Neutrino Physics

Part III: Phenomenology of Massive Neutrinos

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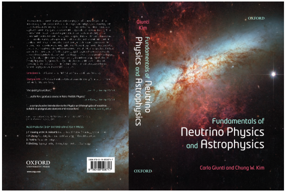
Torino, 30 November - 4 December 2020

http://personalpages.to.infn.it/~giunti/slides/2019/giunti-201130-phdto3.pdf



C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Three-Neutrino Mixing Paradigm

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \nu_{kL}} \qquad \alpha = e, \mu, \tau$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \delta_{\alpha\beta} \underbrace{- 4 \sum_{k>j} \text{Re}\left[U_{\alpha k}^{*} \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^{*}\right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)}_{\text{CP conserving}}$$

$$\underbrace{+ 2 \sum_{k>j} \text{Im}\left[U_{\alpha k}^{*} \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^{*}\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)}_{\text{CP violating}}$$

- Squared-mass differences: $\Delta m_{kj}^2 = m_k^2 - m_j^2$
- Mixing: $U_{\alpha k}^{*} \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^{*}$ quartic rephasing invariants
- Jarlskog invariant: $J_{\text{CP}} = \text{Im}\left[U_{\alpha k}^{*} \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^{*}\right]$

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

ATM, Acc LBL $\nu_\mu \to \nu_\mu$ — Rea LBL $\bar{\nu}_e \to \bar{\nu}_e$, Acc LBL $\nu_\mu \to \nu_e$ — SOL, KamLAND — $\beta\beta_{0\nu}$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS:
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent Δm^2_{kj}: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes $(\beta\beta_{0\nu})$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, Borexino; Super-Kamiokande; GALLEX/GNO, SAGE; Homestake, Kamiokande)

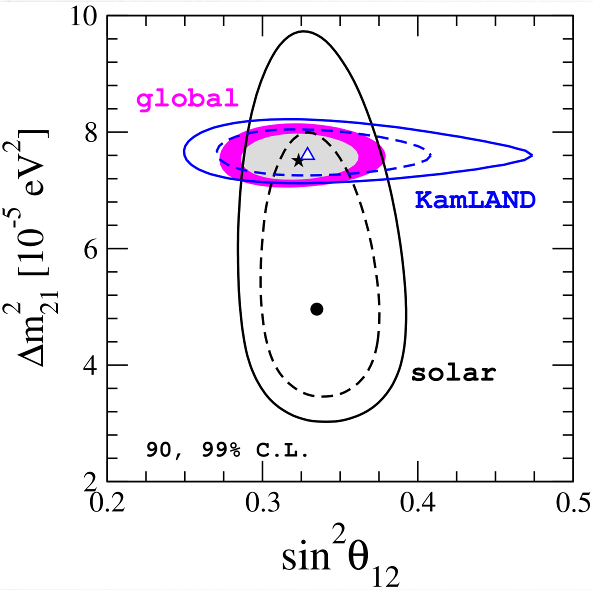
VLBL Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\rightarrow$

$$\Delta m^2_{\text{S}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$$

$$\sin^2\vartheta_{\text{S}} = \sin^2\vartheta_{12} \simeq 0.30$$



[M. Tortola @ Neutrino 2018]

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

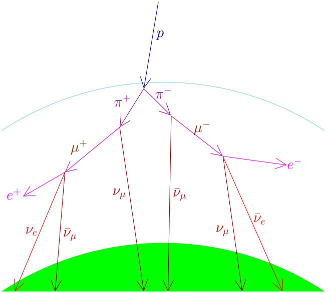
Atmospheric $\nu_\mu \to \nu_\tau$ — (Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2, IceCube, ANTARES)

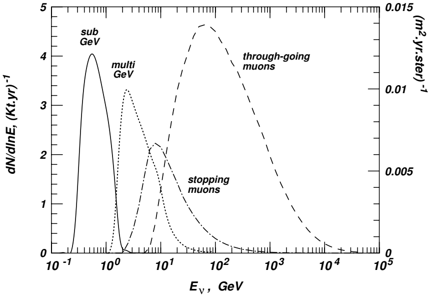
LBL Accelerator ν_μ disappearance — (K2K, MINOS, T2K, NOνA)

LBL Accelerator $\nu_\mu \to \nu_\tau$ — (OPERA)

$$\to \begin{cases} \Delta m^2_{\text{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23} \simeq 0.50 \end{cases}$$

Atmospheric Neutrinos





$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1\,\text{GeV}$$

uncertainty on ratios: $\sim 5\%$

uncertainty on fluxes: $\sim 30\%$

ratio of ratios

$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

$$R^{\text{K}}_{\text{sub-GeV}} = 0.60 \pm 0.07 \pm 0.05$$

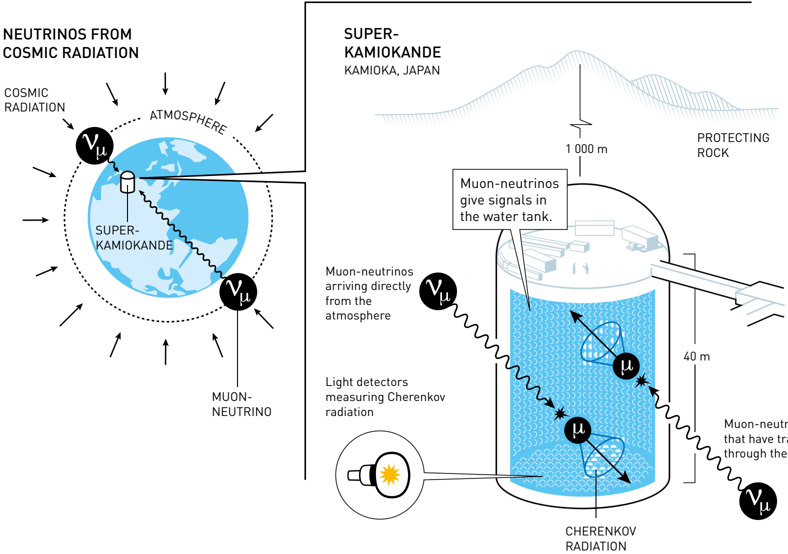
[Kamiokande, PLB 280 (1992) 146]

$$R^{\text{K}}_{\text{multi-GeV}} = 0.57 \pm 0.08 \pm 0.07$$

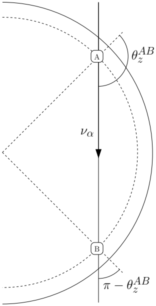
[Kamiokande, PLB 335 (1994) 237]

The Super-Kamiokande Experiment

50 ktons of water, Cherenkov detector, 1000 m underground



The Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\theta_z^{AB})$$

$$\phi^{(A)}_{\nu_\alpha}(\theta_z^{AB}) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z^{AB})$$

$$\Downarrow$$

$$\phi^{(B)}_{\nu_\alpha}(\theta_z) = \phi^{(B)}_{\nu_\alpha}(\pi - \theta_z)$$

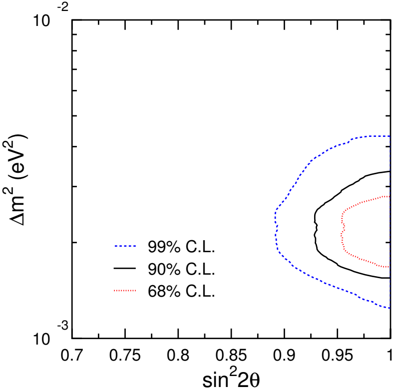
$$A^{\text{up-down}}_{\nu_\mu}(\text{SK}) = \left(\frac{N^{\text{up}}_{\nu_\mu} - N^{\text{down}}_{\nu_\mu}}{N^{\text{up}}_{\nu_\mu} + N^{\text{down}}_{\nu_\mu}}\right) = -0.296 \pm 0.048 \pm 0.01$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

(T. Kajita: 2015 Physics Nobel Prize)

Fit of Super-Kamiokande Atmospheric Data



$\underline{\nu_\mu \to \nu_\tau}$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\text{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\text{th}} = 3.5\,\text{GeV} \Longrightarrow \sim 20\text{events/yr}$
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$N_{\nu_\tau}^{\text{the}} = 78 \pm 26$ @ $\Delta m^2 = 2.4 \times 10^{-3}\,\text{eV}^2$

$N_{\nu_\tau}^{\text{exp}} = 138^{+50}_{-58}$

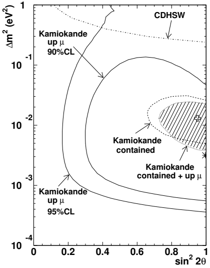
$N_{\nu_\tau} > 0$ @ 2.4σ

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

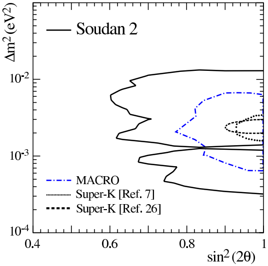
Check: OPERA ($\nu_\mu \to \nu_\tau$)
CERN to Gran Sasso (CNGS)
$L \simeq 732\,\text{km}$ $\langle E \rangle \simeq 18\,\text{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

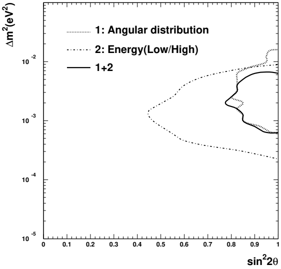
Kamiokande, Soudan-2, MACRO and MINOS



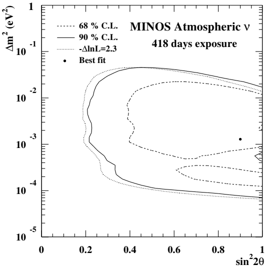
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



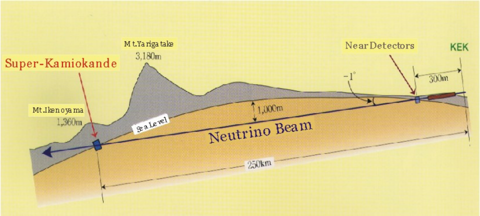
[MACRO, hep-ex/0304037]



[MINOS, hep-ex/0512036]

K2K

confirmation of atmospheric allowed region (June 2002)

