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Response of a purely dissipative incommensurate chain to large driving pulses

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Systems which obey purely dissipative equations of motion can exhibit behavior normally associated with inductive (i.e., inertial) response. An experimentally well-studied example is the transient current oscillations that can occur when a voltage pulse is applied to a sliding charge-density wave. Detailed numerical studies of the nonlinear response of a purely dissipative harmonic chain in a sinusoidal potential are presented in order to understand the effects on the transient current of strong pinning potentials and of sudden large changes in the driving field. The results are compared to the experimental results for the charge-density-wave systems $K_{0.3}MoO_3$, $NbSe_3$, and TaS_3.

I. INTRODUCTION

The dynamics of charge-density waves (CDW's) have been intensely studied for over a decade, and many experiments have shed light on their peculiar conduction properties.[1] The motion has been modeled using the nonlinear first-order differential equation[2]

$$\frac{du(\mathbf{r},t)}{dt} = -K\nabla^2 u(\mathbf{r}) + \frac{d\Phi(\mathbf{r}+\hat{\mathbf{z}}u(\mathbf{r},t))}{dz}\cos(\mathbf{Q}\cdot\mathbf{r}) + F(t) , \quad (1.1)$$

where $\mathbf{Q}$ in the CDW wave vector, $u(\mathbf{r},t)$ describes the distortion at position $\mathbf{r}$ and time t, $\mathbf{K}$ is the CDW elastic constant, Φ is a random pinning potential (presumably a result of impurities), and F is a time-dependent but spatially uniform force from an externally applied electric field. It has been demonstrated that this purely dissipative but spatially extended and nonlinear system in an inhomogeneous environment responds to applied fields in a manner usually associated with inertial systems.[3] One signature of the apparent inertia of the extended deformable model of a CDW is an oscillatory response to sudden changes in the driving voltage (or current), or "ringing." In Ref. 3 it was shown that when the CDW velocity is large, an arbitrarily small change in the voltage can induce an oscillatory current response. This theory indicates that in the limit of weak impurity pinning, application of a pulse results in oscillations for basically all starting configurations, as long as the final field is above the threshold field for CDW conduction. Suggestive experimental results have been reported for $K_{0.3}MoO_3$ (Ref. 4) as well as TaS_3 (Ref. 5) and $NbSe_3$ (Refs. 6–8), but in these experiments large voltage pulses were applied to pinned CDW's. When pulses were applied starting from steady-state configurations for fields well above threshold, no transient oscillations were observed.[7] These results were interpreted in terms of a "dephasing" picture in which the pinned CDW consists of domains that are all in relative minima of the pinning potential. As the field is turned on, initially these domains all oscillate at the narrow-band noise frequency *in phase*, but as the CDW moves, they lose phase coherence and the oscillations disappear. In this picture, one expects transient oscillations only if the initial CDW state is below threshold.

The purpose of this paper is to examine the relation between the perturbation theory of Ref. 3 and the dephasing picture. The two pictures are different visualizations of the same phenomenological equation of motion, so they are not necessarily incompatible. Since ringing has been observed for starting fields that are slightly above threshold,[7] simple dephasing is not a completely satisfying explanation of all the data.

The theory in Ref. 3 is only valid for small changes of the field well above the threshold field F_t, and it predicts oscillations that are extremely small, in this limit. Since the magnitude of the oscillations is roughly proportional to the typical size of the CDW deformations, the failure to observe ringing for starting fields well above threshold may merely reflect that the distortions induced by going below (or very near) threshold are much larger than those induced by starting at very large fields. Whether the starting state being pinned plays a vital role can be addressed by starting the system from strongly perturbed but random configurations and comparing the results with the current characteristic obtained when the system is started from the ground state in zero field. However, this question cannot be answered using a perturbative analysis. In addition, even the linear response near threshold is complicated to calculate[9] (involving high-order perturbation expansions), so that interpretation of the results is not straightforward. Since ringing experiments have been carried out over a large range of pulse amplitudes and final velocities, it is worthwhile to investigate these effects. This paper describes numerical investigations designed to address this situation.

