Sterile Neutrinos in Physics, Astrophysics, Cosmology

Part II: Light Active and Sterile Neutrinos in Cosmology

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Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation

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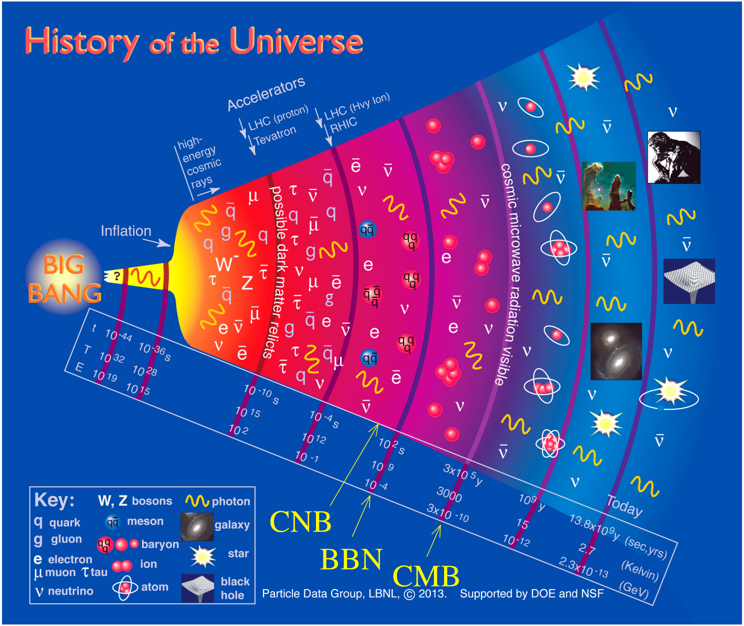

History of the Universe
Accelerators
LHC (proton)
Tevatron
LHC (Hvy Ion)
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-36 s
T 10 32 10 28
E 10 19 10 15
10 -10 s
10 15
10 2
10 -4 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
13.8x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
CNB
BBN
CMB
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
μ muon τ tau
black hole
ν neutrino
atom
Particle Data Group, LBNL, © 2013. Supported by DOE and NSF

Basic Formalism

- Einstein equations of gravity: $\mathcal{R}^{\mu\nu} - \frac{1}{2}\mathcal{R}\, g^{\mu\nu} - \Lambda\, g^{\mu\nu} = 8\pi\, G_{\text{N}}\, T^{\mu\nu}$
- Observations have shown that the Universe is spatially homogeneous and isotropic on large scales: $\gtrsim 100\,\text{Mpc}$.
- The Standard Cosmological Model assumes that there is a frame in which the total matter and radiation of the Universe can be described on large scales by a perfect fluid with the energy momentum tensor

$$T^{\mu\nu} = \text{diag}(\varrho, p, p, p)$$

- In this comoving frame, the geometry of space-time is described by the Friedmann–Robertson–Walker metric

$$d\tau^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1 - k\,r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

- The rate of expansion is given by the Friedmann equation:

$$H^2 = \frac{8\pi G_{\text{N}}}{3}\,\varrho - \frac{k}{R^2} \qquad\qquad H(t) \equiv \frac{\dot{R}(t)}{R(t)}$$

- Redshift: $z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\Delta\lambda}{\lambda} \Longrightarrow \boxed{1 + z = \frac{R(t_0)}{R(t_e)}}$

- Radiation, matter and vacuum energy densities: $\varrho = \varrho_R + \varrho_M + \varrho_\Lambda$

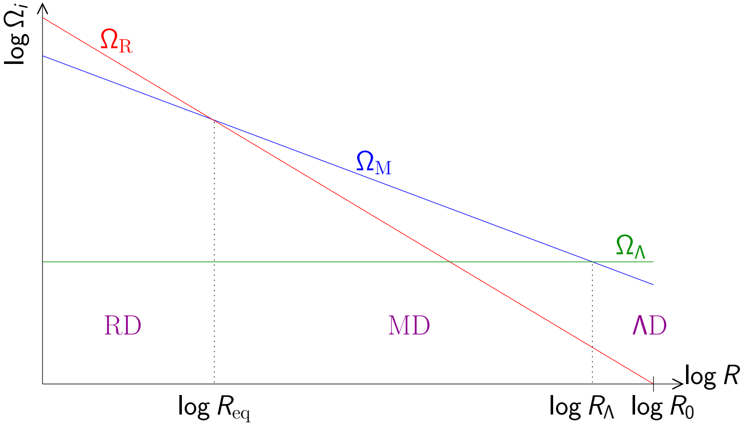
- Equation of state: $p_i = w_i\,\varrho_i$

$$\begin{array}{lll} \text{Radiation:} & w_R = 1/3 \Longrightarrow & \varrho_R \propto R^{-4} \propto (1+z)^4 \\ \text{Matter:} & w_M = 0 \Longrightarrow & \varrho_M \propto R^{-3} \propto (1+z)^3 \\ \text{Vacuum Energy:} & w_\Lambda = -1 \Longrightarrow & \varrho_\Lambda = \text{constant} \end{array}$$

- Flat Universe: $k = 0 \Longrightarrow \varrho = \varrho_c \equiv \frac{3\,H^2}{8\pi\,G_N}$ critical density

$$\varrho_c^0 = \frac{3\,H_0^2}{8\pi\,G_N} = 10.54 h^2\,\text{keV}\,\text{cm}^{-3} \quad \text{with} \quad H_0 = 100\,h\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$$

- $\Omega_i \equiv \frac{\varrho_i}{\varrho_c} \Longrightarrow \Omega_R + \Omega_M + \Omega_\Lambda = 1$ for a flat Universe



$$\Omega_\Lambda^0 \simeq 0.7 \qquad \Omega_\Lambda = \Omega_\Lambda^0$$

$$\Omega_{\rm M}^0 \simeq 0.3 \qquad \Omega_{\rm M} = \Omega_{\rm M}^0 \, (R/R_0)^{-3} = \Omega_{\rm M}^0 \, (1+z)^3$$

$$\Omega_{\rm R}^0 \simeq 10^{-4} \qquad \Omega_{\rm R} = \Omega_{\rm R}^0 \, (R/R_0)^{-4} = \Omega_{\rm M}^0 \, (1+z)^4$$

Friedmann equation for a flat Universe: $H^2 = \frac{8\pi}{3\,M_{\text{P}}^2}\,\varrho$

$$\frac{H^2}{H_0^2} = \frac{\varrho}{\varrho^0} \quad \Longrightarrow \quad H^2 = H_0^2\,\frac{\varrho_\Lambda + \varrho_{\text{M}} + \varrho_{\text{R}}}{\varrho_{\text{c}}^0}$$

$$\varrho_\Lambda = \varrho_\Lambda^0$$

$$\varrho_{\text{M}} = \varrho_{\text{M}}^0 \left(\frac{R_0}{R}\right)^3 = \varrho_{\text{M}}^0\,(1+z)^3$$

$$\varrho_{\text{R}} = \varrho_{\text{R}}^0 \left(\frac{R_0}{R}\right)^4 = \varrho_{\text{R}}^0\,(1+z)^4$$

$$H^2 = H_0^2\,\frac{\varrho_\Lambda^0 + \varrho_{\text{M}}^0\,(1+z)^3 + \varrho_{\text{R}}^0\,(1+z)^4}{\varrho_{\text{c}}^0}$$

$$H^2(z) = H_0^2\left[\Omega_\Lambda^0 + \Omega_{\text{M}}^0\,(1+z)^3 + \Omega_{\text{R}}^0\,(1+z)^4\right]$$

The expansion rate depends on H_0 and on Ω_Λ^0, Ω_{M}^0, Ω_{R}^0

$$t(z) = H_0^{-1}\int_0^{(1+z)^{-1}} \frac{\mathrm{d}x}{\sqrt{\Omega_\Lambda^0\,x^2 + \Omega_{\text{M}}^0\,x^{-1} + \Omega_{\text{R}}^0\,x^{-2}}}$$

Thermodynamics of the Early Universe

- Thermal equilibrium:

$$n_\chi = \frac{g_\chi}{(2\pi)^3} \int f_\chi(\vec{p})\, \mathrm{d}^3 p$$

$$\varrho_\chi = \frac{g_\chi}{(2\pi)^3} \int E_\chi(\vec{p})\, f_\chi(\vec{p})\, \mathrm{d}^3 p$$

$$p_\chi = \frac{g_\chi}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E_\chi(\vec{p})}\, f_\chi(\vec{p})\, \mathrm{d}^3 p$$

- Statistical distribution: $f_\chi(\vec{p}) = \dfrac{1}{e^{(E_\chi(\vec{p})-\mu_\chi)/T_\chi} \pm 1}$