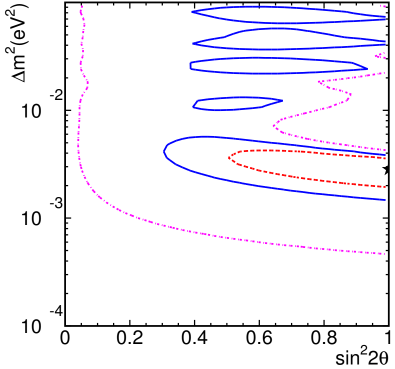


KEK to Kamioka

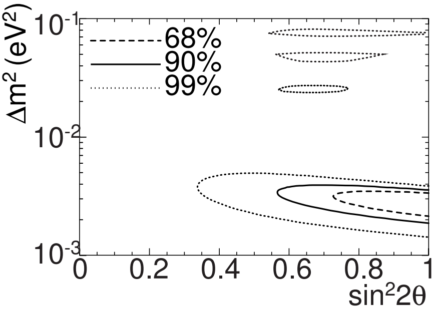
(Super-Kamiokande)

250 km

$\nu_\mu \to \nu_\mu$



[K2K, Phys. Rev. Lett. 90 (2003) 041801]



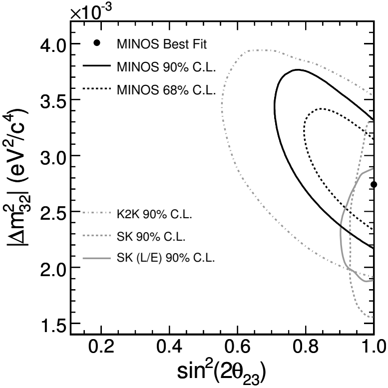
[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

MINOS

May 2005 – Feb 2006

http://www-numi.fnal.gov/

Near Detector: 1 km



$\nu_\mu \to \nu_\mu$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\vartheta > 0.87 \text{ @ } 68\% CL$$

[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]

OPERA

PRL **115,** 121802 (2015) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2015

Discovery of τ Neutrino Appearance in the CNGS Neutrino Beam with the OPERA Experiment

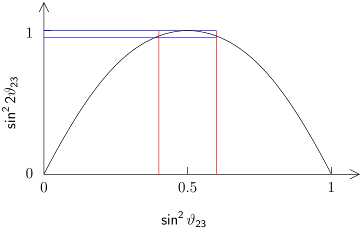
The OPERA experiment was designed to search for $\nu_\mu \to \nu_\tau$ oscillations in appearance mode, i.e., by detecting the τ leptons produced in charged current ν_τ interactions. The experiment took data from 2008 to 2012 in the CERN Neutrinos to Gran Sasso beam. The observation of the $\nu_\mu \to \nu_\tau$ appearance, achieved with four candidate events in a subsample of the data, was previously reported. In this Letter, a fifth ν_τ candidate event, found in an enlarged data sample, is described. Together with a further reduction of the expected background, the candidate events detected so far allow us to assess the discovery of $\nu_\mu \to \nu_\tau$ oscillations in appearance mode with a significance larger than 5σ.

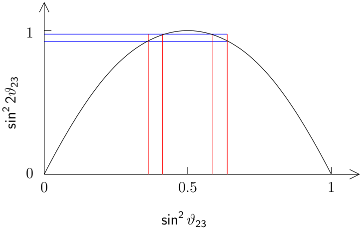
	Expected background					
Channel	Charm	Had. reinterac.	Large μ scat.	Total	Expected signal	Observed
$\tau \to 1h$	0.017 ± 0.003	0.022 ± 0.006		0.04 ± 0.01	0.52 ± 0.10	3
$\tau \to 3h$	0.17 ± 0.03	0.003 ± 0.001		0.17 ± 0.03	0.73 ± 0.14	1
$\tau \to \mu$	0.004 ± 0.001		0.0002 ± 0.0001	0.004 ± 0.001	0.61 ± 0.12	1
$\tau \to e$	0.03 ± 0.01			0.03 ± 0.01	0.78 ± 0.16	0
Total	0.22 ± 0.04	0.02 ± 0.01	0.0002 ± 0.0001	0.25 ± 0.05	2.64 ± 0.53	5

Difficulty of measuring precisely ϑ_{23}

$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\sin^2 2\vartheta_{23} = 4 \sin^2\vartheta_{23} \left(1 - \sin^2\vartheta_{23}\right)$$





The octant degeneracy is resolved by small ϑ_{13} effects:

$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \left[\sin^2 2\vartheta_{23} \cos^2\vartheta_{13} + \sin^4\vartheta_{23} \sin^2 2\vartheta_{13}\right] \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Three-Neutrino Mixing Ingredients

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

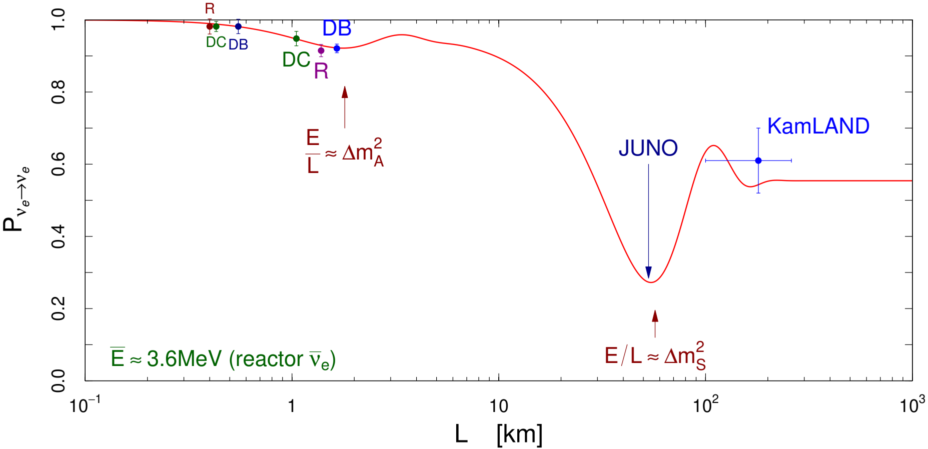
LBL Accelerator $\nu_\mu \to \nu_e$ (T2K, MINOS, NOνA)

LBL Reactor $\bar{\nu}_e$ disappearance (Daya Bay, RENO, Double Chooz)

$\to$

$$\Delta m^2_{\text{A}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{13} \simeq 0.022$$



R
DC
DB
DC
R
DB
E/L ≈ Δm²_A
JUNO
KamLAND
E/L ≈ Δm²_S
Ē ≈ 3.6MeV (reactor ν̄_e)
P_{νe→νe}
1.0
0.8
0.6
0.4
0.2
0.0
10^-1
1
10
10^2
10^3
L [km]

Towards a precise determination of neutrino mixing

well determined: $c_{12}c_{13}$, $s_{12}c_{13}$, s_{13}

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

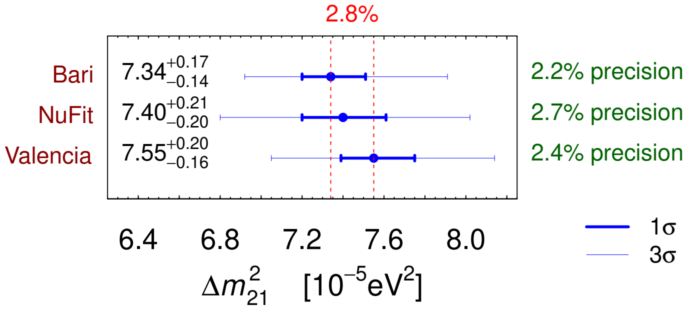
totally unknown: the diagonal phase matrix

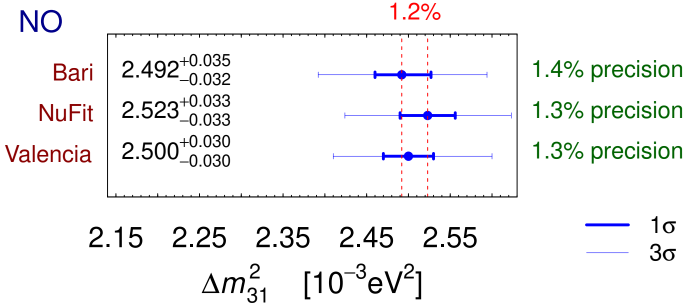
large uncertainty due to ϑ_{23} and δ_{13}: the elements in rows 2 and 3 of columns 1 and 2

medium uncertainty due to ϑ_{23}: $s_{23}c_{13}$, $c_{23}c_{13}$

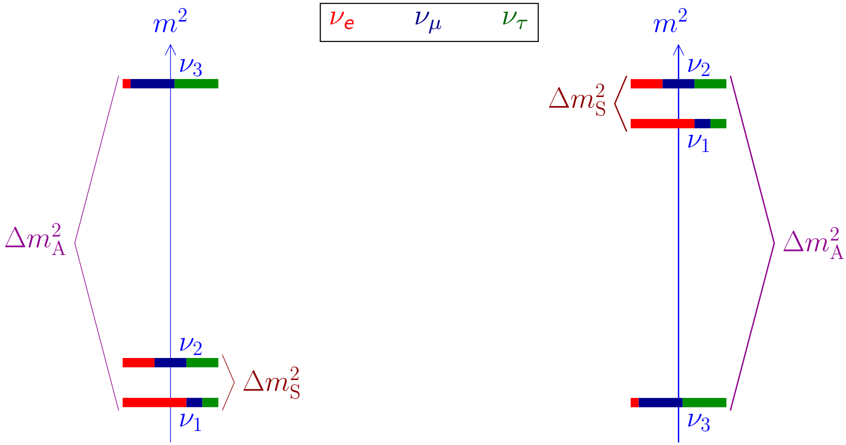
$|U|_{3\sigma} =$

only the mass composition of ν_e is well determined


2.8%
Bari
$7.34^{+0.17}_{-0.14}$
NuFit
$7.40^{+0.21}_{-0.20}$
Valencia
$7.55^{+0.20}_{-0.16}$
2.2% precision
2.7% precision
2.4% precision
6.4
6.8
7.2
7.6
8.0
Δm^2_{21} $[10^{-5} eV^2]$
1σ
3σ