For computer work it is advantageous to simulate a one-dimensional discrete model rather than the three-dimensional continuum model (Eq. 1.1), so many workers have examined variations of the coupled equations of motion[10–14]

$$d\phi_j/dt=\phi_{j+1}-2\phi_j+\phi_{j-1}-U\sin(\beta_j+\phi_j)+F(t)\ , \tag{1.2}$$

where ϕ_j can be interpreted as $Qu(R_j)$ at the position R_j of the jth impurity, U is the strength of the pinning potential, and F is the applied force which is assumed to be spatially uniform but time dependent. (The equation has been scaled so that the elasticity coefficient is unity.) The impurities tend to lock the phases in at random values—β_j. The assumptions necessary to obtain these discrete equations of motion from the continuum equation (1.1) are described in Refs. 10 and 11. Actually, Eqs. (1.1) and (1.2) differ not only because (1.2) is discrete, but also because (1.1) is written in the frame of reference of the CDW, while (1.2) is in the laboratory frame of reference. In addition, using a one-dimensional system rather than a three-dimensional one can substantially affect the results. These differences affect the oscillations (as calculated using linear response theory) in ways that are discussed in the Appendix. However, the gross features that are of primary interest for experiment are expected to be reproduced faithfully.

We study the response to a pulsed force: $F(t)=F_1+\Delta F\theta(t)$. The questions we wish to answer are the following.

(i) How do the oscillations change when the final force is near threshold?

(ii) How does increasing the pulse amplitude ΔF affect the oscillations?

(iii) How does the "ringing" described here relate to the "dephasing" picture described by Zettl,[6] Parilla and Zettl,[7] and Brown, Grüner, and Mihaly?[8]

Even using the simplified model (1.2), answering these questions involves substantial computational difficulties because any finite simulation exhibits narrow-band noise (NBN), or an oscillatory current response to a dc voltage. Distinguishing the transient oscillations from the steady-state one is difficult and tedious, since they both occur at the same frequency (as shown below). Therefore, in this work a modified model was used for which the NBN has a much smaller amplitude and a frequency different from that of the transients.

We consider the response of an overdamped extended harmonic chain in the presence of an incommensurate potential to a spatially uniform but time-dependent force. The (again, purely dissipative) equation of motion is given by Eq. (1.2) with a particular choice of the preferred phases β_j:

$$\beta_j=2\pi[\omega j\ (\mathrm{mod}\ 1)]\ , \tag{1.3}$$

where ω is an irrational number. Although real CDW's are almost certainly pinned by impurities, this incommensurate-pinning model appears to reproduce many features of the randomly pinned case.[9,15,16] The most important advantage here is that the transient oscillations can be distinguished from the NBN, which occurs at a much higher frequency. Also, the NBN amplitude is much smaller than that of a randomly pinned system (its size is typically 10^{-5} of the total current in these simulations, compared to about 20% for a comparable system with random pinning). Other advantages are that the numerical work proceeds more quickly than for a randomly pinned system and that finite-size effects can be characterized systematically and reliably.[15]

Thus, we will answer the questions outlined above for this incommensurate pinning model. The results indicate that nonlinear effects qualitatively alter the transient oscillations. Close to threshold, one often observes harmonics in the ringing. However, harmonic production appears to depend on details of the pinning potential. Using large force pulses enhances the oscillations substantially, especially if the system is initially pinned. It appears that this phenomenon reflects the fact that, in general, the ringing is much more pronounced when the configuration at the start of the pulse is far from steady state. Although large oscillations can be obtained by using arbitrary initial conditions, it appears that experimentally the only way to induce large oscillations is to start the pulse from a metastable configuration below threshold. Larger pulses can also enhance the production of harmonics, though again this may depend on details of the pinning potential. The oscillations discussed here are transients in the voltage-driven configuration and are unrelated to the dynamic oscillatory instability reported in the current-driven configuration by Sneddon and Cox.[17]

The paper is organized as follows. In Sec. II we discuss the oscillations that are present within linear-response theory well above threshold. This section uses methods very similar to those previously described for the randomly pinned model, and the results are very similar to those obtained in Ref. 3. Predictions of how the oscillations decay as a function as the commensurability [the number ω in Eq. (1.3)] is changed are also made. Section III describes the numerical results; the predictions of perturbation theory are verified where they apply, and nonperturbative features are investigated. Finally, in Sec. IV the results are summarized and their implications for experiment discussed. The Appendix discusses the approximations made while going from (1.1) to (1.3) and how they might limit applicability of this work to CDW experiments.

