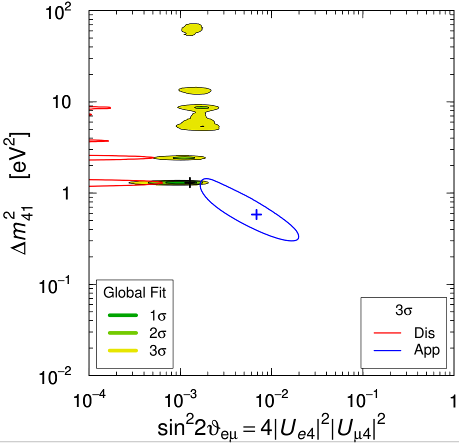


Global Appearance-Disappearance Tension

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu 4}|^2$

$\nu_\mu \to \nu_e$ APP
$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- Global Fit:
  $\chi^2/\text{NDF} = 831.7/797$
  $\text{GoF} = 19\%$
  $\chi^2_{\text{PG}}/\text{NDF}_{\text{PG}} = 42.8/2$
  $\text{GoF}_{\text{PG}} = 5 \times 10^{-10}$ $\leftarrow$ ☹
- Similar tension in
  $3+2, \quad 3+3, \quad \ldots, \quad 3+N_s$

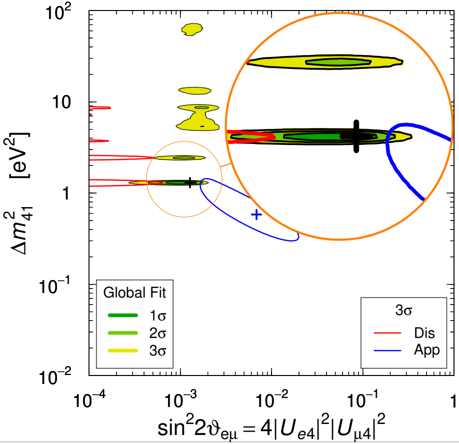
[Giunti, Zavanin, MPLA 31 (2015) 1650003]

Global Appearance-Disappearance Tension

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$

$\nu_\mu \to \nu_e$ APP
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$



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  $\chi^2/\text{NDF} = 831.7/797$
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- Similar tension in
  $3+2, \quad 3+3, \quad \ldots, \quad 3+N_s$

[Giunti, Zavanin, MPLA 31 (2015) 1650003]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

N_{D} = Number of Data $\qquad$ N_{P} = Number of Fitted Parameters

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF}$ $\qquad$ $\text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, dz$ $\qquad$ $p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

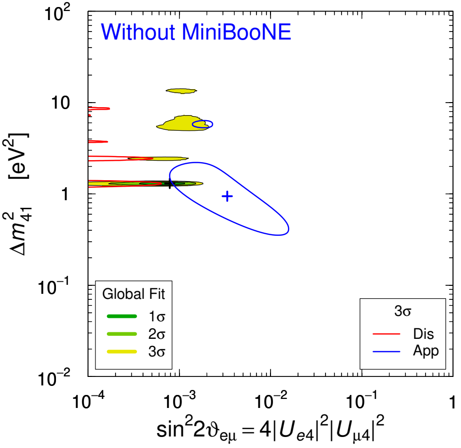
Maltoni, Schwetz, PRD 68 (2003) 033020 (arXiv:hep-ph/0304176)

- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, dz$

Global Fit Without MiniBooNE



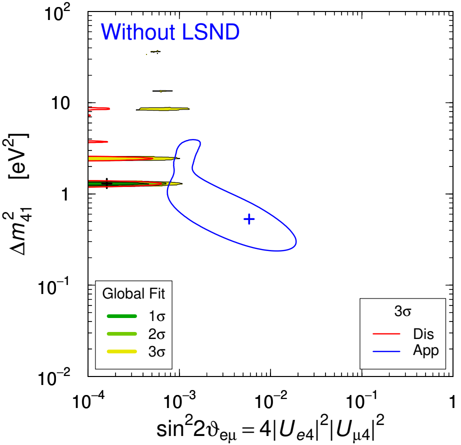
$\chi^2/\text{NDF} = 768.9/763$

$\text{GoF} = 43\%$

$\chi^2_{\text{PG}}/\text{NDF}_{\text{PG}} = 28.7/2$

$\text{GoF}_{\text{PG}} = 6 \times 10^{-7}$ ← ☹

Global Fit Without LSND



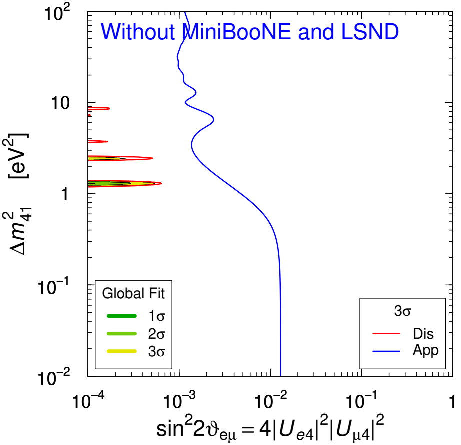
$\chi^2/\mathrm{NDF} = 802.9/793$

$\mathrm{GoF} = 40\%$

$\chi^2_{\mathrm{PG}}/\mathrm{NDF}_{\mathrm{PG}} = 22.1/2$

$\mathrm{GoF}_{\mathrm{PG}} = 2 \times 10^{-5}$ ← ☹

Global Fit Without LSND and MiniBooNE



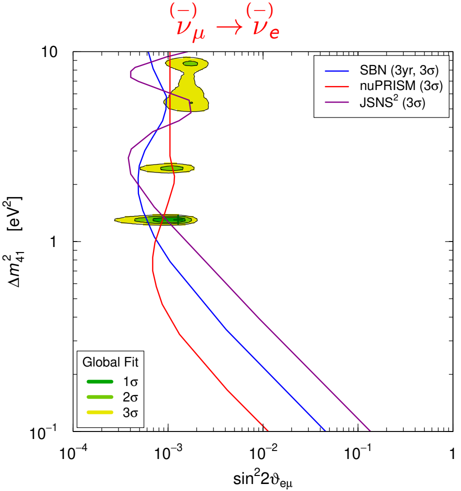
$\chi^2/\text{NDF} = 727.4/759$

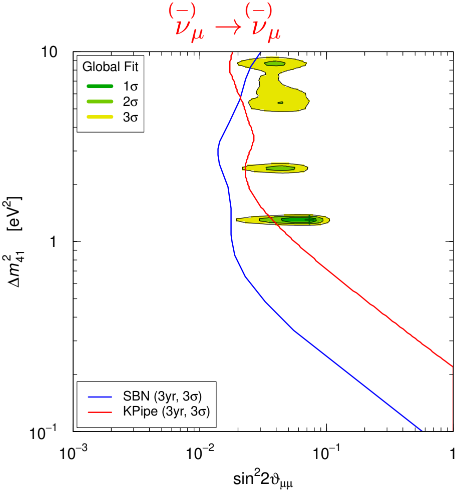
$\text{GoF} = 79\%$

$\chi^2_{\text{PG}}/\text{NDF}_{\text{PG}} = 0/2$

$\text{GoF}_{\text{PG}} = 1 \quad \leftarrow$ ☺

New Dedicated Experiments





Effective 3+1 LBL Oscillation Probabilities

[de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, JHEP 1602 (2016) 111, JHEP 1609 (2016) 016, PRL 118 (2017) 031804; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122; Capozzi et al, PRD 95 (2017) 033006]

$$|U_{e3}| \simeq \sin\vartheta_{13} \simeq 0.15 \sim \varepsilon \quad \Longrightarrow \quad \varepsilon^2 \sim 0.03$$

$$|U_{e4}| \simeq \sin\vartheta_{14} \simeq 0.17 \sim \varepsilon$$

$$|U_{\mu 4}| \simeq \sin\vartheta_{24} \simeq 0.11 \sim \varepsilon$$

$$\alpha \equiv \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq \frac{7 \times 10^{-5}}{2.4 \times 10^{-3}} \simeq 0.031 \sim \varepsilon^2$$

At order ε^3: [Klop, Palazzo, PRD 91 (2015) 073017] $\Delta_{kj} \equiv \Delta m^2_{kj} L/4E$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq\ & 4\sin^2\vartheta_{13}\sin^2\vartheta_{23}\sin^2\Delta_{31} && \sim \varepsilon^2 \\ & +2\sin\vartheta_{13}\sin 2\vartheta_{12}\sin 2\vartheta_{23}(\alpha\Delta_{31})\sin\Delta_{31}\cos(\Delta_{32}+\delta_{13}) && \sim \varepsilon^3 \\ & +4\sin\vartheta_{13}\sin\vartheta_{14}\sin\vartheta_{24}\sin\vartheta_{23}\sin\Delta_{31}\sin(\Delta_{31}+\delta_{13}-\delta_{14}) && \sim \varepsilon^3 \end{aligned}$$

Alternative Explanations of MiniBooNE

- Generation by a particle X produced in the MiniBooNE target is excluded by the angular distribution of the ν_e-like events, that is not strongly forward peaked.