- Chemical potential:
  - $a + b \leftrightarrows c + d \quad \Longrightarrow \quad \mu_a + \mu_b = \mu_c + \mu_d$
  - $\mu_\gamma = 0$ and $\chi + \bar{\chi} \to \gamma\gamma \quad \Longrightarrow \quad \mu_\chi = -\mu_{\bar{\chi}}$
  - Conserved charge $\Longrightarrow \quad \mu_\chi \neq 0$ if $n_\chi \neq n_{\bar{\chi}}$

- Relativistic limit: $T_\chi \gg m_\chi$ and $T_\chi \gg \mu_\chi \Longrightarrow f_\chi(\vec{p}) \simeq \frac{1}{e^{|\vec{p}|/T_\chi} \pm 1}$

$$n_\chi \simeq \begin{cases} \frac{\zeta(3)}{\pi^2} g_\chi T_\chi^3 & (\chi = \text{boson}) \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_\chi T_\chi^3 & (\chi = \text{fermion}), \end{cases}$$

$$\varrho_\chi \simeq \begin{cases} \frac{\pi^2}{30} g_\chi T_\chi^4 & (\chi = \text{boson}) \\ \frac{7}{8} \frac{\pi^2}{30} g_\chi T_\chi^4 & (\chi = \text{fermion}), \end{cases}$$

$$p_\chi \simeq \frac{1}{3} \varrho_\chi ,$$

- Average energy:

$$\langle E_\chi \rangle \simeq \langle |\vec{p}_\chi| \rangle \simeq \begin{cases} \frac{\pi^4}{30\,\zeta(3)} T_\chi \simeq 2.701\, T_\chi & (\chi = \text{boson}) \\ \frac{7\pi^4}{180\,\zeta(3)} T_\chi \simeq 3.151\, T_\chi & (\chi = \text{fermion}) \end{cases}$$

Radiation Temperature Scaling

$$\left.\begin{array}{l} \varrho_{\mathrm{R}} \propto R^{-4} \\ \varrho_{\mathrm{R}} \propto T^{4} \end{array}\right\} \Longrightarrow \boxed{T \propto R^{-1}}$$

The Universe cools during expansion!

Neutrino Decoupling

- Active neutrinos are in equilibrium in the early Universe through weak interactions $(\alpha = e, \mu, \tau)$:

$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrows e^+ e^- \qquad \overset{(-)}{\nu}_\alpha e \leftrightarrows \overset{(-)}{\nu}_\alpha e \qquad \overset{(-)}{\nu}_\alpha N \leftrightarrows \overset{(-)}{\nu}_\alpha N$$

$$\nu_e n \leftrightarrows p e^- \qquad \bar{\nu}_e p \leftrightarrows n e^+ \qquad n \leftrightarrows p e^- \bar{\nu}_e$$

- Interaction rate: $\Gamma_{\nu_\alpha} = n_{\nu_\alpha} \langle \sigma_{\nu_\alpha} v \rangle \sim G_F^2 T^5$

$$n_{\nu_\alpha} \sim T^3 \qquad \sigma_{\nu_\alpha} \sim G_F^2 T^2 \qquad v \simeq 1$$

- In the radiation-dominated era: $H^2 \simeq \frac{8\pi}{3 M_P^2} \varrho_R$ with $\varrho_R = \frac{\pi^2}{30} g_* T^4$

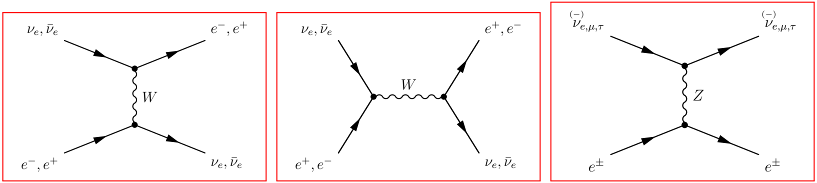
$$H \simeq \frac{2\pi^{3/2}}{3\sqrt{5} M_P} \sqrt{g_*}\, T^2 \qquad g_* = \sum_{\substack{\chi=\text{relativistic}\\ \text{bosons}}} g_\chi + \frac{7}{8} \sum_{\substack{\chi=\text{relativistic}\\ \text{fermions}}} g_\chi$$

- Before ν decoupling: $g_* = g_*^{(\gamma)} + g_*^{(e^\pm)} + g_*^{(\nu)} = 2 + \frac{7}{8} 4 + \frac{7}{8} 6 = 10.75$

- Neutrino decoupling: $\Gamma_{\nu_\alpha} \sim H \quad \Longrightarrow \quad T^{\nu_\alpha\text{-dec}} \sim \left(M_{\rm P}\, G_{\rm F}^2\right)^{-1/3} \sim 1\,\text{MeV}$

- A more precise calculation takes into account that the dominant processes for $T \lesssim 100\,\text{MeV}$ are

$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrows e^+ e^- \qquad \overset{(-)}{\nu}_\alpha e \leftrightarrows \overset{(-)}{\nu}_\alpha e$$



- Since the rates of these processes depend on neutrino energy $E \simeq p$, the decoupling temperature is not instantaneous and depends on p:

$$T^{\nu_e\text{-dec}}(p) \simeq 2.7 \left(\frac{p}{T}\right)^{-1/3} \qquad T^{\nu_{\mu,\tau}\text{-dec}}(p) \simeq 4.5 \left(\frac{p}{T}\right)^{-1/3}$$

- Taking into account that $\langle E \rangle \simeq 3T$, one obtains:

$$T^{\nu_e\text{-dec}} \simeq 1.9\,\text{MeV} \qquad T^{\nu_{\mu,\tau}\text{-dec}} \simeq 3.1\,\text{MeV}$$

- Hot relics: relativistic at decoupling $\Longrightarrow f_{\nu}^{\nu\text{-dec}}(\vec{p}) \simeq \dfrac{1}{e^{|\vec{p}|/T^{\nu\text{-dec}}} + 1}$

- After decoupling: $f_{\nu}(\vec{p}) = f_{\nu}(\vec{p})|_{\nu\text{-dec}} = f_{\nu}^{\nu\text{-dec}}(\vec{p}_{\nu\text{-dec}})$

- Momentum scaling with expansion: $\vec{p} = \vec{p}_{\nu\text{-dec}} \left(\dfrac{R}{R_{\nu\text{-dec}}} \right)^{-1}$

$$f_{\nu}(\vec{p}) \simeq \left[\exp\left(\frac{|\vec{p}|\,(R/R_{\nu\text{-dec}})}{T^{\nu\text{-dec}}} \right) + 1 \right]^{-1} = \frac{1}{e^{|\vec{p}|/T_{\nu}} + 1}$$

Effective temperature scales with expansion: $\boxed{T_{\nu} = T^{\nu\text{-dec}} \left(\dfrac{R}{R_{\nu\text{-dec}}} \right)^{-1}}$

Electron-Positron Annihilation

- After neutrino decoupling at $T \simeq 1\,\text{MeV}$ $e^{\pm}$ and γ are the only relativistic particles in thermal equilibrium.

- At $m_e/3 \simeq 0.2\,\text{MeV}$ electrons and positrons became nonrelativistic: out-of-equilibrium $e^- e^+ \to \gamma\gamma$ heat the photon distribution.

- During this phase the photon temperature does not scale as R^{-1}.

- Entropy density: $s = \dfrac{\varrho + p}{T} = \dfrac{2\pi^2}{45}\, g_s\, T_\gamma^3$

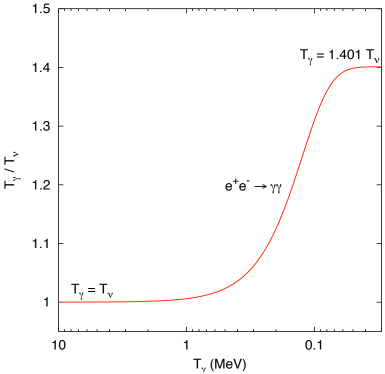
$$g_s = \sum_{\substack{\chi=\text{interacting}\\ \text{relativistic}\\ \text{bosons}}} g_\chi + \frac{7}{8} \sum_{\substack{\chi=\text{interacting}\\ \text{relativistic}\\ \text{fermions}}} g_\chi$$

- Entropy conservation: $s \propto R^{-3} \quad \Longrightarrow \quad T_\gamma \propto g_s^{-1/3}\, R^{-1}$

- Before and after $e^- e^+$ annihilation: $\dfrac{T_\nu^{\text{after}}}{T_\nu^{\text{before}}} = \left(\dfrac{R^{\text{after}}}{R^{\text{before}}}\right)^{-1}$

$$\frac{T_\gamma^{\text{after}}}{T_\gamma^{\text{before}}} = \left(\frac{g_s^{\text{after}}}{g_s^{\text{before}}}\right)^{-1/3} \left(\frac{R^{\text{after}}}{R^{\text{before}}}\right)^{-1} = \left(\frac{g_s^{\text{after}}}{g_s^{\text{before}}}\right)^{-1/3} \frac{T_\nu^{\text{after}}}{T_\nu^{\text{before}}}$$

