Neutrino Physics

Part II: Neutrino Oscillations

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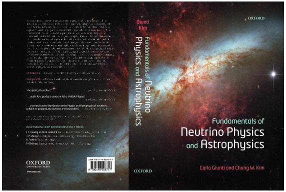
Torino, 30 November - 4 December 2020

http://personalpages.to.infn.it/~giunti/slides/2019/giunti-201130-phdto2.pdf



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part II: Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- Neutrino Oscillations in Matter

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 MeV$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{gathered} \nu + A \to B + C \\ \Downarrow \\ s = 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2} \end{gathered}}$$

$$\begin{array}{ll}
\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\
\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\
\bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\
\nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\
\nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV}
\end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0 \, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO), $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

Neutrino Mixing

Left-handed Flavor Neutrinos produced in Weak Interactions

$$|\nu_e, -\rangle \qquad |\nu_\mu, -\rangle \qquad |\nu_\tau, -\rangle$$

$$\mathcal{H}_{\text{CC}} = \frac{g}{\sqrt{2}} W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right) + \text{H.c.}$$

Fields $\quad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha, -\rangle = \sum_k U^*_{\alpha k} |\nu_k, -\rangle \quad$ States

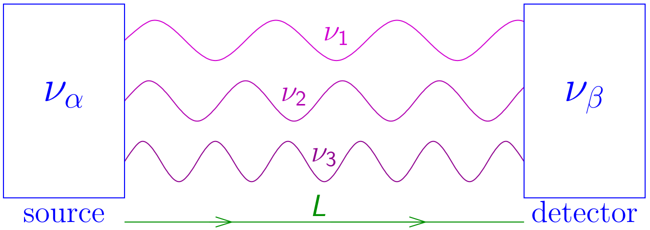
$$|\nu_1, -\rangle \qquad |\nu_2, -\rangle \qquad |\nu_3, -\rangle$$

Left-handed Massive Neutrinos propagate from Source to Detector

3×3 Unitary Mixing Matrix: $\quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

Neutrino Oscillations in Vacuum

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{\alpha 1}^* e^{-iE_1 t} |\nu_1\rangle + U_{\alpha 2}^* e^{-iE_2 t} |\nu_2\rangle + U_{\alpha 3}^* e^{-iE_3 t} |\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

the oscillation probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

Simple Example of Neutrino Production

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\boxed{\nu_\mu = \sum_k U_{\mu k} \nu_k}$$

two-body decay $\Longrightarrow$ fixed kinematics

$$\boxed{E_k^2 = p_k^2 + m_k^2}$$

$$\pi \text{ at rest: } \begin{cases} p_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 - \dfrac{m_k^2}{2}\left(1 + \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \\ E_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 + \dfrac{m_k^2}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \end{cases}$$

0^{th} order: $m_k = 0 \Rightarrow p_k = E_k = E = \dfrac{m_\pi}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$

1^{st} order: $\boxed{E_k \simeq E + \xi \dfrac{m_k^2}{2E}}$ $\boxed{p_k \simeq E - (1 - \xi) \dfrac{m_k^2}{2E}}$

$$\xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{\mathrm{CC}} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\quad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad$ States

initial flavor: $\alpha = e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U_{\alpha k}^* \, e^{-iE_k t + i p_k x} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0,0) = \sum_k U_{\alpha k}^* U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t>0, x>0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha\to\nu_\beta}(t,x) = \left|\mathcal{A}_{\nu_\alpha\to\nu_\beta}(t,x)\right|^2 = \left|\sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k}\right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

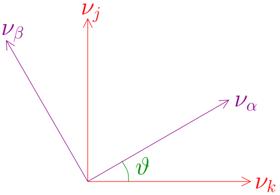
$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(L,E) &= \left|\sum_k U_{\alpha k}^*\, e^{-im_k^2 L/2E}\, U_{\beta k}\right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Effective Two-Neutrino Mixing Approximation

$$\boxed{\begin{aligned} |\nu_\alpha\rangle &= \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle \\ |\nu_\beta\rangle &= -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle \end{aligned}}$$



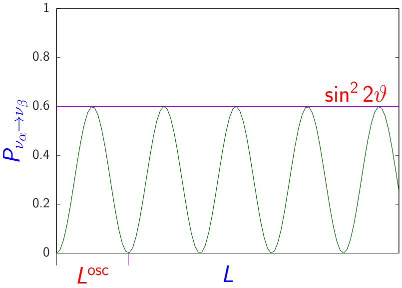
$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m^2_k - m^2_j$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

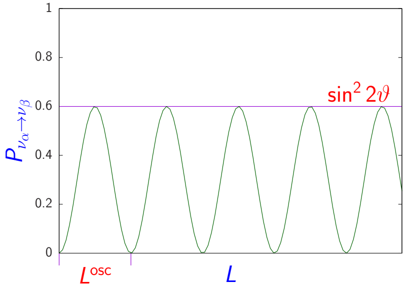
Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \implies L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$



- The effect of a tiny Δm^2 can be amplified by a large distance L.
- A tiny Δm^2 generates oscillations observable at macroscopic distances!
- Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]}\right)$



$$\frac{L}{E} \gtrsim \begin{cases} 10 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1} \, \text{eV}^2 \\ 10^3 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3} \, \text{eV}^2 \\ 10^4 \, \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4} \, \text{eV}^2 \\ 10^{11} \, \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11} \, \text{eV}^2 \end{cases}$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu_{\alpha L}^{\mathsf{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu_{\alpha L}}^{T}$$

$$\begin{array}{lll} \mathsf{C} & \Longrightarrow & \text{Particle} \leftrightarrows \text{Antiparticle} \\ \mathsf{P} & \Longrightarrow & \text{Left-Handed} \leftrightarrows \text{Right-Handed} \end{array}$$

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\text{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $\quad U \leftrightarrows U^* \quad$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $\quad A^{\text{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\quad \Longrightarrow \quad A^{\text{CPT}}_{\alpha\beta} = 0 \quad$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E} \right)$$

is invariant under CPT: $\quad U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$\boxed{A^{\text{CP}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

$$J \neq 0 \iff \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

$$
\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \leftarrow A^{\text{CP}}_{\alpha\beta} \\
&+ P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow -A^{\text{CPT}}_{\beta\alpha} = 0 \\
&+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{CP}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} \qquad \Longrightarrow \qquad \boxed{A^{\text{CP}}_{\alpha\beta} = -A^{\text{CP}}_{\beta\alpha}}
\end{aligned}
$$

T Symmetry

$$P_{\nu_\alpha\to\nu_\beta} \quad \xrightarrow{\mathsf{T}} \quad P_{\nu_\beta\to\nu_\alpha}$$

T Asymmetries: $\quad A^{\mathsf{T}}_{\alpha\beta} = P_{\nu_\alpha\to\nu_\beta} - P_{\nu_\beta\to\nu_\alpha}$

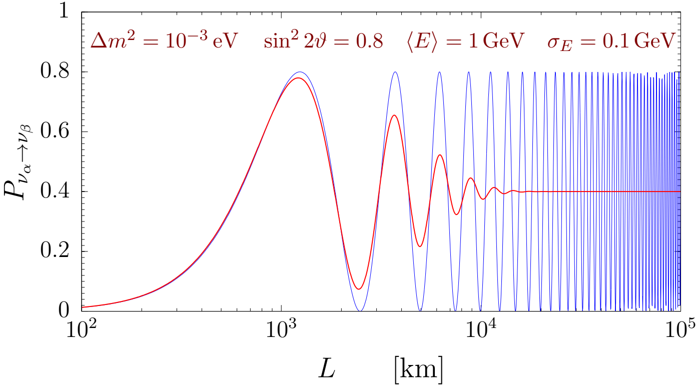
$$\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\mathsf{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\beta\to\bar\nu_\alpha} \\
&= P_{\nu_\alpha\to\nu_\beta} - P_{\nu_\beta\to\nu_\alpha} \leftarrow A^{\mathsf{T}}_{\alpha\beta} \\
&+ P_{\nu_\beta\to\nu_\alpha} - P_{\bar\nu_\beta\to\bar\nu_\alpha} \leftarrow A^{\mathsf{CP}}_{\beta\alpha} \\
&= A^{\mathsf{T}}_{\alpha\beta} + A^{\mathsf{CP}}_{\beta\alpha} \\
&= A^{\mathsf{T}}_{\alpha\beta} - A^{\mathsf{CP}}_{\alpha\beta} \qquad \Longrightarrow \qquad \boxed{A^{\mathsf{T}}_{\alpha\beta} = A^{\mathsf{CP}}_{\alpha\beta}}
\end{aligned}$$