[Jordan, Kahn, Krnjaic, Moschella, Spitz, PRL 122 (2019) 081801]

- Visible decays:

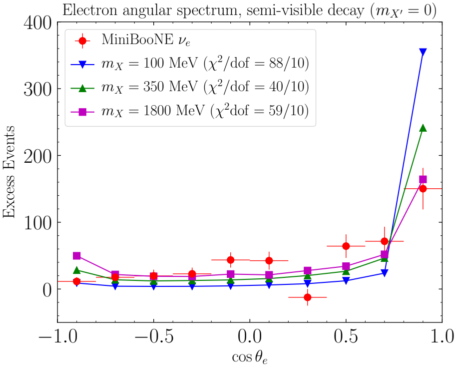
$$X \to e^+e^- \text{ or } X \to \gamma\gamma$$

$$\cos\theta_e > 0.9999$$

- Semi-visible decay:

$$X \to X' + p_{\text{EM}}$$

$\longrightarrow$



Heavy Neutrino Generation in the Detector

- Neutrino Neutral-Current Weak Interaction Lagrangian:

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{NC})} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}} Z_\rho \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \gamma^\rho \nu_{\alpha L}$$

- Sterile neutrinos: $\nu_{\alpha L} = \sum_{k=1}^{3+N_s} U_{\alpha k} \nu_{kL} \qquad (\alpha = e, \mu, \tau, s_1, \ldots, s_{N_s})$

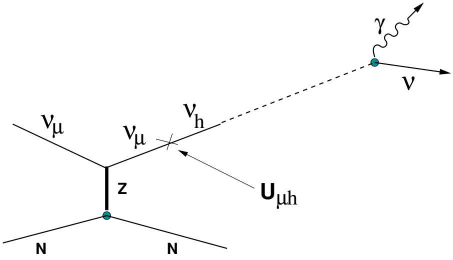
- No GIM: $\mathscr{L}_{\mathrm{I}}^{(\mathrm{NC})} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}} Z_\rho \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \overline{\nu_{jL}} \gamma^\rho \nu_{kL} \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k}$

- $\sum_{\alpha=e,\mu,\tau,s_1,\ldots} U_{\alpha j}^* U_{\alpha k} = \delta_{jk}$ but $\sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \neq \delta_{jk}$

- A heavy neutrino ν_h with $h \geq 4$ can be generated in the detector by neutral-current ν_μ scattering.

Heavy Sterile Neutrino Radiative Decay

[Gninenko, PRL 103 (2009) 241802, PRD 83 (2011) 015015, PRD 83 (2011) 093010, PLB 710 (2012) 86]

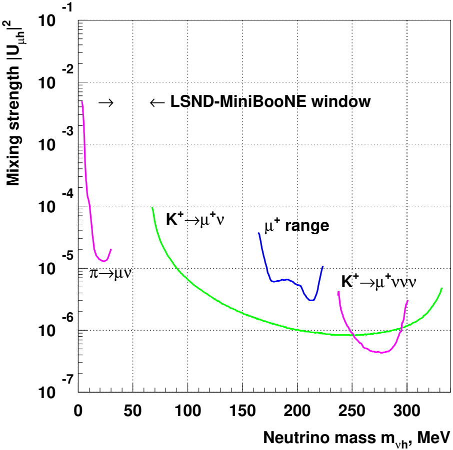


It may explain also LSND with

$$m_{\nu_h} \approx 40 - 80\,\text{MeV}$$

and

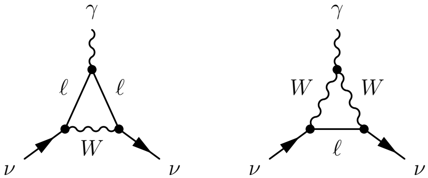
$$|U_{\mu h}|^2 \approx 10^{-3} - 10^{-2}$$



- It needs a fast radiative decay $\tau_{\nu_h} \lesssim 10^{-9}\,\text{s}$ that can be generated by a transition magnetic moment $|\mu_{hi}| \gtrsim 10^{-8}\,\mu_{\text{B}}$:

$$\Gamma_{\nu_h \to \nu_i + \gamma} = \frac{|\mu_{hi}|^2}{8\pi} m_{\nu_h}^3 \left(1 - \frac{m_{\nu_i}^2}{m_{\nu_h}^2}\right)^3$$

