Pinning and *I-V* Characteristics of a Two-Dimensional Defective Flux-Line Lattice

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Using a variational argument, we show that a two-dimensional (2D) crystal becomes defective in the presence of a random pinning potential. The threshold pinning force and flow properties of a plastically deformed crystal are studied using a perturbation theory valid for weak pinning potential and strong driving force. In the case of a 2D flux-line lattice, it is shown that dislocations introduce extra dissipation in the system, thus increasing the total pinning force, and resulting in nonlinear *I-V* characteristics.

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The problem which we consider is that of a two-dimensional (2D) crystal in a random pinning potential. The study of the threshold pinning force and flow properties of a deformable lattice is a general problem relevant to many physical systems. Examples include flux-line lattices in type-II superconductors [1], Wigner crystals of electrons on liquid-helium films [2] or in Ga-As heterojunctions in a strong magnetic field [3], and 2D charge-density waves [4]. In the present Letter we will refer to the 2D lattice as a flux-line lattice (FLL). However, we emphasize that the physical picture which emerges is generic to any 2D crystal in a random potential.

For many years, the collective pinning theory of Larkin and Ovchinnikov (LO) [5] has been widely accepted as a useful description of the pinning force on a flux lattice in the mixed state of a disordered type-II superconductor. More recently this theory has been extended to finite temperatures by Feigel'man *et al.* [6] and by Nattermann [7]. There are also predictions of new forms of three-dimensional vortex matter in the high-T_c superconductors, e.g., entangled flux lines [8] and vortex glass [9]. However, a recent series of calculations based on numerical simulations [10] has demonstrated convincingly that, at least for the two-dimensional case, the nature and size of the pinning force is determined by plastic deformations of the FLL which fall outside the region of validity of collective pinning theory. The present paper represents an initial attempt at an analytical treatment of the nature of the plastic deformations of a 2D FLL in a random potential and of the way in which lattice defects alter the flow characteristics of the lattice under the influence of an external driving force. The physical picture which emerges is quite different from that of LO, and its validity is not limited to 2D FLL's but rather is generic to 2D crystals in a random potential.

Collective pinning theory is usually developed in terms of a static scaling argument which is similar to that used by Lee and Rice [11] to describe the pinning of charge-density waves in quasi-one-dimensional conductors. It involves defining a "correlated volume," which is a volume within which translational crystalline order is maintained in the presence of the random potential. The pinning force results from the incomplete averaging of the random potential over the finite correlated volume. The calculation of the correlated volume is based on the interplay between the elasticity of the FLL and the strength of the random potential which drives the distortions [5]. However, as will be shown below and as has been discussed elsewhere in the context of charge-density waves [12], the random potential, however weak, also generates some local strains which exceed the elastic limit of the lattice and which hence generate dislocations and other defects. We find that these defects have important consequences for the pinning force and for dissipation (friction) of the unpinned, moving FLL.

The original treatment of collective pinning by LO [13] derived the critical pinning force by examining the way that a FLL, moving under the influence of a uniform external force, slows down as the force is reduced. For large applied force, the (small) reduction in velocity due to distortions induced by interaction with the random potential can be calculated in perturbation theory, and this was done by LO for the case of elastic distortions only [13]. In two dimensions this velocity shift is a constant, independent of force. In this paper we consider the effect of dislocations on dissipation by a rapidly moving FLL. We find that a fixed density of dislocations induces an extra velocity shift which is independent of velocity. We also find from simulations that the density of dislocations is velocity dependent, with the density going to zero at large velocities. Thus the generation of dislocations gives rise to nonlinear behavior of the velocity-versus-force curve as is observed experimentally [14] and in simulations [15].