- $T_\gamma^{\text{before}} = T_\nu^{\text{before}}$
- $g_s^{\text{before}} = g_s^{(\gamma)} + g_s^{(e^\pm)} = 2 + \frac{7}{8} 4 = \frac{11}{2}$
- $g_s^{\text{after}} = g_s^{(\gamma)} = 2$
- $T_\nu^{\text{after}} = \left(\frac{4}{11}\right)^{1/3} T_\gamma^{\text{after}} \simeq 0.7138\, T_\gamma^{\text{after}}$



- $$\boxed{T_\nu^0 = \left(\frac{4}{11}\right)^{1/3} T_\gamma^0 = 1.945 \pm 0.001\,\text{K} = (1.676 \pm 0.001) \times 10^{-4}\,\text{eV}}$$

Effective Number of Relativistic Degrees Of Freedom

- Radiation density: $$\varrho_{\mathrm{R}} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\mathrm{eff}} \right] \varrho_{\gamma}$$

- Three standard neutrinos: $N_{\mathrm{eff}}^{3\nu} = 3.0440 \pm 0.0002$

  [Bennett, Buldgen, de Salas, Drewes, Gariazzo, Pastor, Wong, arXiv:2012.02726]

  $N_{\mathrm{eff}}^{3\nu} = 3.046$ [Mangano et al, arXiv:hep-ph/0506164] $N_{\mathrm{eff}}^{3\nu} = 3.045$ [de Salas, Pastor, arXiv:1606.06986]

- $N_{\mathrm{eff}}^{3\nu} > 3$ because neutrino decoupling was not instantaneous at $T^{\nu\text{-dec}}$: higher-energy neutrinos decoupled later and were not completely decoupled during $e^{-}e^{+}$ annihilation. The non-thermal distortions of the energy distribution generate an effective $N_{\mathrm{eff}}^{3\nu} > 3$.

- Light sterile neutrinos can be produced by active-sterile oscillations before the decoupling of the active neutrinos, increasing N_{eff}.

- BSM light particles contribute to $\Delta N_{\mathrm{eff}} \equiv N_{\mathrm{eff}} - N_{\mathrm{eff}}^{3\nu}$.

- A completely thermalized sterile neutrino contributes with $\Delta N_{\mathrm{eff}} = 1$.

- It is possible to have partial thermalizations with $\Delta N_{\mathrm{eff}} < 1$.

ν oscillations in the early universe

[SG+, JCAP 07 (2019) 014]

comoving coordinates: $a = 1/T \quad x \equiv m_e\, a \quad y \equiv p\, a \quad z \equiv T_\gamma\, a \quad w \equiv T_\nu\, a$

density matrix: $$\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$$

$$\frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = \sqrt{\frac{3m_{\mathrm{Pl}}^2}{8\pi\rho_{\mathrm{T}}}} \left\{ -i\frac{x^2}{m_e^3}\left[\frac{\mathbb{M}_{\mathrm{F}}}{2y} - \frac{8\sqrt{2}G_{\mathrm{F}}\, y m_e^6}{3x^6}\left(\frac{\mathbb{E}_\ell}{m_W^2} + \frac{\mathbb{E}_\nu}{m_Z^2}\right), \varrho\right] + \frac{m_e^3 G_{\mathrm{F}}^2}{(2\pi)^3 x^4 y^2}\mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[.,.]$ commutator

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m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[.,.]$ commutator

$$\mathbb{M}_{\mathrm{F}} = U \mathbb{M} U^\dagger$$

$$\mathbb{M} = \mathrm{diag}(m_1^2, \ldots, m_N^2)$$

$$U = R^{34} R^{24} R^{14} R^{23} R^{13} R^{12}$$

e.g. $$R^{14} = \begin{pmatrix} \cos\theta_{14} & 0 & 0 & \sin\theta_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_{14} & 0 & 0 & \cos\theta_{14} \end{pmatrix}$$

$$|U|^2 = \begin{pmatrix} \ldots & \ldots & \ldots & \sin^2\theta_{14} \\ \ldots & \ldots & \ldots & \cos^2\theta_{14}\sin^2\theta_{24} \\ \ldots & \ldots & \ldots & \cos^2\theta_{14}\cos^2\theta_{24}\sin^2\theta_{34} \\ \ldots & \ldots & \ldots & \cos^2\theta_{14}\cos^2\theta_{24}\cos^2\theta_{34} \end{pmatrix}$$

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$$\frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = \sqrt{\frac{3m_{\mathrm{Pl}}^2}{8\pi\rho_{\mathrm{T}}}} \left\{ -i\frac{x^2}{m_e^3}\left[\frac{\mathbb{M}_{\mathrm{F}}}{2y} - \frac{8\sqrt{2}G_{\mathrm{F}}\, y m_e^6}{3x^6}\left(\frac{\mathbb{E}_\ell}{m_W^2} + \frac{\mathbb{E}_\nu}{m_Z^2}\right), \varrho\right] + \frac{m_e^3 G_{\mathrm{F}}^2}{(2\pi)^3 x^4 y^2}\mathcal{I}(\varrho) \right\}$$

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$$\mathbb{M}_{\mathrm{F}} = U \mathbb{M} U^\dagger$$

$$\mathbb{E}_\ell = \mathrm{diag}(\rho_e, \rho_\mu, 0, 0) \qquad \mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho\right) S_a \quad \text{with } S_a = \mathrm{diag}(1,1,1,0)$$

lepton densities

neutrino densities

(only for active neutrinos)

take into account matter effects in oscillations

ν oscillations in the early universe

[SG+, JCAP 07 (2019) 014]

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$\mathcal{I}(\varrho)$ collision integrals

take into account neutrino-electron scattering and pair annihilation

2D integrals over the momentum, take most of the computation time

ν oscillations in the early universe

[SG+, JCAP 07 (2019) 014]

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$\mathcal{I}(\varrho)$ collision integrals

from continuity equation $\dot{\rho} = -3H(\rho + P)$

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\sum_{\ell=e,\mu}\left[\frac{r_\ell^2}{r}J(r_\ell)\right] + G_1(r) - \frac{1}{2\pi^2 z^3}\int_0^\infty dy\, y^3 \sum_{\alpha=e}^{s}\frac{\mathrm{d}\varrho_{\alpha\alpha}}{\mathrm{d}x}}{\sum_{\ell=e,\mu}\left[r_\ell^2 J(r_\ell) + Y(r_\ell)\right] + G_2(r) + \frac{2\pi^2}{15}}$$

$r = x/z$, $r_\ell = m_\ell/m_e\, r$ $\quad$ $J(r)$, $Y(r)$ from non-relativistic transition of $e^\pm$, $\mu^\pm$

$G_1(r)$ and $G_2(r)$ from electromagnetic corrections

ν oscillations in the early universe

[SG+, JCAP 07 (2019) 014]

comoving coordinates: $a = 1/T \quad x \equiv m_e\, a \quad y \equiv p\, a \quad z \equiv T_\gamma\, a \quad w \equiv T_\nu\, a$

density matrix: $\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} & \varrho_{es} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} & \varrho_{\mu s} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} & \varrho_{\tau s} \\ \varrho_{se} & \varrho_{s\mu} & \varrho_{s\tau} & \varrho_{ss} \equiv f_{\nu_s} \end{pmatrix}$

$$\frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = \sqrt{\frac{3m_{\mathrm{Pl}}^2}{8\pi\rho_{\mathrm{T}}}} \left\{ -i\frac{x^2}{m_e^3}\left[\frac{\mathbb{M}_{\mathrm{F}}}{2y} - \frac{8\sqrt{2}G_{\mathrm{F}}\, y m_e^6}{3x^6}\left(\frac{\mathbb{E}_\ell}{m_W^2} + \frac{\mathbb{E}_\nu}{m_Z^2}\right), \varrho\right] + \frac{m_e^3 G_{\mathrm{F}}^2}{(2\pi)^3 x^4 y^2}\mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[.,.]$ commutator

$$\mathbb{M}_{\mathrm{F}} = U\mathbb{M}U^\dagger \qquad \mathbb{E}_\ell = \mathrm{diag}(\rho_e, \rho_\mu, 0, 0) \qquad \mathbb{E}_\nu = S_a \left(\int dy y^3 \varrho\right) S_a$$

$\mathcal{I}(\varrho)$ collision integrals

from continuity equation $\dot\rho = -3H(\rho + P)$

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neutrino temperature w: same equation as z, but electrons always relativistic

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neutrino temperature w: same equation as z, but electrons always relativistic

initial conditions: $\varrho_{\alpha\alpha}$ = Fermi-Dirac at $x_{\mathrm{in}} \simeq 0.001$, with $w = z \simeq 1$

ν oscillations in the early universe

[SG+, JCAP 07 (2019) 014]

