Strength functions of ^{4}He using realistic nuclear interaction

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Strength functions of ^{4}He

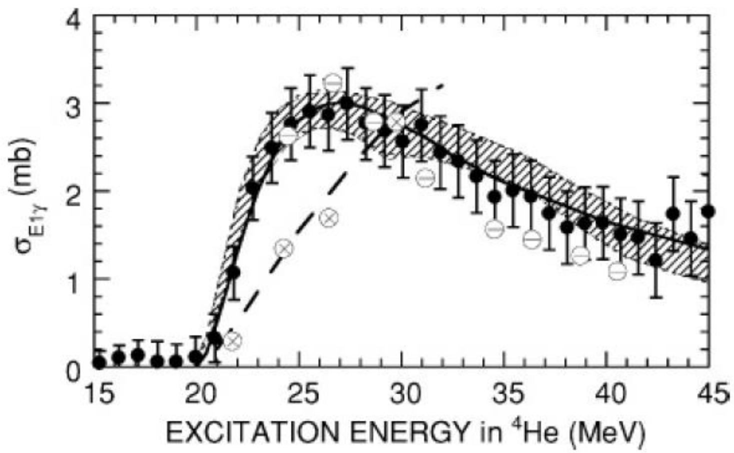
- Photo-absorption of ^{4}He
  - Recent measurements
    - Peak ～27MeV

      S. Nakayama et al., PRC 76, 021305 (2007).
    - Peak ～30 MeV

      T. Shima et al., PRC 72, 044004 (2005).
  - Theoretical studies

    D. Gazit et al. PRL 96, 112302 (2006).
    S. Quaglioni and P. Navratil, PLB652, 370 (2007).

Taken from S. Nakayama et al.
PRC 76, 021305 (2007).



- Excitation of ^{4}He induced by the weak interaction
  - ν-^{4}He reaction (Gamow-Teller, Spin-dipole, etc.)

    → important for the supernova explosion scenario

Reliable model is needed.

Purpose of the study

- Evaluating reliable strength functions induced by electro-weak processes involving ^{4}He
  - Further study for the controversial of photoabsorption cross section
  - Study four-nucleon continuum structure
  - Calculate a reliable neutrino-^{4}He cross section

1. Four-body calculation
2. Use of bare nuclear interaction
3. Continuum -> Complex scaling method

Variational calculation for few-body systems

Hamiltonian $$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} v_{ijk}$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r)P_{1\pi}P_{2\pi} + V_t(r)S_{12} + V_b(r)\boldsymbol{L}\cdot\boldsymbol{S}$$

- AV8 type interactions(AV8', G3RS); central, tensor, spin-orbit
- Three-body interaction (^{3}H, ^{3}He, ^{4}He) E. Hiyama et al. PRC70, 031001(R) (2002)

Basis function

$$\Psi_{(LS)JM_JTM_T} = [\psi_L^{\rm (space)}\psi_S^{\rm (spin)}]_{JM_J}\psi_{TM_T}^{\rm (isospin)}$$

$$\psi_{SM_S}^{\rm (spin)} = \left|[\cdots[[[\tfrac{1}{2}\tfrac{1}{2}]_{S_{12}}\tfrac{1}{2}]_{S_{123}}]\cdots]_{SM_S}\right\rangle$$

$\psi_{LM}^{\rm (space)}$ is expressed in the correlated Gaussians with global vectors

$$F_{(L_1L_2)LM}(u_1,u_2,A,\boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Strength function of ^{4}He

- Ground state: precise wave function

  accuracy～60 keV (GFM, NCSM, …)

  Y. Suzuki, W.H., M. Orabi, K. Arai, Few-body syst. 42 33-72 (2008).

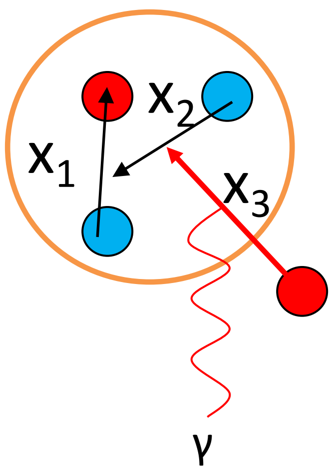
- Continuum state
  - A superposition of many basis functions
    - Important configuration at around 20-40 MeV
      - 3N+N cluster structure W. H. and Y. Suzuki, PRC78, 034305(2008)
      - Distortion of the clusters
  - Discretized <-> Continuous strength function
    - Complex Scaling Method

Configurations for 1⁻ continuum state

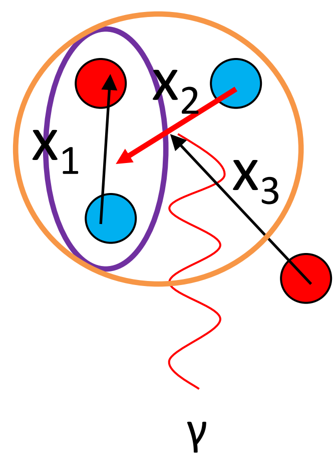
Dipole operator

$$\mathcal{M}(E1,\mu) = \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} e_i \mathcal{Y}_{1\mu}(\boldsymbol{r}_i - \boldsymbol{X})$$

3N+N



3N*+N



3N: three-body cal.
3N-N: p-wave (Gaussians)

2N: two-body cal.
2N-N: p-wave (Gaussians)
3N*-N: s-wave (Gaussians)

Strength function with Complex Scaling Method

$$\boldsymbol{x} \to \mathrm{e}^{i\theta}\boldsymbol{x}$$

$$\Psi_\lambda(\theta) = \sum_i C_i^\lambda(\theta)\Phi_i(\boldsymbol{x}) \qquad H(\theta)\Psi_\lambda(\theta) = E_\lambda(\theta)\Psi_\lambda(\theta)$$

Strength function

$$R(E) = -\frac{1}{\pi}\sum_\lambda \mathrm{Im}\frac{\langle\Psi_0^*(\theta)|W^\dagger(\theta)|\Psi_\lambda(\theta)\rangle\langle\Psi_\lambda^*(\theta)|W(\theta)|\Psi_0(\theta)\rangle}{E - E^\lambda(\theta) + i\epsilon}$$

Dipole strength function
with 3N+N configuration (ϑ=20°)

- Realistic two-body + Three body force
- Effective (central)

