Neutrino-4 anomaly: oscillations or fluctuations?

Carlo Giunti
INFN, Torino, Italy

Center for Neutrino Physics at Virginia Tech University

Virtual Journal Club

3 February 2021

Discussion of
C. Giunti, Y.F. Li, C.A. Ternes, Y.Y. Zhang, arXiv:2101.06785

Mainstream Three Neutrino Mixing Paradigm

- Supported by robust, abundant, and consistent solar, atmospheric and long-baseline (accelerator and reactor) neutrino oscillation data.
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- Neutrino Mixing: a Flavor Neutrino is a superposition of Massive Neutrinos

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{e2}^* & U_{e3}^* \\ U_{\mu1}^* & U_{\mu2}^* & U_{\mu3}^* \\ U_{\tau1}^* & U_{\tau2}^* & U_{\tau3}^* \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

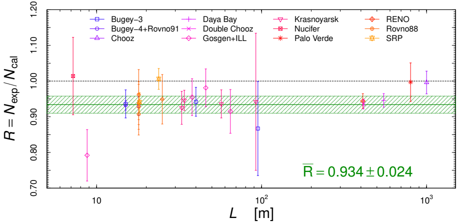
- U is the 3×3 unitary Neutrino Mixing Matrix
- $$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \qquad (\alpha, \beta = e, \mu, \tau)$$
- The oscillation probabilities depend on

U (osc. amplitude) and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ (osc. phase)

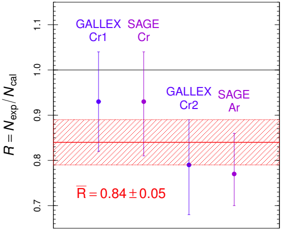
- In the mainstream 3ν mixing paradigm there are two independent Δm^2's:
  - $\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$ Solar Mass Splitting
  - $\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$ Atmospheric Mass Splitting
- The solar and atmospheric mass splittings generate oscillations that are detectable at the distances
  - $L^{\text{osc}}_{\text{SOL}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{SOL}}} \approx 50\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
  - $L^{\text{osc}}_{\text{ATM}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{ATM}}} \approx 1\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
- The solar and atmospheric mass splittings cannot explain flavor neutrino transitions at shorter distances.

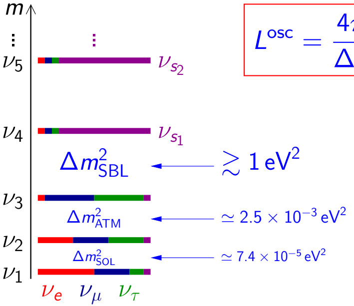
Short-Baseline Neutrino Oscillation Anomalies

Reactor Anomaly: $\bar{\nu}_e \to \bar{\nu}_x$ $(\sim 3\sigma)$



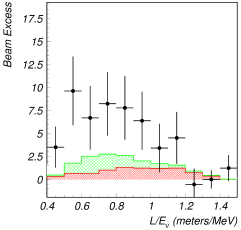
Gallium Anomaly: $\nu_e \to \nu_x$ $(\sim 3\sigma)$





$$\boxed{L^{\rm osc} = \frac{4\pi E}{\Delta m^2}}$$

LSND Anomaly: $\bar{\nu}_\mu \to \bar{\nu}_e$ $(\sim 4\sigma)$



Minimal perturbation of 3ν mixing: effective 3+1 with $|U_{e4}|, |U_{\mu4}|, |U_{\tau4}| \ll 1$

Effective 3+1 SBL Oscillation Probabilities

Appearance ($\alpha \neq \beta$)

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

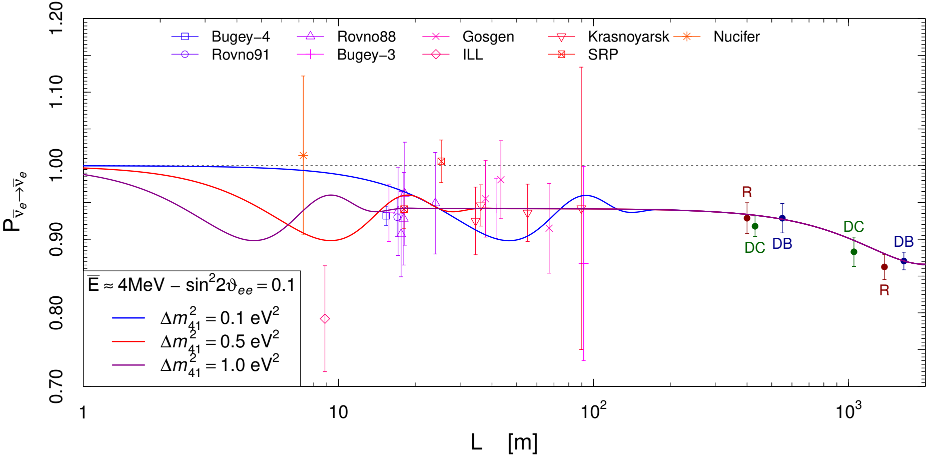
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