NO
1.2%
Bari
$2.492^{+0.035}_{-0.032}$
NuFit
$2.523^{+0.033}_{-0.033}$
Valencia
$2.500^{+0.030}_{-0.030}$
1.4% precision
1.3% precision
1.3% precision
2.15
2.25
2.35
2.45
2.55
Δm^2_{31} $[10^{-3} eV^2]$
1σ
3σ

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

absolute scale is not determined by neutrino oscillation data

Open Problems

- $\vartheta_{23} \lesseqgtr 45^\circ$?
  - T2K (Japan), NOνA (USA), ...
- CP violation ? $\delta_{13} \approx 3\pi/2$?
  - T2K (Japan), NOνA (USA), DUNE (USA), HyperK (Japan), ...
- Mass Ordering ?
  - JUNO (China), PINGU (Antarctica), ORCA (EU), INO (India), ...
- Absolute Mass Scale ?
  - β Decay, Neutrinoless Double-β Decay, Cosmology, ...
- Dirac or Majorana ?
  - Neutrinoless Double-β Decay, ...
- Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

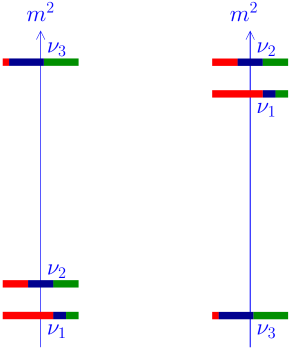
Determination of Mass Ordering

1. Matter Effects: Atmospheric (PINGU, ORCA), Long-Baseline, Supernova Experiments
   - $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \dfrac{\Delta m^2_{31} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{31} > 0$ NO
   - $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\dfrac{\Delta m^2_{31} \cos 2\vartheta_{13}}{2E} \quad \Leftrightarrow \quad \Delta m^2_{31} < 0$ IO
2. Phase Difference: Reactor $\bar{\nu}_e \to \bar{\nu}_e$ (JUNO)

Normal Ordering

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$$

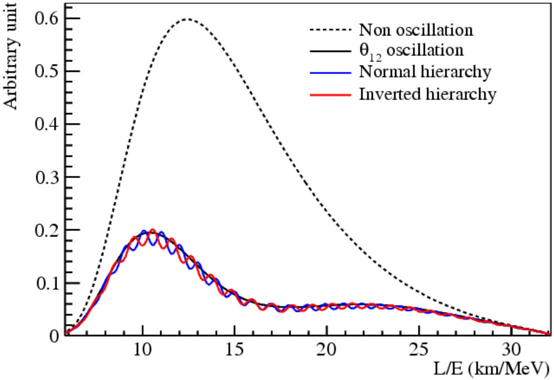
$$|\Delta m^2_{31}| > |\Delta m^2_{32}|$$



Inverted Ordering

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$$

$$|\Delta m^2_{31}| < |\Delta m^2_{32}|$$



Neutrino Physics with JUNO, arXiv:1507.05613

$$
\begin{aligned}
P_{\overset{(-)}{\nu}_e\to\overset{(-)}{\nu}_e} = 1 & - \cos^4\vartheta_{13}\sin^2 2\vartheta_{12}\sin^2\left(\Delta m^2_{21}L/4E\right) \\
& - \cos^2\vartheta_{12}\sin^2 2\vartheta_{13}\sin^2\left(\Delta m^2_{31}L/4E\right) \\
& - \sin^2\vartheta_{12}\sin^2 2\vartheta_{13}\sin^2\left(\Delta m^2_{32}L/4E\right)
\end{aligned}
$$

[Petcov, Piai, PLB 533 (2002) 94; Choubey, Petcov, Piai, PRD 68 (2003) 113006; Learned, Dye, Pakvasa, Svoboda, PRD 78 (2008) 071302; Zhan, Wang, Cao, Wen, PRD 78 (2008) 111103, PRD 79 (2009) 073007]

CP Violation?

$$A^{\mathrm{CP}}_{\alpha\beta} = P_{\nu_\alpha\to\nu_\beta} - P_{\bar{\nu}_\alpha\to\bar{\nu}_\beta}$$

$$= -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

Necessary conditions for observation of CP violation:

- Sensitivity to all mixing angles, including small ϑ_{13}
- Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

LBL $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad A = \frac{2EV}{\Delta m^2_{31}} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad \Delta m^2_{21}/\Delta m^2_{31} \ll 1$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq{}& \sin^2 \underset{\uparrow\,\vartheta_{13}}{2\vartheta_{13}} \sin^2\underset{\uparrow\,\vartheta_{23}\text{ octant}}{\vartheta_{23}} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ &+ \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \underset{\uparrow\,\text{CPV}}{\delta_{13}}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ &+ \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2 \sin^2 2\vartheta_{12} \cos^2\vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

NO: $\Delta m^2_{31} > 0$ IO: $\Delta m^2_{31} < 0$

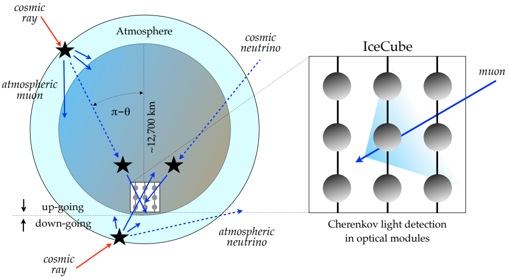
For antineutrinos: $\delta_{13} \to -\delta_{13}$ (CPV) and $A \to -A$ (Matter Effect)

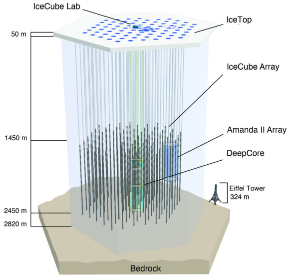
[see: Mezzetto, Schwetz, JPG 37 (2010) 103001]

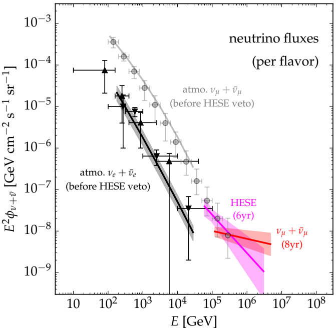
Why it is important to measure accurately the neutrino mixing parameters?

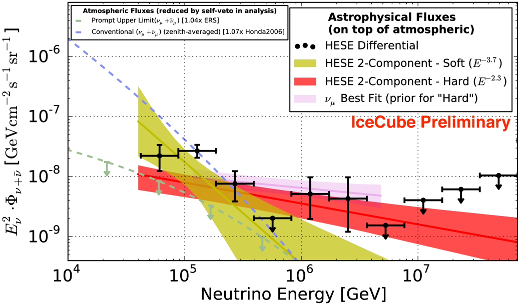
- They are fundamental parameters.
- They lead to selection in huge model space. Examples:
  - Deviation from Tribimaximal Mixing $U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$
  - Violation of μ-τ symmetry $(|U_{\mu k}| = |U_{\tau k}|)$
- They have phenomenological usefulness (e.g. to determine the initial flavor composition of high-energy astrophysical neutrinos).
- CP:
  - CP conservation would need an explanation (a new symmetry?).
  - CP violation may be linked to the CP violation in the sector of heavy neutrinos which generate the matter-antimatter asymmetry in the Universe through leptogenesis (CP-violating decay of heavy neutrinos).

High-Energy Astrophysical Neutrinos

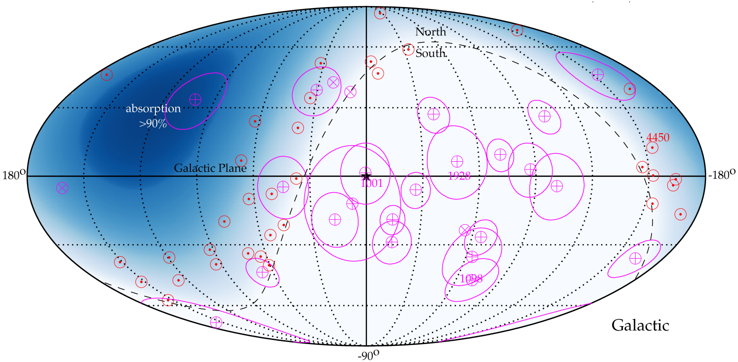








[Ahlers, Halzen, arXiv:1805.11112]



⊙ High-energy ($E \gtrsim 200$ TeV) upgoing tracks: $\text{CC}(\nu_\mu, \bar{\nu}_\mu)$.

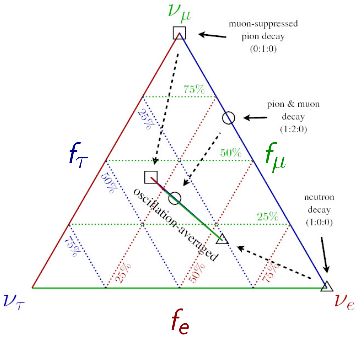
⊗&⊕ HESE (High-Energy Starting Events): high-energy neutrinos ($E \gtrsim 100$ TeV) interacting inside the detector (all-sky directions). ⊗ Tracks: $\text{CC}(\nu_\mu, \bar{\nu}_\mu)$. ⊕ Cascades: $\text{CC}(\nu_e, \bar{\nu}_e, \nu_\tau, \bar{\nu}_\tau) + \text{NC}$. The thin circles indicate the median angular resolution of the cascade events.

- The blue-shaded region indicates the zenith-dependent range where Earth absorption of 100 TeV neutrinos becomes important, reaching more than 90% close to the nadir.
- Dashed line: horizon. Star: Galactic Center.
- The numbers give the energies of the four most energetic events.