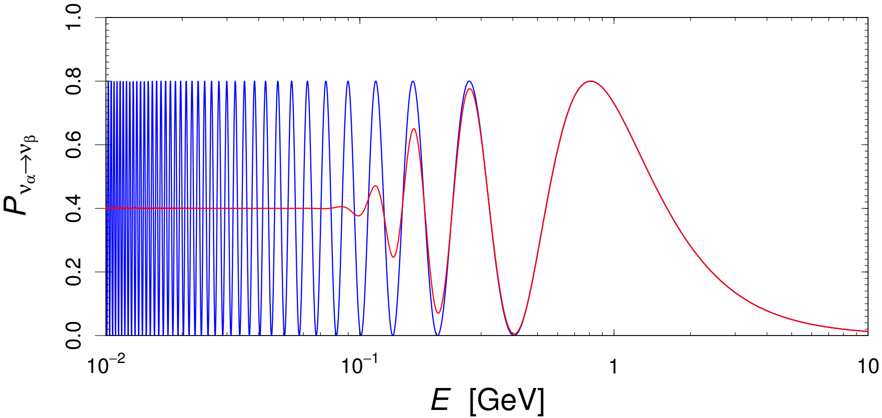
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

$$\Downarrow$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E)\rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\, \mathrm{d}E\right] \qquad (\alpha \neq \beta)$$





$\Delta m^2 = 10^{-3}\,\mathrm{eV}$ $\quad \sin^2 2\vartheta = 0.8$ $\quad L = 10^3\,\mathrm{km}$ $\quad \sigma_E = 0.01\,\mathrm{GeV}$

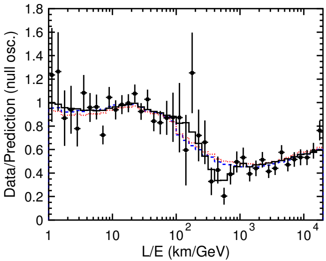
$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E)\, \mathrm{d}E \right] \qquad (\alpha \neq \beta)$$

A Brief History of Neutrino Oscillations

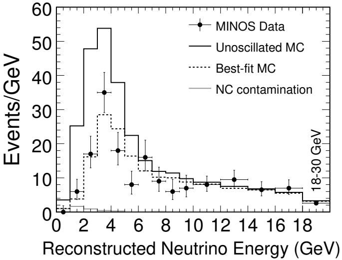
- 1957: Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\Longrightarrow \quad \nu \leftrightarrows \bar{\nu}$
- In 1957 only one neutrino type $\nu = \nu_e$ was known! The possible existence of ν_μ was discussed by several authors. Maybe the first have been Sakata and Inoue in 1946 and Konopinski and Mahmoud in 1953. Maybe Pontecorvo did not know. He discussed the possibility to distinguish ν_μ from ν_e in 1959.
- 1962: Maki, Nakagava, Sakata proposed a model with ν_e and ν_μ and Neutrino Mixing:
  "weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrows \nu_\mu$"
- 1962: Lederman, Schwartz and Steinberger discover ν_μ
- 1967: Pontecorvo: intuitive $\nu_e \leftrightarrows \nu_\mu$ oscillations with maximal mixing. Applications to reactor and solar neutrinos ("prediction" of the solar neutrino problem).
- 1969: Gribov and Pontecorvo: $\nu_e - \nu_\mu$ mixing and oscillations. But no clear derivation of oscillations with a factor of 2 mistake in the phase (misprint?).

- 1975-76: Start of the "Modern Era" of Neutrino Oscillations with a general theory of neutrino mixing and a rigorous derivation of the oscillation probability by Eliezer and Swift, Fritzsch and Minkowski, and Bilenky and Pontecorvo. [Bilenky, Pontecorvo, Phys. Rep. (1978) 225]
- 1978: Wolfenstein discovers the effect on neutrino oscillations of the matter potential ("Matter Effect")
- 1985: Mikheev and Smirnov discover the resonant amplification of solar $\nu_e \to \nu_\mu$ oscillations due to the Matter Effect ("MSW Effect")
- 1998: the Super-Kamiokande experiment observed in a model-independent way the Vacuum Oscillations of atmospheric neutrinos ($\nu_\mu \to \nu_\tau$).
- 2002: the SNO experiment observed in a model-independent way the flavor transitions of solar neutrinos ($\nu_e \to \nu_\mu, \nu_\tau$), mainly due to adiabatic MSW transitions. [see: Smirnov, arXiv:1609.02386]
- 2015: Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (SNO) received the Physics Nobel Prize "for the discovery of neutrino oscillations, which shows that neutrinos have mass".

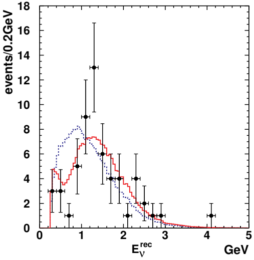
Observations of Neutrino Oscillations



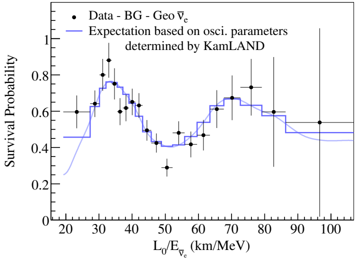
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



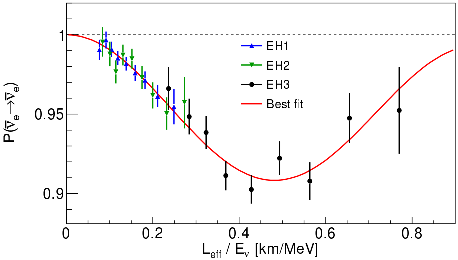
[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]



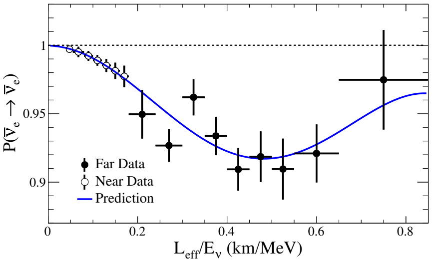
[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]



[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]

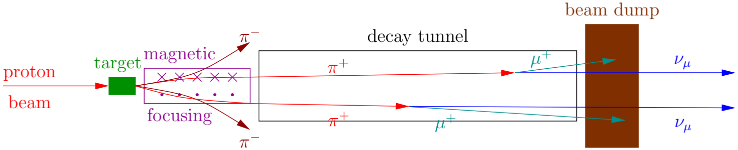


[Daya Bay, PRL, 112 (2014) 061801, arXiv:1310.6732]



[RENO, arXiv:1511.05849]

Accelerator Neutrino Beams



$$\pi^+ \to \mu^+ + \nu_\mu \qquad \tau_{\pi^+} \simeq 2.6 \times 10^{-8}\,\mathrm{s}$$

mainly ν_μ beam contaminated with ν_e and $\bar{\nu}_\mu$ from

$$\pi^+ \to e^+ + \nu_e \qquad \mathrm{B.R.} \simeq 1.2 \times 10^{-4} \quad \text{(helicity suppressed)}$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \qquad \tau_{\mu^+} \simeq 2.2 \times 10^{-6}\,\mathrm{s}$$

since π^+ and μ^+ are ultrarelativistic, they have about the same time for decaying before being absorbed by the beam dump, and

$$\frac{N_{\nu_e}}{N_{\nu_\mu}} \approx \frac{N_{\bar{\nu}_\mu}}{N_{\nu_\mu}} \approx \frac{\tau_{\pi^+}}{\tau_\mu} \approx 0.01$$

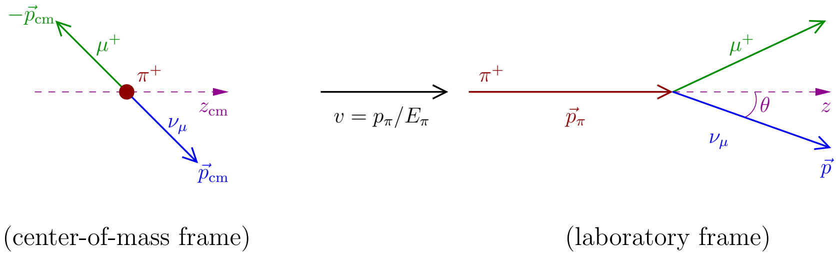
$$N(K^+) \approx 0.1\, N(\pi^+)$$

$$\tau_{K^+} \simeq 1.2 \times 10^{-8}\,\mathrm{s}$$

$$\begin{cases} \mathrm{B.R.}(K^+ \to \mu^+ + \nu_\mu) \simeq 0.64 \\ \mathrm{B.R.}(K^+ \to e^+ + \nu_e) \simeq 1.6 \times 10^{-5} \\ \mathrm{B.R.}(K^+ \to \mu^+ + \nu_\mu + \pi^0) \simeq 0.036 \\ \mathrm{B.R.}(K^+ \to e^+ + \nu_e + \pi^0) \simeq 0.051 \end{cases}$$