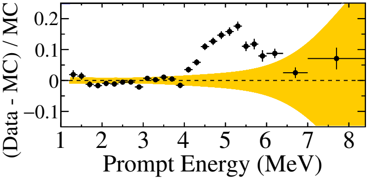
- $\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$
- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, arXiv:1412.1479, arXiv:1507.03986, arXiv:1605.09376; Palazzo et al, arXiv:1412.7524, arXiv:1509.03148; Kayser et al, arXiv:1508.06275, arXiv:1607.02152] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, Giunti, arXiv:1304.2207]

Short-Baseline Reactor Neutrino Oscillations

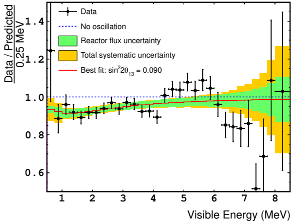


- $\Delta m^2_{\mathrm{SBL}} \gtrsim 0.5\,\mathrm{eV}^2 \gg \Delta m^2_{\mathrm{ATM}}$
- SBL oscillations are averaged at the Daya Bay, RENO, and Double Chooz near detectors $\Longrightarrow$ no spectral distortion
- The reactor antineutrino anomaly is model dependent (depends on the theoretical reactor neutrino flux calculation; is it reliable?).

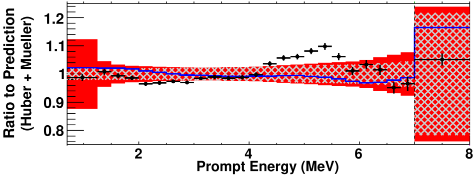
Reactor Antineutrino 5 MeV Bump



[RENO, arXiv:1511.05849]



[Double Chooz, arXiv:1406.7763]



[Daya Bay, arXiv:1508.04233]

- Cannot be explained by neutrino oscillations (SBL oscillations are averaged in RENO, DC, DB).

- If it is due to a theoretical miscalculation of the spectrum, it can have opposite effects on the anomaly:

  [see: Berryman, Huber, arXiv:1909.09267]

  - If it is a 4-6 MeV excess it increases the anomaly: new HKSS flux calculation

    [Hayen, Kostensalo, Severijns, Suhonen, arXiv:1908.08302]

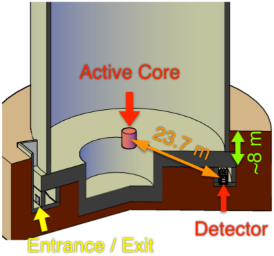
  - If it is a 1-4 MeV suppression it decreases the anomaly: new EF flux calculation

    [Estienne, Fallot, et al, arXiv:1904.09358]

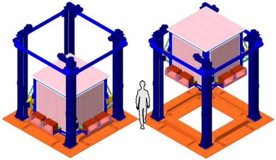
Model Indep. Measurements of Reactor ν Osc.