Neutrino Flavor Composition

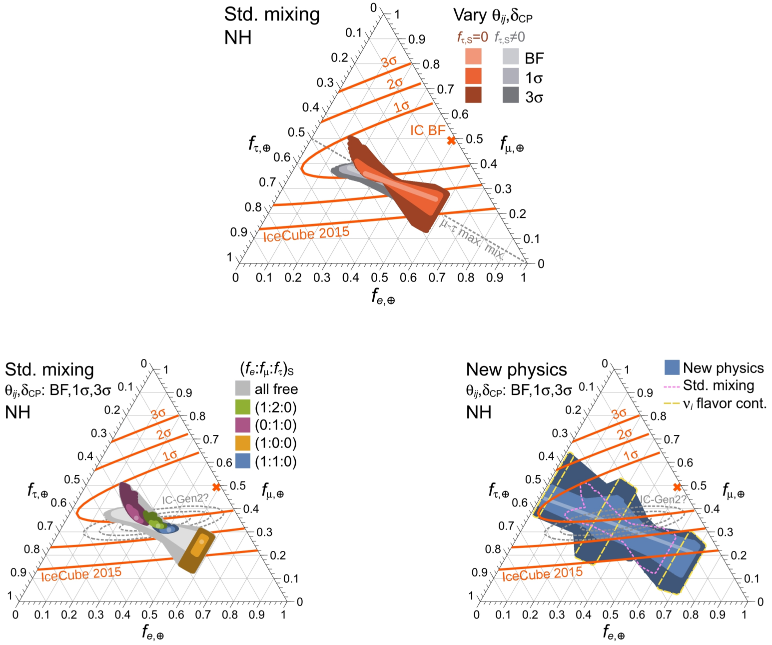
Source: $(f_{e,S} : f_{\mu,S} : f_{\tau,S}) \quad \to \quad$ Earth: $(f_{e,\oplus} : f_{\mu,\oplus} : f_{\tau,\oplus})$

	$f_{e,S}$	$f_{\mu,S}$	$f_{\tau,S}$	$\to$	$f_{e,\oplus}$	$f_{\mu,\oplus}$	$f_{\tau,\oplus}$
Pion and Muon Decay	1/3	2/3	0		1/3	1/3	1/3
Pion only Decay	0	1	0		4/18	7/18	7/18
Charmed Meson Decay	1/2	1/2	0		14/36	11/36	11/36
Neutron Decay	1	0	0		5/9	2/9	2/9



$$f_{\beta,\oplus} = \sum_{\alpha=e,\mu,\tau} f_{\alpha,S} \, \langle P_{\nu_\alpha\to\nu_\beta} \rangle$$

$$\langle P_{\nu_\alpha\to\nu_\beta} \rangle = \sum_{k=1}^{3} |U_{\alpha k}|^2 |U_{\beta k}|^2 \simeq \frac{1}{18} \begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix}$$



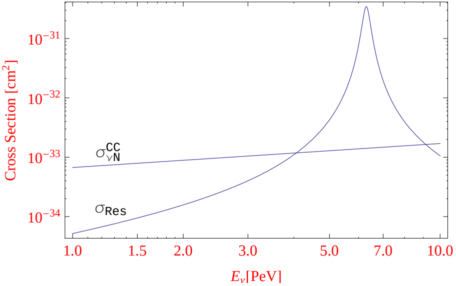
[Bustamante, Beacom, Winter, PRL 115 (2015) 161302 (arXiv:1506.02645)]

The Glashow Resonance

$\bar{\nu}_e + e^- \to W^- \to$ anything at $E_\nu = \frac{m_W^2}{2m_e} = 6.32\,\text{PeV}$ [Glashow, Phys. Rev. 118 (1960) 316]

	$f_{e,S}$	$f_{\mu,S}$	$f_{\tau,S}$	$\to$	$f_{e,\oplus}$	$f_{\mu,\oplus}$	$f_{\tau,\oplus}$	$R_{\bar{\nu}_e}$
Pion and Muon Decay	1/3	2/3	0		1/3	1/3	1/3	0.17
Pion only Decay	0	1	0		4/18	7/18	7/18	0.11
Charmed Meson Decay	1/2	1/2	0		14/36	11/36	11/36	0.19
Neutron Decay	1	0	0		5/9	2/9	2/9	0.56

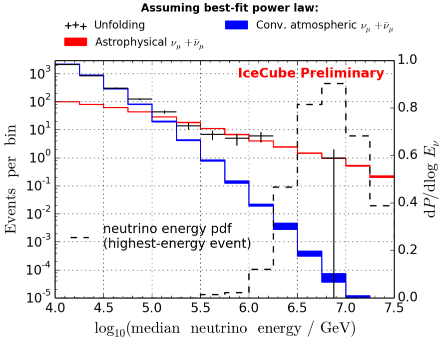
[Barger, Fu, Learned, Marfatia, Pakvasa, Weiler, PRD 90 (2014) 121301 (arXiv:1407.3255)]



- $\Phi_\nu \propto E_\nu^{-\gamma}$
- Standard Fermi shock-acceleration mechanism: $\gamma = 2.0$.
- 2014 IceCube data: events with $E_\nu \lesssim 2\,\text{PeV}$.
- $\gamma \geq 2.3$ at 90% CL.

[Anchordoqui et al, PRD 89 (2014) 083003]

- PeV Energy Partially-contained Events (PEPE) search, with special focus on the Glashow resonance. [IceCube, arXiv:1710.01191]

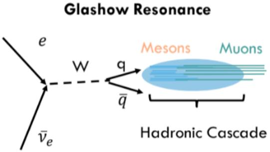


[Ahlers, Halzen, arXiv:1805.11112]

- For the highest energy event the median energy of the parent neutrino is about 7 PeV.
- The energy lost by the muon inside the instrumented detector volume is $2.6 \pm 0.3\,\text{PeV}$.
- The calculation of the probability density function takes into account the additional tracks from charged current interactions of $\nu_\tau + \bar{\nu}_\tau$ and resonant interactions of $\bar{\nu}_e$ with electrons (Glashow resonance).
- Assumption: a democratic composition of neutrino and antineutrino flavors.
- The cosmic neutrino flux is well described by a power law with a spectral index $\gamma = 2.19 \pm 0.10$ and a normalization at 100 TeV neutrino energy of

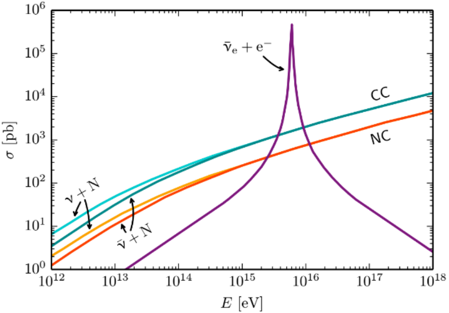
$$(1.01^{+0.26}_{-0.23}) \times 10^{-18}\,\text{GeV}^{-1}\text{cm}^{-2}\text{sr}^{-1}$$

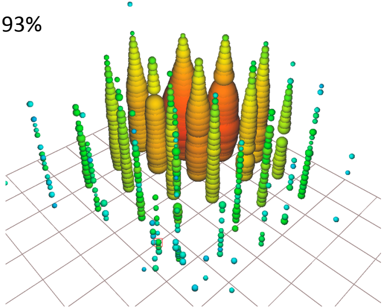
A 5.9 PeV event in IceCube



Resonance: E_ν = 6.3 PeV
Typical visible energy is 93%

Work in progress





Event identified in a partially-contained PeV search (PEPE)
Deposited energy: 5.9±0.18 PeV (stat only)
ICRC 2017 arXiv:1710.01191

Potential hadronic nature of this event under study

I. Taboada | Georgia Inst. of Tech. 20
[Taboada, Neutrino 2018, 6 June 2018]

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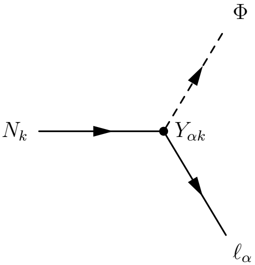
Leptogenesis

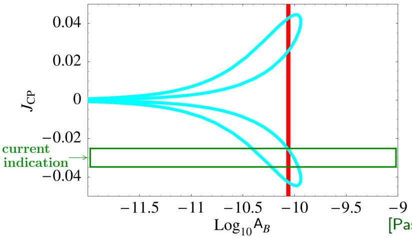
$$\mathcal{L}_I \sim \overline{L_L}\,\Phi^\dagger\,Y\,N_R$$



$$A_L \sim \frac{\sum_{k,\alpha}\left[\Gamma(N_k \to \Phi\ell_\alpha) - \Gamma(N_k \to \bar{\Phi}\bar{\ell}_\alpha)\right]}{\sum_{k,\alpha}\left[\Gamma(N_k \to \Phi\ell_\alpha) + \Gamma(N_k \to \bar{\Phi}\bar{\ell}_\alpha)\right]}$$

Seesaw $\Longrightarrow$ $Y \sim \frac{1}{v}\,\underbrace{M_R^{1/2}\,R}_{\text{inaccessible}}\;\underbrace{m_\nu^{1/2}\,U_{3\times3}}_{\text{measurable}}$ $\qquad (RR^T = \mathbb{1})$

CP-violating $U_{3\times3}$ $\Longrightarrow$ plausible CP-violating Y



$M_{R1} = 5 \times 10^{11}$ GeV

$M_{R1} \ll M_{R2} \ll M_{R3}$

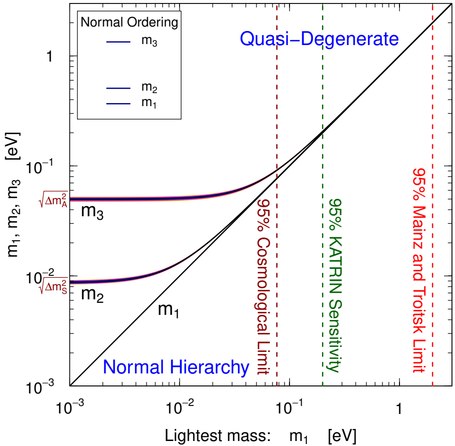
$R_{12} = 0.86$

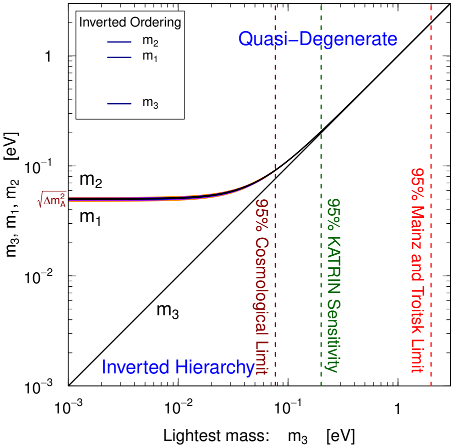
$R_{13} = 0.5$

[Pascoli, Petcov, Riotto, PRD 75 (2007) 083511, arXiv:hep-ph/0609125]

Absolute Scale of Neutrino Masses

Mass Hierarchy or Degeneracy?