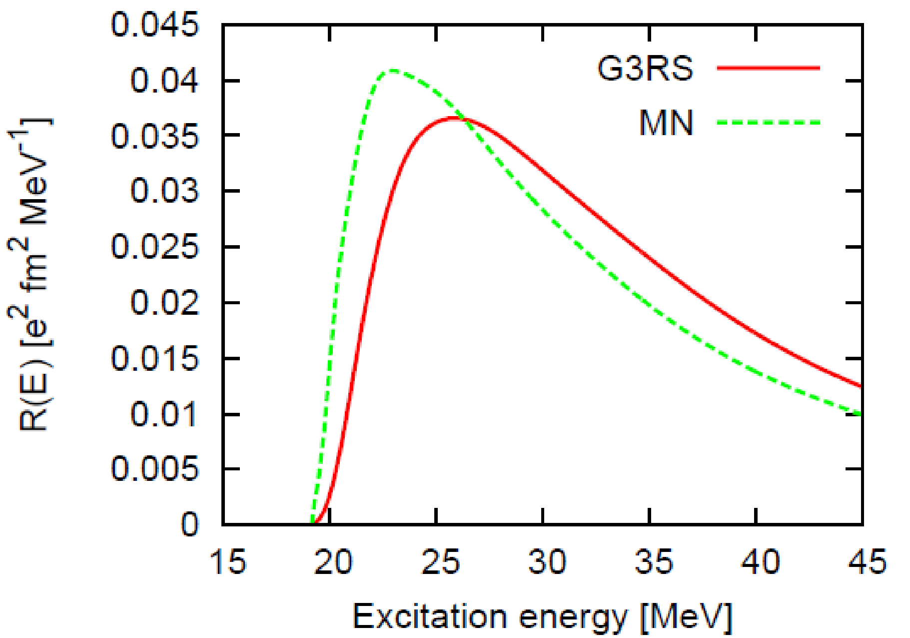
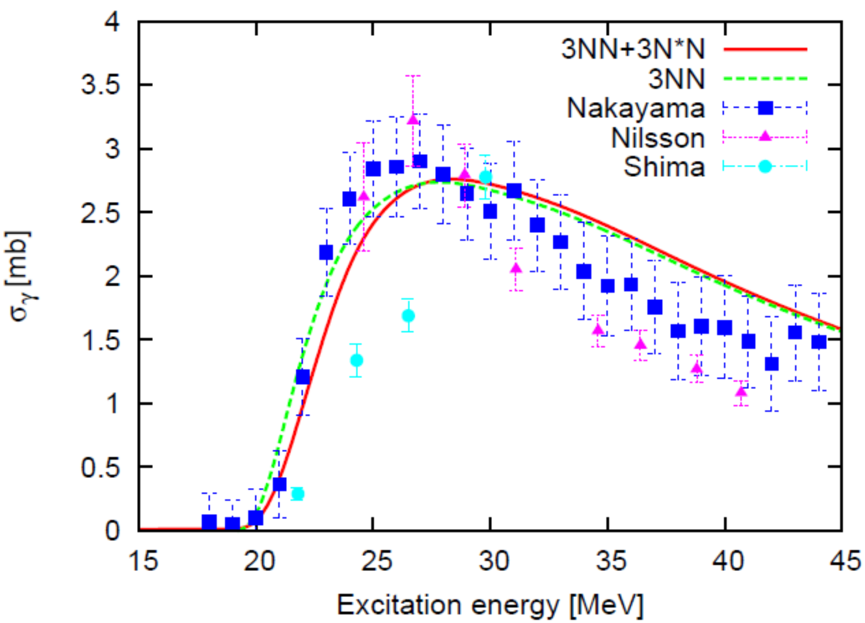


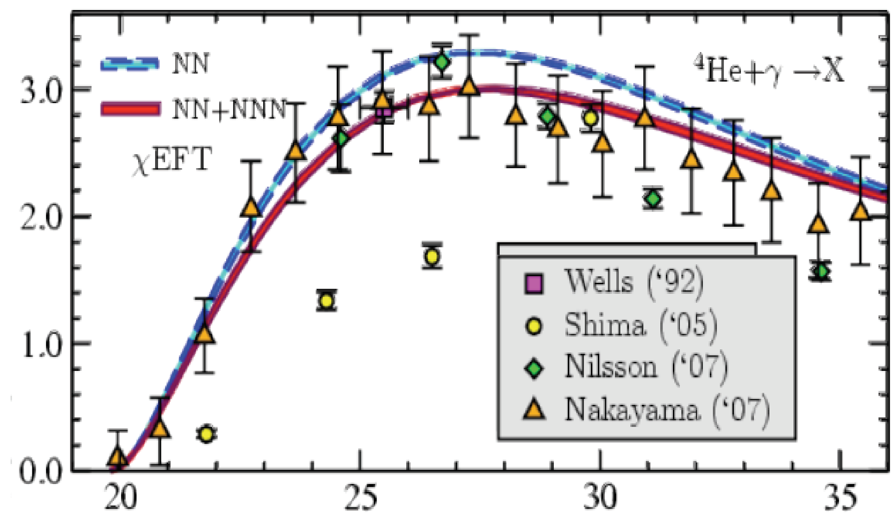
Photo-absorption cross section

Photo-absorption cross section

$$\sigma_\gamma(E) = \frac{4\pi^2}{\hbar c} E R(E)$$



NCSM+LIT



Data taken from
S. Nakayama et al. PRC 76, 021305 (2007).
B. Nilsson et al., PRC 75, 014007 (2007).
T. Shima et al., PRC 72, 044004 (2005).

S. Quaglioni et al., slide presented in FM50
S. Quaglioni et al., Phys. Lett. B652, 370-375(2007)

Operators induced by the weak interaction

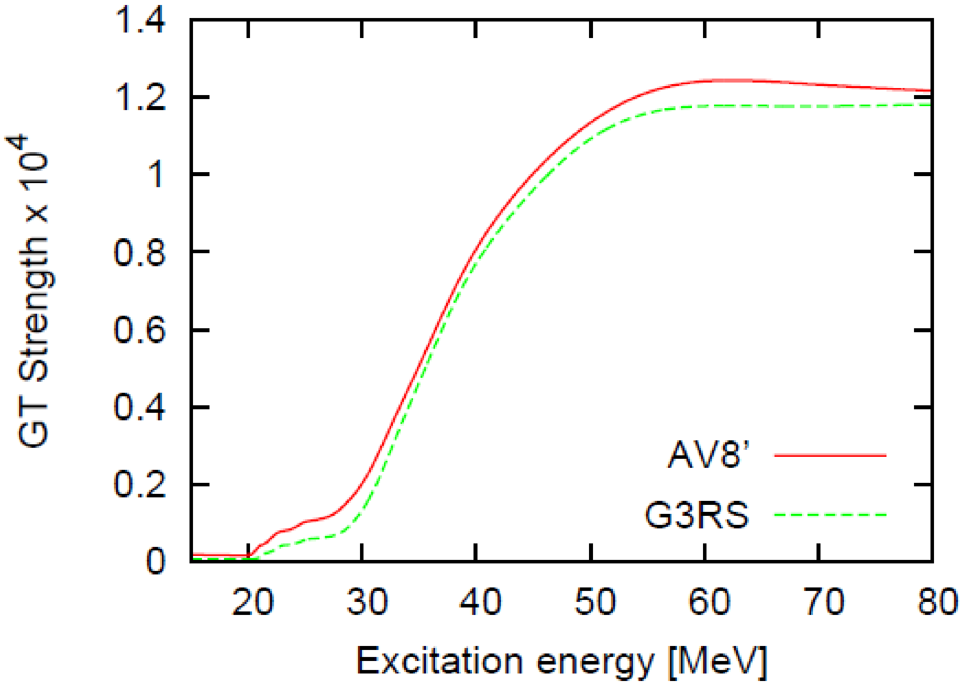
- Allowed transition (2 types)
  - Fermi type: does not contribute to T_z=0 state
  - Gamow-Teller type: $0^+0 \rightarrow 1^+1$
- First forbidden transition (5 types)
  - Dipole (E1) type: $0^+0 \rightarrow 1^-1$
  - Spin-dipole (SD) type (λ=0,1,2): $0^+0 \rightarrow \lambda^-1$
  - SD type in the momentum space $0^+0 \rightarrow 0^-1$

Gamow-Teller strength functions

Definition $$\mathcal{M}(\mathrm{GT},\mu) = \sum_{i=1}^{A} \sigma_\mu^{(i)} \tau_0^{(i)}$$ (neutral current)

Reduced Transition Matrix Element

$$B(\mathrm{GT};0^+0 \to 1^+1) = |\langle \Psi_f \| \mathcal{M}(\mathrm{GT}) \| \Psi_0 \rangle|^2$$