The model system we use is a 2D FLL, which is relevant to experiments on a thin film or a layered superconductor [16], in a pinning potential due to weak randomly distributed atomic defects (e.g., atomic vacancies). These pinning centers induce distortions of the FLL with distortion energy [17]

$$F=\frac{1}{2}\int d^2r\{C_{66}[\partial_j u_i(\mathbf{r})]^2+(C_{11}-C_{66})[\nabla\cdot\mathbf{u}(\mathbf{r})]^2\}+F_p\,, \qquad (1)$$

where $\mathbf{u}(\mathbf{r})$ is the local displacement of the FLL, and C_{11} and C_{66} ($C_{11}\gg C_{66}$ for a FLL) are the bulk and shear moduli of the FLL [17]. The pinning energy is $F_p = \int d^2r \times V(\mathbf{r},\mathbf{u})$, where $V(\mathbf{r},\mathbf{u})$ is the random potential describing the lattice interaction with the pinning centers and is supposed to have zero average and to be short-range correlated:

$$\langle V(\mathbf{r},\mathbf{u})V(\mathbf{r}',\mathbf{u}')\rangle = K(|\mathbf{r}-\mathbf{r}'|,|\mathbf{u}-\mathbf{u}'|)\,,$$

where $K(r,u)$ decreases rapidly for r,u larger than some characteristic length r_p. In the presence of FLL dislocations, the displacement $\mathbf{u}(\mathbf{r})$ has the form $\mathbf{u}(\mathbf{r}) = \phi(\mathbf{r}) + \mathbf{u}^s(\mathbf{r})$, where $\phi(\mathbf{r})$ is the elastic component of the displacement, and $u_i^s(r) = \int d^2r' g_{ij}(\mathbf{r}-\mathbf{r}')b_j(\mathbf{r}')$ is the displacement associated with dislocations of density $\mathbf{b}(\mathbf{r})$ [18]. Here

$$g_{ij}(\mathbf{r}) = \frac{1}{2\pi}\left[\delta_{ij}\tan^{-1}\left(\frac{y}{x}\right) + \frac{C_{66}}{C_{11}}\epsilon_{ij}\ln\left(\frac{r}{a}\right) + \frac{C_{11}-C_{66}}{C_{11}}\epsilon_{jk}\frac{r_i r_k}{r^2}\right],$$

where a is the lattice constant of the ideal FLL and ϵ_{ij} is the completely antisymmetric tensor. (We use the convention that all repeated indices are summed.) In terms of the elastic displacement $\phi(\mathbf{r})$ and the dislocation density $\mathbf{b}(\mathbf{r})$, the distortion energy of the FLL is written as $F = F_{\rm el} + F_{\rm pl} + F_p$ [19]. The elastic energy $F_{\rm el}$ and the plastic energy $F_{\rm pl}$ are given by

$$F_{\rm el} = \tfrac{1}{2}\int d^2r[C_{66}(\partial_j\phi_i)^2 + (C_{11}-C_{66})(\nabla\cdot\phi)^2]\,,$$

$$F_{\rm pl} = -\tfrac{1}{2}\int d^2r\,d^2r'\,b_i(\mathbf{r})D_{ij}(\mathbf{r}-\mathbf{r}')b_j(\mathbf{r}') + E_c\int d^2r[\mathbf{b}(\mathbf{r})]^2\,,$$

where E_c is a phenomenological core energy of the dislocations, and

$$D_{ij}(\mathbf{r}) = (K/4\pi)[\delta_{ij}\ln(r/a) + \epsilon_{ik}\epsilon_{jl}r_k r_l/r^2]\,,$$

with $K = 4C_{66}(C_{11}-C_{66})/C_{11} \simeq 4C_{66}$.