II. PERTURBATION THEORY

We first do an analytic perturbative calculation analogous to that performed previously for Eq. (1.1).[3] The perturbation theory performed for this system involves an expansion about a state with no deformations. This limit can be attained by making either the substrate potential U very small or the applied force F_1, and hence the steady-state velocity v, very large. Expanding to second order in U, the spatially averaged velocity response $\bar{v}(t)$ to the small field pulse $\Delta F\,\Theta(t)$ is

$$\bar{v}(t)=\lim_{N\to\infty}\left[\frac{1}{N}\sum_{j=1}^{N}v_j(t)\right]$$

$$=\Delta F\int_{-\infty}^{\infty}d\omega\frac{e^{i\omega t}}{i\omega-\Sigma(q=0,\omega)}\ , \qquad (2.1)$$

where

$$\Sigma(q,\omega)=\frac{U^2}{4}\left[\left[\frac{1}{i(\omega+v)-D(q-Q)}-\frac{1}{iv-D(Q)}\right]+(v\to -v,\ q\to -q)\right]\ , \qquad (2.2)$$

with $Q=2\pi\omega$. The quantity $D(k)=2[1-\cos(k)]$ describes the stiffness of the springs. The integral yields the spatially averaged velocity:

$$\bar{v}(t)=\Theta(t)\Delta F\left[1+\frac{U^2[v^2-D^2(Q)]}{2[v^2+D^2(Q)]^2}+\frac{U^2}{2[D^2(Q)+v^2]}e^{-\Gamma t}\cos(vt+\phi_0)\right]\ , \qquad (2.3)$$

with $\phi_0=2\tan^{-1}[v/D(Q)]$ and $\Gamma=D(Q)$ (with corrections of order U^2). Thus, within this perturbative linear-response theory, the velocity oscillates at the "washboard" frequency $\omega=v$. The rate of the decay is determined by $D(Q)$, which is smallest for Q near integral multiples of 2π. The oscillations are basically a Fourier transform of the frequency-dependent conductivity $\sigma(\omega)$, and Sneddon has shown that for certain ranges of parameters $\mathrm{Im}\sigma(\omega)$ exhibits inductive dips.[9,18] This feature reflects the fact that the entire CDW moves at velocity v, so the pinning potential causes internal excitations at that frequency. Because the system is purely dissipative, this means that the modes at frequency $\omega=v$ dissipate energy faster than those at other frequencies. Note that the decay rate Γ is smallest for nearly commensurate systems, where $1-\cos Q$ is very small.

In principle, extension of the perturbation theory to higher order in U is straightforward.[3] In analogy to the second-order results, one expects to obtain features in the frequency-dependent conductivity which lead to oscillations at frequencies $\omega\sim n\bar{v}$, decaying at a rate $\Gamma=D(nQ)$, for integral n.

III. NONLINEAR EFFECTS

The perturbation theory outlined above is only valid for small changes in the field well above threshold. By taking more terms in the perturbation expansion, one can study the effects of small changes in the field at low CDW velocities, provided the system is close to dynamic equilibrium. However, if the initial state is far away in configuration space from the equilibrium state at F_2, the perturbation series is not valid, even if infinitely many terms are taken. Since experiments involve large changes in the field, one is led to conduct numerical investigations of the ringing. This section discusses numerical calculations done to verify the results derived above in the regime where perturbation theory applies as well as to investigate nonperturbative effects.

A. Methods

The equations of motion (1.2) were integrated numerically by discretizing the time and using a second-order Taylor approximation with the time step adjusted so the positions were accurate to better than 10^{-6}. The initial conditions were chosen as described below, and periodic boundary conditions were employed. The simulations were performed on systems with 34 degrees of freedom, except for a few runs with 55 degrees of freedom to check for finite-size effects. All calculations were performed on a VAX-11/780 at AT&T Bell Laboratories.

B. Results

Since the effect of the pinning potential can be reduced by moving the chain at a higher velocity, in many calculations the parameter U was fixed at the value 4 and F_1 was varied. [The threshold field F_t for this value of U depends on Q; it ranges from 4 for $Q=0$ to ~ 1.2 for $Q=2\pi(\sqrt{5}+1)/2$.] We exhibit plots of the velocity as a function of time for different values of the parameters Q, F_1, and $\Delta F=F_2-F_1$.