Ratios of spectra at different distances

NEOS

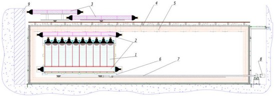


DANSS

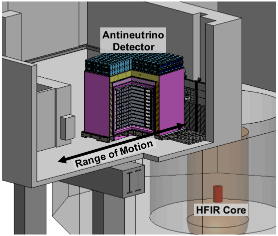


DANSS on a lifting platform

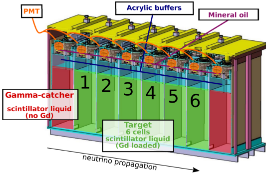
Neutrino-4



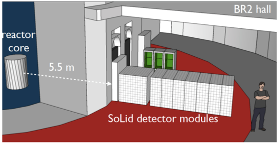
PROSPECT

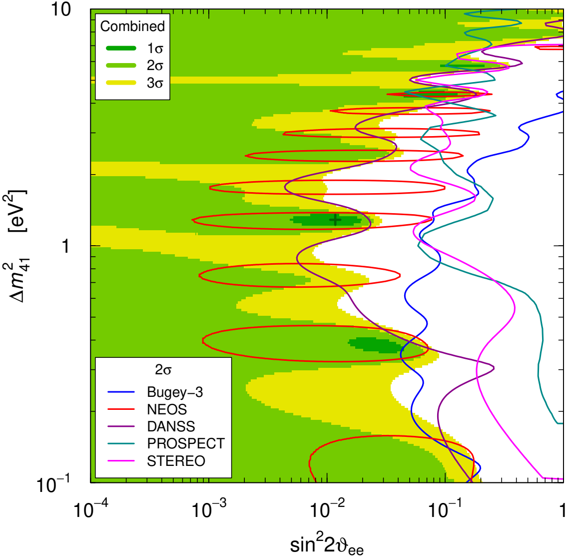


STEREO



SoLid

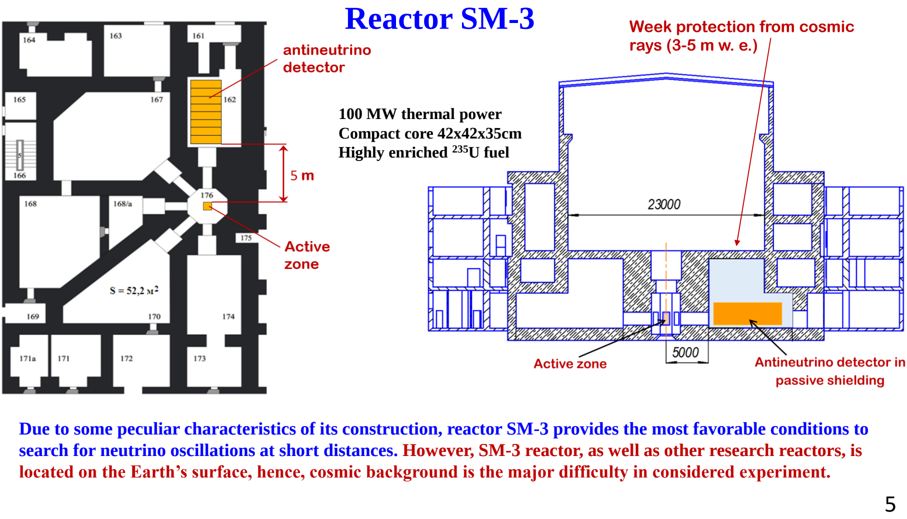




- No compelling indication of oscillations
- In practice these reactor experiments exclude large values of $\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$ for $0.1 \lesssim \Delta m^2_{41} \lesssim 10\,\text{eV}^2$

Neutrino-4

[arXiv:1708.00421, arXiv:1809.10561, arXiv:2003.03199, arXiv:2005.05301, arXiv:2006.13639]



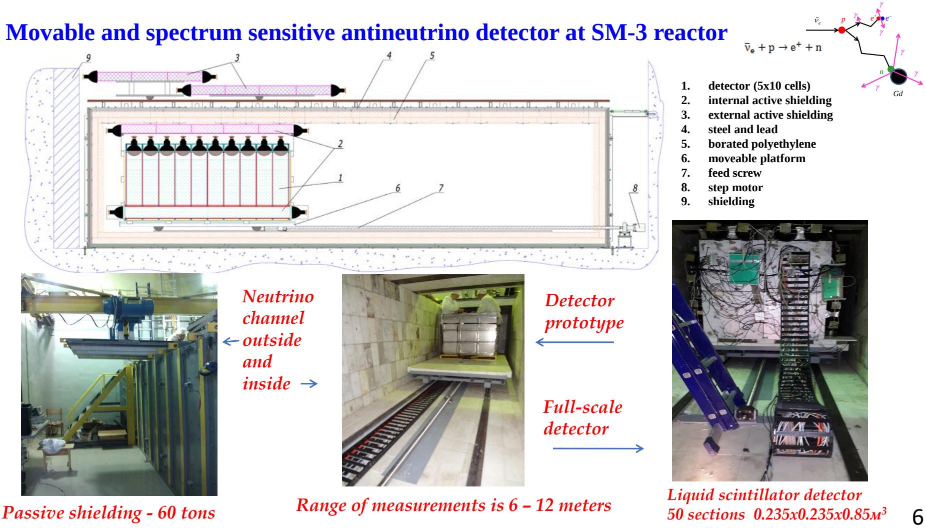
Reactor SM-3

100 MW thermal power
Compact core 42x42x35cm
Highly enriched ^{235}U fuel

Due to some peculiar characteristics of its construction, reactor SM-3 provides the most favorable conditions to search for neutrino oscillations at short distances. However, SM-3 reactor, as well as other research reactors, is located on the Earth's surface, hence, cosmic background is the major difficulty in considered experiment.

5

[A. Serebrov, 17 September 2020]



Movable and spectrum sensitive antineutrino detector at SM-3 reactor

$\bar{\nu}_e + p \rightarrow e^+ + n$

1. detector (5x10 cells)
2. internal active shielding
3. external active shielding
4. steel and lead
5. borated polyethylene
6. moveable platform
7. feed screw
8. step motor
9. shielding