Off-Axis Experiments

high-intensity WB beam
detector shifted by a small angle from axis of beam
almost monochromatic neutrino energy



$$E_{\rm cm} = p_{\rm cm} = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 29.79\,\text{MeV}$$

$$\gamma = \left(1 - v^2\right)^{-1/2} = E_\pi/m_\pi \gg 1$$

$$\begin{cases} E = \gamma\left(E_{\rm cm} + v\,p^z_{\rm cm}\right) \\ p^z = \gamma\left(v\,E_{\rm cm} + p^z_{\rm cm}\right) \end{cases}$$

$$p^z = p\,\cos\theta = E\,\cos\theta \qquad \Longrightarrow \qquad E = \frac{E_{\rm cm}}{\gamma\left(1 - v\,\cos\theta\right)}$$

$$\cos\theta \simeq 1 - \theta^2/2 \qquad \text{and} \qquad v \simeq 1$$

$$E = \frac{E_{\text{cm}}}{\gamma\,(1 - v\cos\theta)} \simeq \frac{\gamma\,(1+v)}{1 + \gamma^2\theta^2 v\,(1+v)\,/2}\,E_{\text{cm}} \simeq \frac{2\gamma}{1+\gamma^2\theta^2}\,E_{\text{cm}}$$

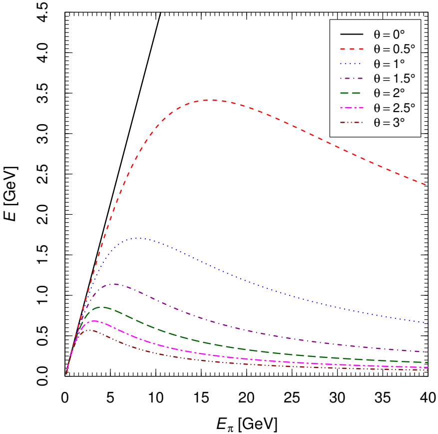
$$E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi}{1+\gamma^2\,\theta^2} = \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi\; m_\pi^2}{m_\pi^2 + E_\pi^2\,\theta^2}$$

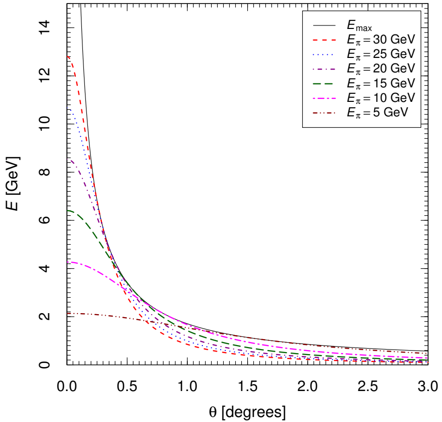
- $E_\pi\theta \ll m_\pi \quad \Longrightarrow \quad E \propto E_\pi$ WB beam
- $E_\pi\theta \gg m_\pi \quad \Longrightarrow \quad E \propto \dfrac{m_\pi^2}{E_\pi\,\theta^2}$ high-energy π^+ give low-energy ν_μ

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) m_\pi^2\,\frac{m_\pi^2 - E_\pi^2\,\theta^2}{\left(m_\pi^2 + E_\pi^2\,\theta^2\right)^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq 0 \quad \text{for} \quad E_\pi = \frac{m_\pi}{\theta} \quad \Longrightarrow \quad E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{m_\pi}{2\theta} \simeq \frac{29.79\ \text{MeV}}{\theta}$$

off-axis angle $\theta \simeq m_\pi / \langle E_\pi \rangle \implies E \simeq \dfrac{29.79\ \text{MeV}}{\theta}$





- E can be tuned on oscillation peak $E_{\text{peak}} = \Delta m^2 L / 2\pi$
- small $E \implies$ short $L_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2} \implies$ sensitivity to small values of Δm^2

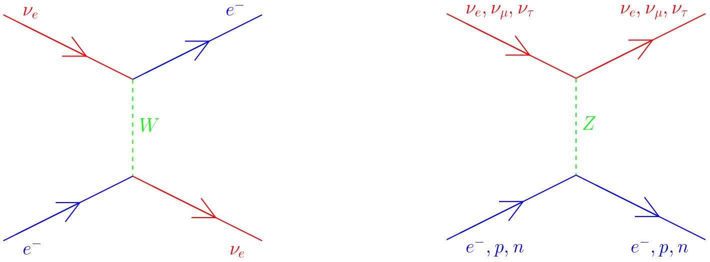
Neutrino Oscillations in Matter

- Neutrino Oscillations in Vacuum

- Neutrino Oscillations in Matter
  - Effective Potentials in Matter
  - Evolution of Neutrino Flavors in Matter
  - Two-Neutrino Mixing
  - Constant Matter Density
  - MSW Effect (Resonant Transitions in Matter)

Effective Potentials in Matter

coherent interactions with medium: forward elastic CC and NC scattering



$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

only $V_{\text{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\text{CC}} = -V_{\text{CC}}$ $\quad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

Evolution of Neutrino Flavors in Matter

- Flavor neutrino ν_α with momentum p: $\quad |\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* \, |\nu_k(p)\rangle$
- Evolution is determined by Hamiltonian
- Hamiltonian in vacuum: $\mathcal{H} = \mathcal{H}_0$

$$\mathcal{H}_0 \, |\nu_k(p)\rangle = E_k \, |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

- Hamiltonian in matter: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \, |\nu_\alpha(p)\rangle = V_\alpha \, |\nu_\alpha(p)\rangle$
- Schrödinger evolution equation: $i \, \frac{\mathrm{d}}{\mathrm{d}t} \, |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle$
- Initial condition: $|\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$
- For $t > 0$ the state $|\nu(p,t)\rangle$ is a superposition of all flavors:

$$|\nu(p,t)\rangle = \sum_\beta \varphi_\beta(p,t) |\nu_\beta(p)\rangle$$

- Transition probability: $P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2$

evolution equation of states

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle \,, \qquad |\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes

$$\varphi_\beta(p,t) = \langle \nu_\beta(p) | \nu(p,t)\rangle \,, \qquad \varphi_\beta(p,0) = \delta_{\alpha\beta}$$

evolution of flavor transition amplitudes

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H} | \nu(p,t)\rangle$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t)\rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t)\rangle$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_\beta(p,t) = \langle\nu_\beta(p)|\mathcal{H}_0|\nu(p,t)\rangle + \langle\nu_\beta(p)|\mathcal{H}_I|\nu(p,t)\rangle$$

$$\langle\nu_\beta(p)|\mathcal{H}_0|\nu(p,t)\rangle =$$

$$\sum_\rho \sum_{k,j} \underbrace{\langle\nu_\beta(p)|\nu_k(p)\rangle}_{U_{\beta k}} \underbrace{\langle\nu_k(p)|\mathcal{H}_0|\nu_j(p)\rangle}_{\delta_{kj}E_k} \underbrace{\langle\nu_j(p)|\nu_\rho(p)\rangle}_{U^*_{\rho j}} \underbrace{\langle\nu_\rho(p)|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho \sum_k U_{\beta k} E_k U^*_{\rho k}\,\varphi_\rho(p,t)$$

$$\langle\nu_\beta(p)|\mathcal{H}_I|\nu(p,t)\rangle = \sum_\rho \underbrace{\langle\nu_\beta(p)|\mathcal{H}_I|\nu_\rho(p)\rangle}_{\delta_{\beta\rho}V_\beta} \underbrace{\langle\nu_\rho(p)|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho \delta_{\beta\rho} V_\beta\,\varphi_\rho(p,t)$$

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k}\,E_k\,U^*_{\rho k} + \delta_{\beta\rho}\,V_\beta \right) \varphi_\rho}$$