We first examine the results in the regime where the perturbative calculation outlined in Sec. II is valid. This limit applies when F_1 is very large and ΔF is small. From Eq. (2.3) one expects the amplitude of the oscillations to be proportional to $U^2\Delta F/[v^2+D^2(Q)]$ and the decay rate Γ to be $D(Q)=2(1-\cos Q)$. Figure 1 compares numeri-

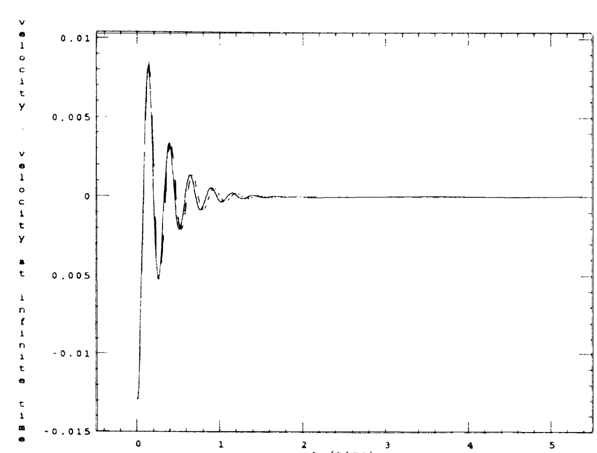

FIG. 1. Comparison of the perturbative expression (2.3) for the velocity response to an applied force (the dashed line) with the numerical result using $U=4$, $F_1=24$, $\Delta F=1$, and $Q=2\pi(55/34)$ (the solid line). The agreement is excellent.

cal results for the spatially averaged velocity as a function of time for $U=4$, $F_1=24$, $\Delta F=1$, and $Q=2\pi(55/34)$ to the second-order in U perturbative result (2.3). It is clear that in this regime the low-order perturbation theory accurately describes the numerical experiment. [The agreement could be improved even more by noting that the oscillation frequency is determined by the final velocity $v(F_2)$ rather than $v(F_1)$.] However, the amplitude of the oscillations here is extremely small ($<0.1\%$ of the mean velocity), so they would be unobservable experimentally.

Figure 2 consists of plots for $U=4$, $F_1=11$, $\Delta F=1$, and different Q values. Although the perturbation theory no longer fits the numerical results quantitatively, it still reproduces the major features of the oscillations. For this case, the amplitude of the oscillations is about 0.5% of the mean velocity.

We next consider the application of small voltage pulses fairly near threshold, where one expects a high-order perturbative expansion to be valid. Although we have not performed this high-order expansion, we qualitatively expect the features outlined above—oscillations at nth harmonics of the washboard frequency ($\omega=nv$), which decay at a rate $\Gamma=2(1-\cos nQ)$. This expectation is verified numerically: Fig. 3 shows the spatially averaged velocity as a function of time for parameter values $U=4$, $F_1=3.8$, $\Delta F=0.2$, and Q values $Q_n=2\pi\omega_n$, with $\omega_1=45/34$, $\omega_2=43/34$, and $\omega_3=41/34$. These values of Q are particularly favorable for observing the third, fourth, and fifth harmonics, respectively, since $D(3Q_1)=D(5Q_3)\sim 0.03$ and $D(4Q_2)\sim 0.14$. Comparison with Fig. 2 shows that the harmonics emerge as the velocity is lowered (this is expected since the higher-order terms in the perturbation theory are larger for low velocity). The observed decay is consistent with the expectations outlined above.

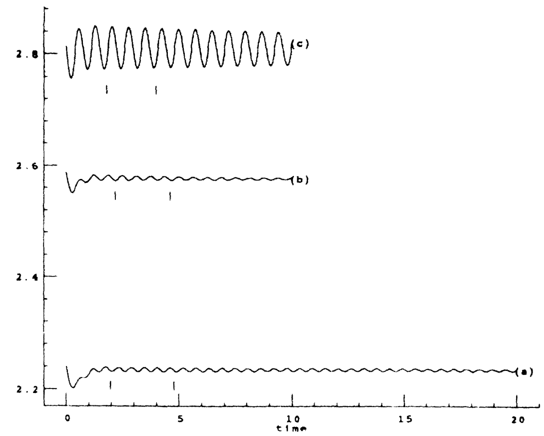

FIG. 3. Spatially averaged velocity versus time after a small change in applied force ($F_1=3.8; F_2=4.0$) for (a) 34 balls, 11 kinks [so $D(3Q)\sim 0$], (b) 34 balls, 9 kinks [so $D(4Q)\sim 0$], and (c) 34 balls, 7 kinks [so $D(5Q)\sim 0$]. The curves are not offset vertically. The vertical lines under each curve denote the time interval $T=2\pi/\bar{v}$, the "washboard" period. The velocity exhibits oscillations at nth harmonics of the washboard frequency $\omega=v$, where $D(nQ)$ is small.

