Finite-temperature symmetry breaking in an anisotropic universe

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We calculate the critical temperature for the restoration of symmetry for a self-interacting scalar field. The symmetry is spontaneously broken either by a negative mass or by its coupling to negative scalar curvature in a homogeneous, anisotropic spacetime.

I. INTRODUCTION

The early Universe is a physical environment where both the quantum properties of matter and of gauge particles and their coupling to gravity must be included in the analysis of physical phenomena. The early Universe is also characterized by nonzero temperature and dynamical evolution in which nonperturbative field configurations arise and multiple stages of phase transitions may occur. Thus to describe physical phenomena here a formulation of quantum field theory in curved spacetime is needed as a first approximation to the ultimate theory of quantum gravity unified with all other interactions. Moreover for applications to the early Universe this formulation must be extended to a statistical quantum field theory in curved spacetime.

Homogeneous but anisotropic spacetimes can be used to model the early Universe possessing a deformation degree of freedom after the Planck era (10^{-43} sec). Quantum fields on a nonmaximally symmetric spacetime background present us with an intricate symmetry structure. At the classical level the scalar curvature acts as an effective mass of the field and thus influences the phase transitions of the system.

In this paper we investigate the symmetry behavior of a self-interacting scalar field in a static homogeneous anisotropic spacetime (Taub) at finite temperature. The zero-temperature theory in Taub spacetime has been considered in Ref. 1, the finite-temperature field theory in flat spacetime has been investigated in Ref. 2, the finite-temperature field theory in static isotropic spacetimes in Ref. 3, and the finite-temperature theory in dynamic spacetimes in Ref. 4. A more complete list of references on finite-temperature field theory in curved spacetimes can be found in Ref. 4.

The spacetime is described by the Taub metric which is an anisotropic generalization of the closed Robertson-Walker metric and corresponds to a mixmaster space with two of its three principal radii of curvature equal. The spacetime background is treated classically since its quantum nature will only appear near the Planck time. The scalar field, on the other hand, is treated as a quantum field at scales much larger than the Planck length. Here we assume that the scalar field is in thermal equilibrium at temperature T, which is the central temperature of the temperature ellipsoid as defined in Ref. 5 for anisotropic spacetimes. The Taub spacetime is assumed static, i.e., a frozen mixmaster universe. For a dynamic spacetime a quasithermal equilibrium can be maintained for slowly varying background and sufficient strong field coupling and interaction.

A possible physical application is during the period before the onset of inflation [grand-unified-theory (GUT) scale: 10^{-35} sec] when the Universe could have been in an anisotropic state. An analogous physical situation with similar mathematical formulation used here is shape transitions of nuclei at finite temperature.[6] In this analogy the deformations that characterize the shape of nuclei correspond to deformations of the Taub space which are parametrized by the variable a. As the temperature is varied different shapes of the nucleus become stable.

Another relevant problem is when the Taub space is considered as an internal space in higher-dimensional unification theories,[7] where the four-dimensional gauge symmetries correspond to isometries of the internal space. Then the critical temperature corresponding to the transition from a deformed to a spherical space signals the restoration of an unbroken gauge symmetry. Finite-temperature Kaluza-Klein theories are treated in Ref. 8. Notice that this goes beyond previous works in which only spheres were considered as internal spaces.[9] Here the anisotropic internal space leads to a more realistic gauge-symmetry group.

The paper is organized as follows. In Sec. II we set up the formalism for evaluating the effective potential and we find the critical temperature for symmetry restoration. The relevant calculations are detailed in the two appendixes.

II. EFFECTIVE POTENTIAL

We want to calculate the effective potential for the background field in an anisotropic spacetime at finite temperature due to the quantum scalar-field fluctuations, in the one-loop approximation. We will use Hawking's zeta-function method.[10] Our aim is to find the finite-temperature contribution to the effective mass of the background field.