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $E = p$ $t = x$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

$$i \, \frac{\text{d}}{\text{d}x} \, \varphi_\beta(p,x) = (p + V_{\text{NC}}) \, \varphi_\beta(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \, \frac{m_k^2}{2E} \, U_{\rho k}^* + \delta_{\beta e} \, \delta_{\rho e} \, V_{\text{CC}} \right) \varphi_\rho(p,x)$$

$$\psi_\beta(p,x) = \varphi_\beta(p,x) \, e^{ipx + i \int_0^x V_{\text{NC}}(x') \, \text{d}x'}$$

$$\Downarrow$$

$$i \, \frac{\text{d}}{\text{d}x} \, \psi_\beta = e^{ipx + i \int_0^x V_{\text{NC}}(x') \, \text{d}x'} \left(-p - V_{\text{NC}} + i \, \frac{\text{d}}{\text{d}x} \right) \varphi_\beta$$

$$\boxed{i \, \frac{\text{d}}{\text{d}x} \, \psi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \, \frac{m_k^2}{2E} \, U_{\rho k}^* + \delta_{\beta e} \, \delta_{\rho e} \, V_{\text{CC}} \right) \psi_\rho}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2}$$

evolution of flavor transition amplitudes in matrix form

$$\boxed{i \, \frac{\mathrm{d}}{\mathrm{d}x} \, \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2EV_{\mathrm{CC}} = 2\sqrt{2} E G_{\mathrm{F}} N_e$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

potential due to coherent forward elastic scattering

Two-Neutrino Mixing

$\nu_e \to \nu_\mu$ transitions with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$

$$U\,\mathbb{M}^2\,U^\dagger = \begin{pmatrix} \cos^2\vartheta m_1^2 + \sin^2\vartheta m_2^2 & \cos\vartheta\sin\vartheta\left(m_2^2 - m_1^2\right) \\ \cos\vartheta\sin\vartheta\left(m_2^2 - m_1^2\right) & \sin^2\vartheta m_1^2 + \cos^2\vartheta m_2^2 \end{pmatrix}$$

$$= \frac{1}{2}\underbrace{\begin{pmatrix} m_1^2 + m_2^2 & 0 \\ 0 & m_1^2 + m_2^2 \end{pmatrix}}_{\uparrow} + \frac{1}{2}\begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}$$

irrelevant common phase

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$

$$\boxed{\begin{array}{l} P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 \\ P_{\nu_e\to\nu_e}(x) = |\psi_e(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x)\end{array}}$$

Constant Matter Density

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta\\ \Delta m^2 \sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x} = 0$$

diagonalization of effective Hamiltonian: $\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$

$$\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta\\ \Delta m^2 \sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix} =$$

$$= \begin{pmatrix}A_{\mathrm{CC}} - \Delta m^2_{\mathrm{M}} & 0\\ 0 & A_{\mathrm{CC}} + \Delta m^2_{\mathrm{M}}\end{pmatrix}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} = \frac{1}{4E}\left[\begin{pmatrix}A_{\mathrm{CC}} & 0\\ 0 & A_{\mathrm{CC}}\end{pmatrix} + \begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

$\uparrow$

irrelevant common phase

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\mathrm{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A^{\mathrm{R}}_{\mathrm{CC}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N^{\mathrm{R}}_{e} = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\mathrm{F}}}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} \psi_1^{\mathrm{M}} \\ \psi_2^{\mathrm{M}} \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2_{\mathrm{M}} & 0 \\ 0 & \Delta m^2_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1^{\mathrm{M}} \\ \psi_2^{\mathrm{M}} \end{pmatrix}$$

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1^{\mathrm{M}} \\ \psi_2^{\mathrm{M}} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \psi_1^{\mathrm{M}} \\ \psi_2^{\mathrm{M}} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}} \\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

$$\nu_e \to \nu_\mu \quad \Longrightarrow \quad \begin{pmatrix} \psi_e(0) \\ \psi_\mu(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix} \psi_1^{\mathrm{M}}(0) \\ \psi_2^{\mathrm{M}}(0) \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} \\ \sin\vartheta_{\mathrm{M}} \end{pmatrix}$$

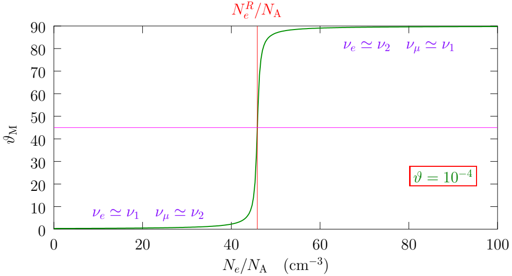
$$\psi_1^{\mathrm{M}}(x) = \cos\vartheta_{\mathrm{M}} \exp\left(i \frac{\Delta m^2_{\mathrm{M}} x}{4E} \right)$$

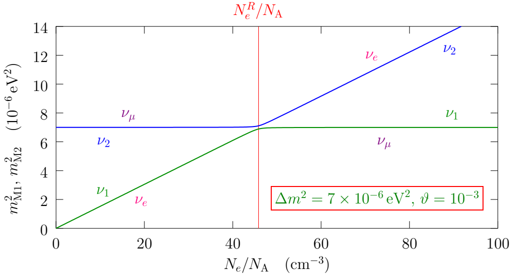
$$\psi_2^{\mathrm{M}}(x) = \sin\vartheta_{\mathrm{M}} \exp\left(-i \frac{\Delta m^2_{\mathrm{M}} x}{4E} \right)$$

$$P_{\nu_e \to \nu_\mu}(x) = |\psi_\mu(x)|^2 = \left| -\sin\vartheta_{\mathrm{M}} \psi_1^{\mathrm{M}}(x) + \cos\vartheta_{\mathrm{M}} \psi_2^{\mathrm{M}}(x) \right|^2$$

$$\boxed{P_{\nu_e \to \nu_\mu}(x) = \sin^2 2\vartheta_{\mathrm{M}} \, \sin^2\left(\frac{\Delta m^2_{\mathrm{M}} x}{4E} \right)}$$

MSW Effect (Resonant Transitions in Matter)





$$\begin{aligned} \nu_e &= \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2 \\ \nu_\mu &= -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2 \end{aligned}$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

tentative diagonalization: $\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} =$$

$$= \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

if matter density is not constant $\mathrm{d}\vartheta_{\mathrm{M}}/\mathrm{d}x \neq 0$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} = \left[\underset{\substack{\uparrow\\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0 \\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix} + \begin{pmatrix}0 & -i\dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} \\ i\dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

$$i\,\frac{d}{dx}\begin{pmatrix}\psi_1^{M}\\ \psi_2^{M}\end{pmatrix} = \left[\frac{1}{4E}\underbrace{\begin{pmatrix}-\Delta m^2_{M} & 0\\ 0 & \Delta m^2_{M}\end{pmatrix}}_{\uparrow\ \text{adiabatic}} + \underbrace{\begin{pmatrix}0 & -i\,\frac{d\vartheta_{M}}{dx}\\ i\,\frac{d\vartheta_{M}}{dx} & 0\end{pmatrix}}_{\substack{\uparrow\\ \text{non-adiabatic}\\ \text{maximum at resonance}}}\right]\begin{pmatrix}\psi_1^{M}\\ \psi_2^{M}\end{pmatrix}$$

initial conditions:

$$\begin{pmatrix}\psi_1^{M}(0)\\ \psi_2^{M}(0)\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{M} & -\sin\vartheta^0_{M}\\ \sin\vartheta^0_{M} & \cos\vartheta^0_{M}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{M}\\ \sin\vartheta^0_{M}\end{pmatrix}$$

solution approximating all non-adiabatic $\nu_1^{M} \leftrightarrows \nu_2^{M}$ transitions in resonance

$$\begin{aligned}
\psi_1^{M}(x) \simeq\ & \left[\cos\vartheta^0_{M}\exp\left(i\int_0^{x_R}\frac{\Delta m^2_{M}(x')}{4E}\,dx'\right)\mathcal{A}^{R}_{11} + \sin\vartheta^0_{M}\exp\left(-i\int_0^{x_R}\frac{\Delta m^2_{M}(x')}{4E}\,dx'\right)\mathcal{A}^{R}_{21}\right]\\
& \times\exp\left(i\int_{x_R}^{x}\frac{\Delta m^2_{M}(x')}{4E}\,dx'\right)\\
\psi_2^{M}(x) \simeq\ & \left[\cos\vartheta^0_{M}\exp\left(i\int_0^{x_R}\frac{\Delta m^2_{M}(x')}{4E}\,dx'\right)\mathcal{A}^{R}_{12} + \sin\vartheta^0_{M}\exp\left(-i\int_0^{x_R}\frac{\Delta m^2_{M}(x')}{4E}\,dx'\right)\mathcal{A}^{R}_{22}\right]\\
& \times\exp\left(-i\int_{x_R}^{x}\frac{\Delta m^2_{M}(x')}{4E}\,dx'\right)
\end{aligned}$$