$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}$ eV

95% Cosmological Limit: Planck TT + lowP + BAO [arXiv:1502.01589]

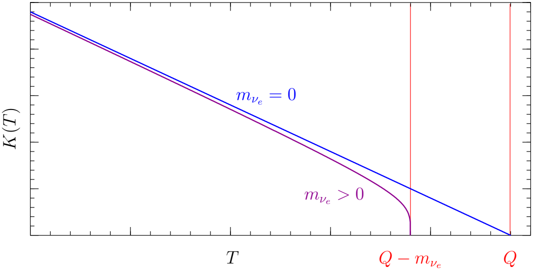
Tritium Beta-Decay

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

Kurie function: $$K(T) = \left[(Q - T) \sqrt{(Q-T)^2 - m_{\nu_e}^2} \right]^{1/2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$



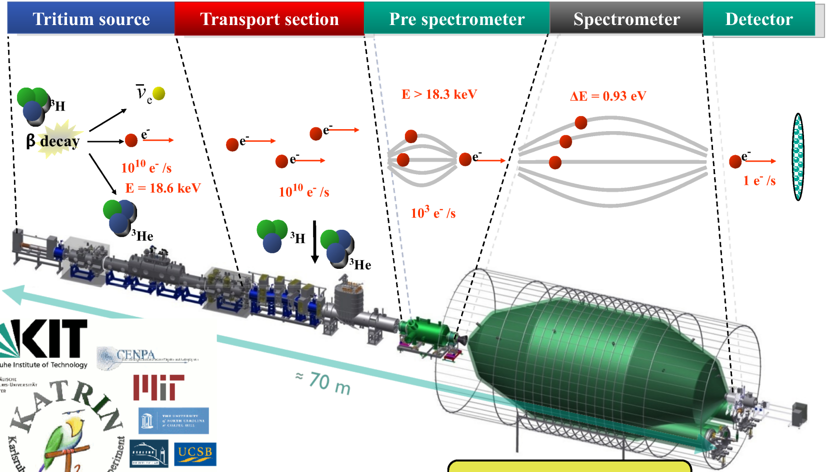
$m_{\nu_e} < 1.1\,\mathrm{eV}$ (90% C.L.)

KATRIN

[PRL 123 (2019) 221802, arXiv:1909.06048]

Expected final sensitivity:
$m_{\nu_e} \approx 0.2\,\mathrm{eV}$

The Karlsruhe Tritium Neutrino Experiment
KATRIN - overview



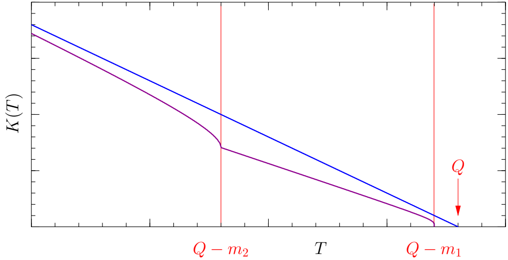
new

Sensitivity on $m(\nu_e)$:

2 eV $\rightarrow$ 200 meV

Transport of the KATRIN spectrometer from the Rhine river to the Karlsruhe Institute of Technology.
(Novembre 2006)

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses ($m_k \ll Q - T$)

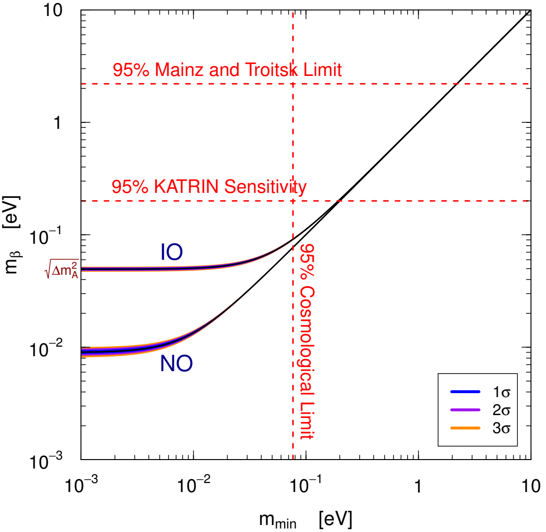
effective mass: $$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Predictions of 3ν-Mixing Paradigm

$$m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$



- Quasi-Degenerate:
$$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- Inverted Hierarchy:
$$m_\beta^2 \simeq (1 - s_{13}^2) \Delta m_A^2 \simeq \Delta m_A^2$$

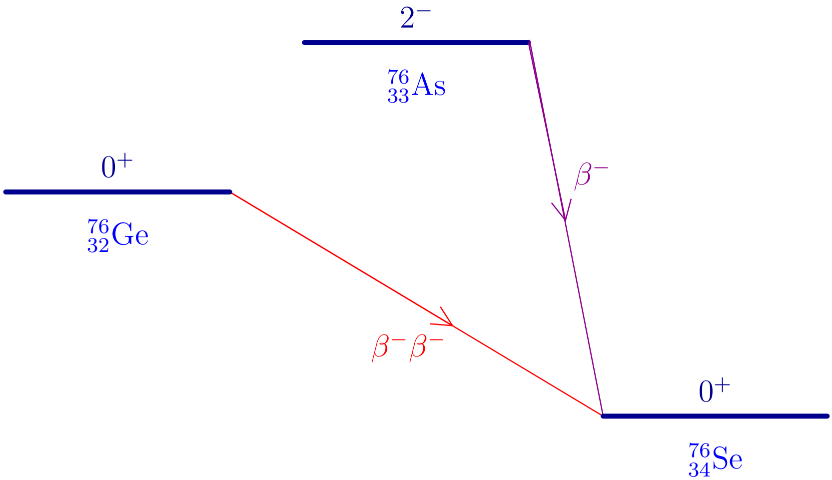
- Normal Hierarchy:
$$m_\beta^2 \simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2$$
$$\simeq 2 \times 10^{-5} + 6 \times 10^{-5} \, \text{eV}^2$$

- If $m_\beta \lesssim 4 \times 10^{-2}$ eV

$\Downarrow$

Normal Spectrum

Neutrinoless Double-Beta Decay



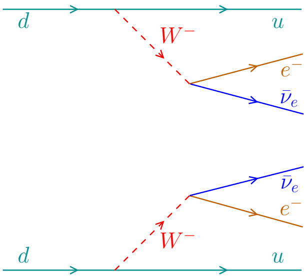
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

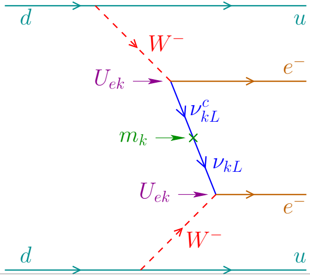


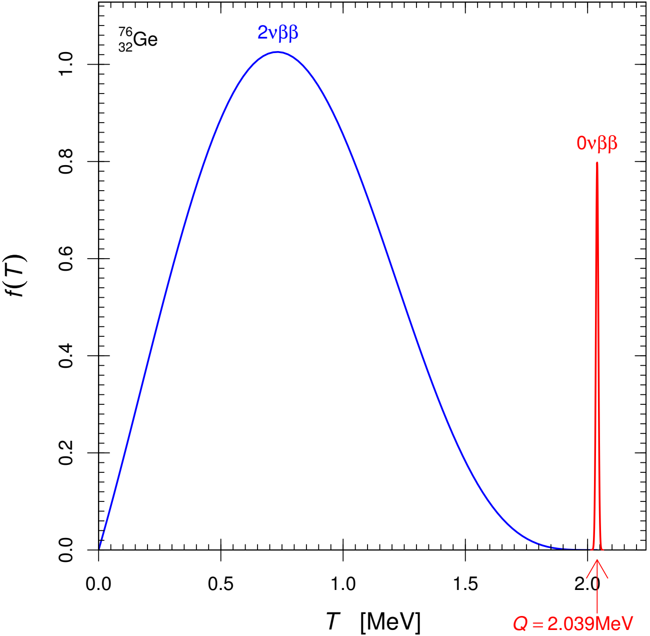
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left| \sum_k U_{ek}^2\, m_k \right|$





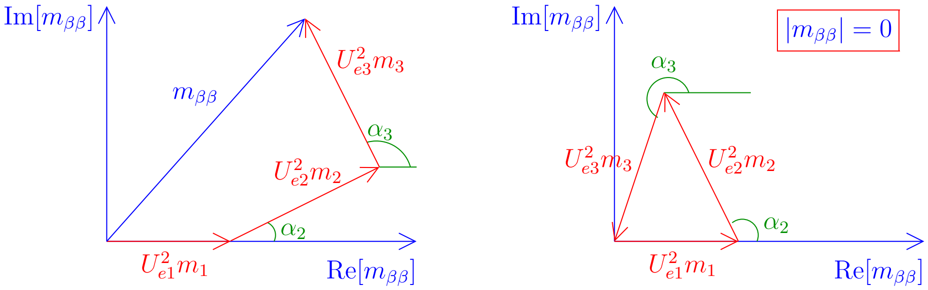
$^{76}_{32}$Ge
2νββ
0νββ
f(T)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
T [MeV]
Q = 2.039MeV

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2\, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

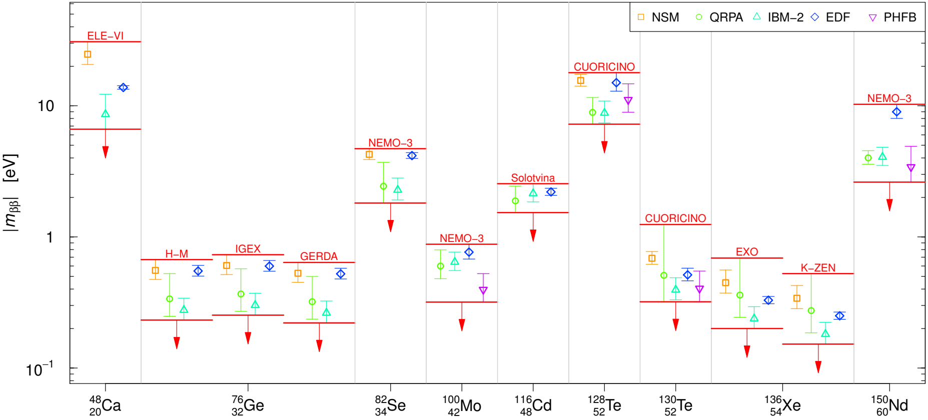
$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$