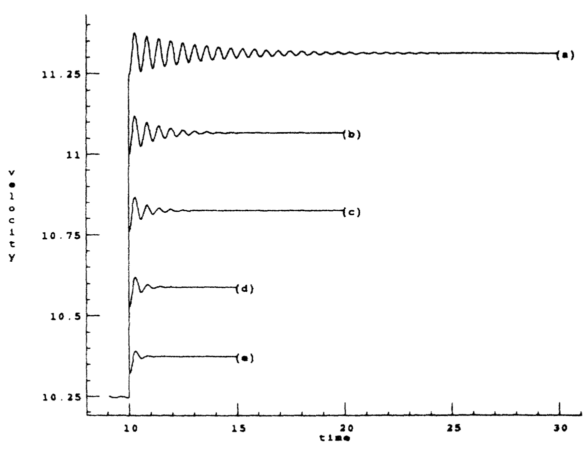

FIG. 2. Spatially averaged velocity versus time after a small change in applied force for (a) 34 balls, 3 kinks [$Q=2\pi(37/34)$], (b) 34 balls, 5 kinks, (c) 34 balls, 7 kinks, (d) 34 balls, 9 kinks, and (e) 34 balls, 21 kinks. All simulations were done with potential strength $U=4$, starting field $F_1=11$, and final field $F_2=12$. The curves are offset vertically by $v=0.25$. The results are described extremely well qualitatively by the perturbative prediction [Eq. (2.3)].

Finally we examine the nonperturbative regime where the starting configuration is far from the final configuration, so that an expansion in powers of small displacements breaks down. One way to make a large change in the configuration is to apply a large pulse ΔF to a steady-state configuration. Figure 4 shows velocity oscillations for $U=4$, $Q=2\pi(55/34)$, and final field $F_2=F_1+\Delta F=12$ for different values of F_1 both above and below the threshold force F_t.[19] The oscillations are much more pronounced if F_1 is below threshold than if F_1 is very large. However, it is not clear whether the difference is that the

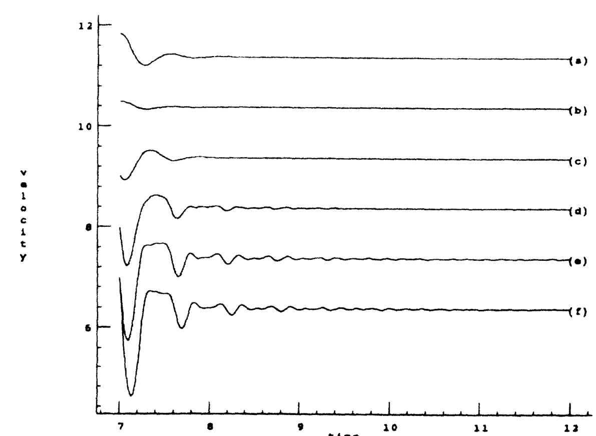

FIG. 4. Oscillations for $Q=2\pi(55/34)$, final field $F_1=12$, and potential strength $U=4$ (so $F_t\sim 1.2$) starting from steady state with (a) $F_1=50$, (b) $F_1=15$, (c) $F_1=6$, (d) $F_1=1.5$, (e) $F_1=1$, and (f) $F_1=0$ (ground state). The curves are offset vertically by $v=1$.

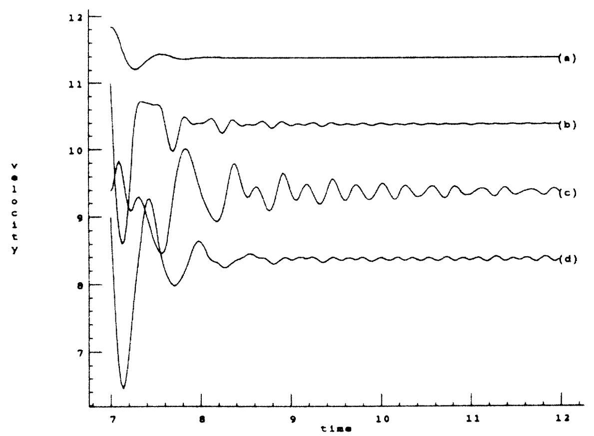

FIG. 5. Oscillations for $Q=2\pi(55/34)$, final field $F_1=12$, and potential strength $U=4$, starting from (a) the steady state for $F_1=50$, (b) the ground state for $F_1=0$, (c) a configuration with the ϕ_j's randomly chosen in the interval $[-2\pi, 2\pi]$, and (d) the metastable state reached by allowing state (c) to evolve with $F_1=0$ for a time interval $t=7$. The curves are offset vertically by $v=1$.