Averaged ν_e Survival Probability

$$\psi_e(x) = \cos\vartheta\,\psi_1^{\mathrm{M}}(x) + \sin\vartheta\,\psi_2^{\mathrm{M}}(x)$$

neglect interference (averaged over energy spectrum)

$$\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = \cos^2\vartheta\,\cos^2\vartheta^0_{\mathrm{M}}\,|\mathcal{A}^{\mathrm{R}}_{11}|^2 + \cos^2\vartheta\,\sin^2\vartheta^0_{\mathrm{M}}\,|\mathcal{A}^{\mathrm{R}}_{21}|^2 + \sin^2\vartheta\,\cos^2\vartheta^0_{\mathrm{M}}\,|\mathcal{A}^{\mathrm{R}}_{12}|^2 + \sin^2\vartheta\,\sin^2\vartheta^0_{\mathrm{M}}\,|\mathcal{A}^{\mathrm{R}}_{22}|^2$$

conservation of probability (unitarity)

$$|\mathcal{A}^{\mathrm{R}}_{12}|^2 = |\mathcal{A}^{\mathrm{R}}_{21}|^2 = P_{\mathrm{c}} \qquad |\mathcal{A}^{\mathrm{R}}_{11}|^2 = |\mathcal{A}^{\mathrm{R}}_{22}|^2 = 1 - P_{\mathrm{c}}$$

$P_{\mathrm{c}} \equiv$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{c}}\right)\cos 2\vartheta^0_{\mathrm{M}}\,\cos 2\vartheta}$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter:
$$\gamma = \left.\frac{\Delta m^2_{\mathrm{M}}/2E}{2|d\vartheta_{\mathrm{M}}/dx|}\right|_{\mathrm{R}} = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A_{\mathrm{CC}}}{dx}\right|_{\mathrm{R}}}$$

$A \propto x$ $\quad F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $\quad F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ $\quad F = 1 - \tan^2\vartheta$

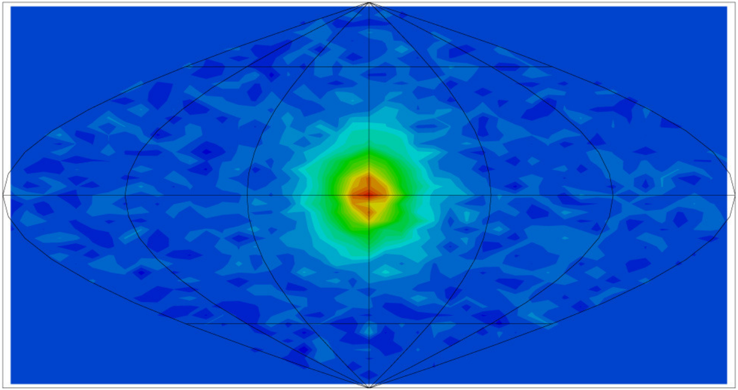
[Pizzochero, PRD 36 (1987) 2293]

[Toshev, PLB 196 (1987) 170]

[Petcov, PLB 200 (1988) 373]

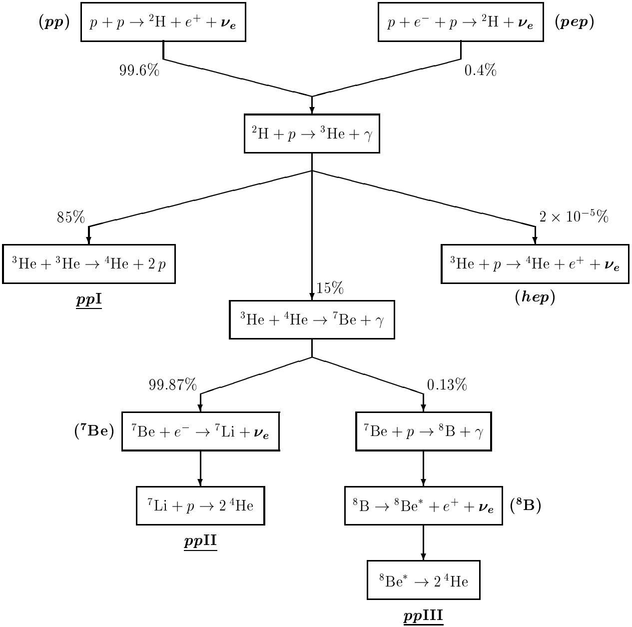
Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

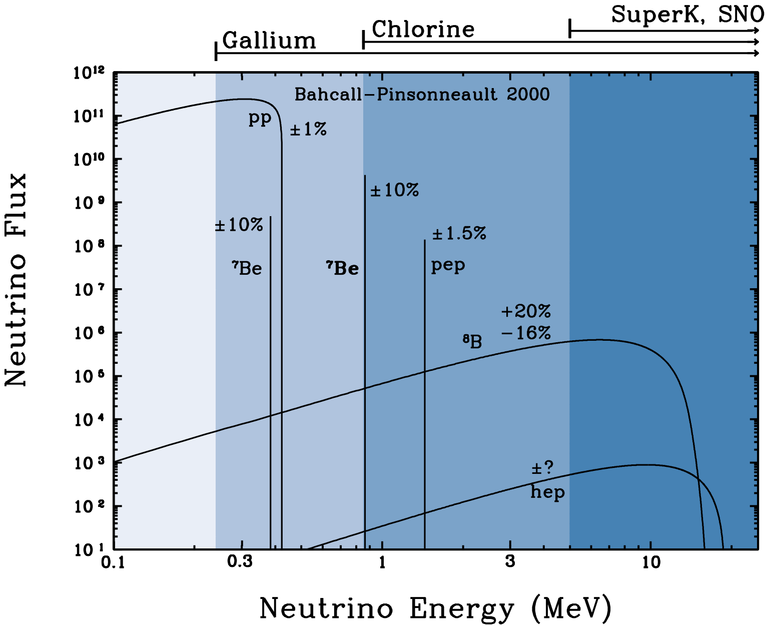
Solar Neutrinos



The sun observed through neutrinos by Super-Kamiokande

Standard Solar Model (SSM): *pp* chain




Gallium
Chlorine
SuperK, SNO
Bahcall−Pinsonneault 2000
pp
±1%
±10%
±10%
^{7}Be
^{7}Be
±1.5%
pep
+20%
−16%
^{8}B
±?
hep
Neutrino Flux
Neutrino Energy (MeV)
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
0.1
0.3
1
3
10

Solar Neutrino Observations

- 1957: Bruno Pontecorvo suggests to observe solar neutrinos using a detector tank containing Clorine through the process

$$\nu_e + {}^{37}_{17}\text{Cl} \to {}^{37}_{18}\text{Ar} + e^-$$

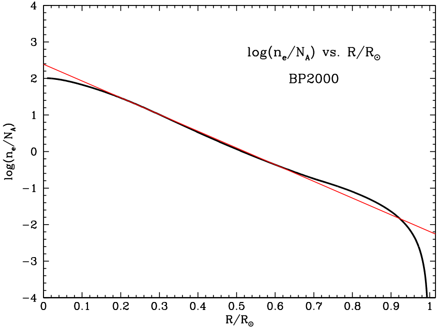
- 1964: John N. Bahcall calculates the cross sections and finds that it is enough to observe solar neutrinos.
- 1964: Raymond Davis proposes the Homestake experiment that is constructed in 1965–1967. It is based in the radiochemical counting of the ${}^{37}\text{Ar}$ produced by solar neutrinos in a tank with 615 tons of tetrachloroethylene (C_2Cl_4).
- 1970: Davis (2002 Physics Nobel Prize) and collaborators observe for the first time solar neutrinos counting ${}^{37}\text{Ar}$ atoms that are produced with a rate of about one every 2 days in the Homestake detector which contains about 2×10^{30} atoms!
- Solar neutrinos have been observed in the experiments Homestake (1970-1994), Kamiokande (1987-1995) SAGE (1990-2010), GALLEX/GNO (1991-2000), Super-Kamiokande (1996-2019), SNO (1999-2008), Borexino (2007-2019).

The solar neutrino problem

- 1968: Bruno Pontecorvo suggests that part of solar ν_e's can disappear into ν_μ (or ν_τ) due to oscillations.
- 1970: Discovery of the solar neutrino problem in the Homestake experiment that counts about 0.5 ^{37}Ar atoms per day with a SSM prediction of about 1.5 ^{37}Ar atoms per day.
- All the other solar neutrino experiments observed a suppression of the solar ν_e signal.
- From 1970 to 2002 experts debated on the possible solutions of the solar neutrino problem.
- The two solutions that were considered more likely are:
  - There is a mistake in the SSM prediction of the solar ν_e flux.
  - Part of the solar ν_e's disappear into ν_μ (or ν_τ) due to oscillations as suggested by Pontecorvo.