3NN+3N*N configuration
θ=15°

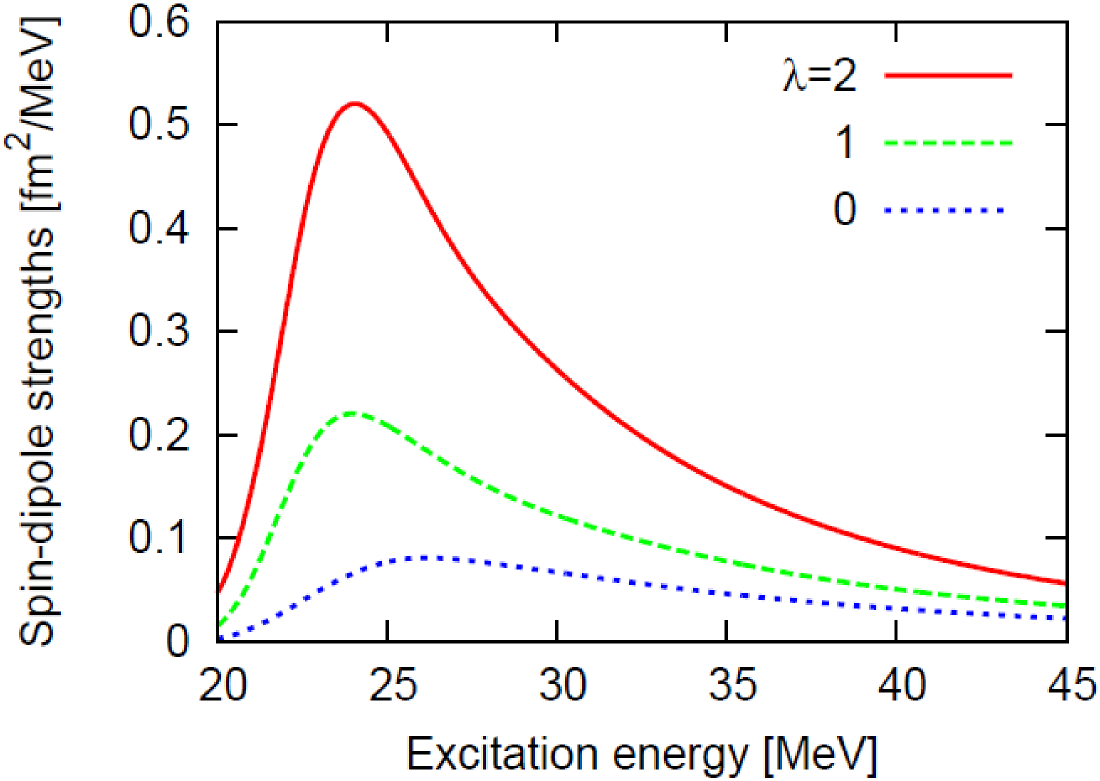
Distributions reflect the D-state probabilities in the ground states
AV8' 14%
G3RS 11%

Spin-dipole strength functions

Definition $$\mathcal{M}(\mathrm{SD},\lambda\mu) = \sum_{i=1}^{A} \left[\mathcal{Y}_1(\boldsymbol{r}_i - \boldsymbol{X}) \times \sigma^{(i)}\right]_{\lambda\mu} \tau_0^{(i)}$$ (neutral current)

Reduced Transition Matrix Element

$$B(\mathrm{SD}; 0^+0 \to \lambda^-1) = |\,\langle \Psi_f |||\mathcal{M}(\mathrm{SD},\lambda)||| \Psi_0 \rangle\,|^2$$

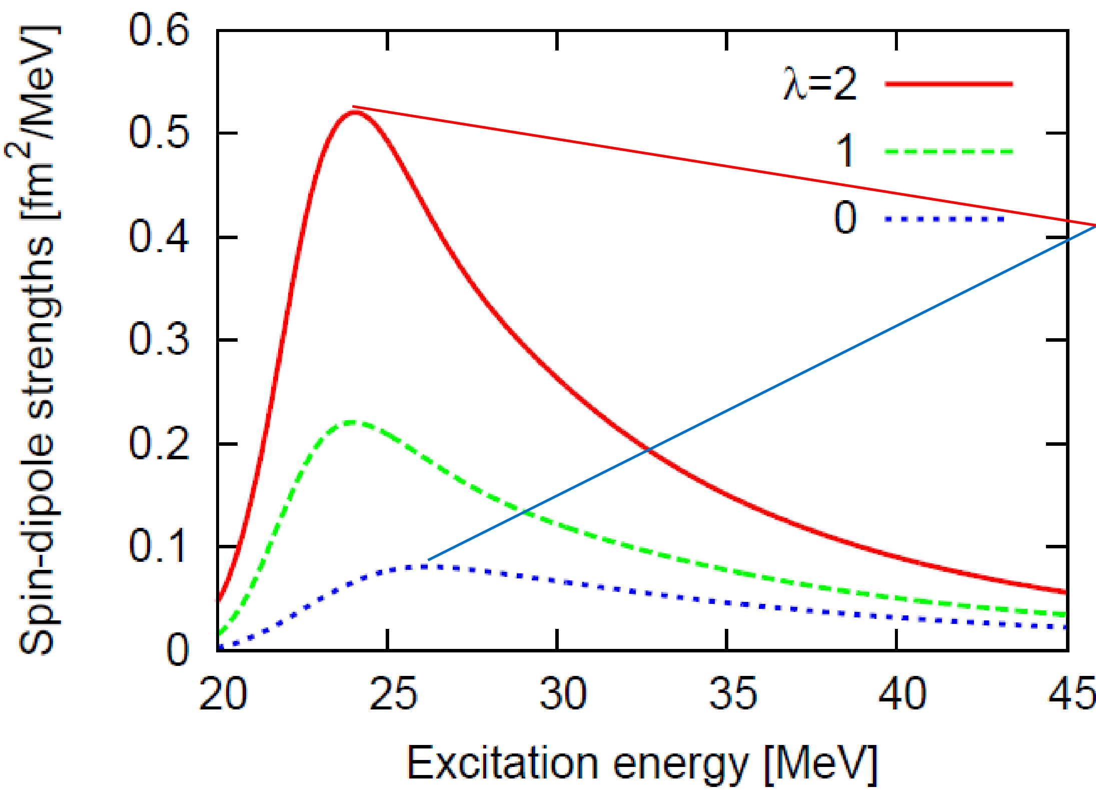


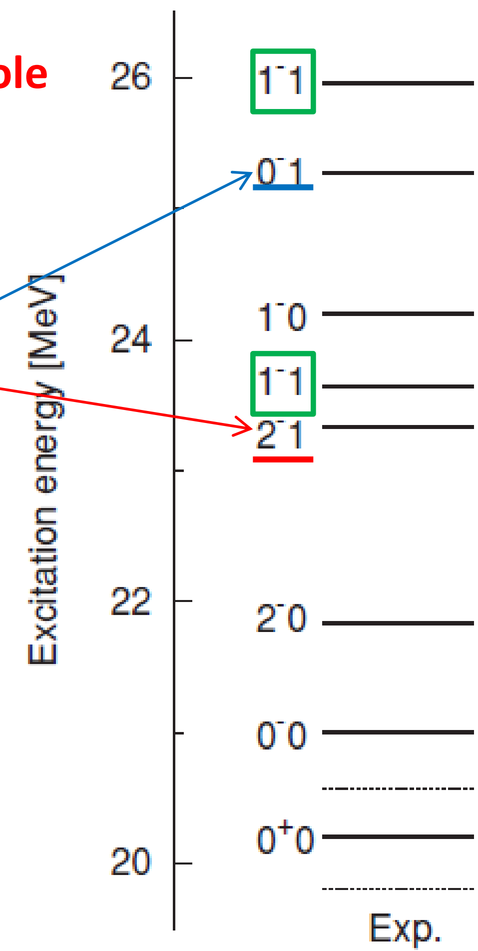
3NN+3N*N configuration
θ=20°

Spin-dipole strength functions

LS components calculated with a bound state approx. W. H. and Y. Suzuki, PRC78, 034305(2008)