Neutrino channel outside and inside

Detector prototype

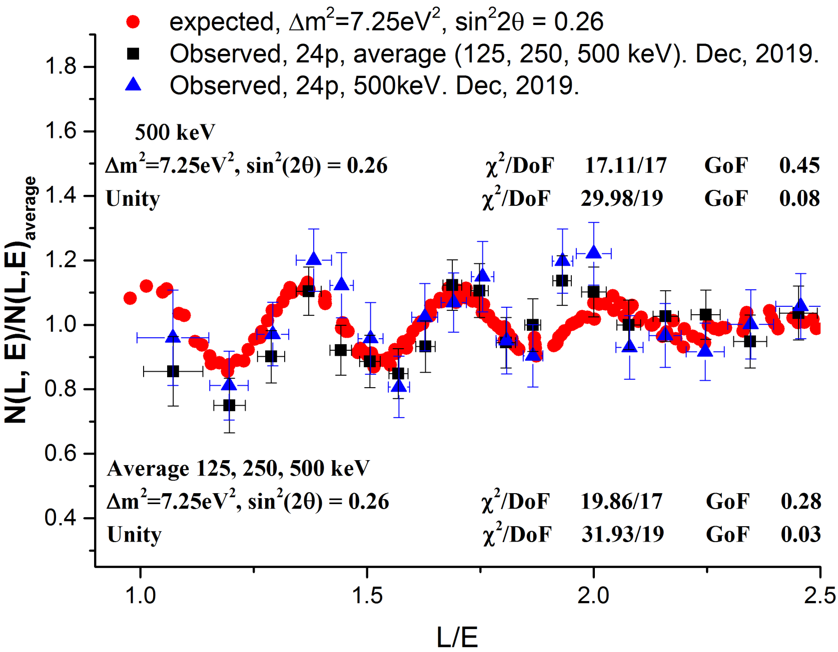
Full-scale detector

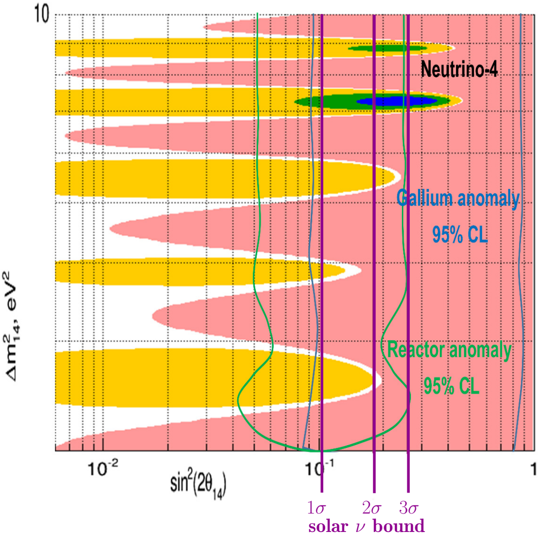
Passive shielding - 60 tons

Range of measurements is 6 – 12 meters

Liquid scintillator detector
50 sections 0.235x0.235x0.85м³

[A. Serebrov, 17 September 2020]


expected, Δm²=7.25eV², sin²2θ = 0.26
Observed, 24p, average (125, 250, 500 keV). Dec, 2019.
Observed, 24p, 500keV. Dec, 2019.
500 keV
Δm²=7.25eV², sin²(2θ) = 0.26 χ²/DoF 17.11/17 GoF 0.45
Unity χ²/DoF 29.98/19 GoF 0.08
Average 125, 250, 500 keV
Δm²=7.25eV², sin²(2θ) = 0.26 χ²/DoF 19.86/17 GoF 0.28
Unity χ²/DoF 31.93/19 GoF 0.03
N(L, E)/N(L,E)average
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
1.0
1.5
2.0
2.5
L/E



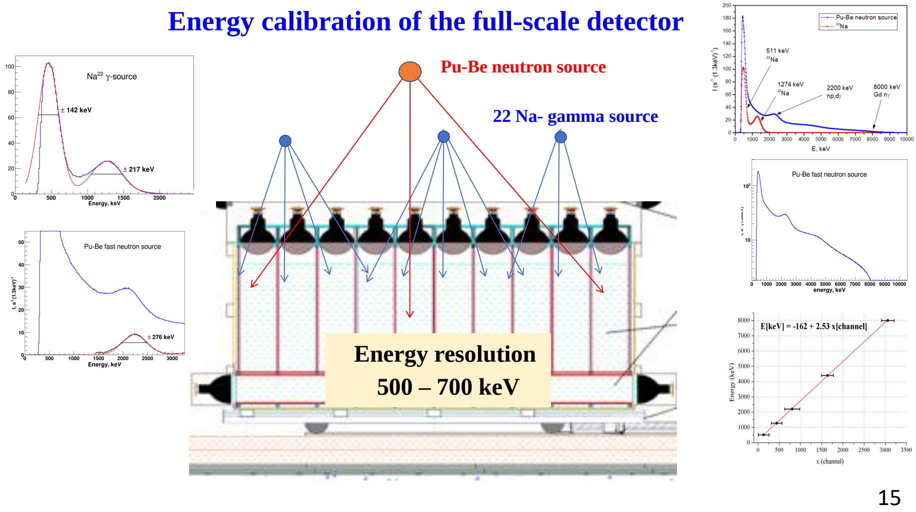
- Neutrino-4 best fit:

$$\sin^2 2\vartheta_{ee} = 0.26$$

$$\Delta m^2_{41} = 7.25\,\text{eV}^2$$

- Very large mixing!
- Not a small perturbation of 3ν mixing.
- Tension with solar neutrino bound.