The ideal flow velocity of the FLL is obtained by balancing the Lorentz force $\mathbf{J}\times\mathbf{B}/c$ and the viscous force $\eta\mathbf{v}$, which leads to a constant flow velocity $\mathbf{v}_0 = \mathbf{J}\times\mathbf{B}/\eta c$ and a flow resistivity ρ_f such that Ohm's law, $\mathbf{J} = \mathbf{E}/\rho_f$, is satisfied [20]. In the presence of lattice deformations, the average velocity of the FLL has an extra negative contribution, $\mathbf{v} = \langle\partial\mathbf{u}/\partial t\rangle = \mathbf{v}_0 + \delta\mathbf{v}$, where $\delta\mathbf{v}$ depends, in general, on the driving force. The velocity-versus-force curve of the FLL corresponds to the I-V characteristic of a type-II superconductor for which the average velocity of the FLL is proportional to the voltage and the driving force is proportional to the current [20]. Extrapolating to zero flow velocity, $\delta\mathbf{v}(\mathbf{v}_0) \simeq -\mathbf{v}_0$, leads to an estimate of the critical current given by $J_c = |\delta\mathbf{v}|B/\rho_f c$.

The equilibrium static properties of the FLL in the presence of a weak pinning potential are obtained by minimizing the total distortion energy of the lattice. The elastic properties of the system are determined by minimizing the elastic energy with respect to the elastic displacement $\phi(\mathbf{r})$, leading to an equilibrium value of $\phi(\mathbf{r})$ for a given random potential. The correlation function of the elastic displacement, $g(r) = \langle|\phi(\mathbf{r}) - \phi(\mathbf{0})|^2\rangle$, can then be obtained [5]. The correlation length R_c is the value of r when $g(r)$ is of the order of the lattice constant of the FLL. This length scale R_c was used by LO [5] to determine the correlated volume, thus the total pinning force.

The observation that the FLL is always defective for macroscopic systems can be explained by a variational argument [21]. To this end, we minimize the distortion energy with respect to the dislocation density $\mathbf{b}(\mathbf{r})$, which leads to an equilibrium dislocation density

$$b_i(\mathbf{r}) = \int d^2r'\,d^2r''\,G_{ij}^d(\mathbf{r}-\mathbf{r}')g_{jl}(\mathbf{r}'-\mathbf{r}'')F_l(r'')\,,$$

where $\mathbf{F}(\mathbf{r},\mathbf{u}) = -\nabla_{\mathbf{u}}V(\mathbf{r},\mathbf{u})$ is the pinning force corresponding to the pinning potential $V(\mathbf{r},\mathbf{u})$, and the Green's function G_{ij}^d which relates the random force to the dislocation density is

$$G_{ij}^d(\mathbf{k}) = \frac{1}{2E_c}\left[\delta_{ij} + \epsilon_{im}\epsilon_{jn}k_m k_n\left(\frac{1}{k^2+k_c^2} - \frac{1}{k^2}\right)\right],$$

where $k_c = (K/2E_c)^{1/2}$ is a cutoff related to the dislocation core energy. The average distortion energy of the ground state is then given by

$$E_0 = -(W/4)\int d^2r\,d^2r'\,d^2r'' \times G_{il}^d(\mathbf{r}-\mathbf{r}'')g_{lk}(\mathbf{r}''-\mathbf{r}')g_{ik}(\mathbf{r}-\mathbf{r}')\,, \quad (2)$$

where $W = \int d^2r[\partial^2K(r,u)/\partial u^2]$ $(u \sim r_p)$ is the mean-squared value of the random force. E_0 is less than zero because the integrand is a perfect square. Thus the presence of the dislocations will lower the total energy of the system, and we conclude that in the presence of the random pinning potential, dislocations will appear in the 2D FLL, as observed in the numerical simulations [10] and experiments [17].