- Simplest extensions of the Standard Model:



$$|\mu_{hi}| \sim 10^{-11}\,\mu_{\text{B}} \frac{m_{\nu_h}}{100\,\text{MeV}} |U_{\ell h}| \sim 10^{-12}\,\mu_{\text{B}}$$ not enough

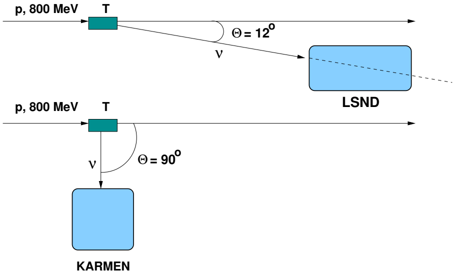
- More exotic extensions of the Standard Model may give the needed

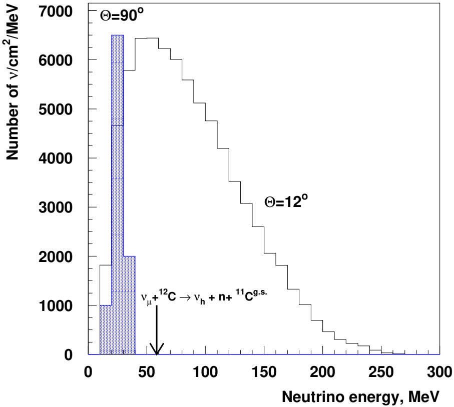
$$|\mu_{hi}| \gtrsim 10^{-8}\,\mu_{\text{B}}$$

- It is interesting that this mechanism can explain why the LSND signal was not observed in KARMEN:

ν_μ from π^+ decay in flight

$$\nu_\mu + {}^{12}\text{C} \to \nu_h + {}^{11}\text{C} + n$$



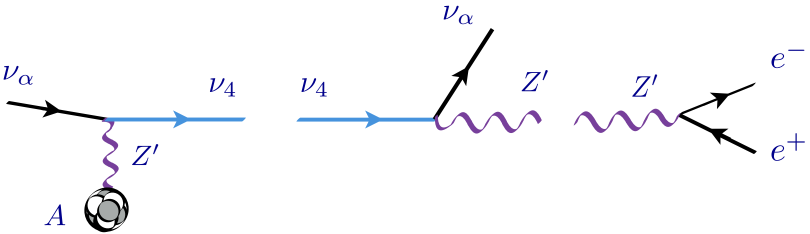


[Gninenko, PRD 83 (2011) 015015]

- This mechanism can be ruled out by Liquid Argon Time Projection Chamber (LArTPC) detectors that distinguish between electrons and photons: MicroBooNE, ICARUS, SBND (Fermilab Short-Baseline Neutrino Oscillation Program).

Interacting Heavy Sterile Neutrino

[Bertuzzo, Jana, Machado, Zukanovich Funchal, PRL 121 (2018) 241801]

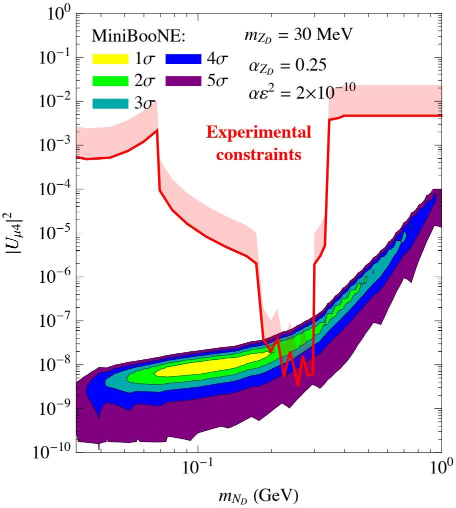


[Arguelles, Hostert, Tsai, arXiv:1812.08768]

