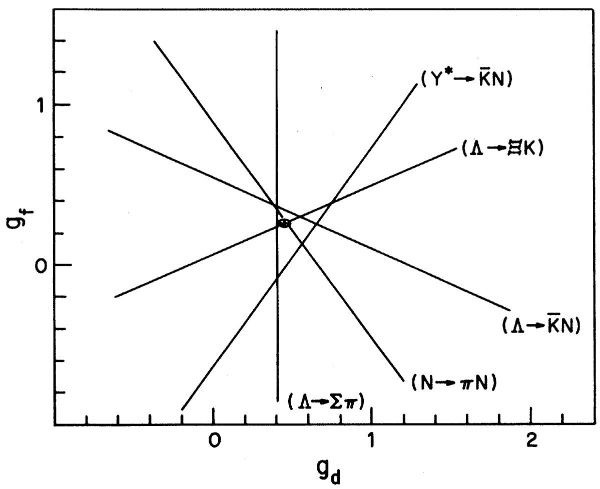

FIG. 2. Best fit to the coupling constants g_d and g_f of the assumed $\frac{7}{2}^-$ octet. The new resonant wave with unknown isospin is tried as a triplet of SU(2) and marked Y^*.

according to Eq. (7), the predicted partial widths of the predicted Ξ', getting

$$\Gamma(\Xi' \to \Xi\pi) \sim 5 \text{ MeV},$$

$$\Gamma(\Xi' \to \Sigma K) \sim 27 \text{ MeV},$$

$$\Gamma(\Xi' \to \Xi\eta) \sim 0.4 \text{ MeV},$$

$$\Gamma(\Xi' \to \Lambda K) \sim 0.2 \text{ MeV}.$$

As a speculation we point out the possibility of identifying Ξ' with the poorly known group of particles summarized by Söding *et al.*[6] under the name $\Xi(1940)$.

We should like to express our gratitude to the CERN Track Chamber Division, and specially to Professor Ch. Peyrou, Dr. R. Armenteros, and Dr. D. E. Plane, for supplying the film and the beam. This work would not have been possible without the encouragement and advices of Dr. R. Barloutaud, and one of us (J.G.) would like to thank Professor A. Berthelot and Dr. A. Leveque for their generous support at the Centre d'Etudes Nucléaires de Saclay where part of this work has been done.

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Approximate Symmetries and Pseudo-Goldstone Bosons*

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In theories with spontaneously broken local symmetries, renormalizability sometimes forces the scalar field interactions to have a larger group of symmetries than the gauge field interactions. Symmetries can then arise in zeroth order which are violated by finite higher-order effects, thus providing a possible natural explanation of the approximate symmetries observed in nature. Such theories contain spinless bosons which behave like Goldstone bosons, but which pick up a small mass from higher-order effects.

Renormalizable field theories with spontaneously broken local symmetries were suggested[1] as a framework for unifying the electromagnetic and weak interactions and for solving the divergence difficulties of the weak interactions. It is becoming increasingly apparent[2,3] that such theories can also provide a solution to one other of the long-outstanding problems of particle physics, the existence of approximate symmetries such as isospin conservation.

The new opportunity for understanding this old problem arises from the circumstance that a Lagrangian which is renormalizable, is invariant under a local symmetry group G, and contains a definite set of fields transforming according to prescribed representations of G, is often so constrained by these conditions that in zeroth order, after spontaneous symmetry breaking, the masses and other physical parameters are found to obey certain symmetry relations, which are *not* merely consequences of some unbroken subgroup of G, but which nevertheless remain valid for all values[4] of the parameters in the Lagrangian. In this case corrections generally appear in high order, but *all corrections to the zeroth-order symmetry relations must be finite,* because the theory is renormalizable, and there are no counterterms available to absorb a divergence, should one occur.

In a previous Letter[2] I discussed one class of zeroth-order fermion mass relations, those arising from limitations on the representation content of the scalar fields coupled to the fermions. Since then, I have tried without success to find models of this type which could explain such approximate symmetries as electron chirality and hadronic $SU(3)\otimes SU(3)$. Meanwhile, Georgi and Glashow[3] described a few examples of theories in which zeroth-order mass relations arise from constraints on the zeroth-order vacuum expectation values of the scalar fields, rather than solely from their representation content. It was not clear, however, whether this was an exceptional or a widespread phenomenon, or whether it could be put to use in constructing models of the real world.

In this note I will describe a broad class of quite natural theories in which the vacuum expectation values of the scalar fields are subject to certain constraints, not merely corresponding to unbroken subgroups of the underlying symmetry group G, for all values[4] of the parameters in the Lagrangian. These theories necessarily contain a new kind of particle, a spinless meson which behaves like a Goldstone boson, but has a small mass, and is not eliminated by the Higgs mechanism. I suggest that the pion and its relatives may be just such "pseudo-Goldstone" bosons.