	0^-1	1_1^-1	1_2^-1	2^-1	
(LS)= (10)	--	51.0	42.9	--	<- **Dipole**
= (11)	96.9	43.0	53.1	93.7	<- **Spin-dipole**



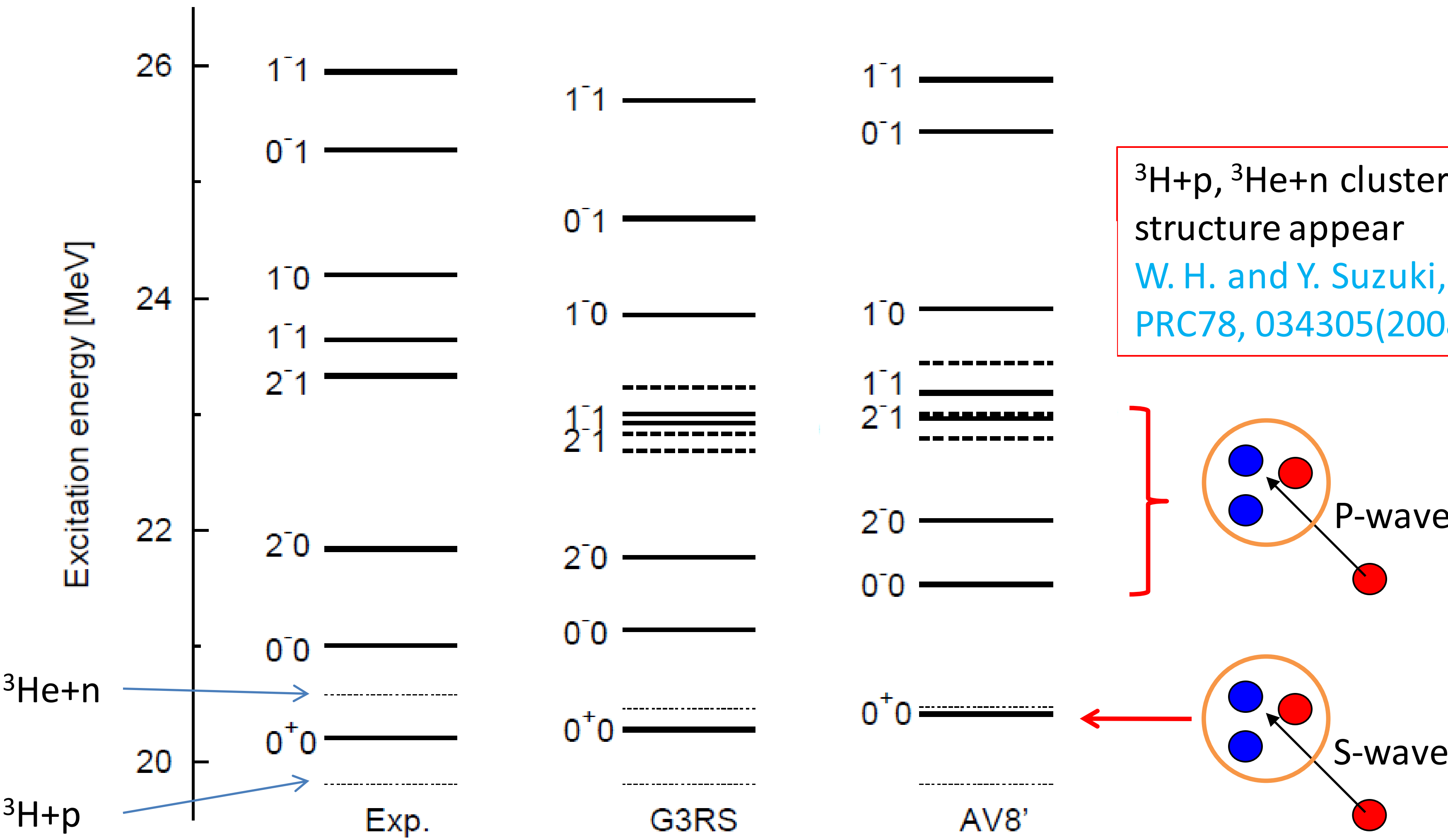


Summary and Future work

- Four-body calculation with bare realistic interactions
  - Important configurations at 20 - 40 MeV
    - 3N+N cluster structure
    - Distortion of the clusters
    - Complex Scaling method
  - Dipole strength functions
    - Good agreement with some of the experiments
    - At low energy: disagree with the experiment by Shima et al.
  - Strength functions induced by the weak interaction
    - GT strengths <-> ground state property
    - Spin-dipole and dipole strengths <-> continuum structure of ^{4}He

      Future: ν-^{4}He cross section

^{4}He spectrum



^{3}H+p, ^{3}He+n cluster structure appear

W. H. and Y. Suzuki, PRC78, 034305(2008)

Ground state energy also agrees with the other precise methods within 60 keV

Y. Suzuki, W.H., M. Orabi, K. Arai, Few-body syst. 42 33-72 (2008).

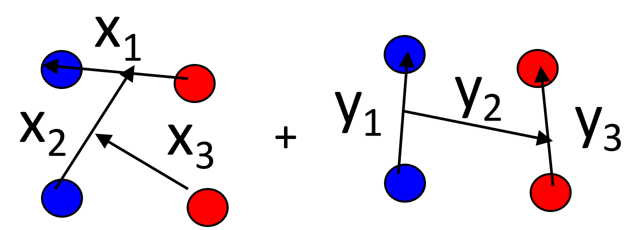
Test of GVR

Potential	G3RS		AV8′		
Method	GVR	PWE	GVR	PWE	Faddeev
$^3\mathrm{H}(\frac{1}{2}^+)$					
E	−7.73	−7.72	−7.76	−7.76	−7.767
$\langle T\rangle$	40.24	40.22	47.59	47.57	47.615
$\langle V_c\rangle$	−26.80	−26.79	−22.50	−22.49	−22.512
$\langle V_t\rangle$	−21.13	−21.13	−30.85	−30.84	−30.867
$\langle V_b\rangle$	−0.03	−0.03	−2.00	−2.00	−2.003
$\sqrt{\langle r^2\rangle}$	1.79	1.79	1.75	1.75	
$P(0,1/2)$	92.95	92.94	91.38	91.37	91.35
$P(2,3/2)$	7.01	7.02	8.55	8.57	8.58
$P(1,1/2)$	0.03	0.03	0.04	0.04	}0.07
$P(1,3/2)$	0.02	0.02	0.02	0.02	
$^4\mathrm{He}(0^+)$					
E	−25.29	−25.26	−25.08	−25.05	
$\langle T\rangle$	86.93	86.77	101.59	101.36	
$\langle V_c\rangle$	−66.24	−66.11	−54.93	−54.73	
$\langle V_{\mathrm{Coul}}\rangle$	0.76	0.76	0.77	0.77	
$\langle V_t\rangle$	−46.62	−46.55	−67.85	−67.79	
$\langle V_b\rangle$	−0.13	−0.12	−4.65	−4.66	
$\sqrt{\langle r^2\rangle}$	1.51	1.51	1.49	1.49	
$P(0,0)$	88.46	88.50	85.76	85.79	
$P(2,2)$	11.30	11.26	13.87	13.85	
$P(1,1)$	0.25	0.24	0.36	0.36	