Solar Neutrino MSW Transitions

SUN: $N_e(x) \simeq N_e^{\mathrm{c}} \exp\left(-\frac{x}{x_0}\right)$ $\quad N_e^{\mathrm{c}} = 245\, N_{\mathrm{A}}/\mathrm{cm}^3$ $\quad x_0 = \frac{R_\odot}{10.54}$



$$\overline{P}^{\mathrm{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{c}}\right) \cos 2\vartheta^0_{\mathrm{M}}\, \cos 2\vartheta$$

$$P_{\mathrm{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{\mathrm{d}\ln A_{\mathrm{CC}}}{\mathrm{d}x}\right|_{\mathrm{R}}}$$

$$F = 1 - \tan^2\vartheta$$

$$A_{\mathrm{CC}} = 2\sqrt{2} E G_{\mathrm{F}} N_e$$

practical prescription:

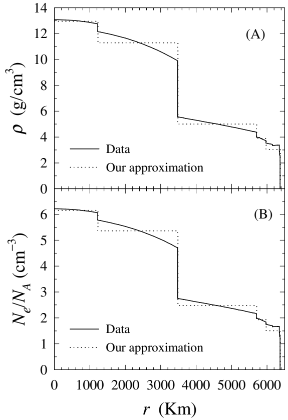
[Lisi et al., PRD 63 (2001) 093002]

$$\begin{cases} \text{numerical } \left|\mathrm{d}\ln A_{\mathrm{CC}}/\mathrm{d}x\right|_{\mathrm{R}} & \text{for } x \leq 0.904 R_\odot \\ \left|\mathrm{d}\ln A_{\mathrm{CC}}/\mathrm{d}x\right|_{\mathrm{R}} \to \frac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{cases}$$

Electron Neutrino Regeneration in the Earth

$$P^{\text{sun+earth}}_{\nu_e \to \nu_e} = \overline{P}^{\text{sun}}_{\nu_e \to \nu_e} + \frac{\left(1 - 2\overline{P}^{\text{sun}}_{\nu_e \to \nu_e}\right)\left(P^{\text{earth}}_{\nu_2 \to \nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



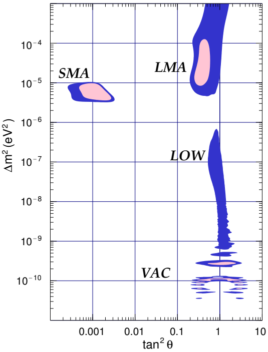
$P^{\text{earth}}_{\nu_2 \to \nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

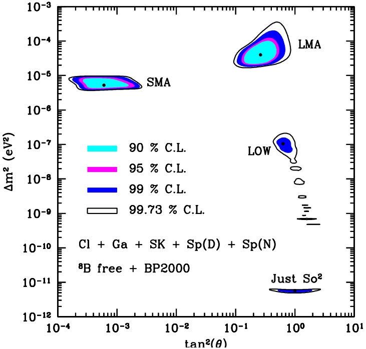
Wave functions of flavor neutrinos are joined at the boundaries of steps.

Solar Neutrino Oscillations

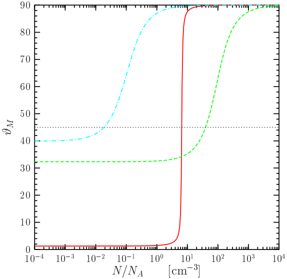
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\text{eV}^2$,	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



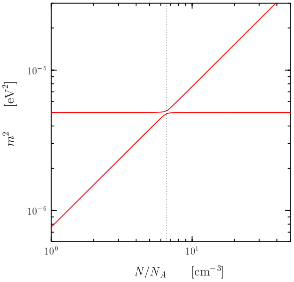
[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



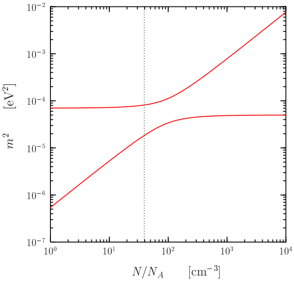
solid line: $\Delta m^2 = 5 \times 10^{-6}\,\mathrm{eV}^2$
(typical SMA) $\tan^2\vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\mathrm{eV}^2$
(typical LMA) $\tan^2\vartheta = 0.4$

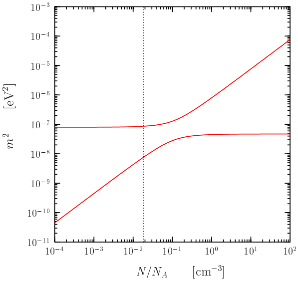
dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\mathrm{eV}^2$
(typical LOW) $\tan^2\vartheta = 0.7$



typical SMA

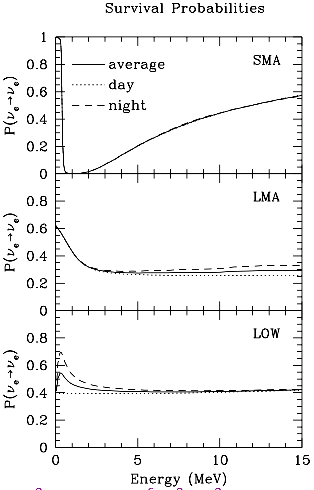


typical LMA



typical LOW

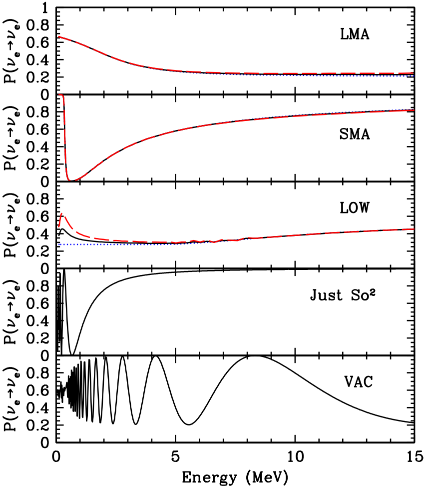
[Bahcall, Krastev, Smirnov, PRD 58 (1998) 096016]



SMA: $\Delta m^2 = 5.0 \times 10^{-6}\,\text{eV}^2$ $\sin^2 2\vartheta = 3.5 \times 10^{-3}$

LMA: $\Delta m^2 = 1.6 \times 10^{-5}\,\text{eV}^2$ $\sin^2 2\vartheta = 0.57$

LOW: $\Delta m^2 = 7.9 \times 10^{-8}\,\text{eV}^2$ $\sin^2 2\vartheta = 0.95$



[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]

LMA: $\Delta m^2 = 4.2 \times 10^{-5}\,\text{eV}^2$ $\tan^2\vartheta = 0.26$

SMA: $\Delta m^2 = 5.2 \times 10^{-6}\,\text{eV}^2$ $\tan^2\vartheta = 5.5 \times 10^{-4}$

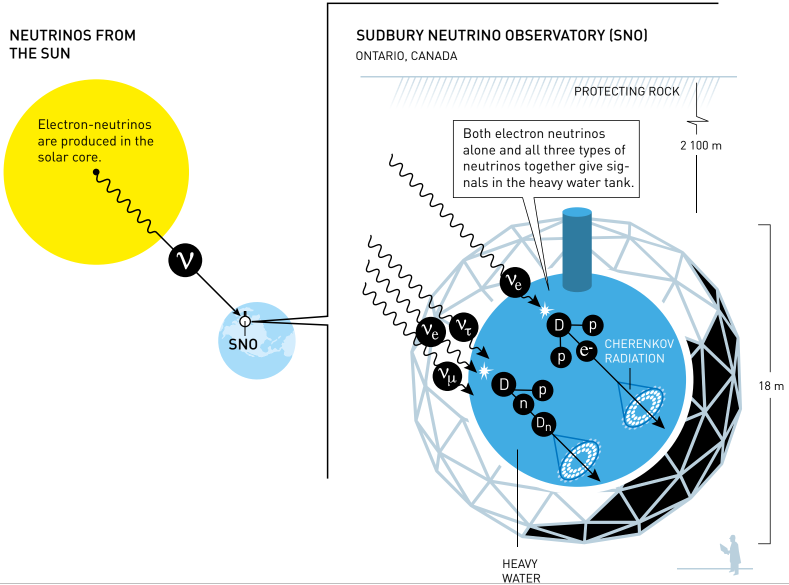
LOW: $\Delta m^2 = 7.6 \times 10^{-8}\,\text{eV}^2$ $\tan^2\vartheta = 0.72$

Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}\,\text{eV}^2$ $\tan^2\vartheta = 1.0$

VAC: $\Delta m^2 = 1.4 \times 10^{-10}\,\text{eV}^2$ $\tan^2\vartheta = 0.38$

The SNO Experiment

1 kton of D_2O, Cherenkov detector, 2100 m underground



- Observed SNO rates relative to the SSM predictions:

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02 \qquad \frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.02 \pm 0.13$$

- The CC measurements confirms the solar neutrino problem: ν_e disappear.