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FORTran-Evolved PrimordIAl Neutrino Oscillations (FortEPiaNO)

https://bitbucket.org/ahep_cosmo/fortepiano_public

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$\dot{\rho} = -3H(\rho + P)$

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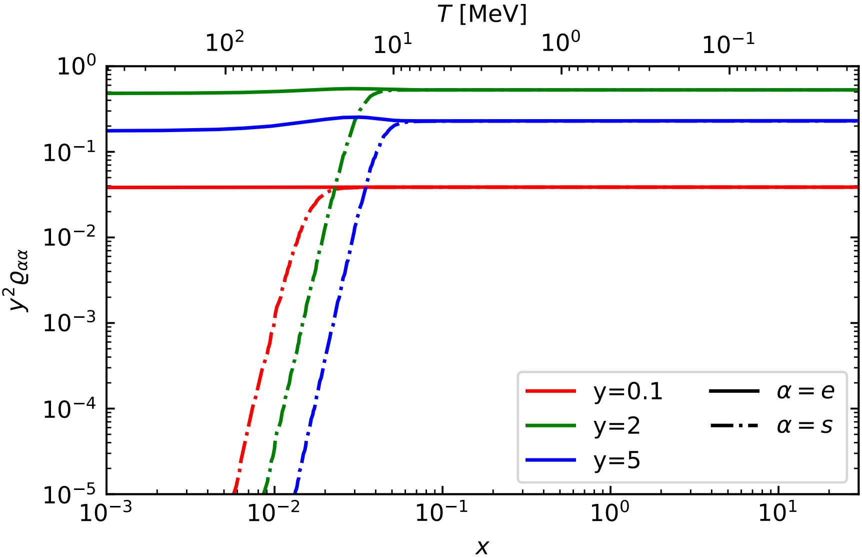
Momentum distributions

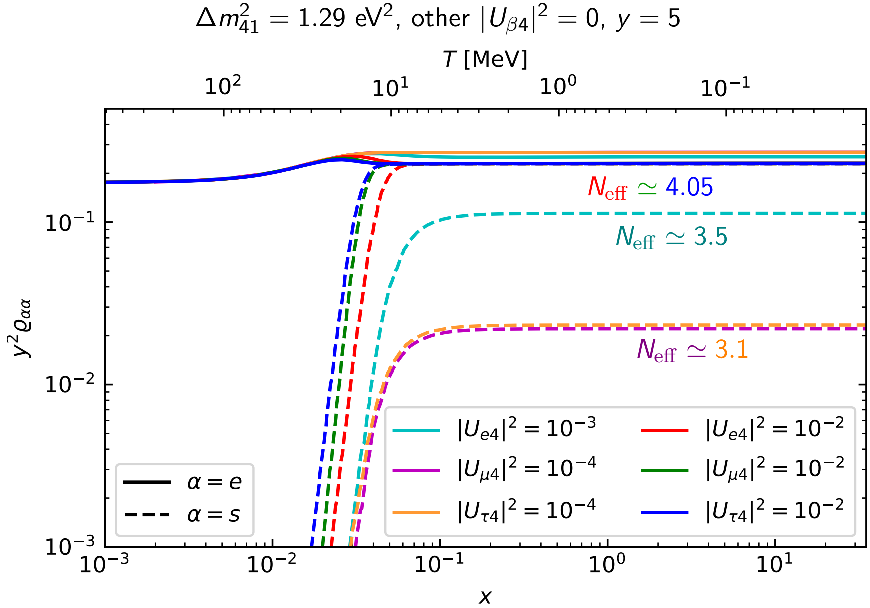
[SG+, JCAP 07 (2019) 014]

$$\Delta m^2_{41} = 1.29 \text{ eV}^2,\ |U_{e4}|^2 = 10^{-2},\ |U_{\mu 4}|^2 = |U_{\tau 4}|^2 = 0,\ N_{\rm eff} \simeq 4.05$$



Momentum distributions

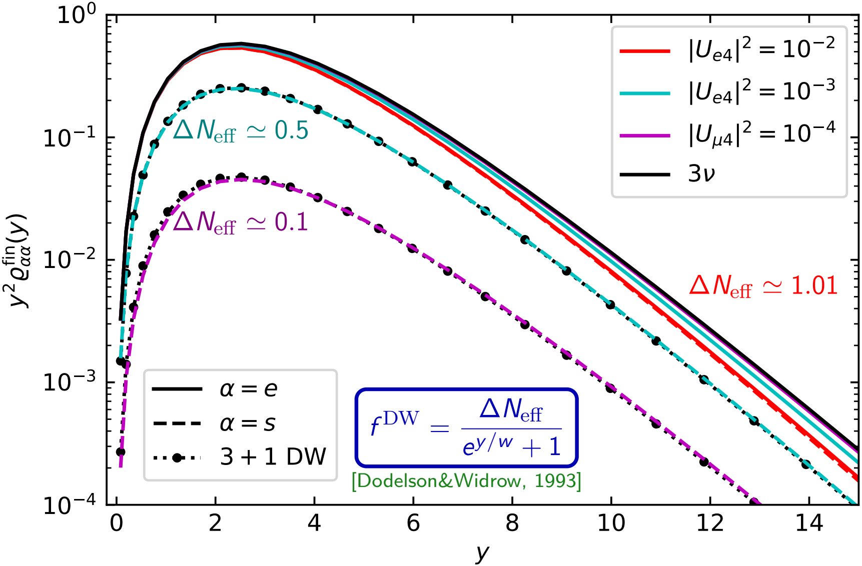
[SG+, JCAP 07 (2019) 014]



Momentum distributions

[SG+, JCAP 07 (2019) 014]

$\Delta m^2_{41} = 1.29$ eV^2, other $|U_{\beta 4}|^2 = 0$, $\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{active}}$



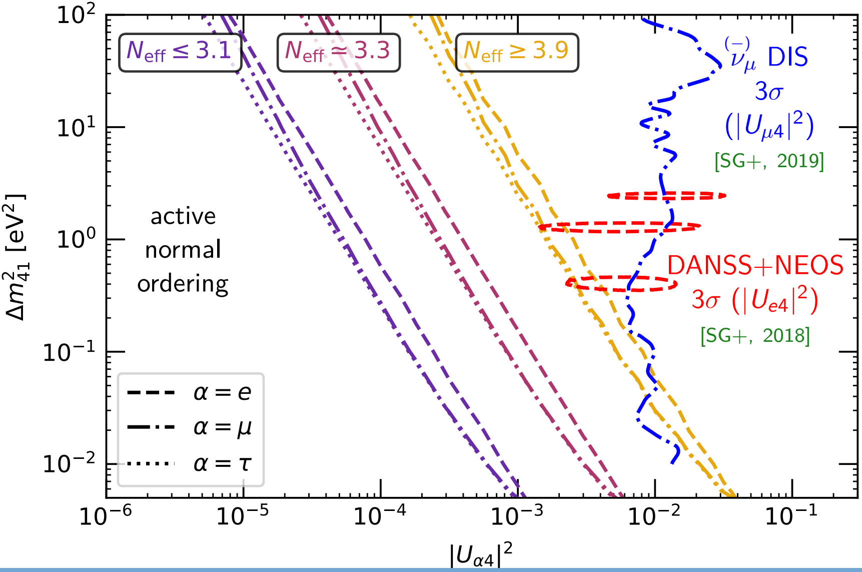
$$f^{\text{DW}} = \frac{\Delta N_{\text{eff}}}{e^{y/w} + 1}$$

[Dodelson&Widrow, 1993]

$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

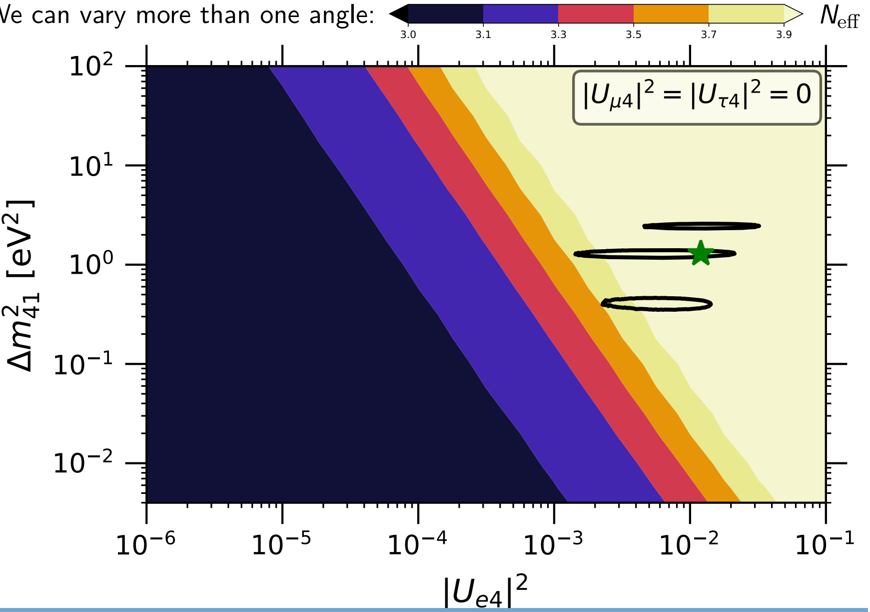
Only vary one angle and fix two to zero: do they have the same effect?