Comparison with Partial Wave Expansion (PWE)

$$\exp\left(-\frac{1}{2}a_1\boldsymbol{x}_1^2-\frac{1}{2}a_2\boldsymbol{x}_2^2\cdots\right)\times[[[\mathcal{Y}_{\ell_1}(\boldsymbol{x}_1)\mathcal{Y}_{\ell_2}(\boldsymbol{x}_2)]_{L_{12}}\mathcal{Y}_{\ell_3}(\boldsymbol{x}_3)]_{L_{123}}\cdots]$$

→ combine rearrangement channels



Ground state energy agrees with the other precise methods within 60 keV.
H. Kamada et al., PRC64, 044001 (2001)

Y. Suzuki, W. H., M. Orabi, K. Arai, Few-Body Systems **42**, 33 (2008).

Four-body calculation for 1⁻ continuum state

Increase basis size one by one

Dipole operator

$$\mathcal{M}(E1,\mu)=\sqrt{\frac{4\pi}{3}}\sum_{i=1}^{A}e_i\mathcal{Y}_{1\mu}(\boldsymbol{r}_i-\boldsymbol{X})$$

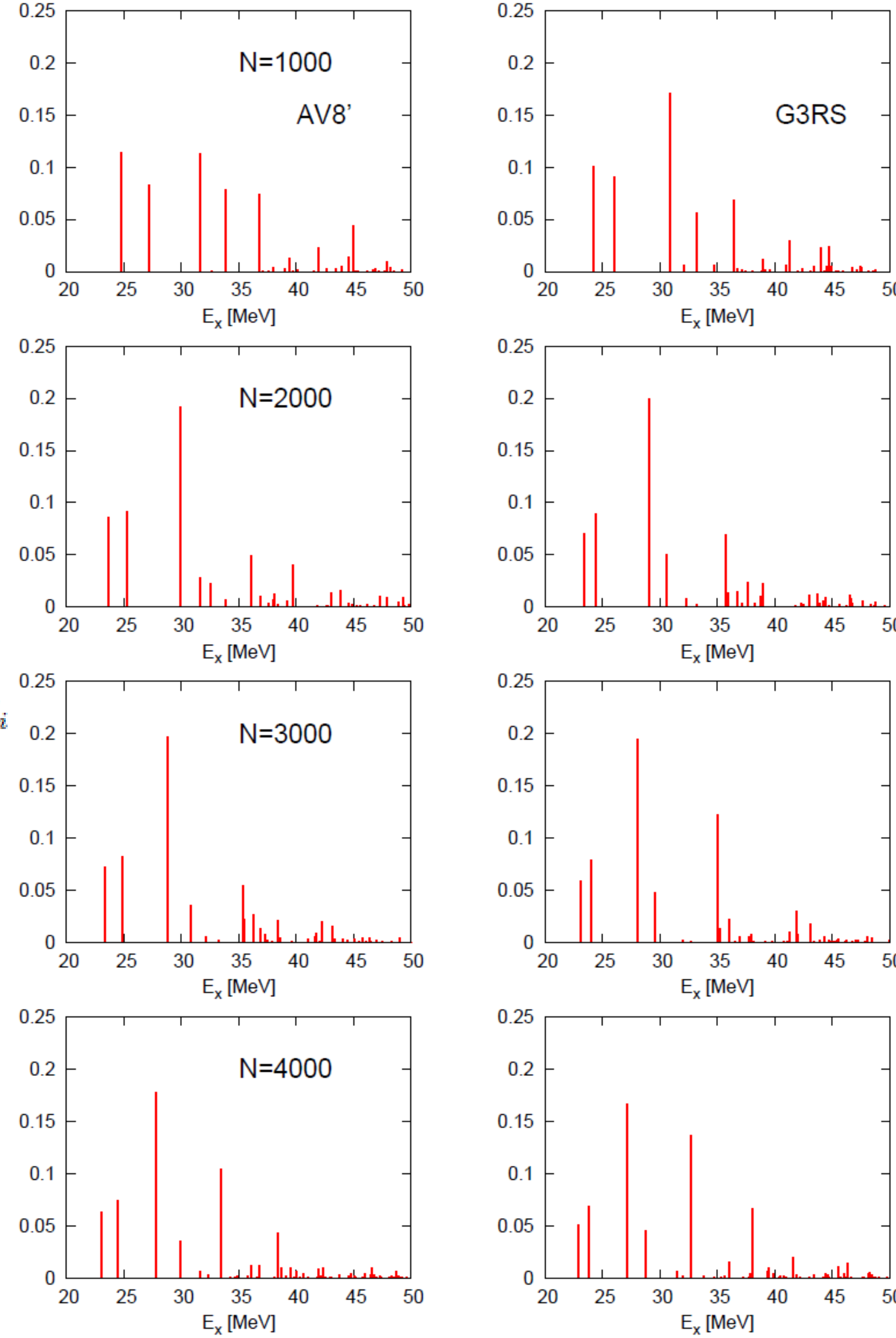
Transition strength

$$R(E_i)=\sum_k\left|\langle\Psi_{E_k}|\mathcal{O}|\Psi_0\rangle\right|^2\delta_{E_k-E_0,E_i}$$

Sum rule $\langle\Psi_0|\mathcal{M}^\dagger\mathcal{M}|\Psi_0\rangle\sim\frac{ZN}{3(A-1)}\langle r_p^2\rangle$