90% C.L. Experimental Bounds

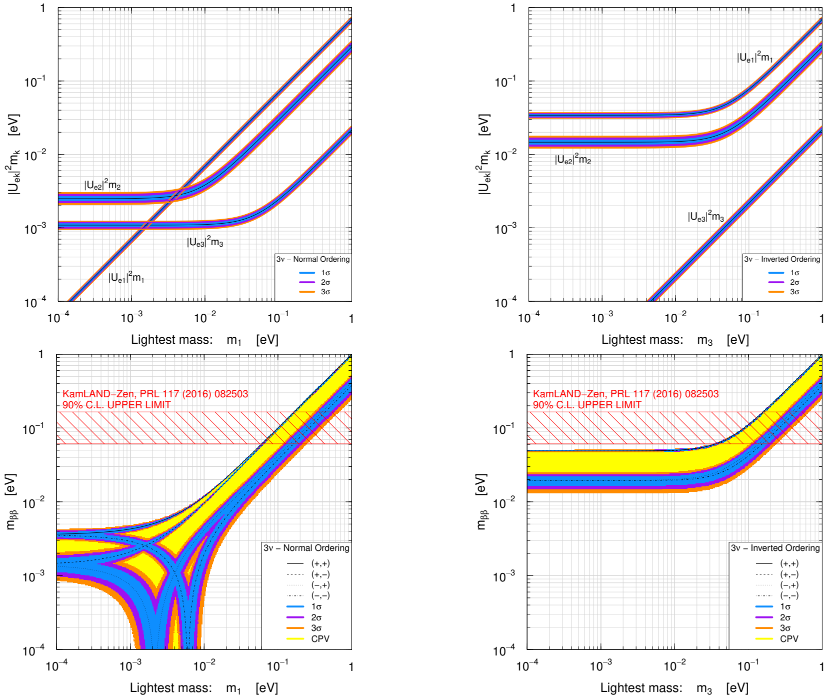
$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\mathrm{Ca} \to {}^{48}_{22}\mathrm{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\mathrm{Ge} \to {}^{76}_{34}\mathrm{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	Majorana	$> 4.8 \times 10^{25}$	$< 0.20 - 0.43$
	GERDA	$> 8.0 \times 10^{25}$	$< 0.12 - 0.26$
${}^{82}_{34}\mathrm{Se} \to {}^{82}_{36}\mathrm{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\mathrm{Mo} \to {}^{100}_{44}\mathrm{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\mathrm{Cd} \to {}^{116}_{50}\mathrm{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\mathrm{Te} \to {}^{128}_{54}\mathrm{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\mathrm{Te} \to {}^{130}_{54}\mathrm{Xe}$	CUORE	$> 1.5 \times 10^{25}$	$< 0.11 - 0.52$
${}^{136}_{54}\mathrm{Xe} \to {}^{136}_{56}\mathrm{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.17 - 0.49$
	KamLAND-Zen	$> 1.1 \times 10^{26}$	$< 0.06 - 0.16$
${}^{150}_{60}\mathrm{Nd} \to {}^{150}_{62}\mathrm{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



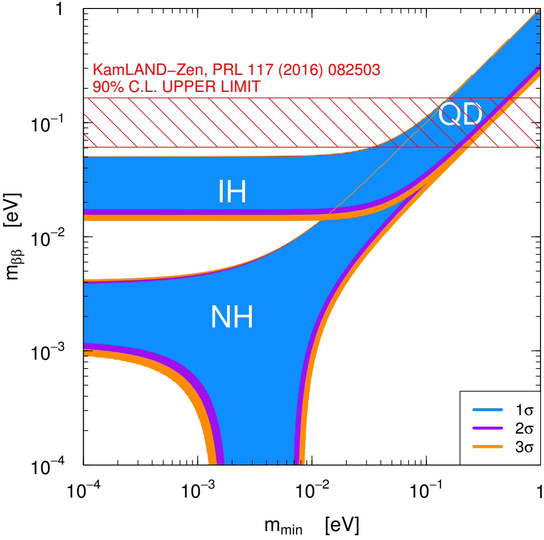
[Bilenky, CG, IJMPA 30 (2015) 0001]

Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$


$|U_{ek}|^2 m_k$ [eV]
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
3ν – Normal Ordering
1σ
2σ
3σ
Lightest mass: m_1 [eV]
3ν – Inverted Ordering
Lightest mass: m_3 [eV]
$m_{\beta\beta}$ [eV]
KamLAND–Zen, PRL 117 (2016) 082503
90% C.L. UPPER LIMIT
(+,+)
(+,−)
(−,+)
(−,−)
CPV

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\mathrm{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\mathrm{S}}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\mathrm{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3} \, \mathrm{eV}$$

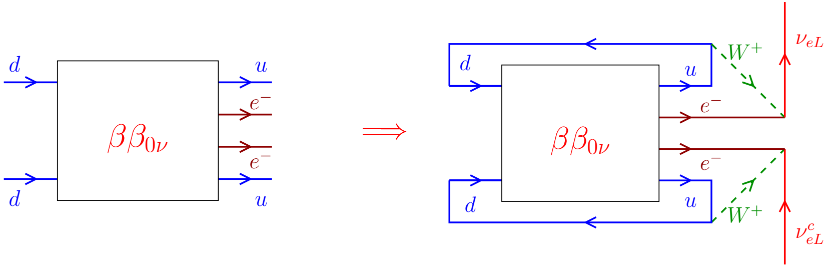
- If $|m_{\beta\beta}| \lesssim 10^{-2}$ eV

$\Downarrow$

Normal Spectrum

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

- $|m_{\beta\beta}|$ can vanish because of unfortunate cancellations among the ν_1, ν_2, ν_3 contributions or because neutrinos are Dirac particles.
- However, $\beta\beta_{0\nu}$ decay can be generated by another mechanism beyond the Standard Model.
- In this case, a Majorana mass for ν_e is generated by radiative corrections:



[Schechter, Valle, PRD 25 (1982) 2951; Takasugi, PLB 149 (1984) 372]

- Majorana Mass Term: $\mathcal{L}^{\text{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right)$
- Very small four-loop diagram contribution: $m_{ee} \sim 10^{-24}\,\text{eV}$

[Duerr, Lindner, Merle, JHEP 06 (2011) 091 (arXiv:1105.0901)]

- In any case finding $\beta\beta_{0\nu}$ decay is important for
  - Finding total Lepton number violation $(\Delta L = \pm 2)$.
  - Establishing the Majorana (or pseudo-Dirac) nature of neutrinos.
- On the other hand, even if $\beta\beta_{0\nu}$ decay is not found, it is not possible to prove experimentally that neutrinos are Dirac particles, because
  - A Dirac neutrino is equivalent to 2 Majorana neutrinos with the same mass.
  - It is impossible to prove experimentally that the mass splitting is exactly zero.

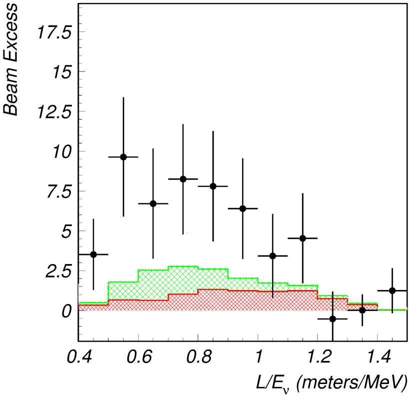
Short-Baseline Neutrino Oscillation Anomalies

- In the standard framework of three-neutrino mixing there are two independent Δm^2's:
  - $\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$
  - $\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$
- Atmospheric and solar neutrino oscillations are detectable at the distances
  - $L^{\text{osc}}_{\text{ATM}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{ATM}}} \approx 1\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
  - $L^{\text{osc}}_{\text{SOL}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{SOL}}} \approx 50\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
- The atmospheric and solar neutrino oscillations cannot explain flavor neutrino transitions at shorter distances.

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad$ $20\,\text{MeV} \leq E \leq 52.8\,\text{MeV}$



$\Delta m^2_{\text{SBL}} \gtrsim 0.1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$

- Well-known and pure source of $\bar{\nu}_\mu$

$$p + \text{target} \to \pi^+ \xrightarrow{\text{at rest}} \mu^+ + \nu_\mu$$

800 MeV

$$\mu^+ \xrightarrow[\text{at rest}]{} e^+ + \nu_e + \bar{\nu}_\mu$$

$L \simeq 30\,\text{m}$

$$\bar{\nu}_e + p \to n + e^+$$

Well-known detection process of $\bar{\nu}_e$

- $\approx 3.8\sigma$ excess
- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

[PRD 65 (2002) 112001]

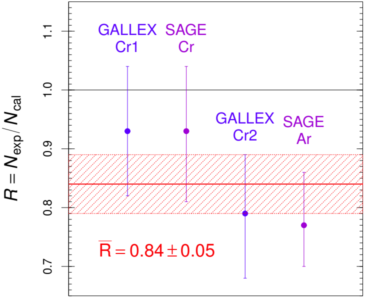
Gallium Anomaly

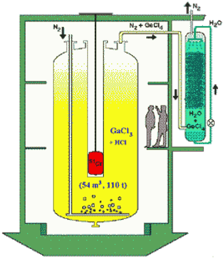
Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

$E \simeq 0.75\,\mathrm{MeV}$ $\quad E \simeq 0.81\,\mathrm{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$





$\approx 2.9\sigma$ deficit

$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$ $\quad \langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$

$\Delta m^2_{\mathrm{SBL}} \gtrsim 1\,\mathrm{eV}^2 \gg \Delta m^2_{\mathrm{ATM}}$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807; Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344, MPLA 22 (2007) 2499, PRD 78 (2008) 073009, PRC 83 (2011) 065504]