The background geometry has the Taub space form with the metric

$$ds^2=-dt^2+\sum_{a=1}^{3} l_a^2(\sigma^a)^2 \ . \tag{2.1}$$

Here $l_1=l_2\neq l_3$ and the σ^a's form a basis one-form on S^3 satisfying the structure relations $d\sigma^a=\frac{1}{2}\epsilon^a_{bc}\sigma^b\wedge\sigma^c$. In the Euler-angle parametrization[11] ($0\leq\theta\leq\pi$, $0\leq\phi$, $\psi\leq z\pi$) the σ^a's are given by

$$\sigma^a=\cos\psi\, d\theta+\sin\psi\sin\theta\, d\phi\ ,$$
$$\sigma^b=-\sin\psi\, d\theta+\cos\psi\sin\theta\, d\phi\ , \quad (2.2)$$
$$\sigma^c=d\psi+\cos\theta\, d\phi\ .$$

The l_a's are the three principal curvature radii of the homogeneous space and are constants for a static universe. The general case with three different l_a's corresponds to the diagonal mixmaster space. The case when all three l_a's are equal is the closed Friedmann-Robertson-Walker (FRW) universe.

The homogeneous anisotropic background geometry depends on the scale a and the deformation a defined by

$$a=2l_1,\quad \alpha=(l_1/l_3)^2-1\ . \quad (2.3)$$

The range of α is $-1<\alpha<\infty$ and $\alpha=0$ corresponds to the "round" S^3. We call the configuration with $\alpha>0$ oblate and that with $\alpha<0$ prolate. The curvature scalar of the geometry (2.1) can be reduced to[12]

$$R=\frac{4l_1^2-l_3^2}{4l_1^2}=\frac{6(1+\frac{4}{3}\alpha)}{a^2(1+\alpha)}\ . \quad (2.4)$$

The volume of the Taub space is given by

$$\Omega=\frac{2\pi^2a^3}{\sqrt{1+\alpha}}\ . \quad (2.5)$$

A massive (m) scalar field $\tilde{\phi}$ with quartic self-interaction (λ) coupled to a static Taub space is described by the Lagrangian density

$$\mathcal{L}[\tilde{\phi},g_{ab}]=-\frac{1}{2}\tilde{\phi}\left[-\Box+m^2+(1-\xi)\frac{R}{6}\right]\tilde{\phi}-\frac{\lambda}{4!}\tilde{\phi}^4\ , \quad (2.6)$$

where

$$\Box=g^{\mu\nu}\nabla_\mu\nabla_\nu=\frac{1}{(-g)^{1/2}}\frac{\partial}{\partial x^\mu}\left[(-g)^{1/2}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right] \quad (2.7)$$

is the Laplace-Beltrami operator on $R^1\times S^3$ and the coupling constant $\xi=0,1$ denotes conformal and minimal coupling, respectively.

The action has a minimum of $\tilde{\phi}=\phi$, which is a solution of the classical equations of motion. We consider quantum fluctuations h around the classical background field ϕ and determine the effective action due to these fluctuations

$$e^{-\Gamma[\phi,g_{ab}]}=\int[dh]e^{-S[\phi+h,g_{ab}]}\ . \quad (2.8)$$

When $\mathcal{L}[\phi+h,g_{ab}]$ is expanded about the background field, the one-loop effects will be governed by those terms which are bilinear in the quantum field h. We denote the corresponding Lagrangian by L:

$$L=-\frac{h}{2}\left[-\Box+m^2+(1-\xi)\frac{R}{6}+\frac{1}{2}\lambda\phi^2\right]h\ . \quad (2.9)$$

The quantum field h satisfies, to the lowest order in h, the equation

$$Ah=0\ , \quad (2.10)$$

where A is the operator

$$A=-\Box+\mathcal{M}^2 \quad (2.11)$$

with

$$\mathcal{M}^2=m^2+(1-\xi)\frac{R}{6}+\frac{1}{2}\lambda\phi^2\ . \quad (2.12)$$

Now the effective action which is related to the effective Lagrangian $\mathcal{L}_{\text{eff}}$ by

$$\Gamma[\phi,g_{ab}]=\int d^4x\sqrt{-g}\,\mathcal{L}_{\text{eff}} \quad (2.13)$$

is expanded in powers of h:

$$\Gamma[\phi]=S[\phi]+\Gamma^{(1)}+\Gamma'\ . \quad (2.14)$$

Here $S[\phi]$ is the classical action

$$S[\phi]=\int d^4x\sqrt{-g}\,\mathcal{L}^{(0)}[\phi]\ , \quad (2.15)$$

with

$$\mathcal{L}^{(0)}[\phi]=-\frac{1}{2}\phi\left[-\Box+m^2+(1-\xi)\frac{R}{6}\right]\phi-\frac{\lambda}{4!}\phi^4\ . \quad (2.16)$$