- The NC measurement shows that the total flux of ν_e, ν_μ, ν_τ in agreement with the SSM prediction.

- The only possible explanation of the two measurements is that solar ν_e's transform into ν_μ and/or ν_τ. (A. McDonald: 2015 Physics Nobel Prize)

- The simplest and most plausible mechanism are neutrino oscillations.

- The oscillations of solar neutrinos have been confirmed in 2002 by the KamLAND very-long-baseline reactor neutrino experiment.

KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

53 nuclear power reactors in Japan and Korea

average distance from reactors: 180 km

- 6.7% of flux from one reactor at 88 km
- 79% of flux from 26 reactors at 138–214 km
- 14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector: $\bar{\nu}_e + p \to e^+ + n$, energy threshold: $E_{\text{th}}^{\bar{\nu}_e p} = 1.8\,\text{MeV}$

data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.): $N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$

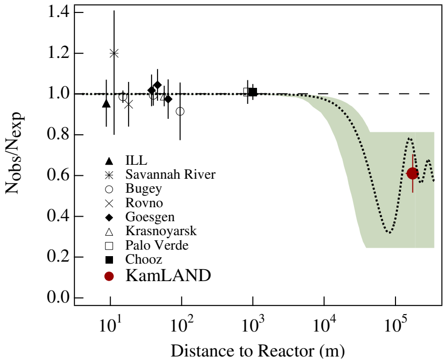
expected number of background events: $N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$

observed number of neutrino events: $N_{\text{observed}}^{\text{KamLAND}} = 54$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

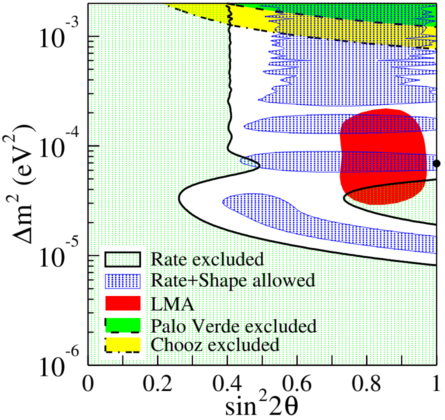
99.95% C.L. evidence of $\bar{\nu}_e$ disappearance

confirmation of LMA (December 2002)



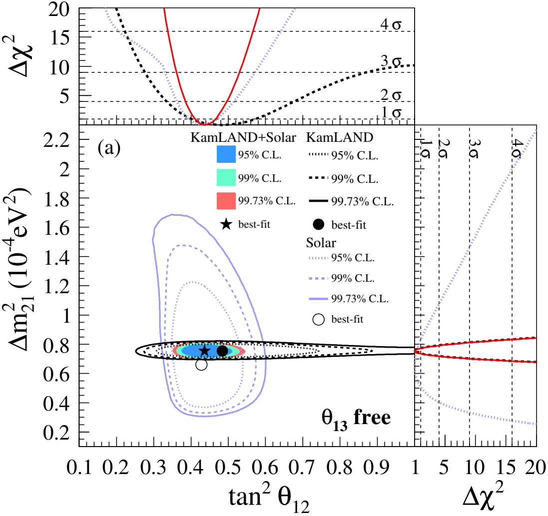
Shade: 95% C.L. LMA

Curve: $\begin{cases} \Delta m^2 = 5.5 \times 10^{-5}\,\text{eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$



95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

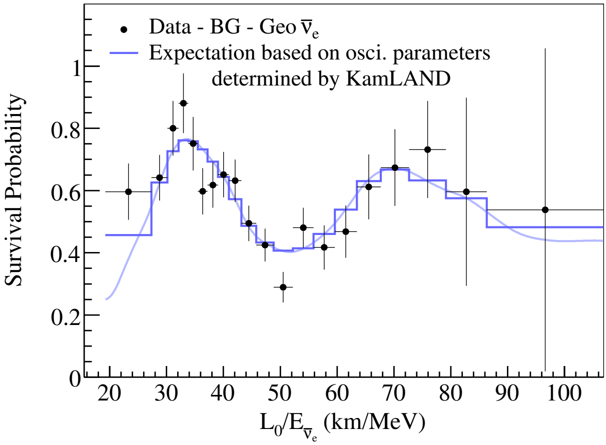


$$\Delta m^2_{21} = 7.53^{+0.19}_{-0.18} \times 10^{-5}\,\text{eV}^2$$

$$\tan^2\vartheta_{12} = 0.437^{+0.029}_{-0.026}$$

$$\sin^2\vartheta_{13} = 0.023 \pm 0.015$$

[KamLAND, PRL 100 (2008) 221803]



[KamLAND, PRL 100 (2008) 221803]

LMA Solar Neutrino Oscillations

best fit of reactor + solar neutrino data: $\Delta m^2 \simeq 7 \times 10^{-5}\,\text{eV}^2 \quad \tan^2\vartheta \simeq 0.4$

$$\boxed{\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\vartheta^0_M \, \cos 2\vartheta}$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)} \qquad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A}{dx}\right|_R} \qquad F = 1 - \tan^2\vartheta$$

$$A_{CC} \simeq 2\sqrt{2} E G_F N_e^c \exp\left(-\frac{x}{x_0}\right) \quad \Longrightarrow \quad \left|\frac{d\ln A}{dx}\right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15}\,\text{eV}$$

$$\tan^2\vartheta \simeq 0.4 \quad \Longrightarrow \quad \sin^2 2\vartheta \simeq 0.82\,,\ \cos 2\vartheta \simeq 0.43 \qquad \gamma \simeq 2 \times 10^4 \left(\frac{E}{\text{MeV}}\right)^{-1}$$

$$\gamma \gg 1 \quad \Longrightarrow \quad P_c \ll 1 \quad \Longrightarrow \quad \boxed{\overline{P}^{\text{sun,LMA}}_{\nu_e\to\nu_e} \simeq \frac{1}{2} + \frac{1}{2}\cos 2\vartheta^0_M \, \cos 2\vartheta}$$

$$\boxed{\cos 2\vartheta_{\mathrm{M}}^{0} = \frac{\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}}{\sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}^{0}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}}}$$

critical parameter [Bahcall, Peña-Garay, JHEP 0311 (2003) 004]

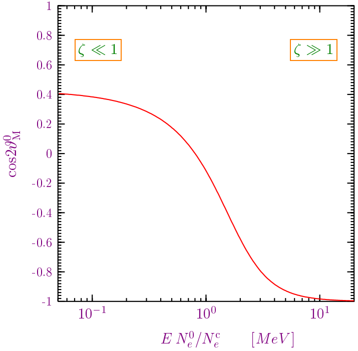
$$\zeta = \frac{A_{\mathrm{CC}}^{0}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}}\right) \left(\frac{N_e^0}{N_e^c}\right)$$

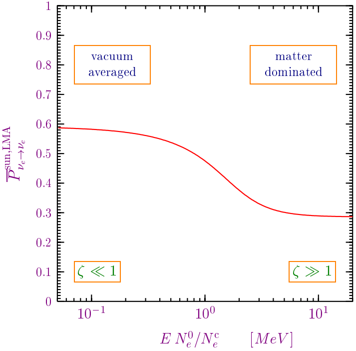
$$\zeta \ll 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \vartheta \implies \overline{P}^{\mathrm{sun}}_{\nu_e \to \nu_e} \simeq 1 - \frac{1}{2} \sin^2 2\vartheta$$

vacuum averaged survival probability

$$\zeta \gg 1 \implies \vartheta_{\mathrm{M}}^{0} \simeq \pi/2 \implies \overline{P}^{\mathrm{sun}}_{\nu_e \to \nu_e} \simeq \sin^2\vartheta$$

matter dominated survival probability



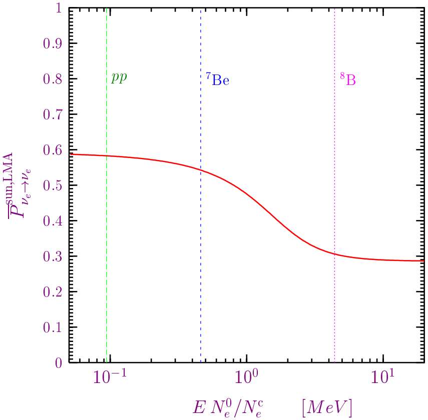


$$\zeta = \frac{A_{\mathrm{CC}}^{0}}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2} E G_{\mathrm{F}} N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left(\frac{E}{\mathrm{MeV}} \right) \left(\frac{N_e^0}{N_e^{\mathrm{c}}} \right)$$