N	AV8’	G3RS
1000	0.841	0.877
2000	0.849	0.883
3000	0.852	0.885
4000	0.853	0.886

With all configurations
0.855 ～95%

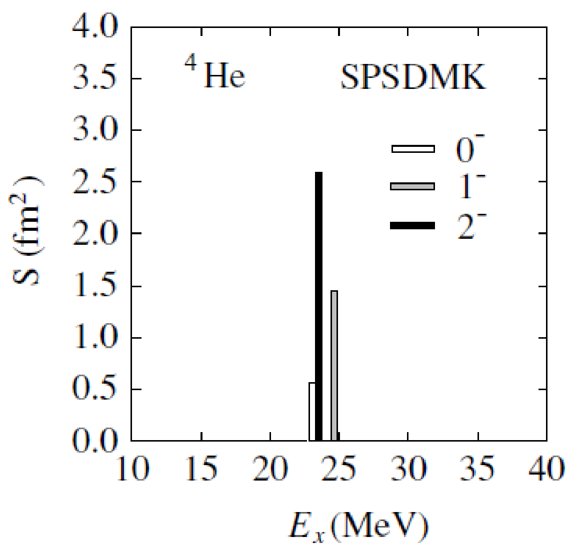


Spin-dipole strengths

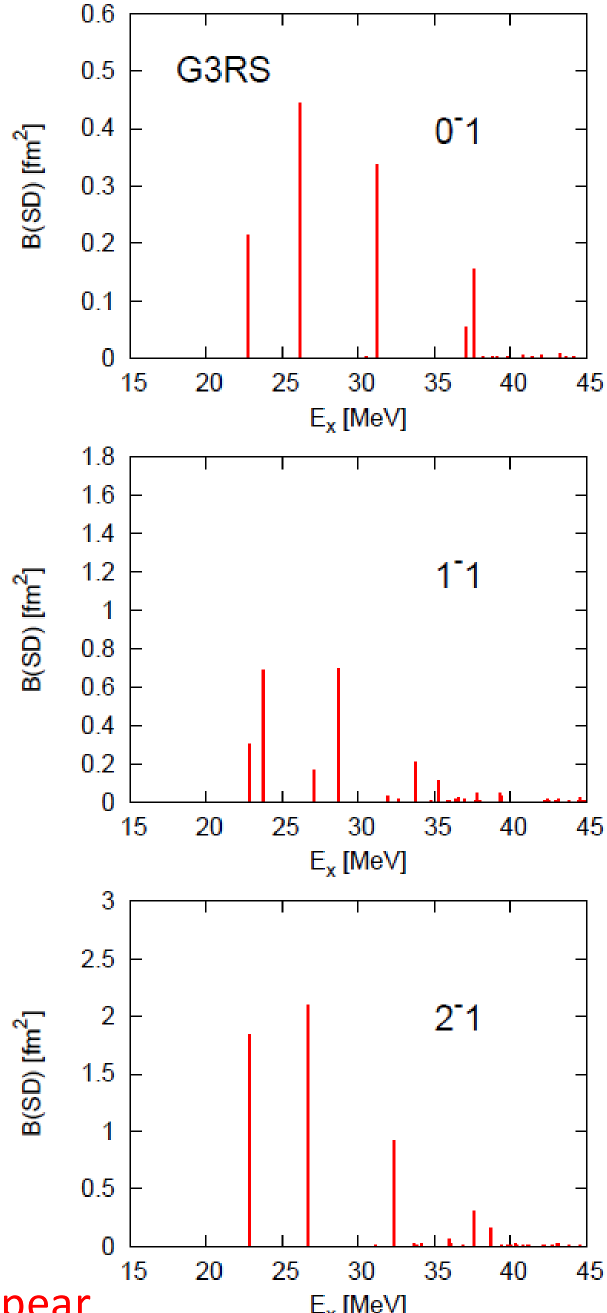
$$\mathcal{M}(\mathrm{SD}, \lambda\mu) = \sum_{i=1}^{A} \left[\mathcal{Y}_1(\boldsymbol{r}_i - \boldsymbol{X}) \times \sigma^{(i)}\right]_{\lambda\mu} t_{\pm}^{(i)}$$

$$B(\mathrm{SD}; 0^+0 \to \lambda^- 1) = |\langle \Psi_f ||| \mathcal{M}(\mathrm{SD}, \lambda) ||| \Psi_0 \rangle|^2$$

Shell model calculation
T. Suzuki et al., Phys. Rev. C 74, 034307 (2006)



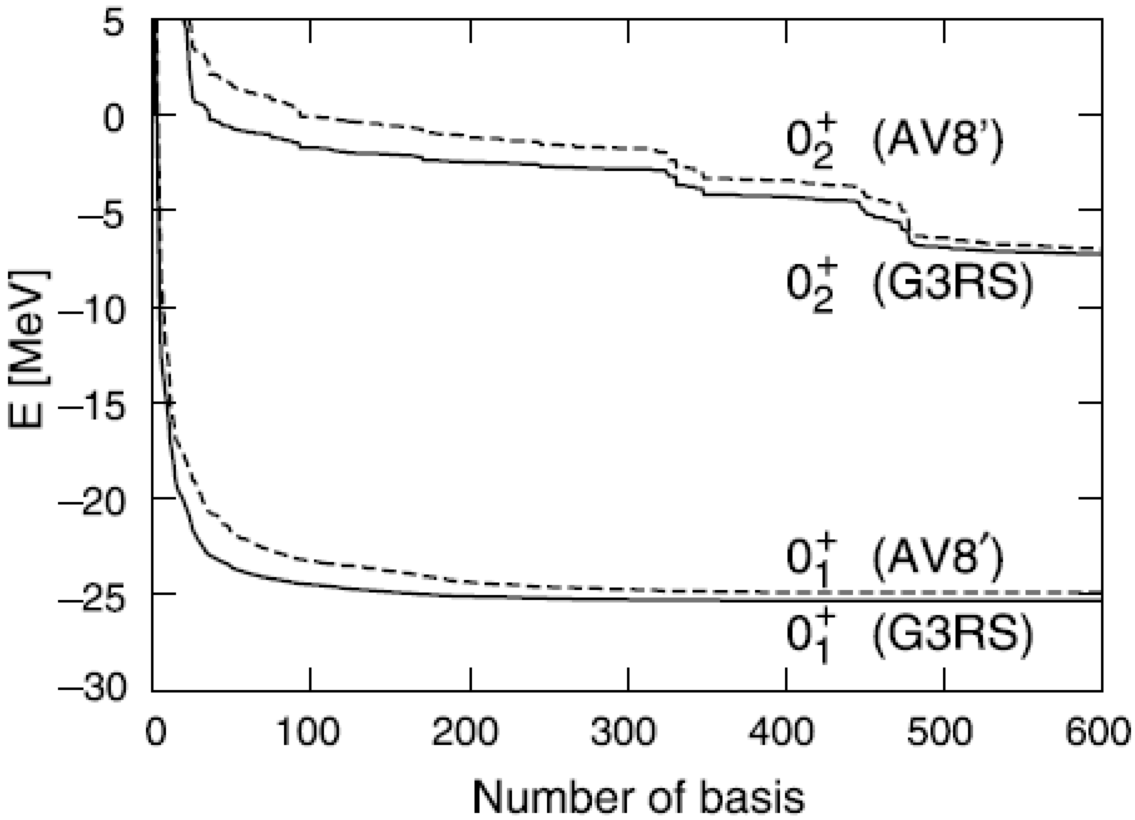
many states appear



Energy convergence for ^{4}He

Optimization of a basis set by the Stochastic Variational Method

K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).