$\Gamma^{(1)}$ is the one-loop effective action

$$\Gamma^{(1)}=\frac{i\hbar}{2}\ln\text{Det}(\mu^{-2}iA)\equiv\int d^4x\sqrt{-g}\,\mathcal{L}^{(1)}\ . \quad (2.17)$$

Here A is the operator encountered in (2.11) and Γ' denotes higher-loop contributions. The mass scale μ is needed to render the measure $d[h]$ in (2.8) dimensionless. For a static homogeneous spacetime where ϕ is a constant field we can define the effective potential by $V(\phi)=-(\text{vol})^{-1}\Gamma(\phi)$ where vol denotes the spacetime volume.

For an interacting scalar field the classical potential is

$$V^{(0)}(\phi)=\frac{1}{2}m^2\phi^2+(1-\xi)\frac{R}{12}\phi^2+\frac{\lambda}{4!}\lambda\phi^4\ . \quad (2.18)$$

We are interested in the case of broken symmetry. This means that the coefficient of ϕ^2 in the classical is negative. For example, for a massless field this requires $R<0$; that is, α must be in the range $-1<\alpha<-\frac{3}{4}$. The question we want to address is at what temperature the symmetry is restored such that $\phi=0$ becomes the energetically preferred state of the system.

We want now to calculate the one-loop finite-temperature contribution to the effective mass of the background field. We deal with renormalized parameters throughout and we assume that the form of possible finite-temperature divergences is the same as those of the zero-temperature theory.[2]

We evaluate using Hawking's zeta-function[10] method which gives finite results (no poles present):

$$V_\beta^{(1)} = -\frac{\hbar}{2(\mathrm{vol})}\zeta'(0) \tag{2.19}$$

with

$$\zeta(\nu) = \sum_M (\mu^{-2}\lambda_M)^{-\nu} ,$$

where M labels the eigenvalues λ_M of the fluctuation operator and vol$=\beta V$ with V the volume of the Taub space. Here λ_M are the eigenvalues of the operator A on the Euclideanized metric obtained from (2.1) by a Wick rotation to imaginary time $\tau = it$. The finite-temperature T is introduced by imposing periodic boundary conditions on τ with period $\beta = T^{-1}$. The topology of the spacetime is now $S^1 \times S^3$. The eigenvalues of the operator A in the Euclideanized Taub space with topology $S^1\times S^3$ are[13]

$$\lambda_M = \left[\frac{2\pi p}{\beta}\right]^2 + \frac{J(J+1)}{l_1^2} + \left[\frac{1}{l_3^2} - \frac{1}{l_1^2}\right]K^2 + m^2 + (1-\xi)\frac{R}{6} + \frac{\lambda}{2}\phi^2 . \tag{2.20}$$

Here $M = \{p,J,K,L\}; p = 0,1,\ldots; J$ takes all values of positive integers and half integers and $K,L = -J, -J+1,\ldots,J-1,J$. Notice λ_M has an L degeneracy, and the first term in λ_M comes from the S^1 factor. We make the following redefinitions where the discrete variables n and q take only integer values:

$$n = 2J+1, \quad q = J-k ,$$
$$\sigma = ma^2 + \tfrac{1}{6}(\xi - 1)Ra^2 + \tfrac{1}{2}\lambda\phi^2 a^2 - 1 \tag{2.21}$$

with $a = 2l_1, \alpha = (l_1/l_2)^2 - 1$. We then have

$$\lambda_M = [n^2 + \sigma + \alpha(n-1-2q)^2]/a^2 + \left[\frac{2\pi p}{\beta}\right]^2 \tag{2.22}$$

and

$$\zeta(\nu) = \sum_M (\mu^{-2}\lambda_M)^{-\nu} = (\mu)^{2\nu}\sum_M \lambda_M^{-\nu} = (\mu)^{2\nu}\sum_p \sum_{n=1}^{\infty}\sum_{q=0}^{n-1}\left[[n^2+\sigma+\alpha(n-1-2q)^2]/a^2 + \left[\frac{2\pi p}{\beta}\right]^2\right]^{-\nu} . \tag{2.23}$$

To evaluate the finite sum we will use the Plana formula (see Appendix A). We restrict our calculations to the prolate configuration $\alpha < 0$ since in this case the curvature can be negative and thus the scalar field can be in the symmetry-breaking state ($\xi > 0$). Also the ($p = 0$) mode will give a subdominant contribution ($\sim 1/\beta$) to the effective potential at high temperatures and can be neglected. We are interested in a high-temperature expansion and the terms we keep to calculate the effective mass are proportional to higher powers of $1/\beta$ than the ($p=0$)-mode contribution.