[Palazzo, arXiv:1105.1705, arXiv:1201.4280]
[Giunti, Laveder, Li, Liu, Long, arXiv:1210.5715]
[Gariazzo, Giunti, Laveder, Li, arXiv:1703.00860]



Energy calibration of the full-scale detector

Pu-Be neutron source

22 Na- gamma source

Energy resolution

500 – 700 keV

15

- We approximate the energy resolution with the function

$$R(E_{\rm p}, E_{\rm p}') = \frac{1}{\sqrt{2\pi}\,\sigma_{E_p}} \exp\left(-\frac{(E_{\rm p} - E_{\rm p}')^2}{2\sigma_{E_p}^2}\right) \qquad \text{with} \qquad \sigma_{E_p} = 0.19\sqrt{\frac{E_p}{\text{MeV}}}\ \text{MeV}$$

- Neutrino-4 data:
  - $n_L = 24$ distances between 6.4 m and 11.9 m from the center of the reactor, at intervals of 23.5 cm.
  - $n_E = 9$ bins of prompt energy $E_{\rm p}$ from 1.5 to 6 MeV with uniform $\Delta E_p = 500\,\text{keV}$ width ($E_\nu = E_{\rm p} + m_n - m_p - m_e \simeq E_{\rm p} + 0.78\,\text{MeV}$)
- $24 \times 9 = 216$ ratios $\quad R_{ik}^{\text{exp}} = \dfrac{N_{ik}^{\text{exp}} L_k^2}{n_L^{-1} \sum_{k'=1}^{n_L} N_{ik'} L_{k'}^2}$
- Theoretical event rates:

$$N_{ik}^{\text{the}} = \frac{N_i^0}{L_k^2} \left[1 - \sin^2 2\vartheta_{ee} \left\langle \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \right\rangle_{ik} \right]$$

- Theoretical ratios:

$$R_{ik}^{\text{the}} = \frac{1 - \sin^2 2\vartheta_{ee} \left\langle \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \right\rangle_{ik}}{1 - \sin^2 2\vartheta_{ee}\, n_L^{-1} \sum_{k'=1}^{n_L} \left\langle \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \right\rangle_{ik'}}$$

Independent of uncertain reactor neutrino flux!

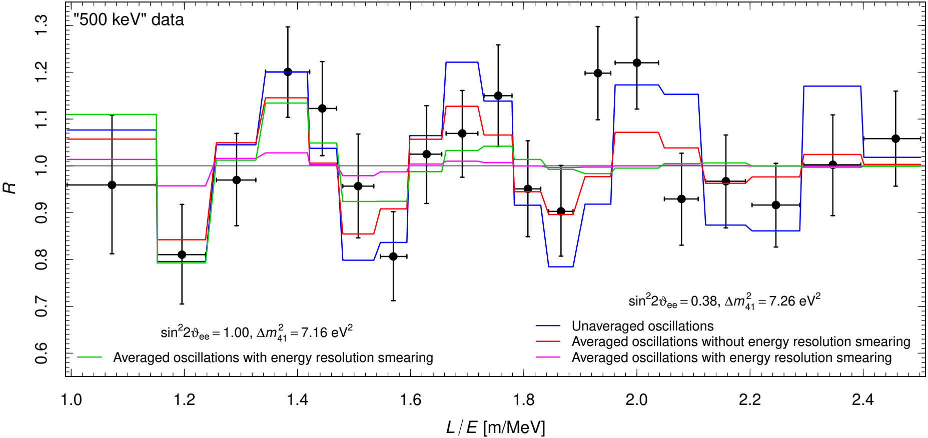
- The data were analized and published in bins of L/E collecting the 216 R_{ik}^{exp}'s in groups of $n_g = 8$ values that correspond to neighboring L/E intervals:

$$R_j^{\text{exp}} = \frac{1}{n_g} \sum_{i,k \in g(j)} R_{ik}^{\text{exp}} = \frac{1}{n_g} \sum_{i,k \in g(j)} \frac{N_{ik}^{\text{exp}} L_k^2}{n_L^{-1} \sum_{k'=1}^{n_L} N_{ik'} L_{k'}^2}$$