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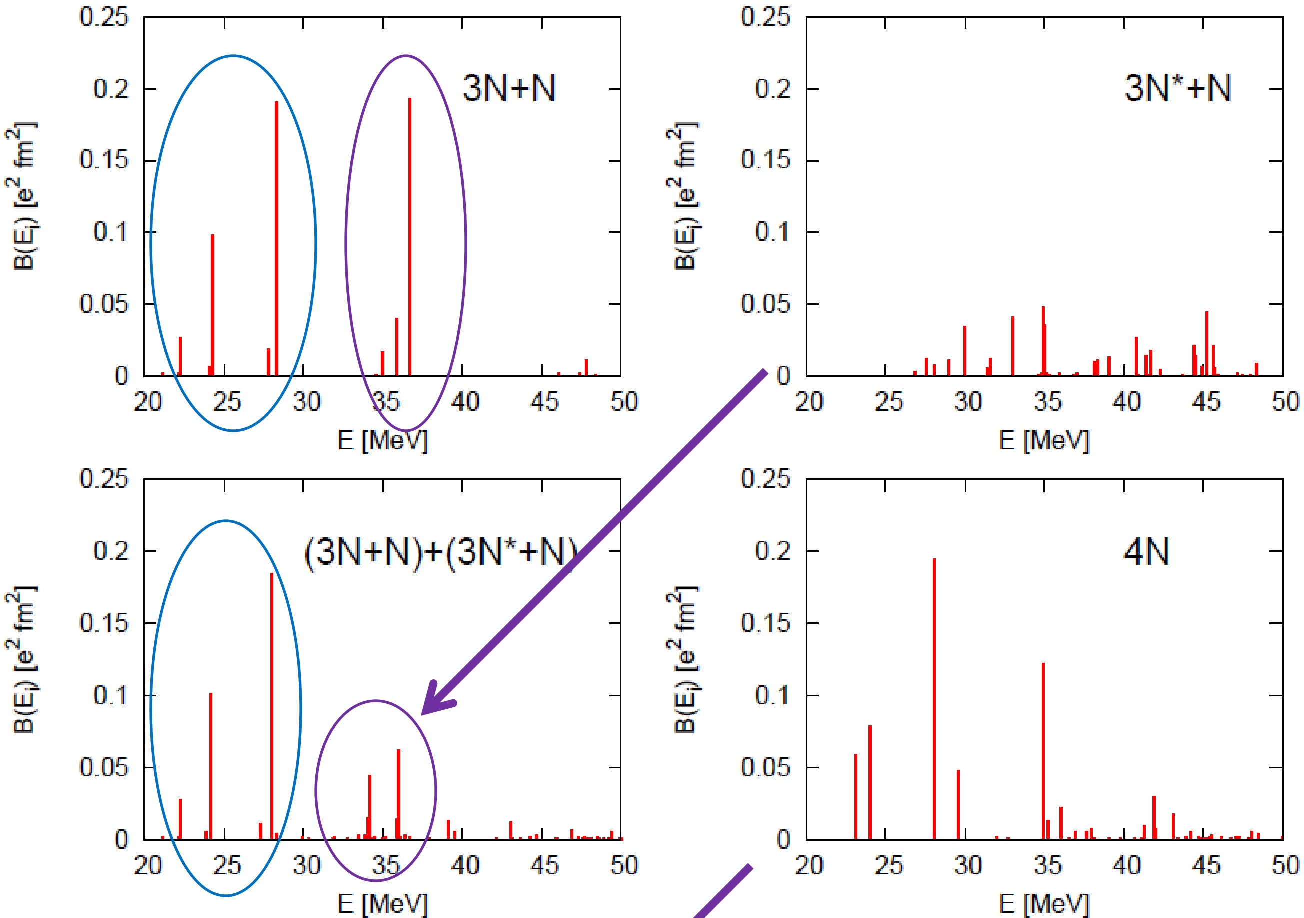
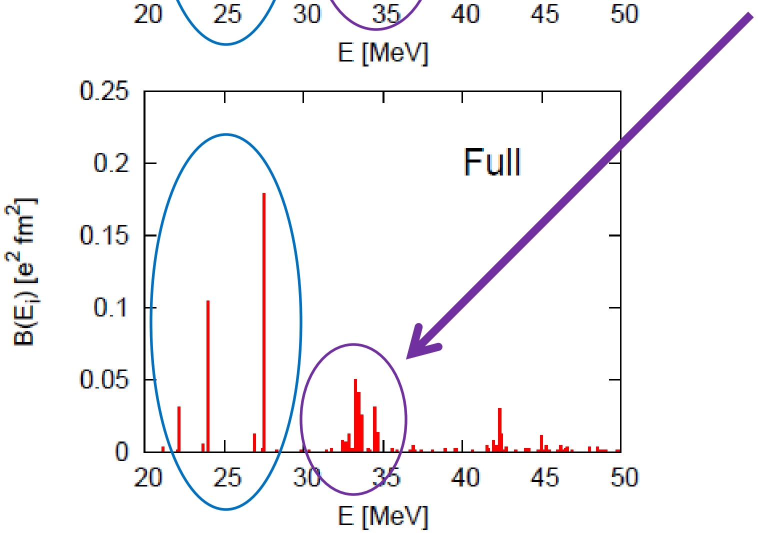


~25 MeV

3N+N configurations

~35 MeV

2N+N+N and ...



Dipole operator

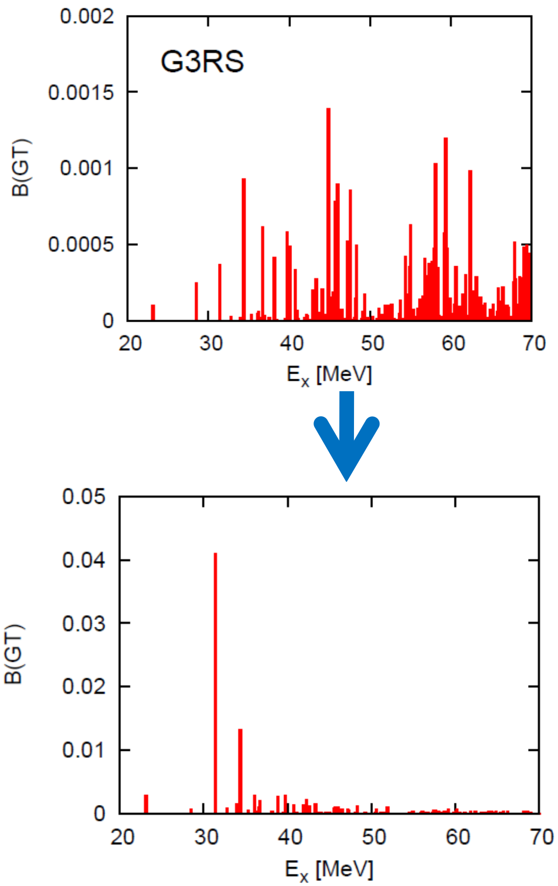
$$\mathcal{M}(E1,\mu) = \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} e_i \mathcal{Y}_{1\mu}(\boldsymbol{r}_i - \boldsymbol{X})$$