We find, from the calculation in Appendix A,

$$\zeta(\nu) = \zeta(\nu)_1 + \zeta(\nu)_2 , \tag{2.24}$$

where

$$\zeta(\nu)_1 = (a\mu)^{2\nu}\int_0^1 dy\, M^{-\nu}\sum_p F \tag{2.25}$$

with

$$F(y) = \sum_n n^2(n^2 + A^2)^{-\nu} ,$$

$$M(y) = 1 + \alpha(1-2y)^2 ,$$

$$A^2 = \frac{\sigma}{M} + \frac{a^2}{M}\left[\frac{2\pi p}{\beta}\right]^2 , \tag{2.26}$$

and

$$\zeta(\nu)_2 = (2i)(\mu a)^{2\nu}(1+\alpha)^{-\nu}\sum_p \int_0^\infty \frac{dy}{e^{2\pi y}+1} G \tag{2.27}$$

with

$$G(y) = \sum_n n\{[(n+iB)^2 - E^2]^{-\nu} - [(n-iB)^2 - E^2]^{-\nu}\} ,$$

$$B = \frac{2ya}{1+\alpha} , \tag{2.28}$$

$$E^2 = C^2 - \frac{\sigma}{1+\alpha} - \frac{a^2}{1+\alpha}\left[\frac{2\pi p}{\beta}\right]^2 ,$$

$$C^2 = \frac{4ay^2}{(1+\alpha)^2} .$$

To evaluate $F(y)$ we express it in the form of a Sommerfeld-Watson integral (see Appendix A),

$$F(y) = \frac{i}{2}\int_C dz\, z^2(z^2 + A^2)^{-\nu}\cot(\pi z) , \tag{2.29}$$

and we find

$$F(y) = -\sin(\pi\nu)\left[\frac{(-1)^\nu}{2}A^{3-2\nu}\frac{\Gamma(1-\nu)\Gamma(\nu-\frac{3}{2})}{\Gamma(-\frac{1}{2})} + 2\int_A^\infty dx\, x^2\frac{(A^2-x^2)^{-\nu}}{e^{2\pi x}-1}\right] . \tag{2.30}$$

Next we evaluate $G(y)$ in a similar fashion as $F(y)$ (Appendix A). The result is given by (A8).

For a second-order phase transition the effective mass vanishes at the critical temperature. We want now to calculate the dominant contribution for high temperature to effective mass. From the calculations in Appendix B we find that only the first term in $\zeta(0)_1$ is relevant. All other terms give subdominant contributions. We find

$$V^{(1)}_{\beta(m)}=\frac{\hbar\lambda}{4}\zeta(-1)\left[\frac{a}{V}\right]\left[\frac{2\pi a}{\beta}\right]^2 I\phi^2 . \qquad (2.31)$$

With $V=2\pi^2a^2/(1+\alpha)^{1/2}$, the volume of the Taub space, and $I(\alpha)$ depends on α, its explicit form given in Appendix B. The subscript m denotes contribution to the effective mass term of V_β and is positive. Thus the finite-temperature correction to the effective mass can lead to symmetry restoration. Here we are assuming that the classical mass $m^2_{\rm cl}=m^2+(1-\xi)R/6$ is negative. Notice that the scale μ does not appear at this order. It will appear at higher order (in β) terms which involve a product of a lower power of $1/\beta$ with a logarithmic factor. The μ dependence will reside in the argument of the logarithm. Thus to leading order in $1/\beta$ the mass scale μ does not appear in $V^{(1)}$. However, the total energy $V^{(0)}+V^{(1)}$ should be μ independent. Including the classical contribution to the effective mass we have, for the total energy,

$$V_{\beta(m)}=\left[\tfrac{1}{2}m^2+(1-\xi)\frac{R}{12} - \frac{\hbar\lambda}{4}\zeta(-1)\left[\frac{a}{V}\right]\left[\frac{2\pi a}{\beta}\right]^2 I\right]\phi^2$$