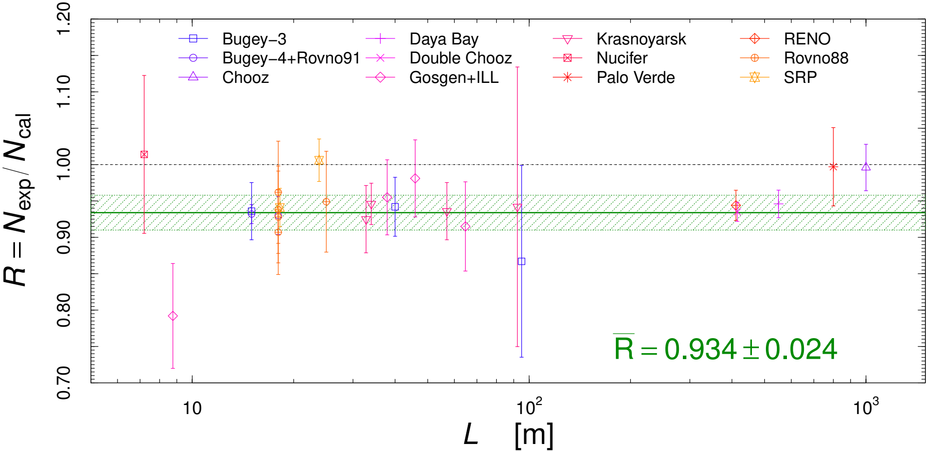
- ${}^{3}\mathrm{He} + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + {}^{3}\mathrm{H}$ cross section measurement [Frekers et al., PLB 706 (2011) 134]

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

New reactor $\bar{\nu}_e$ fluxes: Huber-Mueller (HM)

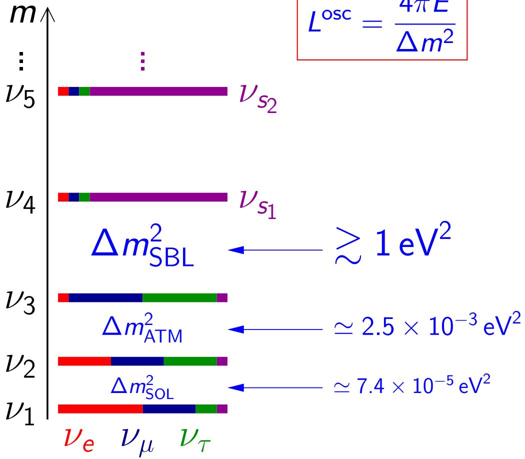
[Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]

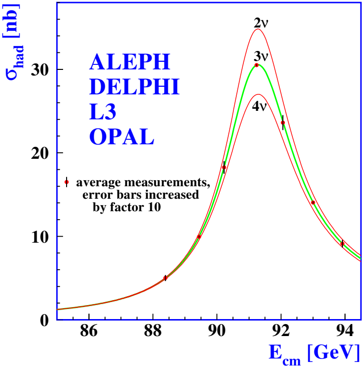


$\approx 2.8\sigma$ deficit

Beyond Three-Neutrino Mixing: Sterile Neutrinos

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$





$$N^{\text{LEP}}_{\nu_{\text{active}}} = 2.9840 \pm 0.0082$$

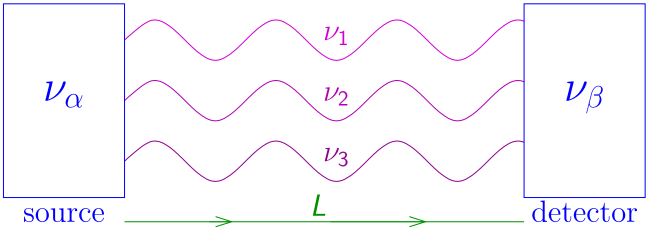
Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

Short-Baseline Neutrino Oscillations

Three-Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle$$



$$|\nu_{\text{detector}}\rangle \simeq U_{\alpha 1}\,e^{-iEL}\,|\nu_1\rangle + U_{\alpha 2}\,e^{-iEL}\,|\nu_2\rangle + U_{\alpha 3}\,e^{-iEL}\,|\nu_3\rangle = e^{-iEL}|\nu_\alpha\rangle$$

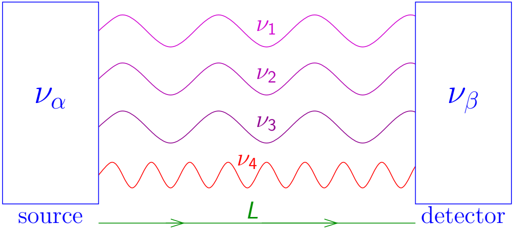
$$P_{\nu_\alpha\to\nu_\beta}(L) = |\langle\nu_\beta|\nu_{\text{detector}}\rangle|^2 \simeq |e^{-iEL}\langle\nu_\beta|\nu_\alpha\rangle|^2 = \delta_{\alpha\beta}$$

No Observable Short-Baseline Neutrino Oscillations!

Short-Baseline Neutrino Oscillations

3+1 Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle + U_{\alpha 4}\,|\nu_4\rangle$$



$$|\nu_{\text{detector}}\rangle \simeq e^{-iEL}\left(U_{\alpha 1}\,|\nu_1\rangle + U_{\alpha 2}\,|\nu_2\rangle + U_{\alpha 3}\,|\nu_3\rangle\right) + U_{\alpha 4}\,e^{-iE_4 L}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \neq \delta_{\alpha\beta}$$

Observable Short-Baseline Neutrino Oscillations!

The oscillation probabilities depend on U and

$$\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$$

- Some authors that probably did not think about the quantum mechanics of neutrino oscillations present $\nu_\mu \to \nu_e$ short-baseline transitions due to sterile neutrinos as

$$\nu_\mu \to \nu_s \to \nu_e$$

- This is wrong!

THERE IS NO INTERMEDIATE ν_s !

Two possible interpretations of $\nu_\mu \to \nu_s \to \nu_e$:

1. There is a transition from ν_μ to ν_s, and then to ν_e: wrong!
   Because the intermediate determination of the neutrino flavor interrupts the quantum evolution.
   Moreover, ν_s is not detectable!

2. There is an intermediate linear combination of massive neutrinos that corresponds to $|\nu_s\rangle$: wrong!
   This is possible only with the mixing $(|a|^2 + |b|^2 + |c|^2 = 1)$

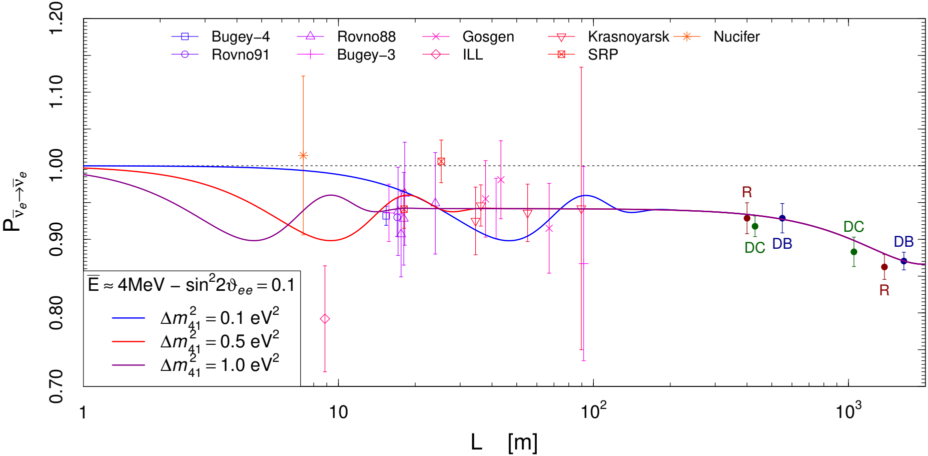
$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \\ |\nu_s\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cdots & \cdots & \cdots & 0 \\ a & b & c & 1 \\ \cdots & \cdots & \cdots & 0 \\ -a & -b & -c & 1 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \end{pmatrix}$$

$$|\nu(L)\rangle = \frac{e^{-iEL}}{\sqrt{2}} \left[a\,|\nu_1\rangle + b\,|\nu_2\rangle + c\,|\nu_3\rangle + e^{-i(E_4-E)L}\,|\nu_4\rangle \right]$$

$$|\nu(L)\rangle = |\nu_\mu\rangle \quad \text{for} \quad L = 0 \quad \text{and} \quad |\nu(L)\rangle \propto |\nu_s\rangle \quad \text{for} \quad e^{-i(E_4-E)L} = -1$$

but in this case there are no SBL $\nu_\mu \to \nu_e$ transitions!

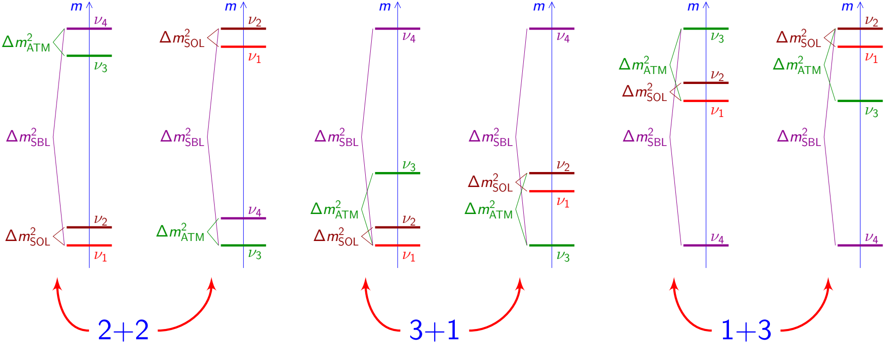
Short-Baseline Reactor Neutrino Oscillations



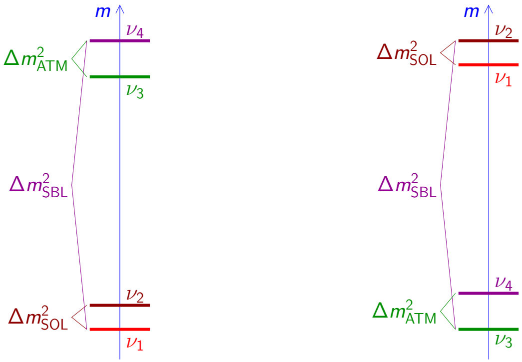
$$\Delta m^2_{\text{SBL}} \gtrsim 0.5\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$$

- SBL oscillations are averaged at the Daya Bay, RENO, and Double Chooz near detectors $\Longrightarrow$ no spectral distortion

Four-Neutrino Schemes: 2+2, 3+1 and 1+3



2+2 Four-Neutrino Schemes

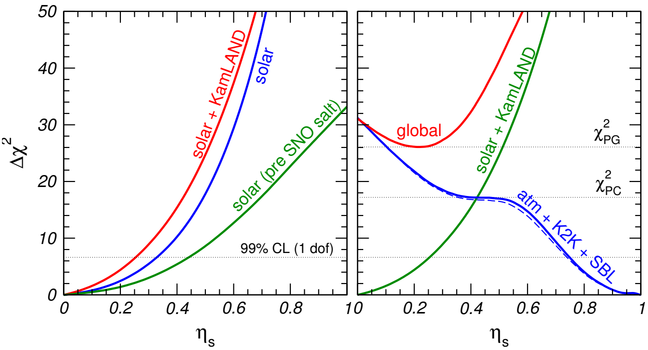


- After LSND (1995) 2+2 was preferred to 3+1, because of the 3+1 appearance-disappearance tension

  [Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

- This is not a perturbation of 3-ν Mixing $\Longrightarrow$ Large active–sterile oscillations for solar or atmospheric neutrinos!