starting and ending configurations are farther apart in configuration space if F_1 is below threshold compared to F_1 very large, or if the large oscillations occur only if the starting configuration is pinned.

In order to investigate whether any large perturbation causes large oscillations or if starting from a metastable configuration is necessary, the following numerical experiment was conducted. First, a configuration was constructed in which the initial distortions $\phi_j(t=0)$ were randomly chosen in the interval $[-2\pi,2\pi]$, and this state was allowed to evolve in time according to Eq. (1.2) with parameter values $F_2=12$ (F_1 is irrelevant here), $U=4$, and $Q=2\pi(55/34)$. Then, the same configuration was allowed to relax in zero field for a time interval $T_1=7$ before the force was applied. The system then has a chance to conform to the impurity potential, which would enhance the oscillations greatly if they were a result of dephasing only. These two velocity traces are compared to the results using the $F=0$ ground-state and $F=50$ steady-state configuration as starting conditions in Fig. 5. The high-field steady-state configuration exhibits much smaller oscillations than any of the other cases, but the oscillations for the random configuration are about as large as those for the metastable states.

IV. DISCUSSION

We have examined the response of an incommensurate harmonic chain to voltage pulses. The response to small pulses is well described by a perturbation expansion for small deformations of the chain, and application of large pulses from pinned configurations dramatically enhances the current oscillations.

The fact that the ringing is much more pronounced when the starting configuration is a metastable state below threshold than when it is a high-field steady state is consistent with the suggestion that the transient oscillations can be viewed at a dephasing of the narrow-band noise, which starts off with an induced coherence from the pinning and then gradually becomes incoherent.[6–8] This point of view is compatible with the ideas presented here, if only because both pictures arise from the *same* model consisting of many elastically coupled degrees of freedom in an external potential. However, the treatments presented here and in Ref. 3 make it clear that theoretically this model leads to ringing even when the starting state is totally dephased (e.g., a uniform state). The experimental observation of no ringing when the field is decreased suddenly is consistent with the numerical result that starting the CDW from a nearly uniform state involves a much smaller perturbation than starting it from a metastable configuration. Experimentally, it appears the only way to induce the large distortions necessary to see sizeable oscillations is to start the CDW below threshold. Sudden temperature changes could be used to change the CDW steady-state configuration substantially, but it is unlikely that they could be made quickly enough to see the ringing response.

A major question is whether these results apply without major modification to randomly pinned systems. A few simulations using randomly pinned systems were conducted, and though some enhancement of the transients was seen, the results appear to depend in nontrivial ways on the system size, the realization of the impurity configuration, and the starting configuration. At this stage it seems that generalization of the conclusions to randomly pinned systems is quite nontrivial, with the results appearing to depend on details of the impurity potential. Further work is clearly called for to elucidate this issue.

In principle, the work described here could be used to determine whether coupling to the lattice is at all substantial for sliding CDW systems. For instance, in TaS_3 the CDW is very nearly commensurate [i.e., ω, in Eq. (1.3) is very nearly 1/4]. One would expect the nearly commensurate pinning potential to induce anomalously large transient oscillations at a frequency $\omega=4v$. This oscillation would occur even in the presence of random impurities, which would induce oscillations at $\omega=v$ and harmonics. However, observation of this phenomenon must be considered extremely unlikely, since TaS_3 appears to lock in at low temperatures, and one would also expect to observe narrow-band noise at this frequency in the commensurate phase. This narrow-band noise has never been observed experimentally.

Since the deformable model used to describe the CDW here is very similar to that used to describe the dynamics of flux lattices in type-II superconductors,[20] some of the considerations described here may also be relevant to that experimental system.

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APPENDIX

This appendix discusses the approximations involved in going from Eq. (1.1) to Eq. (1.2) (Ref. 12) and the differences between random pinning and incommensurate pinning.