The spontaneous breakdown of a symmetry group G is in general manifested in the appearance of nonzero vacuum expectation values of a multiplet (perhaps reducible) of Hermitian spin-0 fields φ_i. The zeroth-order vacuum expectation value λ_i of φ_i is determined by the condition that the G-invariant polynomial $-P(\varphi)$ appearing in the Lagrangian be stationary at $\varphi=\lambda$:

$$\partial P(\varphi)/\partial\varphi_i = 0 \text{ at } \varphi=\lambda. \tag{1}$$

In general, we would not expect the solutions of Eq. (1) to exhibit any particular symmetry, except that λ might be invariant under some unbroken subgroup S of G; in which case no higher-order corrections could arise. There are, however, theories in which constraints on λ arise because G invariance *requires* $P(\varphi)$ (but not the rest of the Lagrangian) to be invariant under a larger group of pseudosymmetries $\bar{G}$.

To see how this is possible, note that every G-invariant polynomial in φ may be expressed[5] as a polynomial function of certain "typical basic polynomials" in φ. For some φ representations, the table of typical basic invariants for G is the same as for a larger group $\bar{G}$, in which case *any* polynomial in φ, invariant under G, is also invariant under $\bar{G}$. This is the case[6] for the seven-dimensional representation of the exceptional Lie group G_2; the only invariant is the "length" $\varphi_i\varphi_i$, so any G_2-invariant function of φ is also invariant under the larger group O(7). More often, the existence of pseudosymmetries depends upon the renormalizability requirement, that $P(\varphi)$ must be at most *quartic*. If enough of the typical basic invariants are of higher than fourth order in φ, and are thereby excluded from $P(\varphi)$, then we may find that the ones remaining are all invariant under some larger group $\bar{G}$. For instance, consider the adjoint representation of U(3), furnished by Hermitian traceless 3×3 matrices Φ. If we adjoin to U(3) a reflection $R:\Phi\to-\Phi$, then the typical basic invariants are $\mathrm{Tr}\Phi^2$ and $\mathrm{Det}\Phi^2$. However, $\mathrm{Det}\Phi^2$ is of sixth order in Φ, and cannot appear in a quartic polynomial. Hence, every quartic polynomial in Φ which is invariant under $G=\mathrm{U}(3)\otimes R$ must be a function of $\mathrm{Tr}\Phi^2$ alone, and therefore must be invariant[7] under the larger group $\bar{G}=\mathrm{SO}(8)$.

When $P(\varphi)$ is invariant under a pseudosymmetry group G, it is natural to find solutions of Eq. (1) for which λ is invariant under any given subgroup $\bar{S}$ of $\bar{G}$, for all values[4] of the parameters in the Lagrangian. In particular, $P(\varphi)$ does not know that the whole Lagrangian is only invariant under G, not $\bar{G}$, so there is no reason why $\bar{S}$ must be contained within G. Those transformations that are in both G and $\bar{S}$ are the only true unbroken symmetries in the theory; presumably these are restricted to electromagnetic gauge transformations in the real world. On the other hand, those transformations that are in $\bar{S}$ but not in G are symmetries of the zeroth-order vacuum expectation values but not of the whole theory, and therefore may account for the approximate symmetries observed in nature.

The breaking of the pseudosymmetry group $\bar{G}$ down to $\bar{S}$ gives rise to a spinless boson with vanishing zeroth-order mass for every independent generator of $\bar{G}$ not in $\bar{S}$. Those bosons corresponding to generators of the true symmetry group G are true Goldstone bosons, and are eliminated by the Higgs mechanism.[8] However, there will also be a boson for each independent generator of $\bar{G}$ which is not in G or $\bar{S}$; these are massless in zeroth order, but since they do not correspond to true symmetries of the whole Lagrangian, they pick up a finite mass from higher-order effects. These are the pseudo-Goldstone bosons.

For instance, in the $U(3)\otimes R$ example discussed above, the only possible symmetry-breaking solution of Eq. (1) must have λ invariant under an SO(7) subgroup $\bar{S}$ of $\bar{G}$. In this case the true symmetry group formed by the overlap between G and $\bar{S}$ can be either $U(1)\otimes U(1)\otimes U(1)$ or $U(2)\otimes U(1)$, leaving either $21-3=18$ or $21-5=16$ "approximate" symmetry generators belonging to $\bar{S}$ but not G. There are $28-21=7$ generators of $\bar{G}$ not in $\bar{S}$, to which correspond seven bosons with vanishing zeroth-order mass. Of these, either $9-3=6$ or $9-5=4$ are true Goldstone bosons, and are removed by the Higgs mechanism,[8] leaving behind either $7-6=1$ or $7-4=3$ pseudo-Goldstone bosons.