$$\equiv \tfrac{1}{2}m^2_{\rm eff}\phi^2 \qquad (2.32)$$

with

$$R=\frac{6}{a^2}\frac{1+\frac{4}{3}\alpha}{1+\alpha}$$

the curvature. We can find now the critical temperature by setting $m^2_{\rm eff}=0$, when we have symmetry breaking at the classical level either due to negative mass ($m<0$) or to negative curvature ($R<0$). Notice $\beta_{\rm crit}\sim\lambda^{1/2}$ which will be small for weak coupling λ. Thus high temperature is a consistent approximation. From our result we can see the interplay of curvature effects and finite temperature. Negative curvature leads to symmetry breaking ($\xi>0$) and finite temperature leads to restoration of the broken symmetry.

III. CONCLUSION

We have found that a self-interacting scalar in a broken-symmetry state due to its coupling to the curvature of the Taub space can be restored to the symmetric state if it is heated to a high enough temperature. The dependence of the critical temperature on the radius a and the deformation parameter α can be deduced by setting $m_{\rm eff}=0$ in Eq. (2.32). Notice that as we approach the transition point ($\alpha=-\frac{3}{4}$) from the prolate to the oblate configuration the critical temperature tends to zero since the curvature becomes positive. The critical temperature also tends to zero as we approach the extreme prolate configuration ($\alpha=-1$) because the factor $I(\alpha)/V(\alpha)$ grows faster than $R(\alpha)$.

To investigate what type of phase transition occurs we need to calculate higher-power contributions of the background field to the effective potential. These additional contributions can be deduced from the expressions in the appendixes. In particular we will need to include higher-order contributions than $1/\beta^2$ to determine higher-power contributions than ϕ^2 to the effective potential.

These higher-power terms of the effective potential become relevant, for example, when we want to examine the order of the phase transition at the critical temperature and thus deduce its applicability to the early Universe, i.e., if it produces inflation consistent with observation. We know that an inflationary phase can happen if the system stays in a false vacuum for a long time (in the Hubble time scale). There are two ways to achieve this: (a) a barrier between the true vacuum and the false vacuum (old scenario),[14] (b) a slow evolution of the state from the false vacuum to the true vacuum (new scenario).[9] If the phase transition is of second order neither condition can be satisfied and inflation will not occur.

In the higher-dimensional unification context we can consider the seven-dimensional spacetime $R^3\times$Taub $\times S^1$. Here we consider a free scalar field ($\lambda=0$) in the seven-dimensional space (also zero background scalar field) whose vacuum fluctuations support the background spacetime via the Casimir effect. Now the effective potential $V(\alpha,T)$ can be easily obtained by summing over the infinite tower of massive states that the scalar field generates when observed in the four-dimensional spacetime.[7] The only difference from the expression (2.23) is that $\zeta(\nu)$ in this case is multiplied by a function of ν and the exponent in the summation changes from $-\nu$ to $-\nu+\frac{3}{2}$. At different temperatures T the minimum of V will shift to different values of α, i.e., $\alpha_{\rm min}(T)$. Since the coupling constant depends on a (Ref. 15) we will have variation of the coupling constant of the gauge theory with temperature. There will also be a temperature T_c at which the internal manifold will explode and this should be independent of the shape of the internal space.[16] Finally with $V(\alpha,T)$, we can examine the stability of the internal space at least with respect to squashing deformations at finite temperature.

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APPENDIX A

To evaluate the finite sum in (2.27) we use the Plana formula

$$\sum_{j=n}^{m}\phi(j)=\int_{n-1/2}^{m+1/2}\phi(x)dx -i\int_0^{\infty}\frac{dy}{e^{2\pi y}+1}[\phi(n-\tfrac{1}{2}+iy)-\phi(n-\tfrac{1}{2}-iy) -\phi(m+\tfrac{1}{2}+iy) +\phi(m+\tfrac{1}{2}-iy)] \ . \tag{A1}$$

Here $\phi(z)$ is analytic in the region bounded by $(n-\frac{1}{2}\pm i\infty)$ and $(m+\frac{1}{2}\pm i\infty)$ and for $z=a+ib$ with a in $[n-\frac{1}{2},m+\frac{1}{2}]$, $\phi(z)e^{-2\pi b}\to 0$ when $b\to+\infty$ (Ref. 17).