We now turn to the dynamic response of the FLL to the applied driving force (i.e., the electric-current-induced Lorentz force). The equation of motion for the elastic displacement is obtained by balancing the force acting on the flux lines [13]. In the reference frame which is moving with a velocity $\mathbf{v}$, we have

$$\eta(\partial\phi/\partial t) - C_{66}\nabla^2\phi - (C_{11}-C_{66})\nabla(\nabla\cdot\phi) = \mathbf{F}(\mathbf{r},\mathbf{v}t+\phi+\mathbf{u}^s)\,, \quad (3)$$

whose solution can be obtained by iteration. To lowest order in the pinning potential, the average excess velocity of the FLL is found to be [13]

$$\delta v_i = -(1/16\eta C_{66})\sum_{\mathbf{q}} q_i q^2 K(0,q)\,\mathrm{sgn}(\mathbf{q}\cdot\mathbf{v}_0)\,,$$

which is independent of the magnitude of the driving force $\mathbf{v}_0$. Thus elastic deformations in the 2D FLL lead to a linear I-V curve, whereas nonlinear I-V characteristics are often observed for thin films in experiments [14].

In order to understand the effects of the FLL dislocations on the pinning and the critical current, it is necessary to study the dynamic response of dislocations to the applied force. We assume that there are a small number of free dislocations in the FLL with an average number density n_d [22]. For a triangular lattice, the basic Burgers vectors should be one of the six elementary lattice vectors $\{\mathbf{b}^\alpha\}$ $(\alpha=1,2,\ldots,6)$. For the motion of the dislocations, we assume that each dislocation drifts independently under the influence of the stress in the FLL. Under these assumptions, the current of dislocation type α, moving along the ith direction, is given by

$$j_i^\alpha = \frac{Dn_d}{6a^2} b_i^\alpha b_j^\alpha b_l^\alpha \epsilon_{mj}\sigma_{lm} ,$$

where D is a phenomenological diffusion constant for dislocations, and σ_{ij} is the stress in the FLL due to elastic deformation. We have assumed that only glide of the dislocations is allowed. The total dislocation current is obtained by summing over the six possible Burgers vectors $J_j^i = \sum_{\alpha=1}^6 b_i^\alpha j_j^\alpha$. The average flow velocity of the FLL due to the motion of the dislocations is then given by [22]

$$\langle \partial u_i^s/\partial t\rangle = -\int d^2r'\, g_{ij}(\mathbf{r}-\mathbf{r}')\langle \partial_k J_k^j(\mathbf{r}')\rangle .$$

Inserting the stress due to the elastic deformation [solution of Eq. (3)] and averaging over the random potential, we find that the motion of the dislocations induces an extra drift velocity of the FLL given by

$$\delta v_i^s = -(1/16C_{66}\Delta\eta)\sum_{\mathbf{q}} \epsilon_{mi} q_m q^2 K(0,q)\,\mathrm{sgn}(\mathbf{q}\cdot\mathbf{v}) ,$$

where $1/\Delta\eta = (Dn_d a^2/8)\pi R_c^2$. This extra drift velocity is independent of the magnitude of the driving force for a fixed dislocation density n_d.

In principle, the number density of dislocations should be determined from some first-principles theory which takes dynamical nucleation and annihilation of dislocations into account. Unfortunately, at the present time there is no known theory of how to calculate the nucleation and annihilation rates for our model. Therefore, in order to estimate how the density of dislocations varies with the flow velocity, we appeal to numerical simulations.

For N_v particles with position $\mathbf{r}_i(t)=(x_i(t),y_i(t))$ and N_p attractive identical pins with random fixed positions $\mathbf{r}_i^p$ in an area $A=L_xL_y$ with periodic boundary conditions [10,15], the potential energy of the system is given by

$$U = \tfrac{1}{2}\sum_{i\neq j} A_v v(|\mathbf{r}_i-\mathbf{r}_j|/R_v) - \sum_{i,j} A_p v(|\mathbf{r}_i-\mathbf{r}_j^p|/R_p) - \sum_i \mathbf{r}_i\cdot\mathbf{F}_{\mathrm{dr}} ,$$