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

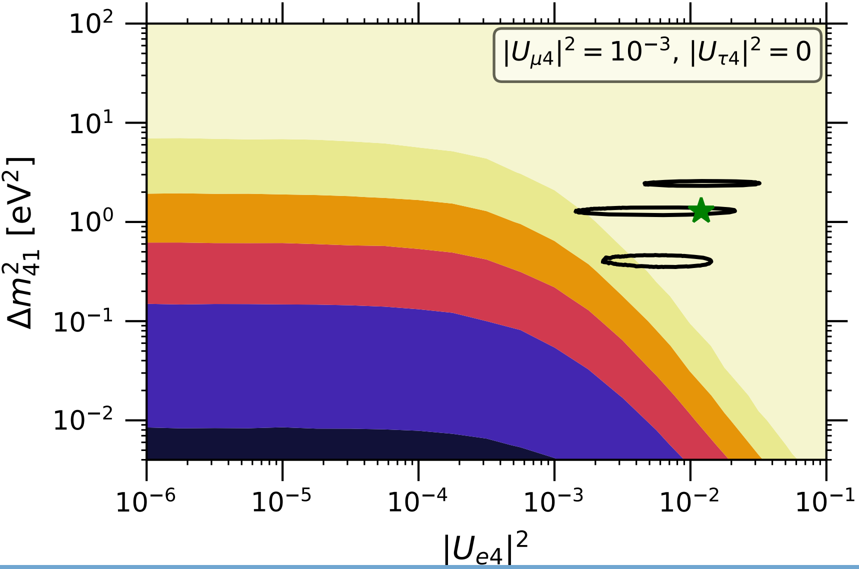
We can vary more than one angle:



N_{eff} and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

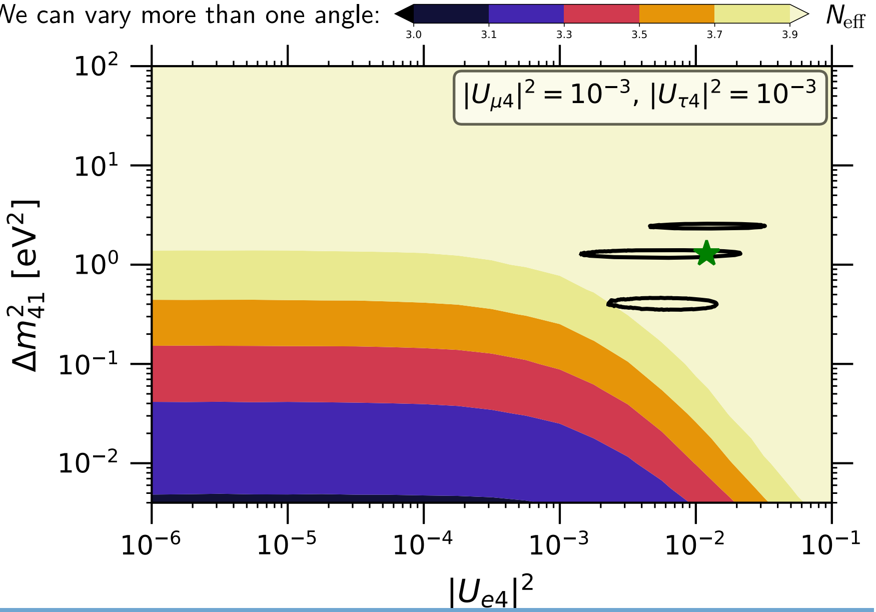
We can vary more than one angle:



$N_{\rm eff}$ and the new mixing parameters

[SG+, JCAP 07 (2019) 014]

We can vary more than one angle:



Nonrelativistic Transition

- After decoupling $T_\nu \propto R^{-1}$ $\Longrightarrow$ $T_\nu = T_\nu^0 \left(\frac{R_0}{R} \right) = T_\nu^0 (1+z)$

- Nonrelativistic transition: $T_{\nu_i}^{\text{nr}} \simeq 3 m_i \Rightarrow z_{\nu_i}^{\text{nr}} \simeq \frac{m_i}{3\,T_\nu^0} \simeq 2.0 \times 10^3 \left(\frac{m_i}{\text{eV}} \right)$

  $m_3 \gtrsim 5 \times 10^{-2}\,\text{eV} \Rightarrow z_{\nu_3}^{\text{nr}} \gtrsim 100$ $\quad$ $m_2 \gtrsim 8 \times 10^{-3}\,\text{eV} \Rightarrow z_{\nu_2}^{\text{nr}} \gtrsim 16$

- After the nonrelativistic transition: $\varrho_{\nu_i} \simeq m_i n_{\nu_i}$

- $n_\nu^0 + n_{\bar\nu}^0 \simeq \frac{3}{2} \frac{\zeta(3)}{\pi^2} (T_\nu^0)^3 \simeq \frac{6}{11} \frac{\zeta(3)}{\pi^2} (T_\gamma^0)^3 = \frac{3}{11} n_\gamma^0 \simeq 112\,\text{cm}^{-3}$

- $\varrho_c^0 \equiv \frac{3\,H_0^2}{8\pi\,G_N} \simeq 10.54\,h^2\,\text{keV}\,\text{cm}^{-3} \Rightarrow \Omega_{\nu_i}^0 \simeq \frac{m_i(n_\nu^0 + n_{\bar\nu}^0)}{\varrho_c^0} \simeq \frac{m_i}{94.1\,h^2\,\text{eV}}$

- Nonthermal distortions $\Longrightarrow$ $\boxed{\Omega_{\nu_i}^0 \simeq \frac{m_i}{93.1\,h^2\,\text{eV}}}$

  $\Omega_{\nu_3}^0 \gtrsim 5 \times 10^{-4}$

  $\Omega_{\nu_2}^0 \gtrsim 9 \times 10^{-5}$

- Total contribution of SM neutrinos to the current energy density of the Universe: [Gershtein, Zeldovich, JETP Lett. 4 (1966) 120; Cowsik, McClelland, PRL 29 (1972) 669]

$$\boxed{\Omega^0_{3\nu} \simeq \frac{\sum_i m_i}{93.1\, h^2\, \text{eV}}}$$

$$\left.\begin{array}{l} \Omega^0_{3\nu} \leq \Omega^0_{\text{M}} - \Omega^0_{\text{B}} \simeq 0.25 \\ h \simeq 0.7 \end{array}\right\} \Longrightarrow \sum_{i=1}^{3} m_i \lesssim 10\,\text{eV}$$

- This bound is not competitive with the current kinematical laboratory limit from the KATRIN experiment:

$$m_1, m_2, m_3 \lesssim m_\beta \lesssim 1\,\text{eV} \Longrightarrow \sum_{i=1}^{3} m_i \lesssim 3\,\text{eV}$$

- For a completely thermalized non-standard (mainly sterile) massive neutrino ν_4:

$$\Omega^0_{\nu_4} \simeq \frac{m_4}{94.1\, h^2\, \text{eV}} \Longrightarrow m_4 \lesssim 10\,\text{eV}$$

Matter-Radiation Equality

- Matter-radiation equality is important because subhorizon matter density fluctuations can grow only during the matter-dominated era.
- Therefore structure formation starts at matter-radiation equality.
- Where neutrino still relativistic at matter-radiation equality?
- The answer to this question is important in order to determine the effect of neutrinos on structure formation.
- Redshift of matter-radiation equality:

$$\left.\begin{array}{l} \varrho_{\mathrm{M}} \propto R^{-3} \\ \varrho_{\mathrm{R}} \propto R^{-4} \end{array}\right\} \Rightarrow \frac{\varrho_{\mathrm{M}}}{\varrho_{\mathrm{R}}} = \frac{\varrho_{\mathrm{M}}^{0}}{\varrho_{\mathrm{R}}^{0}} \frac{R}{R_0} = \frac{\varrho_{\mathrm{M}}^{0}}{\varrho_{\mathrm{R}}^{0}} (1+z)^{-1} \Rightarrow 1 + z_{\mathrm{eq}} = \frac{\varrho_{\mathrm{M}}^{0}}{\varrho_{\mathrm{R}}^{0}} = \frac{\Omega_{\mathrm{M}}^{0}}{\Omega_{\mathrm{R}}^{0}}$$

- This relation assumes that the number of relativistic particles is not changed.