To apply the Plana formula we need the pole structure of

$$\phi(z)=\left[[n^2+\sigma+\alpha(n-1-2z)^2]/a^2+\left[\frac{2\pi p}{\beta}\right]^2\right]^{-\nu} \tag{A2}$$

for this we need to specify the sign of α. For the prolate case $(\alpha<0)$ the poles are at

$$z=\frac{n-1}{2}\pm\frac{1}{2}\left[\frac{1}{|\alpha|}\right]^{1/2}\left[n^2+\sigma+a^2\left[\frac{2\pi p}{\beta}\right]^2\right]^{1/2} . \tag{A3}$$

The poles are outside the region of integration $(-\frac{1}{2},n-\frac{1}{2})$ if $\sigma+a^2(2\pi b/\beta)^2>0$ which is true for high temperature provided $p\neq 0$. This condition is sufficient for our purposes [see comment following Eq. (2.23)].

To make the limits of integration in the Plana formula independent of n we perform a change of variables $[y=(1+2x)/2n]$ in the first integral. The integrand now becomes $n\phi(x)$ and $\phi(x)$ can be rewritten as

$$\phi(x)=(a)^{2\nu}M^{-\nu}(n^2+A^2)^{-\nu} , \tag{A4}$$

where $M(y)$ and $A(y)$ are given in (2.30).

The $\zeta(\nu)_2$ contribution comes from the second integral in (A1). It can be simplified since $\phi(-\frac{1}{2}+iy)=\phi(n-\frac{1}{2}-iy)$ and similar equality for the other two terms. We can rewrite them as

$$\begin{aligned}\phi(-\tfrac{1}{2}+iy)&=(a)^{2\nu}(1+\alpha)^{-\nu}[(n-iB)^2-E^2]^{-\nu} ,\\ \phi(-\tfrac{1}{2}-iy)&=(a)^{2\nu}(1+\alpha)^{-\nu}[(n+iB)^2-E^2]^{-\nu}\end{aligned} \tag{A5}$$

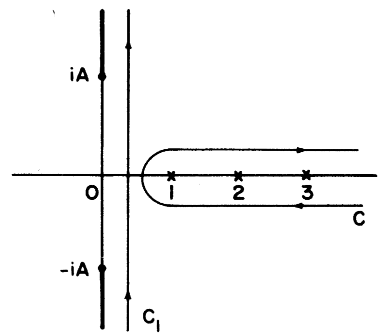

FIG. 1. Paths C and C_1 for evaluating $F(y)$.

with B and E defined in (2.32)

Now we proceed with the evaluation of $F(y)$. The parameter A^2 is positive since M is positive for the prolate case and we consider only the case $p\neq 0$. Expressing $F(y)$ as a Sommerfeld-Watson integral

$$F(y)=\frac{i}{2}\int_C dz\, z^2(z^2+A^2)^{-\nu}\cot(\pi z) \ . \tag{A6}$$

We deform the contour C to C_1 (see Fig. 1) and using the C_1 path we find

$$\begin{aligned}F(y)=\frac{i}{2}\Bigg[&e^{i\pi\nu}\int_{\infty}^{A}d(-ix)(-ix)^2 \times[(-ix)^2+A^2]^{-\nu}\cot(-i\pi x)\\ &+e^{-i\pi\nu}\int_{A}^{\infty}d(ix)(ix)^2 \times[(ix)^2+A^2]^{-\nu}\cot(i\pi x)\Bigg]\end{aligned} \tag{A7}$$

from this the expression (2.30) can be easily derived.

Next we consider $G(y)$. For the prolate case B is positive, C^2 is negative, and for high temperature (with $p\neq 0$) E^2 is also negative. Expressing the two terms in $G(y)$ as Sommerfeld-Watson integrals, as we did for the evaluation of $F(y)$ above, we find, after some changes of variables in the integrals (below $E=|E|$),