where $v(\rho)$ is a Gaussian potential and $\mathbf{F}_{\mathrm{dr}}$ is an external driving force. The units are fixed by setting $A_v=1$ and the ideal triangular lattice spacing $a_0=1$. Simulations were done for a system with $N_v=1020$, $N_p=438$, $R_v=0.6$, and $R_p=0.25$. The particles follow a diffusive equation of motion $\eta\, d\mathbf{r}_i/dt = -\partial U/\partial\mathbf{r}_i$, with the time scale fixed by setting $\eta=1$. The simulations are run as follows: Starting from an ideal triangular lattice, we apply a constant homogeneous driving force $\mathbf{F}_{\mathrm{dr}}$ to the particles and follow the system as it evolves. The center-of-mass velocity $\langle v\rangle$ is measured in the asymptotic long-time stationary regime. The defects in the system are analyzed by computing the coordination number of all the particles [19]. Defects in the lattice show up as particles with coordination numbers different from six. We have found that the number of defects, which is zero for large driving forces, increases with decreasing driving force (see Fig. 1). We have also observed that the I-V curve (i.e., the $\langle v\rangle$-F_{dr} curve) deviates from linear behavior at the point where dislocations start to be generated in the system (Fig. 2). These observations show that the plastic contribution to the drift velocity of the FLL increases with decreasing driving force, thus making the I-V characteristics nonlinear. Examples of such nonlinear I-V curves in simulations for small driving forces have been published previously [15].

In conclusion, we have shown that in the presence of a pinning potential, a 2D FLL is unstable against the formation of the FLL dislocations. The motion of dislocations in the FLL introduces extra dissipation in the system, thus increasing the critical current. The resulting I-V characteristics are nonlinear due to the nonlinear behavior of the dislocation density. At large current (i.e., large driving force), there are no plastic deformations in the system, and the collective pinning theory applies, resulting in a linear I-V curve. As the current is decreased, defects such as dislocations start to be generated in the system, and the I-V curve starts to deviate from linear.

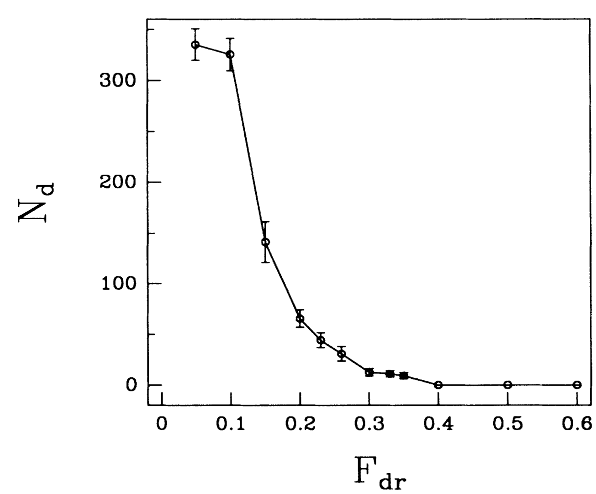

FIG. 1. Number of defects in a 2D lattice as a function of the driving force for a given pinning potential $A_p=0.1$.

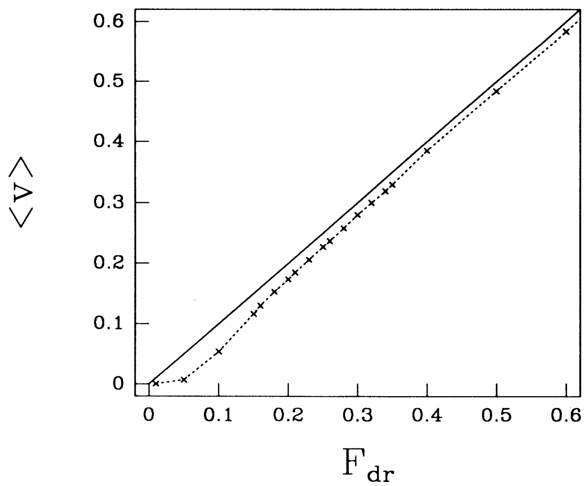

FIG. 2. Velocity-vs-driving force curve for $A_p = 0.1$. The solid line is given by $\langle v \rangle = F_{dr}$. The dotted line is a guide for the eye.