- If neutrinos were relativistic at matter-radiation equality:

$$1 + z_{\text{eq}} = \frac{\Omega_{\text{M}}^{0}}{\Omega_{\text{R}}^{0}}(m_\nu = 0) = \frac{\Omega_{\text{M}}^{0}}{\Omega_{\gamma}^{0} + \Omega_{\nu}^{0}(m_\nu = 0)}$$

$$\Omega_{\text{R}}^{0}(m_\nu = 0) = \left[1 + 3\left(\frac{4}{11}\right)^{4/3}\right]\Omega_{\gamma}^{0} \simeq 4.4 \times 10^{-5}\, h^{-2}$$

$$\simeq 8.9 \times 10^{-5} \quad \text{for} \quad h \simeq 0.7$$

$$z_{\text{eq}} \simeq 2.4 \times 10^{4} \left(\Omega_{\text{M}}^{0}\right) h^{2} \simeq 3.5 \times 10^{3} \quad \text{for} \quad \Omega_{\text{M}}^{0} \simeq 0.3$$

$$z_{\nu_i}^{\text{nr}} \simeq 2.0 \times 10^{3} \left(\frac{m_i}{\text{eV}}\right) < z_{\text{eq}} \quad \text{for} \quad m_i \lesssim 1.75\,\text{eV}$$

- Therefore light massive neutrinos with $m_i \lesssim 1\,\mathrm{eV}$ became nonrelativistic after matter-radiation equality.

- Before $t_{\nu_i}^{\mathrm{nr}}$ neutrinos free stream $\Longrightarrow$ subhorizon matter density fluctuations are suppressed by neutrino free streaming from t_{eq} to $t_{\nu_i}^{\mathrm{nr}}$.

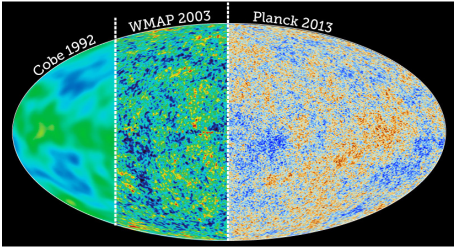
- Current physical free-streaming scale: $\lambda_{\nu_i\text{-fs}}^{0} \simeq z_{\nu_i\text{-nr}}\, d_{\mathrm{H}}(z_{\nu_i\text{-nr}})$

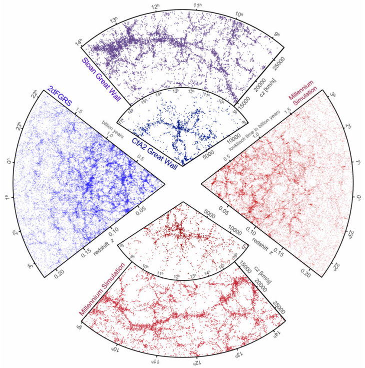
- Matter-dominated era: $d_{\mathrm{H}}(z) \simeq 2\, H_0^{-1}\, z^{-3/2}\, (\Omega_{\mathrm{M}}^{0})^{-1/2}$

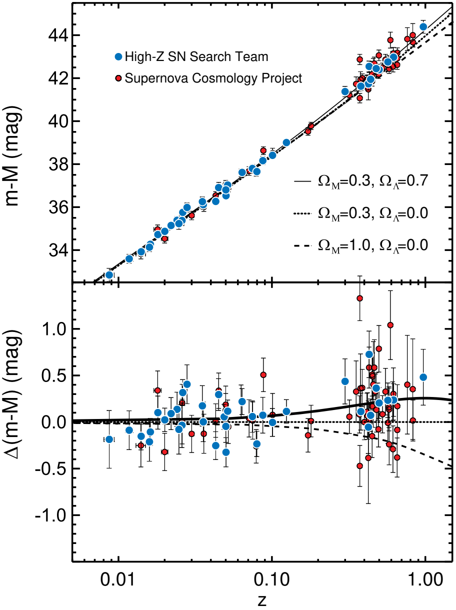
$$\lambda_{\nu_i\text{-fs}}^{0} \simeq 0.013 \left(\frac{m_i}{\mathrm{eV}}\right)^{-1/2} (\Omega_{\mathrm{M}}^{0})^{-1/2}\, h^{-1}\, \mathrm{Mpc}$$

$$k_{\nu_i\text{-fs}}^{0} \simeq \frac{2\pi}{\lambda_{\nu_i\text{-fs}}^{0}} \simeq 0.047 \left(\frac{m_i}{\mathrm{eV}}\right)^{1/2} \sqrt{\Omega_{\mathrm{M}}^{0}}\, h\, \mathrm{Mpc}^{-1}$$

Cosmological Data







Power Spectrum of Density Fluctuations

- Density fluctuations: $\delta(t,\vec{x}) \equiv \dfrac{\varrho(t,\vec{x}) - \langle \varrho(t) \rangle}{\langle \varrho(t) \rangle} = \int \dfrac{d^3k}{(2\pi)^3}\, \delta(t,\vec{k})\, e^{i\vec{k}\cdot\vec{x}}$

- The Fourier transform converts differential equations into algebraic equations.

- In the linear theory, the algebraic equations for the amplitude of each fluctuation mode with wavenumber $\vec{k}$ are independent.

- The amplitude $\delta(t,\vec{k})$ of each fluctuation mode evolves in time independently of the others and can be conveniently studied separately.

- In a cosmological model, it is not possible to calculate the exact amount of perturbations in the observable Universe, because we do not know the initial conditions.
- One can calculate the statistical properties of the perturbations, extracted from a statistical ensemble of possible universes.
- Since we do not have experimental access to an ensemble of universes the statistical properties of perturbations are obtained by averaging over large volumes or different directions in the sky.

- Two-point correlation function:

$$\xi(t, y) \equiv \langle \delta(t, \vec{x})\, \delta(t, \vec{x} + \vec{y}) \rangle$$

- If the fluctuations are statistically homogeneous in space, the two-point correlation function does not depend on $\vec{x}$.

- If the fluctuations are statistically isotropic, $\xi(t, y)$ depends only on $y \equiv |\vec{y}|$.

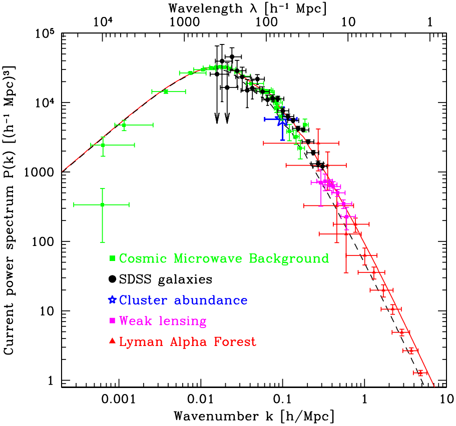
- Two-point correlation function in Fourier space:

$$\langle \delta(t, \vec{k})\, \delta(t, \vec{k}') \rangle = (2\pi)^3\, \delta^3(\vec{k} + \vec{k}')\, P(k, t)$$

- Power spectrum: $\boxed{P(k, t) = \langle |\delta(t, \vec{k})|^2 \rangle}$

- The power spectrum is the variance of the distribution of fluctuations in Fourier space.

- Gaussian fluctuations are completely characterized by their variance, i.e. by the power spectrum.



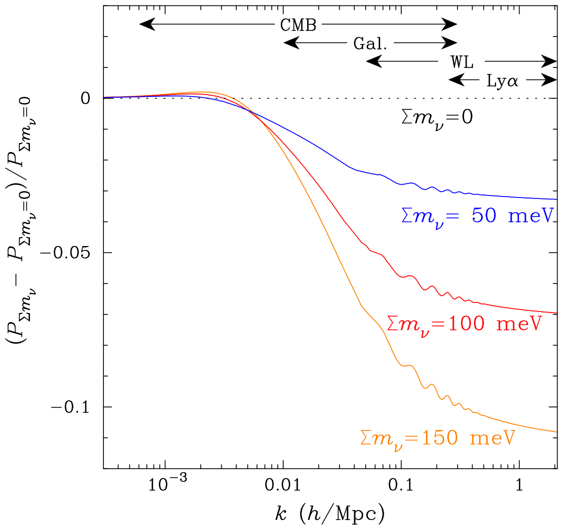
[Tegmark, arXiv:hep-ph/0503257]

Solid Curve: flat ΛCDM model

$$h = 0.72$$
$$\Omega_M^0 = 0.28$$
$$\Omega_B^0 / \Omega_M^0 = 0.16$$

Dashed Curve: $\sum_{i=1}^{3} m_i = 1\,\text{eV}$

$$f_\nu \equiv \frac{\Omega_\nu^0}{\Omega_M^0} \simeq \frac{\sum_i m_i}{93.1\,h^2\,\text{eV}\,\Omega_M^0} \simeq 0.07$$



[Abazajian et al, arXiv:1309.5383]

Cosmic Microwave Background Radiation

- Temperature fluctuations: $$\frac{\Delta T_\gamma(\theta,\phi)}{T_\gamma} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} \, Y_\ell^m(\theta,\phi)$$
- Two-point correlation function:

$$C^{TT}(\theta) \equiv \left\langle \frac{\Delta T_\gamma(\theta_1,\phi_1)}{T_\gamma} \, \frac{\Delta T_\gamma(\theta_2,\phi_2)}{T_\gamma} \right\rangle$$

where θ is the angle between the directions (θ_1,ϕ_1) and (θ_2,ϕ_2).