Going from (1.1) to (1.2) involves a discretization of the spatial gradient (which one expects to be unimportant for long-wavelength excitations) as well as changing from a three-dimensional system to a one-dimensional one and a modification of the impurity pinning term [the distortion $u(\mathbf{r})$ enters differently in the two equations]. The change in dimensionality and the difference in the pinning terms can affect the transient oscillations in a manner that can be examined within perturbation theory.

We first discuss the difference in the pinning term. Assume that the impurity potential is rigid, and that the CDW is deformable. In the frame of reference that is comoving with the CDW, the charge density $\rho(\mathbf{r})$ is just $\cos\mathbf{Q}\cdot\mathbf{r}$, but the bit of the CDW at point $\mathbf{r}$ "sees" the impurity potential Φ at the actual (distorted) position, so the impurity force $U(\mathbf{r})$ is $-d\Phi(\mathbf{r}+u(\mathbf{r}))/dz$. This argument yields Eq. (1.1).[2,18,20] Now compare with Eq. (1.2), where the deformation $u(\mathbf{r})$ appears inside the cosine term.[21] One can try to derive a three-dimensional analog of (1.2) by considering a coordinate system that is fixed in the frame of the impurities. The impurity potential is $\Phi(\mathbf{r})$, but the charge density at point $\mathbf{r}$ is $\rho_0\cos(\mathbf{Q}\cdot\mathbf{r}')$, where $\mathbf{r}'$ is determined by the condition $\mathbf{r}'+u(\mathbf{r}')\hat{\mathbf{z}}=\mathbf{r}$. In general, this is a complicated nonlinear equation for $\mathbf{r}'$. However, if the CDW is very stiff, so only very long-wavelength excitations are important, then $\mathbf{r}'$ is expected to be very nearly $\mathbf{r}-u(\mathbf{r})\hat{\mathbf{z}}$ and the two equations are equivalent.

Although the two equations have the differences outlined above, no unphysical artifacts are expected to result from using Eq. (1.2), since it describes the real physical system of a chain connected by springs of random length with equal stiffness in a sinusoidal potential.

The use of a one-dimensional equation rather than a three-dimensional one has fairly dramatic consequences for the perturbative results. The first concerns the validity of the perturbation theory. For Eq. (1.2), if one calculates $\phi_j(t)$ to fourth order in powers of U, one finds the result contains integrals of the form $\int d^dq/(Aq^4+\omega^2)$, even when v is finite. These integrals diverge at small q and $\omega=0$ for $d\le 4$. Clearly, the divergence becomes significantly stronger as d decreases. For Eq. (1.1) the divergence is weaker; the relevant integral is $d^dq/(q_z^2+Aq^4)$, which diverges for $d\le 3$. Therefore, strictly speaking, the perturbation theory is merely a qualitative guide rather than a quantitative calculation of the oscillations. Fisher[22] has discussed the role of dimensionality in (1.2) within perturbation theory.

Ignoring the subtleties of the perturbation theory's convergence, one can compare the perturbative predictions for the two equations (1.1) and (1.2), allowing for variations in the dimensionality d. We will assume that the impurity potential is Gaussian distributed, so that

$$\left[\frac{1}{2\pi}\right]^d \int d^dq\, e^{i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{r}}\Phi(\mathbf{q})\Phi(\mathbf{q}')=\Phi_0^2\delta(\mathbf{q}-\mathbf{q}') \quad (\mathrm{A1})$$

holds. The voltage oscillations in response to a current pulse are still determined by Eq. (2.1). Equation (1.1) leads to a self-energy $\Sigma(\mathbf{q},\omega)$ of the form[3,18]

$$\Sigma_1(\mathbf{k},\omega)=\frac{|\Phi(Q)|^2Q^4}{(2\pi)^d}\int d^dq[G(\mathbf{k}+\mathbf{q},(q_z-Q_z)v+\omega)-G(\mathbf{q},(q_z-Q_z)v)]\ , \quad (\mathrm{A2})$$

with $G^{-1}(\mathbf{k},\omega)=i\omega+K(\mathbf{k})^2$, whereas Eq. (1.2) yields

$$\Sigma_2(\mathbf{k},\omega)=|U(Q)|^2(2\pi)^{-d}\int d^dq[G(\mathbf{k}+\mathbf{q},\omega-v)-G(\mathbf{q},-v)]\ , \quad (\mathrm{A3})$$