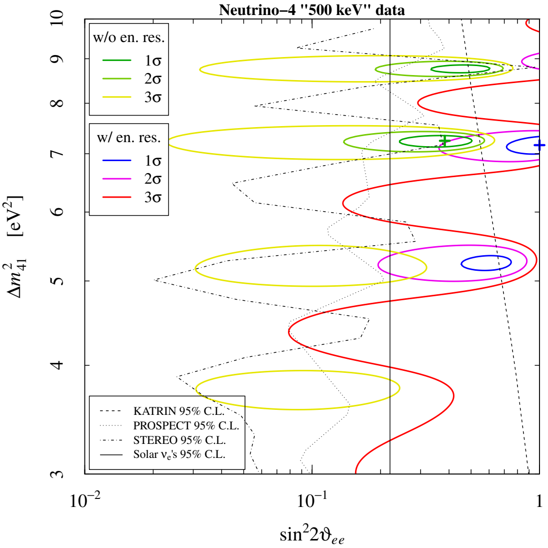
- Corresponding theoretical averages:

$$R_j^{\text{the}} = \frac{1}{n_g} \sum_{i,k \in g(j)} R_{ik}^{\text{the}}$$

$$= \frac{1}{n_g} \sum_{i,k \in g(j)} \frac{1 - \sin^2 2\vartheta_{ee} \left\langle \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \right\rangle_{ik}}{1 - \sin^2 2\vartheta_{ee} \, n_L^{-1} \sum_{k'=1}^{n_L} \left\langle \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right) \right\rangle_{ik'}}$$



$$\left\langle \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)\right\rangle_{ik} = \frac{\int_{L_k^{\mathrm{min}}}^{L_k^{\mathrm{max}}} dL\, L^{-2} \int_{E_i^{\mathrm{min}}}^{E_i^{\mathrm{max}}} dE'_{\mathrm{p}} \int dE_{\mathrm{p}}\, R(E_{\mathrm{p}}, E'_{\mathrm{p}}) \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \phi_{\bar{\nu}_e}(E)\, \sigma_{\bar{\nu}_e p}(E)}{\int_{L_k^{\mathrm{min}}}^{L_k^{\mathrm{max}}} dL\, L^{-2} \int_{E_i^{\mathrm{min}}}^{E_i^{\mathrm{max}}} dE'_{\mathrm{p}} \int dE_{\mathrm{p}}\, R(E_{\mathrm{p}}, E'_{\mathrm{p}})\, \phi_{\bar{\nu}_e}(E)\, \sigma_{\bar{\nu}_e p}(E)}$$



$$\chi^2 = \sum_{j=1}^{19} \left(\frac{R_j^{\text{the}} - R_j^{\text{exp}}}{\Delta R_j^{\text{exp}}} \right)^2$$

	without en. res.	with en. res.
χ^2_{min}	14.9	18.2
GoF	60%	37%
$(\sin^2 2\vartheta_{ee})_{\text{bf}}$	0.38	1.0
$(\Delta m^2_{41})_{\text{bf}}$	7.2	7.2
$\Delta\chi^2_{\text{NO}}$	13.1	9.8
p-value	0.0014	0.0075
σ-value	3.2	2.7

Wilks Theorem (1938)

THE LARGE-SAMPLE DISTRIBUTION OF THE LIKELIHOOD RATIO FOR TESTING COMPOSITE HYPOTHESES[1]

By S. S. Wilks

……

Let $P_\Omega(O_n)$ be the least upper bound of P for the simple hypotheses in Ω, and $P_\omega(O_n)$ the least upper bound of P for those in ω. Then

$$\lambda = \frac{P_\omega(O_n)}{P_\Omega(O_n)} \tag{2}$$

……

which *optimum* estimates of the θ's exist. That is, we shall assume the existence of functions $\bar{\theta}_i(x_1, \cdots x_n)$(maximum likelihood estimates of the θ_i) such that[3] their distribution is

$$\frac{|c_{ij}|^{\frac{1}{2}}}{(2\pi)^{h/2}} e^{-\frac{1}{2}\sum_{i,j=1}^{h} c_{ij} z_i z_j} (1 + \phi)\, dz_1 \cdots dz_h \tag{3}$$

where $z_i = (\bar{\theta}_i - \theta_i)\sqrt{n}$, $c_{ij} = -E\left(\dfrac{\partial^2 \log f}{\partial\theta_i\,\partial\theta_j}\right)$, E denoting mathematical expectation, and ϕ is of order $1/\sqrt{n}$ and $||c_{ij}||$ is positive definite. Denoting (3) by

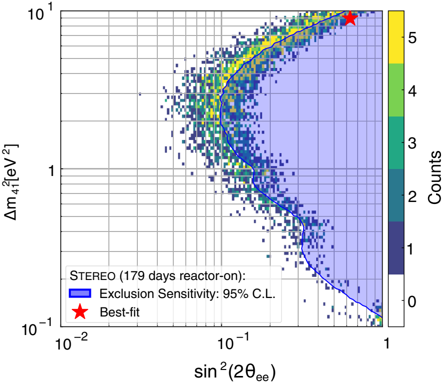
……

Theorem: If a population with a variate x is distributed according to the probability function $f(x, \theta_1, \theta_2 \cdots \theta_h)$, such that optimum estimates $\bar{\theta}_i$ of the θ_i exist which are distributed in large samples according to (3), then when the hypothesis H is true that $\theta_i = \theta_{0i}$, $i = m + 1, m + 2, \cdots h$, the distribution of $-2 \log \lambda$, where λ is given by (2) is, except for terms of order $1/\sqrt{n}$, distributed like χ^2 with $h - m$ degrees of freedom.