Even when λ is constrained by some subgroup $\bar{S}$ of a group of pseudosymmetries $\bar{G}$, this does not immediately lead to a constraint on the fermion masses unless the Yukawa interaction is also invariant under $\bar{G}$. Indeed, for every solution λ of Eq. (1) there are an infinity of other solutions λ_g obtained by acting on λ with any transformation g in $\bar{G}$. The λ_g for g in G are physically equivalent, but the others are not, and in order to determine which λ is the physical solution it is necessary to take higher-order corrections into account.[9] When the Yukawa interaction is $\bar{G}$ invariant these complications have no effect on the zeroth-order fermion mass relations, though they do affect the mass splittings.

There is in fact a large class of pseudosymmetries which naturally leave the Yukawa interactions invariant. These arise from a phenomenon I would like to call the *unlocking* of representations. Suppose the boson field multiplet φ_i may be decomposed into two separate multiplets χ_a and η_k, transforming under the representations D_χ and D_η (perhaps themselves reducible) of G. In general, the table of "typical basic invariant" polynomials includes some involving both χ and η, which are invariant under simultaneous G transformations on both χ and η, but not under G transformations on χ or η separately. It sometimes happens that all of these invariants, which lock the transformations of χ and η together, are higher than fourth order in the boson fields, and so are excluded from $P(\varphi)$. In this case D_χ and D_η become unlocked, and the polynomial $P(\varphi)$ becomes necessarily invariant under a pseudosymmetry group $\bar{G}=G_\chi\otimes G_\eta$, consisting of independent transformations on χ and η. If all of the scalar fields which are allowed by G invariance to have Yukawa couplings are contained in one of the scalar multiplets, say χ, then the Yukawa interactions will be invariant under both G_χ and (trivially) G_η, and the zeroth-order fermion mass matrix will therefore exhibit invariance under any subgroup $\bar{S}$ of $\bar{G}$ which leaves λ invariant. But the gauge couplings are only G invariant, not $\bar{G}$ invariant, and so any symmetry in $\bar{G}$ which is not in G will be broken by higher-order weak and electromagnetic effects, which also give a mass to the pseudo-Goldstone bosons.

As an example of unlocking, consider an $SU(4)\otimes SU(4)$ model, in which the left- and right-handed parts of a quark quartet form the representations $(\underline{4},\underline{1})$ and $(\underline{1},\underline{4})$. Let χ be a complex $(\underline{4},\underline{4}^*)$ boson multiplet, which is *strongly* coupled to the quarks and itself, and consequently has a rather small vacuum expectation value, of the order of the quark masses. Let η be a single real $(\underline{20},\underline{1})\oplus(\underline{1},\underline{20})$ multiplet, which cannot couple to the quarks, and has a large vacuum expectation value, of the order of 300 GeV. Unlocking occurs because the products $\chi_a^*\chi_b$ and $\chi_a\chi_b$ contain only the representations $(\underline{15},\underline{15})$, $(\underline{15},\underline{1})$, $(\underline{1},\underline{15})$, $(\underline{1},\underline{1})$, $(\underline{6},\underline{6})$, and $(\underline{10},\underline{10}^*)$, while the symmetric products $\eta_k\eta_l$ contain only the representations $(\underline{20},\underline{20})$, $(\underline{20},\underline{1})$, $(\underline{84},\underline{1})$, $(\underline{175},\underline{1})$, $(\underline{1},\underline{20})$, $(\underline{1},\underline{84})$, $(\underline{1},\underline{175})$, and $(\underline{1},\underline{1})$; the only representation in common is $(\underline{1},\underline{1})$, so any invariant formed from $\chi_a^*\chi_b\eta_k\eta_l$ or $\chi_a\chi_b\eta_k\eta_l$ must be the product of an invariant function of χ times an invariant function of η. It will therefore be natural to find solutions of Eq. (1) such that λ^χ breaks $SU(4)\otimes SU(4)$ down, say, to U(3), while λ^η breaks $SU(4)\otimes SU(4)$ all the way down to electromagnetic gauge invariance.[10] With $\lambda^\chi \ll \lambda^\eta$, the vector-meson mass matrix is dominated by λ^η terms, and weak and electromagnetic effects can produce finite corrections to U(3) invariance. The true Goldstone bosons are linear combinations of χ and η fields, but are nearly pure η, and so the Higgs mechanism has only a small effect on the χ propagators. The pseudo-Goldstone bosons here are essentially just those χ fields corresponding to symmetries broken by λ^χ, i.e., to a pseudoscalar octet, triplet, and singlet and a scalar triplet. It remains to be seen whether such models have anything to do with reality, but at least they open up the possibility of an integration of current algebras, including soft-pion theorems, with the new picture of weak and electromagnetic interactions.

Inspection of the one-loop contributions to the fermion mass matrix shows explicitly that cancelations eliminate the divergences in corrections to all types of zeroth-order mass relations, including those discussed here as well as those discussed in Refs. 2 and 3. This and other matters will be discussed in detail in a more comprehensive article on approximate symmetries now in preparation.

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[4]"All values" here means all values within at least some finite range, as requried for strict renormalizability. The physical solutions of Eq. (1) are those for which $P(\lambda)$ is a minimum.

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[6]B. Kostant, private communication.

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