with $G^{-1}(\mathbf{k},\omega)=i\omega+D(k)$. If long-wavelength distortions dominate, the cosine term implicit in (A3) can be expanded; the two expressions are identical up to rescaling of v and K in (A3) except when $Q_zv\cong\omega$, where $G^{-1}(\mathbf{k}+\mathbf{q},(q_z-Q_z)v+\omega)$ goes like Kq^2+q_zv rather than Kq^2. The two expressions coincide except in a region of phase space where $Kq^2<vq_z$, whose volume vanishes as $K\rightarrow\infty$. The physical arguments given above lead one to expect this. However, for finite K, the additional term in (A1) has the effect of reducing the effectiveness of the very long-wavelength modes. Since the decay rate of the oscillations arising from scattering off potential components with wave vector q increases as Kq^2, suppressing the long-wavelength contributions inhibits ringing.

We will assume that the characteristic length scale of deformation is much greater than the CDW wavelength. The integral (A3) can be done explicitly and the time dependence as $t\rightarrow\infty$ can be obtained by scaling the resulting form for the current response (2.3); for $\mathbf{v}$ along $\mathbf{Q}$ one finds

$$\bar{v}(t)\sim\begin{cases}|U(\mathbf{Q})|^2\mathbf{A}_1t^{-d/2}\sin(vt+\phi_1)\ , & d>2\ ,\\ |U(\mathbf{Q})|^2A_2t^{-(2-d/2)}\sin(vt+\phi_2)\ , & d<2\ ,\end{cases} \quad (\mathrm{A4})$$

where the A_i and ϕ_i are constants of order unity. The voltage response to a current pulse resulting from the self-energy (A2) is not always oscillatory; for small CDW stiffness K, a smooth decay after an initial overshoot is found, as discussed in Ref. 3 for current driving. A measure of the stiffness is the Lee-Rice length[23] $L_0\sim(K/\Phi_0)^{2/(4-d)}$. In $NbSe_3$ L_0 is much greater than the inverse of the CDW wave vector Q^{-1}, so well-defined oscillations are expected. The dependence on the dimensionality d and the change of character when $d=2$ indicates that 1-d models cannot be used to quantitatively fit CDW's with three-dimensional fluctuations. However, 1-d chains are good qualitative guides to CDW behavior.

We now examine how the incommensurate pinning results compare to those for the random models. We will restrict ourselves to one dimension. The incommensurate pinning model here involves scattering off a potential with components only at Q and $-Q$. In the sense that low-frequency components of the impurity components are missing altogether, this model mimics the suppression of the effects of the $q\rightarrow 0$ modes that occurs for the correct equation of motion with random impurities that is incorrect in Eq. (A3). Whether ringing occurs within perturbation theory depends on how the oscillation decay rate

$D(Q)$ compares to the oscillation frequency v.

The physical process giving rise to the transient oscillations is anomalous dissipation for modes whose frequency corresponds to the "washboard" frequency $\omega_0=\mathbf{Q}\cdot\mathbf{v}$ for (1.1) and $\omega_0=v$ for (1.2). This is true for both random equations of motion as well as the incommensurate case. Therefore, it is not unreasonable to hope that the nonlinear effects found for the incommensurate chain provide a good guide to the behavior of a randomly pinned CDW.

The discrete incommensurate model, because the pinning is so regular, does not seem to develop large polarizations ("bubbles") that are a hallmark of the randomly pinned system.[12,24,25] However, this difference does not seem to effect the oscillations in any fundamental way, because in a unipolar pulse sequence the polarization, once it is built up, does not change much.[12,26] The whole CDW moves at basically the same velocity, so the enhanced dissipation at the washboard frequency that leads to the oscillations still operates. Therefore, one expects to see qualitatively similar phenomena, though the perturbation theory is relevant only at extremely high fields when the polarizations are small.

[1]See, e.g., *Proceedings of the International Conference on Charge-Density Waves in Solids*, Vol. 217 of *Lecture Notes in Physics* (Springer-Verlag, Berlin, 1985).
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[19]The parameters chosen for this simulation are not physically realistic, but the incommensurate pinning model is sufficiently different from the randomly pinned one, that in any case only the qualitative result that large perturbations enhance the transient oscillations should be taken seriously (see the Appendix).
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