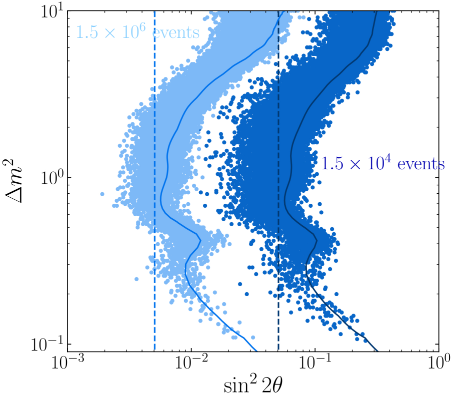
Deviations from χ^2 Distribution (Wilks' Theorem)

[Agostini, Neumair, arXiv:1906.11854; Silaeva, Sinev, arXiv:2001.10752; Giunti, arXiv:2004.07577]
[PROSPECT+STEREO, arXiv:2006.13147; Coloma, Huber, Schwetz, arXiv:2008.06083]

Even in the absence of real oscillations, binned data can often be fitted better by oscillations that reproduce the statistical fluctuations of the bins.

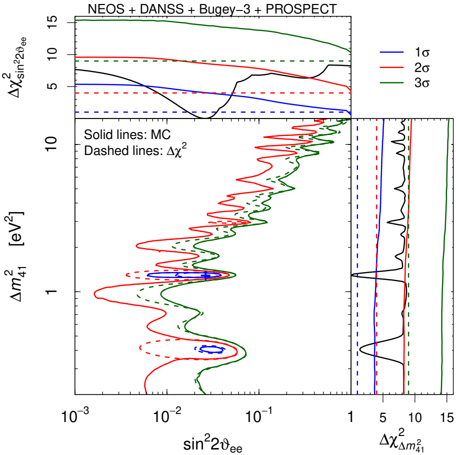


[STEREO, arXiv:1912.06582]



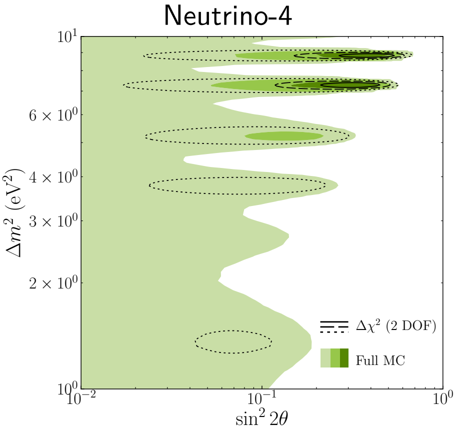
[Coloma, Huber, Schwetz, arXiv:2008.06083]

MC evaluation of test statistic distribution



$2.4\sigma\,(\Delta\chi^2) \longrightarrow 1.8\sigma\,(\text{MC})$

[Giunti, arXiv:2004.07577]



$3.2\sigma\,(\Delta\chi^2) \longrightarrow 2.6\sigma\,(\text{MC})$

[Coloma, Huber, Schwetz, arXiv:2008.06083]

- MC calculations are unfortunately difficult and require a lot of computer time.
- They must be completely redone for each combination of experiments.

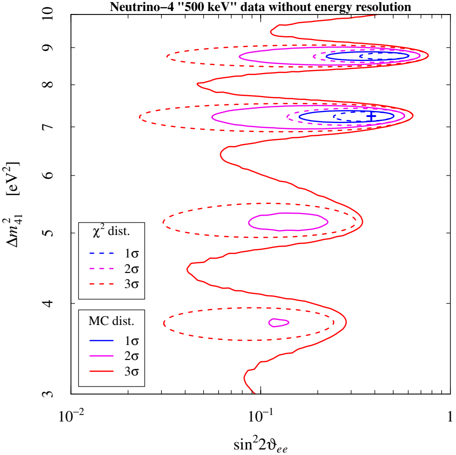
Monte Carlo confidence intervals

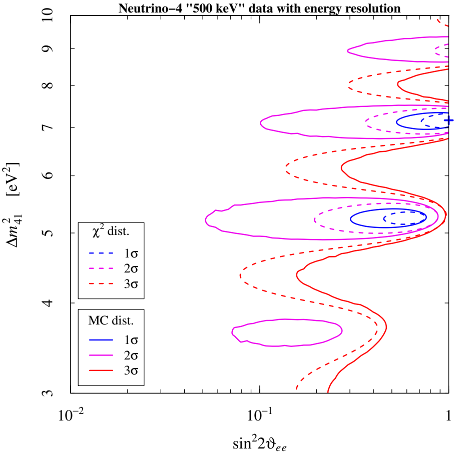
- For each point on a grid in the $(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$ plane we generated a large number of random data sets (of the order of 10^5) with the uncertainties of the Neutrino-4 data set.