When the current is further decreased, the FLL becomes highly defective, and the motion of flux lines follows a channel-like pattern [10,15]. The curvature changes sign when the current is close to a threshold value (the critical current). When the current is smaller than the critical current, the FLL is pinned and the voltage is zero. Because the theory is a zero-temperature one, these results are relevant to the temperature and field region in which thermal fluctuations can be neglected, i.e., the low-temperature and intermediate-field ($H_{c1} \ll H \ll H_{c2}$) region [14]. We should emphasize that the type of behavior described in this Letter is generic for two-dimensional lattices moving under the influence of an applied force through a random potential. In particular, similar considerations should apply to the motion of a 2D Wigner crystal of electrons on liquid-helium films [2] and in GaAs heterojunctions in a strong magnetic field [3].

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[1] See, e.g., A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium Superconductivity,* edited by D. N. Langenberg and A. I. Larkin (Elsevier, Amsterdam, 1986), pp. 493–542.

[2] H.-W. Jiang and A. J. Dahm, Phys. Rev. Lett. **62**, 1396 (1989).

[3] V. J. Goldman, M. Stantos, M. Shayegan, and J. E. Cunningham, Phys. Rev. Lett. **65**, 2189 (1990).

[4] For a review of charge-density waves, see, e.g., G. Gruner, Rev. Mod. Phys. **60**, 1129 (1988).

[5] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[6] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).

[7] T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990).

[8] D. R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).

[9] M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).

[10] H. J. Jensen, A. Brass, and A. J. Berlinsky, Phys. Rev. Lett. **60**, 1676 (1988); A. Brass, H. J. Jensen, and A. J. Berlinsky, Phys. Rev. B **39**, 102 (1989).

[11] P. A. Lee and T. M. Rice, Phys. Rev. B **19**, 3970 (1979).

[12] S. N. Coppersmith, Phys. Rev. Lett. **65**, 1044 (1990).

[13] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **65**, 1704 (1973) [Sov. Phys. JETP **38**, 854 (1974)]. See also, A Schmid and W. Hauger, J. Low Temp. Phys. **11**, 667 (1973).

[14] See, e.g., P. Berghuis, A. L. F. van der Slot, and P. H. Kes, Phys. Rev. Lett. **65**, 2583 (1990). These authors explore the melting of the FLL. Our results apply to, presumably, the region well below the melting line, i.e., the lowest field data in Fig. 1 of this reference.

[15] H. J. Jensen, A. Brass, Y. Brechet, and A. J. Berlinsky, Phys. Rev. B **38**, 9235 (1988).

[16] Specifically, this model applies to the cases where the film thickness d is smaller than the longitudinal correlation length L_c (see Ref. [5]) and comparable to the penetration depth λ. In the 3D limit, $d > L_c$, the flux-line bending cannot be neglected. If $d \ll \lambda$, the vortices interact via a long-range logarithmic potential, thus nonlocal elastic moduli should be used. For the cases of layered superconductors, the interlayer coupling should be weak enough. For further discussions of layered superconductors, see, e.g., S. Doniach, in *High Temperature Superconductivity (Proceedings),* edited by K. S. Bedell *et al.* (Addison-Wesley, New York, 1990).

[17] E. H. Brandt and U. Essman, Phys. Status Solidi B **144**, 13 (1987).

[18] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, New York, 1970).

[19] D. R. Nelson and B. I. Halperin, Phys. Rev. B **19**, 2457 (1979).

[20] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[21] An early attempt to study this instability of the FLL against the formation of dislocations is given by S. J. Mullock and J. E. Evetts, J. Appl. Phys. **57**, 2588 (1985). These authors minimized the distortion energy for a particular dislocation arrangement.

[22] We follow closely the treatment of dislocation dynamics in a 2D crystal by A. Zippelius, B. I. Halperin, and D. R. Nelson, Phys. Rev. B **22**, 2514 (1980).