$\langle E \rangle_{pp} \simeq 0.27\,\mathrm{MeV}\,,\ \langle r_0 \rangle_{pp} \simeq 0.1\,R_\odot \implies \langle E\, N_e^0 / N_e^{\mathrm{c}} \rangle_{pp} \simeq 0.094\,\mathrm{MeV}$

$E_{^7\mathrm{Be}} \simeq 0.86\,\mathrm{MeV}\,,\ \langle r_0 \rangle_{^7\mathrm{Be}} \simeq 0.06\,R_\odot \implies \langle E\, N_e^0 / N_e^{\mathrm{c}} \rangle_{^7\mathrm{Be}} \simeq 0.46\,\mathrm{MeV}$

$\langle E \rangle_{^8\mathrm{B}} \simeq 6.7\,\mathrm{MeV}\,,\ \langle r_0 \rangle_{^8\mathrm{B}} \simeq 0.04\,R_\odot \implies \langle E\, N_e^0 / N_e^{\mathrm{c}} \rangle_{^8\mathrm{B}} \simeq 4.4\,\mathrm{MeV}$



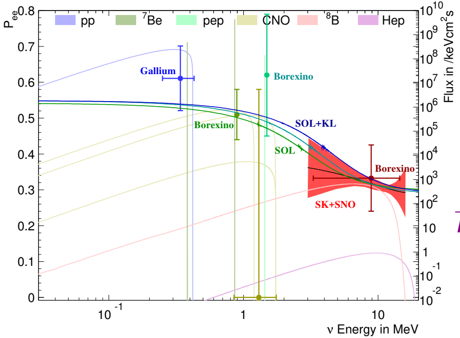
Solar Neutrino Spectrum

$$\overline{P}_{ee}^{\rm SOL} = \sum_{k=1}^{3} |U_{ek}|^2 |U_{ek}^0|^2 = \left(\frac{1}{2} + \frac{1}{2} \cos 2\vartheta_{12}^0 \cos 2\vartheta_{12} \right) \cos^4\vartheta_{13} + \sin^4\vartheta_{13}$$

Averaged Vacuum Oscillations

$\theta_{12}^0 \simeq \theta_{12}$

$$\overline{P}_{ee}^{\rm SOL} \simeq \left(1 - \frac{1}{2}\sin^2\vartheta_{12}\right) \times \left(1 - \sin^2\vartheta_{13}\right)$$



[SK, PRD 94 (2016) 052010, arXiv:1606.07538]

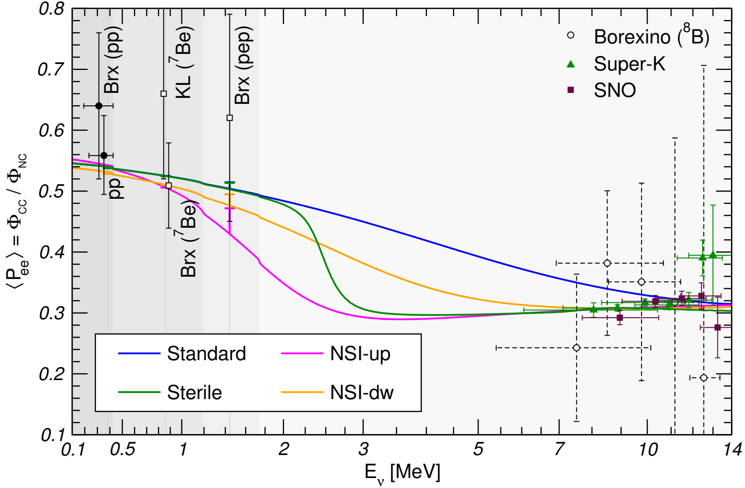
Adiabatic MSW Transitions

$\theta_{12}^0 \simeq \pi/2$

$$\overline{P}_{ee}^{\rm SOL} \simeq \sin^2\vartheta_{12} \times \left(1 - \sin^2\vartheta_{13}\right)$$

$$\tan 2\vartheta_{12}^0 = \frac{\tan 2\vartheta_{12}}{1 - \dfrac{2EV_{\rm CC}^0}{\Delta m_{21}^2 \cos\vartheta_{12}}}$$

- Small (solar) $\Delta m_{21}^2 \Longrightarrow$ Low-E transition
- Large (KamLAND) $\Delta m_{21}^2 \Longrightarrow$ High-E transition
- Non-Standard Interactions (NSI)?
- Very light sterile neutrinos?



[Maltoni, Smirnov, EPJA 52 (2016) 87, arXiv:1507.05287]

In Neutrino Oscillations Dirac = Majorana

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495; Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Evolution of Amplitudes: $$i \frac{d\psi_\alpha}{dx} = \frac{1}{2E} \sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \psi_\beta$$

difference: $\begin{cases} \text{Dirac:} & U^{(D)} \\ \text{Majorana:} & U^{(M)} = U^{(D)} D(\lambda) \end{cases}$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad D M^2 = M^2 D \quad \Longrightarrow \quad D M^2 D^\dagger = M^2$$

$$U^{(M)} M^2 (U^{(M)})^\dagger = U^{(D)} D M^2 D^\dagger (U^{(D)})^\dagger = U^{(D)} M^2 (U^{(D)})^\dagger$$

Common Question: Do Charged Leptons Oscillate?

- Mass is the only property which distinguishes e, μ, τ.
- The flavor of a charged lepton is defined by its mass!
- By definition, the flavor of a charged lepton cannot change.

$\boxed{\text{THE FLAVOR OF CHARGED LEPTONS DOES NOT OSCILLATE}}$

[CG, Kim, FPL 14 (2001) 213] [CG, hep-ph/0409230] [Akhmedov, JHEP 09 (2007) 116]

a misleading argument

[Sassaroli, Srivastava, Widom, hep-ph/9509261, EPJC 2 (1998) 769] [Srivastava, Widom, hep-ph/9707268]

in $\pi^+ \to \mu^+ + \nu_\mu$ the final state of the antimuon and neutrino is entangled

$$\Downarrow$$

if the probability to detect the neutrino oscillates as a function of distance, also the probability to detect the muon must oscillate

WRONG!

the probability to detect the neutrino (as ν_μ or ν_τ or ν_e) does not oscillate as a function of distance, because

$$\sum_{\beta=e,\mu,\tau} P_{\nu_\mu \to \nu_\beta} = 1$$

conservation of probability (unitarity)

[Dolgov, Morozov, Okun, Shchepkin, NPB 502 (1997) 3] [CG, Kim, FPL 14 (2001) 213]

Λ oscillations from $\pi^- + p \to \Lambda + K^0$

[Widom, Srivastava, hep-ph/9605399] [Srivastava, Widom, Sassaroli, PLB 344 (1995) 436]

refuted in [Lowe et al., PLB 384 (1996) 288] [Burkhardt, Lowe, Stephenson, Goldman, PRD 59 (1999) 054018]

Correct definition of Charged Lepton Oscillations

[Pakvasa, Nuovo Cim. Lett. 31 (1981) 497]

Analogy

- Neutrino Oscillations: massive neutrinos propagate unchanged between production and detection, with a difference of mass (flavor) of the charged leptons involved in the production and detection processes.
- Charged-Lepton Oscillations: massive charged leptons propagate unchanged between production and detection, with a difference of mass of the neutrinos involved in the production and detection processes.

NO FLAVOR CONVERSION!

The propagating charged leptons must be ultrarelativistic, in order to be produced and detected coherently (if τ is not ultrarelativistic, only e and μ contribute to the phase).

Practical Problems

- The initial and final neutrinos must be massive neutrinos of known type: precise neutrino mass measurements.

- The energy of the propagating charged leptons must be extremely high, in order to have a measurable oscillation length

$$\frac{4\pi E}{(m_\mu^2 - m_e^2)} \simeq \frac{4\pi E}{m_\mu^2} \simeq 2 \times 10^{-11} \left(\frac{E}{\mathrm{GeV}} \right) \mathrm{cm}$$

detailed discussion: [Akhmedov, JHEP 09 (2007) 116, arXiv:0706.1216]