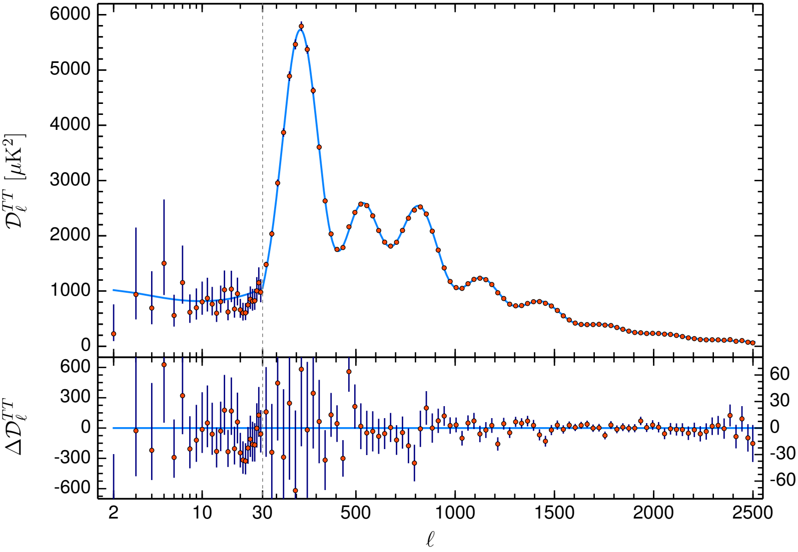
- If the multipoles are independent random variables:

$$C^{TT}(\theta) = \sum_\ell \frac{2\ell+1}{4\pi} \, C_\ell^{TT} \, P_\ell(\cos\theta)$$

- Angular power spectrum: $$C_\ell^{TT} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \langle |a_{\ell m}|^2 \rangle$$
- The C_ℓ^{TT} are the variances of the multipole moments $a_{\ell m}$.
- Gaussian fluctuations are completely characterized by the variances C_ℓ^{TT}.
- Angular variations: $\Delta\theta \sim \pi/\ell$

Planck

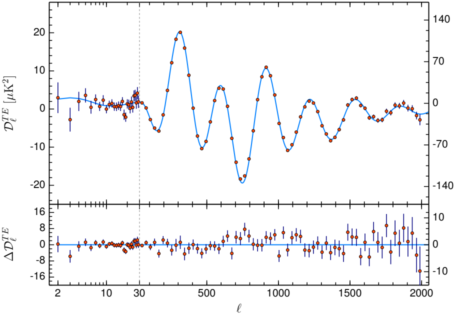
[Planck, arXiv:1807.06209]

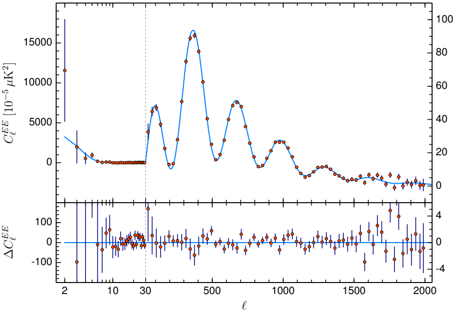


$$\mathcal{D}_\ell^{TT} = \ell(\ell+1)C_\ell^{TT}/2\pi$$

Planck Polarization Data

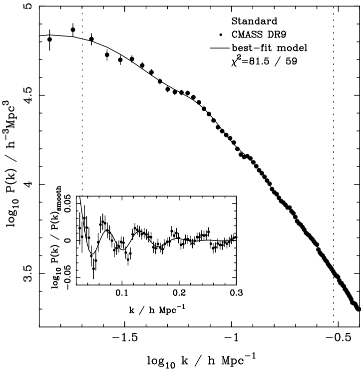
[Planck, arXiv:1807.06209]





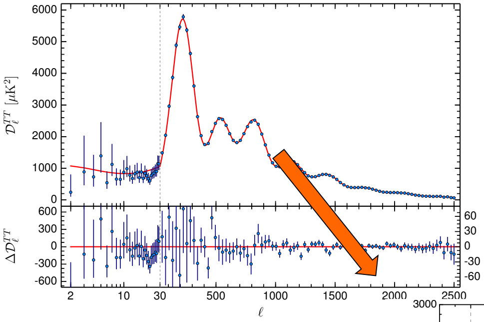
Planck Terminology

- TT: the Planck TT data (low-ℓ for $\ell < 30$ and high-ℓ for $\ell \geq 30$).
- lowE (lowP): the Planck polarization data at multipoles $\ell < 30$ (low-ℓ).
- TE: the Planck TE data at $\ell \geq 30$.
- EE: the Planck EE data at $\ell \geq 30$.
- Lensing: the Planck weak lensing data.
- BAO: the Baryon Acoustic Oscillation data.



Baryon Oscillation Spectroscopic Survey (BOSS)
part of the Sloan Digital Sky Survey III (SDSS-III)
Data Release 9 (DR9) CMASS sample

[arXiv:1203.6594]

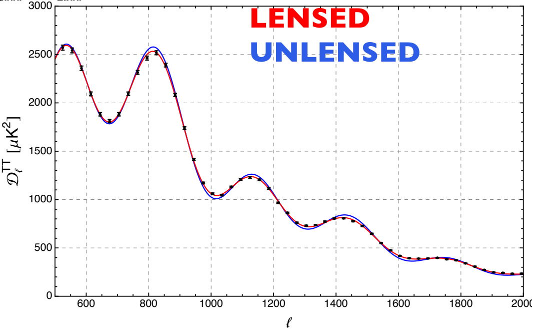


Lensing smooths the peaks of the CMB power spectrum…

… and introduces nongaussianities in the map (nonzero 4-point c.f.)

Neutrino free streaming damps matter perturbations and *reduces* lensing

The effect is proportional to ν energy density



[M. Lattanzi @ Moriond EW 2018]

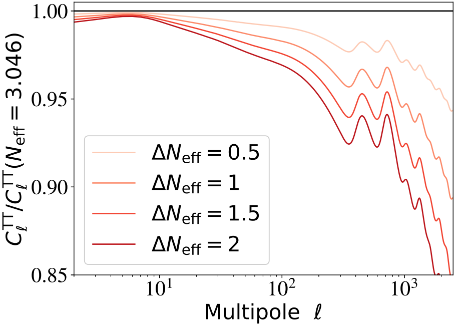
Planck Limits on $\sum_{k=1}^{3} m_\nu$

[Planck, arXiv:1807.06209]

Cosmological data set	$\sum_{k=1}^{3} m_\nu$ (95% B.P.)
Planck TT + lowE	< 0.54 eV
Planck TT,TE,EE + lowE	< 0.26 eV
Planck TT + lowE + lensing	< 0.44 eV
Planck TT,TE,EE + lowE + lensing	< 0.24 eV
Planck TT + lowE + BAO	< 0.16 eV
Planck TT,TE,EE + lowE + BAO	< 0.13 eV
Planck TT + lowE + lensing + BAO	< 0.13 eV
Planck TT,TE,EE + lowE + lensing + BAO	< 0.12 eV

B.P.: Bayesian Probability

Very Light Sterile Neutrinos: Dark Radiation

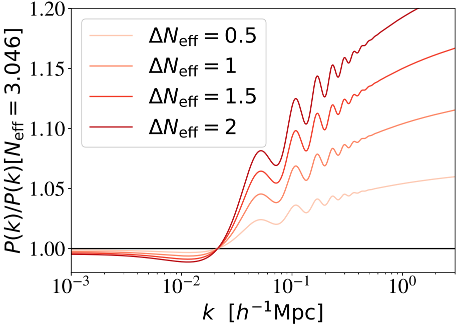


[Lesgourgues, Verde, Review of Particle Physics 2017]

- Photons feel gravitational forces from a denser neutrino component.
- Decreases the acoustic peaks because the distribution of free-streaming neutrinos is smoother than that of the photons.