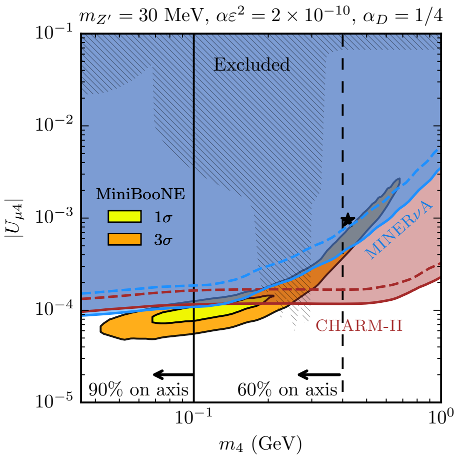
$$\mathcal{L} \supset \frac{m_{Z'}^2}{2} Z'_\mu Z'^\mu + g_{\mathcal{D}} Z'_\mu \overline{\nu}_s \gamma^\mu \nu_s + e\epsilon\, Z'^\mu\, J_\mu^{\mathrm{em}} + \frac{g}{c_W} \epsilon'\, Z'^\mu\, J_\mu^{\mathrm{Z}}$$

$$\Gamma_{\nu_4 \to Z' + \nu_\mu} = \frac{\alpha_{\mathcal{D}}}{2} |U_{\mu 4}|^2 \frac{m_{\nu_4}^3}{m_{Z'}^2} \left(1 - \frac{m_{Z'}^2}{m_{\nu_4}^2}\right) \left(1 + \frac{m_{Z'}^2}{m_{\nu_4}^2} - 2\frac{m_{Z'}^4}{m_{\nu_4}^4}\right)$$

$$\Gamma_{Z' \to e^+ e^-} \approx \frac{\alpha\, \epsilon^2}{3}\, m_{Z'}$$



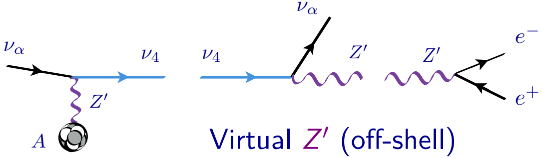
[Bertuzzo et al, PRL 121 (2018) 241801]



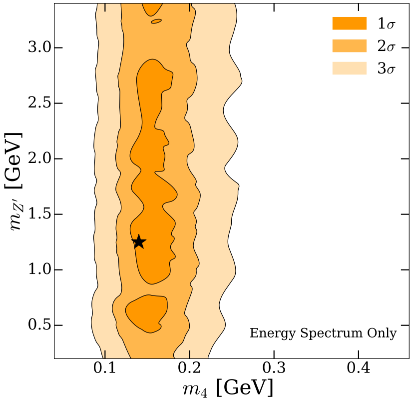
[Arguelles, Hostert, Tsai, arXiv:1812.08768]

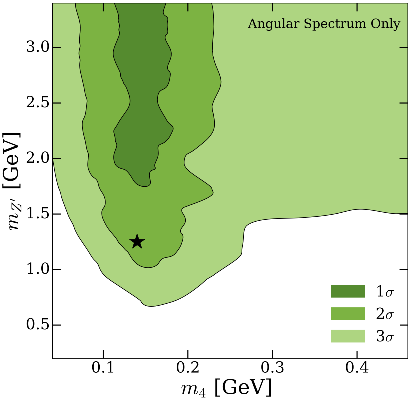
Heavy New Gauge Boson

[Ballett, Pascoli, Ross-Lonergan, PRD 99 (2019) 071701]



Virtual Z' (off-shell)





Conclusions I

- Neutrinos can be powerful messengers of new physics beyond the SM as the existence of light sterile neutrinos indicated by the reactor, Gallium and LSND anomalies.

- Exciting 2018 model-independent indication of light sterile neutrinos at the eV scale from the NEOS and DANSS experiments in approximate agreement with the reactor and Gallium anomalies.

- 2019 DANSS data do not confirm the 2018 indication and the reactor indications in favor of SBL oscillations seem to be fading away.

- Important checks in the near future by the reactor experiments PROSPECT, STEREO, SoLid. (Neutrino-4?)

- Independent tests through the effect of m_4 in β-decay (KATRIN), electron-capture (ECHo, HOLMES) and $\beta\beta_{0\nu}$-decay experiments.

Conclusions II

- In principle, the simplest explanation of the LSND and MiniBooNE ν_e-like excesses is neutrino oscillations, that requires a new Δm^2_{SBL} associated with a sterile neutrino.
- Unfortunately, the LSND and MiniBooNE ν_e-like excesses are too large to be compatible with the existing bounds on ν_e and ν_μ disappearance in the framework of $3 + N_s$ active-sterile neutrino mixing:

  APPEARANCE-DISAPPEARANCE TENSION
- Alternative explanations exist with a heavy sterile neutrino produced and decayed in the detector.
- Promising Fermilab SBN program aimed at a conclusive solution of the mystery with three Liquid Argon Time Projection Chamber (LArTPC): a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-T600).
- It is important that LArTPC detectors can distinguish a single ν_e-induced electron from a γ or a collimated e^+e^- pair.