$$\begin{aligned}G(y)=(-i)(\sin\pi\nu)\Bigg[&\frac{(-i)^{-\nu}}{\Gamma(\nu)}[2(2E)^{2-2\nu}\Gamma(2-\nu)\Gamma(2\nu-2)+2E(2E)^{1-2\nu}\Gamma(1-\nu)\Gamma(2\nu-1)]\\ &-(-1)^{-\nu}\int_0^{B-E}dy[y+(E-B)]y^{-\nu}(y+2E)^{-\nu}\\ &+2\int_{B+E}^{\infty}dx(x)\frac{[E^2-(x-B)^2]^{-\nu}}{e^{2\pi x}-1}+2\int_0^{\infty}dx(x)\frac{[E^2-(x+B)^2]^{-\nu}}{e^{2\pi x}-1}\Bigg]\\ -(\cos\pi\nu)&\left[\int_0^{B-E}dx(x)[E^2-(x-B)^2]^{-\nu}+2\int_0^{B-E}dx(x)\frac{[E^2-(x-B)^2]^{-\nu}}{e^{2\pi x}-1}\right]\\ -&\left[\int_{B-E}^{B+E}dx(x)[E^2-(x-B)^2]^{-\nu}+2\int_{B-E}^{B+E}dx(x)\frac{[E^2-(x-B)^2]^{-\nu}}{e^{2\pi x}-1}\right] .\end{aligned} \tag{A8}$$

APPENDIX B

We now evaluate the temperature sums. First we consider $\zeta(\nu)_1$. Since we are going to make a high-temperature expansion we drop the second term in the expression (2.30) because it is exponentially suppressed as $\beta \to 0$. We denote the ζ function with the $(p \neq 0)$ mode deleted by ζ_0:

$$\zeta_0(\nu)_1 = 2(a\mu)^{2\nu} \int_0^1 dy\, M \sum_{p=1}^{\infty} F \qquad \text{(B1)}$$

with

$$\sum_{p=1}^{\infty} F \simeq -\frac{(-1)^\nu}{2} \frac{\Gamma(1-\nu)\Gamma(\nu-\frac{3}{2})}{\Gamma(-\frac{1}{2})} (\sin\pi\nu) \sum_{p=1}^{\infty} A^{3-2\nu} . \qquad \text{(B2)}$$

We rewrite A^2 as $A^2 = b^2 p^2 (1 - c^2/p^2)$ with $b^2 = (1/M)(2\pi a/B)^2$ and $c^2 = |\sigma|(B/2\pi a)^2$ and we make a small β (high-temperature) expansion of A^2:

$$\begin{aligned} S \equiv \sum_{p=1}^{\infty} A^{3-2\nu} &= b^{3-2\nu} \sum_p p^{3-2\nu} (1 - C^2/p^2)^{3/2-\nu} \\ &\simeq \zeta(-3+2\nu) - \tfrac{3}{2} c^2 \zeta(-1+2\nu) \\ &\quad + \tfrac{1}{2}(\tfrac{3}{2}-\nu)(\tfrac{1}{2}-\nu) c^4 \zeta(1+2\nu) + \cdots . \end{aligned} \qquad \text{(B3)}$$

Here ζ is the usual Riemann zeta function. Since we are interested in terms in $\zeta'(0)_1$ with background-field dependence we keep only the second term in the last expression of (B3). This will give a contribution to the effective mass term in the effective potential. We denote this by a subscript m:

$$\zeta_0'(0)_{1(m)} \simeq -\pi \zeta(-1) \int_0^1 dy (b^3 c^2) . \qquad \text{(B4)}$$

For the broken-symmetry case

$$|\sigma| = -ma^2 - (1-\xi)\frac{R}{6} a^2 + 1 - \frac{\lambda}{2} \phi^2 a^2$$

(λ is small) and we get a contribution to $V^{(1)}$:

$$V^{(1)}_{B(m)} \simeq \frac{\hbar}{4} \lambda \zeta(-1) \left[\frac{2\pi a}{\beta} \right]^2 \left[\frac{a}{V} \right] I \phi^2 , \qquad \text{(B5)}$$

with

$$I(\alpha) = \int_0^1 \frac{dy}{M^3} = \frac{1}{4} \left[\frac{1}{(1+\alpha)^2} + \frac{3}{2} \frac{1}{1+\alpha} + \frac{3}{2\sqrt{-\alpha}} \operatorname{arctanh} \sqrt{-\alpha} \right] . \qquad \text{(B6)}$$

It turns out that this is the dominant contribution to $V^{(1)}$ $(\sim 1/\beta^2)$ because $\zeta(\nu)_2$ gives contributions to $V^{(1)}$ starting with terms proportional to $1/\beta$.

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