- For each random data set:
  - We calculated the value of χ^2 corresponding to the generating values of $\sin^2 2\vartheta_{ee}$ and Δm^2_{41}: $\chi^2_{\text{MC}}(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$.
  - We found the minimum value of χ^2 in the $(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$ plane: $\chi^2_{\text{MC,min}}(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$.

- In this way, we obtained the distribution of $\Delta\chi^2_{\text{MC}}(\sin^2 2\vartheta_{ee}, \Delta m^2_{41}) = \chi^2_{\text{MC}}(\sin^2 2\vartheta_{ee}, \Delta m^2_{41}) - \chi^2_{\text{MC,min}}(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$.

- This distribution allows us to determine if the value of $\Delta\chi^2(\sin^2 2\vartheta_{ee}, \Delta m^2_{41}) = \chi^2(\sin^2 2\vartheta_{ee}, \Delta m^2_{41}) - \chi^2_{\text{min}}(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$ obtained with the analysis of the actual Neutrino-4 data is included or not in a region with a fixed confidence level.

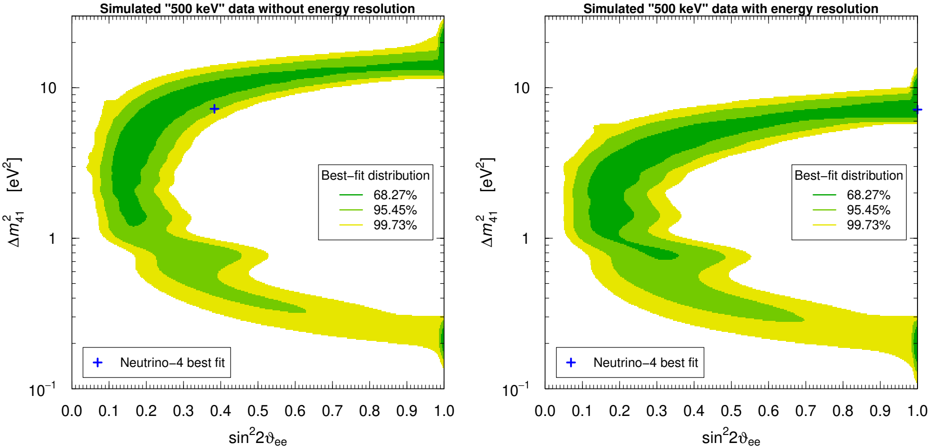




	χ^2 dist.	MC dist.
p-value	0.0014	0.011
σ-value	3.2	2.5

	χ^2 dist.	MC dist.
p-value	0.0075	0.028
σ-value	2.7	2.2

Best-fit distribution in absence of oscillations



	without en. res.	with en. res.
$P(\sin^2 2\vartheta_{ee} < 0.1)$	0.009	0.008
$P(0.1 < \sin^2 2\vartheta_{ee} < 0.5)$	0.680	0.625
$P(0.5 < \sin^2 2\vartheta_{ee} < 0.9)$	0.152	0.184
$P(\sin^2 2\vartheta_{ee} > 0.9)$	0.159	0.183

Summary and Conclusions

- The Neutrino-4 collaboration caimed a discovery of large-mixing short-baseline neutrino oscillations at more than 3σ.
- There is a strong tension between the Neutrino-4 large mixing and the exclusion curves of KATRIN, PROSPECT, STEREO, and solar ν_e's.
- We found that the results of the Neutrino-4 collaboration can be reproduced approximately only by neglecting the effects of the energy resolution of the detector.
- Including these effects, the best-fit point and the surrounding 1σ allowed region in the $(\sin^2 2\vartheta_{ee}, \Delta m^2_{41})$ plane lie at even larger values of the mixing.
- The 3σ allowed region is much larger than that claimed by the Neutrino-4 collaboration and include the case of zero mixing, i.e. the absence of oscillations.
- The statistical significance of short-baseline neutrino oscillations decreases from 3.2σ to 2.7σ.

- With a Monte Carlo evaluation of the distribution of $\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}}$, the statistical significance of short-baseline neutrino oscillations further decreases to about 2.2σ.
- Monte Carlo simulations of a large set of Neutrino-4-like data show that it is not unlikely to obtain a best-fit point that has a large mixing, even maximal, in the absence of oscillations.
- We conclude that the claimed Neutrino-4 indication in favor of short-baseline neutrino oscillations with very large mixing is rather doubtful.