- $\varrho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \varrho_\gamma$
- $\Delta N_{\rm eff} = N_{\rm eff} - 3.044$
- Fixed $z_{\rm eq}$, z_Λ, ω_B^0
- $z_{\rm eq} \simeq \dfrac{\Omega_M^0 h^2}{\omega_\gamma^0 \left(1 + 0.227 N_{\rm eff}\right)}$
- $z_\Lambda \simeq \left(\dfrac{\Omega_\Lambda^0}{\Omega_M^0}\right)^{1/3} \simeq \left(\dfrac{1-\Omega_M^0}{\Omega_M^0}\right)^{1/3}$
- Therefore fixed Ω_M^0
- $\omega_B^0 = \Omega_B^0 h^2$
- It can be done by increasing h^2 and decreasing Ω_B^0 with an increase of $\Omega_{\rm CDM}^0 = \Omega_M^0 - \Omega_B^0$

Very Light Sterile Neutrinos: Dark Radiation



[Lesgourgues, Verde, Review of Particle Physics 2017]

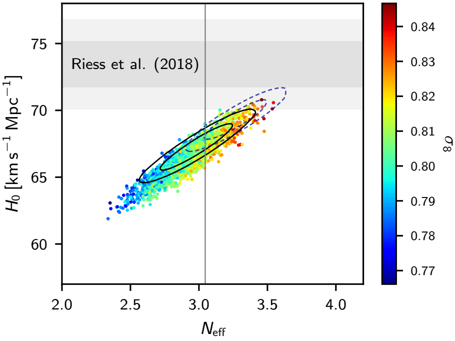
- Increased fluctuations due to increased $\Omega^0_{\rm CDM}$.
- Decreased BAO due to decreased $\Omega^0_{\rm B}$.

- $\varrho_{\rm R} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\varrho_\gamma$
- $\Delta N_{\rm eff} = N_{\rm eff} - 3.044$
- Fixed $z_{\rm eq}$, z_Λ, $\omega^0_{\rm B}$
- $z_{\rm eq} \simeq \dfrac{\Omega^0_{\rm M} h^2}{\omega^0_\gamma\,(1 + 0.227\,N_{\rm eff})}$
- $z_\Lambda \simeq \left(\dfrac{\Omega^0_\Lambda}{\Omega^0_{\rm M}}\right)^{1/3} \simeq \left(\dfrac{1-\Omega^0_{\rm M}}{\Omega^0_{\rm M}}\right)^{1/3}$
- Therefore fixed $\Omega^0_{\rm M}$
- $\omega^0_{\rm B} = \Omega^0_{\rm B}\, h^2$
- It can be done by increasing h^2 and decreasing $\Omega^0_{\rm B}$ with an increase of $\Omega^0_{\rm CDM} = \Omega^0_{\rm M} - \Omega^0_{\rm B}$

Planck Limits on Dark Radiation

[Planck, arXiv:1807.06209]

Cosmological data set	N_{eff} (95% B.P.)
Planck TT + lowE	$3.00^{+0.57}_{-0.53}$
Planck TT,TE,EE + lowE	$2.92^{+0.36}_{-0.37}$
Planck TT + lowE + lensing + BAO	$3.11^{+0.44}_{-0.43}$
Planck TT,TE,EE + lowE + lensing + BAO	$2.99^{+0.34}_{-0.33}$



Planck TT,TE,EE + lowE + lensing + BAO + R18 (68% B.P.)

$$N_{\text{eff}} = 3.27 \pm 0.15$$

$$H_0 = (69.32 \pm 0.97)\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$$

Tension with $\sigma_8 = 0.8101 \pm 0.0061$

Massive Sterile Neutrinos

- Sterile neutrinos can be produced by $\nu_{e,\mu,\tau} \to \nu_s$ oscillations before active neutrino decoupling $(t_{\nu\text{-dec}} \sim 1\,\text{s})$
- Energy density of radiation before matter-radiation equality:

$$\varrho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \varrho_\gamma \qquad (t < t_{\text{eq}} \sim 6 \times 10^4\,\text{y})$$

$$N_{\text{eff}}^{\text{SM}} = 3.044 \qquad \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$$

- Contribution of sterile neutrinos with a thermal FD distribution with temperature T_s: $\varrho_s = (T_s/T_\nu)^4 \varrho_\nu \implies \Delta N_{\text{eff}} = (T_s/T_\nu)^4$
- A sterile neutrino $\nu_s \simeq \nu_4$ with mass $m_s \equiv m_4 \sim 1\,\text{eV}$ becomes non-relativistic at $T_\nu \sim m_s/3$, that is at $t_{\nu_s\text{-nr}} \sim 2.0 \times 10^5\,\text{y}$, before recombination at $t_{\text{rec}} \sim 3.8 \times 10^5\,\text{y}$
- Current energy density of sterile neutrinos:

$$\Omega_s = \frac{n_s m_s}{\varrho_c} \simeq \frac{(T_s/T_\nu)^3 m_s}{94.1\,h^2\,\text{eV}} = \frac{\Delta N_{\text{eff}}^{3/4} m_s}{94.1\,h^2\,\text{eV}} = \frac{m_s^{\text{eff}}}{94.1\,h^2\,\text{eV}}$$

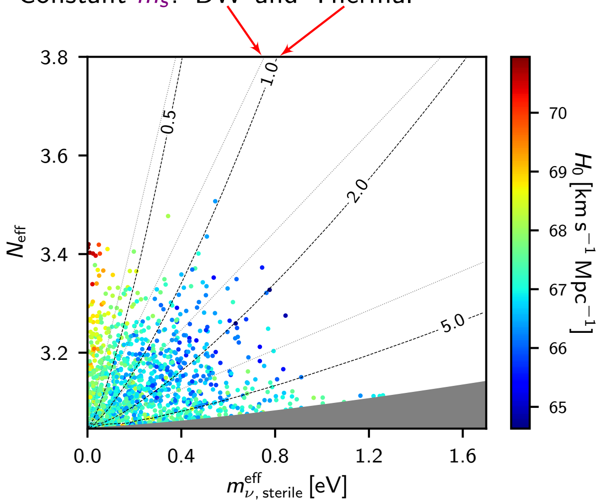
$$m_s^{\text{eff}} = \Delta N_{\text{eff}}^{3/4} m_s = (T_s/T_\nu)^3 m_s = \Omega_s\, 94.1\,h^2\,\text{eV}$$

Planck Limits on Massive Sterile Neutrinos

[Planck, arXiv:1807.06209]

Planck TT,TE,EE + lowE + lensing

Constant m_s: DW and Thermal



- $m_s^{\rm eff} \equiv 94.1\,\Omega_s\,h^2\,\text{eV}$
- Thermal distribution:

$$f_s(E) = \frac{1}{e^{E/T_s}+1}$$

$$m_s^{\rm eff} = \left(\frac{T_s}{T_\nu}\right)^3 m_s = (\Delta N_{\rm eff})^{3/4} m_s$$

- Dodelson-Widrow (DW):

$$f_s(E) = \frac{\Delta N_{\rm eff}}{e^{E/T_\nu}+1}$$

$$m_s^{\rm eff} = \Delta N_{\rm eff}\, m_s$$

The limits on N_{eff} and m_s^{eff} depend on the prior on m_s, that is necessary to exclude the parameter space that is degenerate with a change in the CDM density.

Planck TT,TE,EE + lowE + lensing + BAO

For prior m_s bounds in the thermal distribution scenario, at 95% B.P.:

$$m_s < 10\,\text{eV} \quad \Longrightarrow \quad N_{\text{eff}} < 3.29 \quad \text{and} \quad m_s^{\text{eff}} < 0.65\,\text{eV}$$

$$m_s < 2\,\text{eV} \quad \Longrightarrow \quad N_{\text{eff}} < 3.34 \quad \text{and} \quad m_s^{\text{eff}} < 0.23\,\text{eV}$$

$\Delta N_{\text{eff}} = 1$ is excluded at about 6σ for any m_s!

Main proposed mechanisms to avoid the thermalization of the sterile neutrinos:

- A large lepton asymmetry [Hannestad, Tamborra, Tram, JCAP 1207 (2012) 025; Mirizzi, Saviano, Miele, Serpico, PRD 86 (2012) 053009; Saviano et al., PRD 87 (2013) 073006; Hannestad, Hansen, Tram, JCAP 1304 (2013) 032]

- A secret interaction in the sterile neutrino sector [Hannestad, Hansen, Tram, PRL 112 (2014) 031802; Dasgupta, Kopp et al, PRL 112 (2014) 031803, JCAP 1510 (2015) 011; Bringmann, Hasenkamp, Kersten, JCAP 1407 (2014) 042; Ko, Tang, PLB 739 (2014) 62; Archidiacono, Hannestad et al, PRD 91 (2015) 065021, PRD 93 (2016) 045004, JCAP 1608 (2016) 067, JCAP 2012 (2020) 029; Mirizzi, Mangano, Pisanti, Saviano, PRD 90 (2014) 113009, PRD 91 (2015) 025019; Tang, PLB 750 (2015) 201; Cherry, Friedland, Shoemaker, arXiv:1411.1071]

Conclusions

- Light massive neutrinos are Hot Dark Matter.
- Their effects on cosmological observables depend on their abundances and their masses.
- Cosmological data give information on neutrino physics, but it is model-dependent.
- Neutrino physics may contribute to solve tensions in the Cosmological data.
- Light sterile neutrinos are allowed only if their thermalization is suppressed.
- Heavy sterile neutrinos with mass of the order of keV can contribute to the Dark Matter (not discussed).