2+2 Schemes are Strongly Disfavored



Solar: Matter Effects + SNO NC

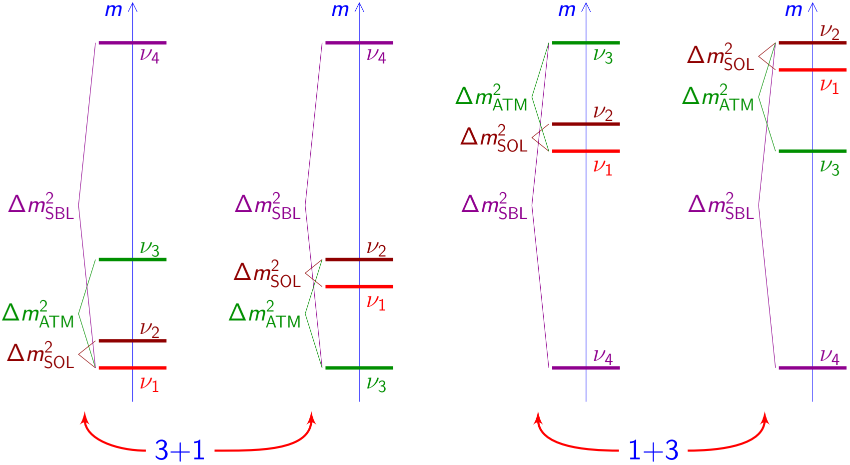
Atmospheric: Matter Effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 = 1 - |U_{s3}|^2 + |U_{s4}|^2$$

$$\text{99\% CL:} \quad \begin{cases} \eta_s < 0.25 & \text{(Solar + KamLAND)} \\ \eta_s > 0.75 & \text{(Atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122]

3+1 and 1+3 Four-Neutrino Schemes



- Perturbation of 3-ν Mixing: $|U_{e4}|^2, |U_{\mu 4}|^2, |U_{\tau 4}|^2 \ll 1 \quad |U_{s4}|^2 \simeq 1$
- 1+3 schemes are disfavored by cosmology (ΛCDM):

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV}$$

[Planck, Astron. Astrophys. 594 (2016) A13 (arXiv:1502.01589)]

Effective 3+1 SBL Oscillation Probabilities

$$|\nu_\alpha\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} |\nu_k\rangle \quad \xrightarrow{t} \quad |\nu_\alpha(t)\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} e^{-iE_k t} |\nu_k\rangle$$

$$A_{\nu_\alpha \to \nu_\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \qquad (\langle \nu_\beta | \nu_k \rangle = U_{\beta k})$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 \\ &= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2 \\ &= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2 \end{aligned}$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^* U_{\beta k} e^{-i(E_k - E_1)t} \right|^2$$

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} \quad \Longrightarrow \quad E_k - E_1 \simeq \frac{\Delta m_{k1}^2}{2p}$$

$$E = p \qquad\qquad t \simeq L$$

$$P_{\nu_\alpha \to \nu_\beta} \simeq \left| \sum_{k=1}^{4} U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} \exp\left(-i \frac{\Delta m_{21}^2 L}{2E} \right) + U_{\alpha 3}^* U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m_{41}^2 L}{2E} \right) \right|^2$$

SBL $\Longrightarrow$ $\frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1$

$$P_{\nu_\alpha \to \nu_\beta}^{\text{SBL}} \simeq \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m_{41}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^* U_{\beta 4}$$

$$P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq \left|\delta_{\alpha\beta} - U^*_{\alpha4} U_{\beta4}\left[1 - \exp\left(-i\frac{\Delta m^2_{41} L}{2E}\right)\right]\right|^2$$

$$= \delta_{\alpha\beta} + |U_{\alpha4}|^2 |U_{\beta4}|^2 \left(2 - 2\cos\frac{\Delta m^2_{41} L}{2E}\right)$$

$$- 2\delta_{\alpha\beta}|U_{\alpha4}|^2 \left(1 - \cos\frac{\Delta m^2_{41} L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 2|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right) \left(1 - \cos\frac{\Delta m^2_{41} L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 4|U_{\alpha4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta4}|^2\right) \sin^2\frac{\Delta m^2_{41} L}{4E}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq 4|U_{\alpha4}|^2 |U_{\beta4}|^2 \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\alpha} \simeq 1 - 4|U_{\alpha4}|^2 \left(1 - |U_{\alpha4}|^2\right) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Appearance $(\alpha \neq \beta)$ — Disappearance

$$P^{\rm SBL}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

$$P^{\rm SBL}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- $\Delta m^2_{\rm SBL} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$
- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to $\Delta m^2_{\rm ATM}$ [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122] and solar exp. sensitive to $\Delta m^2_{\rm SOL}$ [Long, Li, CG, PRD 87, 113004 (2013) 113004]

Common Parameterization of 4 × 4 Mixing Matrix

$$U = \left[W^{34} R^{24} W^{14} R^{23} W^{13} R^{12}\right] \text{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}, e^{i\lambda_{41}}\right)$$

$$= \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ \cdots & \cdots & \cdots & c_{14}s_{24} \\ \cdots & \cdots & \cdots & c_{14}c_{24}s_{34}e^{-i\delta_{34}} \\ \cdots & \cdots & \cdots & c_{14}c_{24}c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 & 0 \\ 0 & 0 & e^{i\lambda_{31}} & 0 \\ 0 & 0 & 0 & e^{i\lambda_{41}} \end{pmatrix}$$

$$|U_{e4}|^2 = \sin^2\vartheta_{14} \quad \Rightarrow \quad \sin^2 2\vartheta_{ee} = 4|U_{e4}|^2\left(1 - |U_{e4}|^2\right) = \sin^2 2\vartheta_{14}$$

$$|U_{\mu 4}|^2 = \cos^2\vartheta_{14}\sin^2\vartheta_{24} \simeq \sin^2\vartheta_{24} \Rightarrow \sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2\left(1 - |U_{\mu 4}|^2\right) \simeq \sin^2 2\vartheta_{24}$$

3+1: Appearance vs Disappearance

- SBL Oscillation parameters: Δm^2_{41} $|U_{e4}|^2$ $|U_{\mu 4}|^2$ $(|U_{\tau 4}|^2)$
- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2 \left(1 - |U_{\mu 4}|^2\right) \simeq 4|U_{\mu 4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

quadratically suppressed for small $|U_{e4}|^2$ and $|U_{\mu 4}|^2$

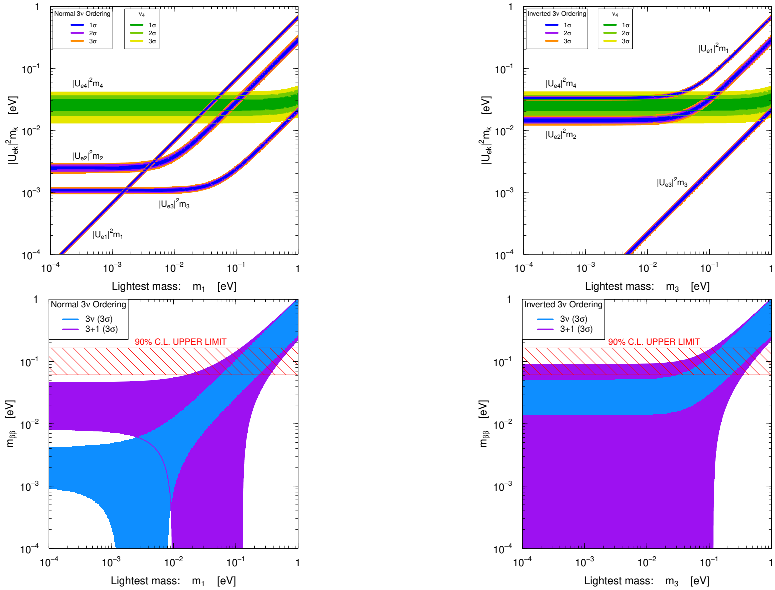
$\Downarrow$

Appearance-Disappearance Tension

[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

Neutrinoless Double-Beta Decay

$$m_{\beta\beta} = \left| |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3 + |U_{e4}|^2 \, e^{i\alpha_{41}} \, m_4 \right|$$



Conclusions

- Mainstream 3ν-mixing research: precise measurements of mass ordering, masses, mixing angles and CP violating phases with neutrino oscillations, β decay, $\beta\beta_{0\nu}$ decay.

- Neutrinos provide a Window to the New Physics beyond the Standard Model through:
  - Small (Majorana) Masses.
  - Sterile Neutrinos.
  - Non-Standard Interactions. [see Ohlsson, RPP 76 (2013) 044201, arXiv:1209.2710]
  - Electromagnetic Interactions. [see CG, Studenikin, RMP 87 (2015) 531, arXiv:1403.6344]
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