Transition strength

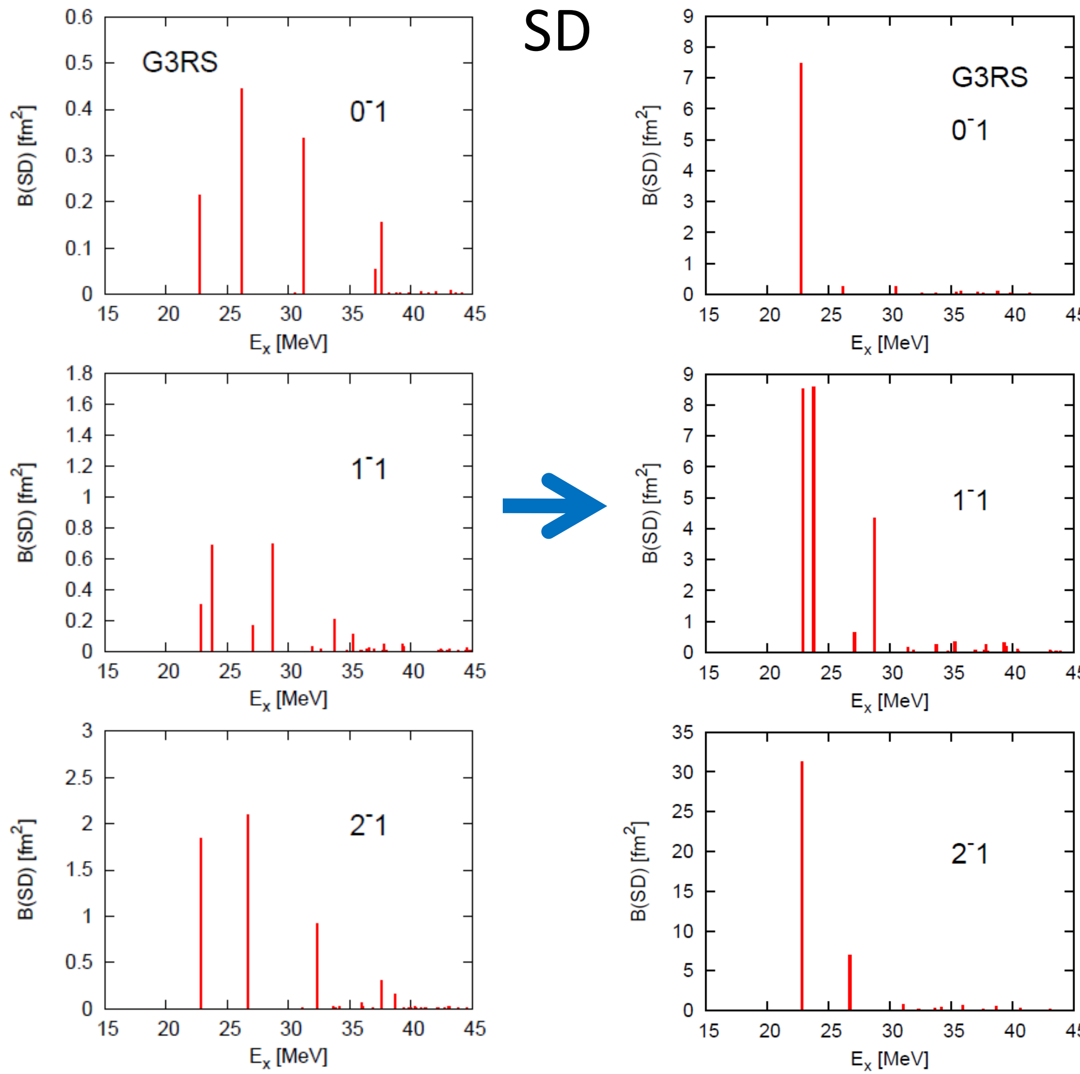
$$R(E_i) = \sum_k \left| \langle \Psi_{E_k} | \mathcal{O} | \Psi_0 \rangle \right|^2 \delta_{E_k - E_0, E_i}$$

Transitions from the first excited 0^+

GT



SD



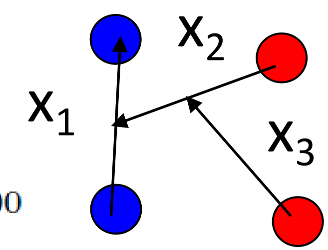
3N+N structure

Correlated Gaussian and global vector

Correlated Gaussian

$$\exp\left(-\frac{1}{2}ar^2\right) \to \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right) = \exp\left(-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right)$$

$$\exp\left(A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right) \sim \sum_n (\boldsymbol{x}_i\cdot\boldsymbol{x}_j)^n \sim \sum_{\ell=n,n-2,\ldots} [\mathcal{Y}_\ell(\boldsymbol{x}_i)\mathcal{Y}_\ell(\boldsymbol{x}_j)]_{00}$$



Global vector

$$r^l Y_{lm}(\hat{\boldsymbol{r}}) \equiv \mathcal{Y}_{lm}(\boldsymbol{r}) \to \mathcal{Y}_{LM_L}(\tilde{u}\boldsymbol{x}) = \mathcal{Y}_{LM_L}\left(\sum_{i=1}^{A-1} u_i\boldsymbol{x}_i\right)$$

$$\mathcal{Y}_{LM_L}(u_1\boldsymbol{x}_1+u_2\boldsymbol{x}_2) = \sum_{\ell=0}^{L}\sqrt{\frac{4\pi(2L+1)!}{(2\ell+1)!(2L-2\ell+1)!}}\; u_1^\ell u_2^{L-\ell}[\mathcal{Y}_\ell(\boldsymbol{x}_1)\mathcal{Y}_{L-\ell}(\boldsymbol{x}_2)]_{LM_L}$$

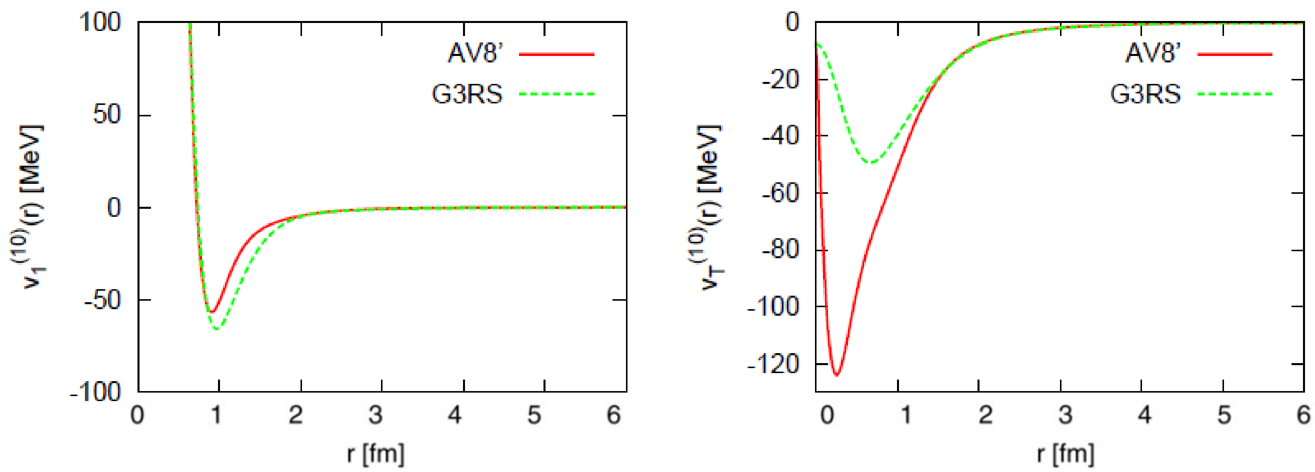
Global Vector Representation (GVR)

Parity $(-1)^{L_1+L_2}$

$$F_{(L_1L_2)LM}(u_1,u_2,A,\boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Hard to obtain a precise solution using a realistic interaction

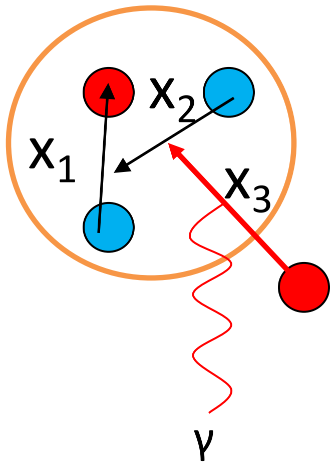
^3E



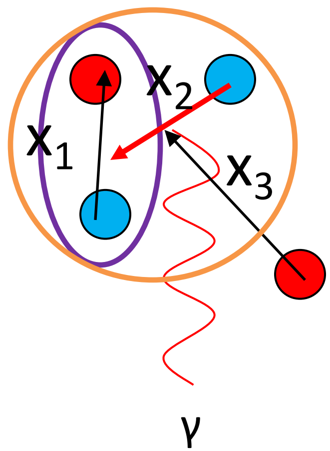
- Short-range repulsion
  - → A superposition of many basis states
- Strong tensor component
  - → Angular momentum coupling

Configurations for 1^- continuum state

3N+N



3N*+N



Dipole operator

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Transition strength

$$R(E_i) = \sum_k \left| \langle \Psi_{E_k} | \mathcal{O} | \Psi_0 \rangle \right|^2 \delta_{E_k - E_0, E_i}$$

3N: three-body cal.
3N-N: p-wave

2N: two-body cal.
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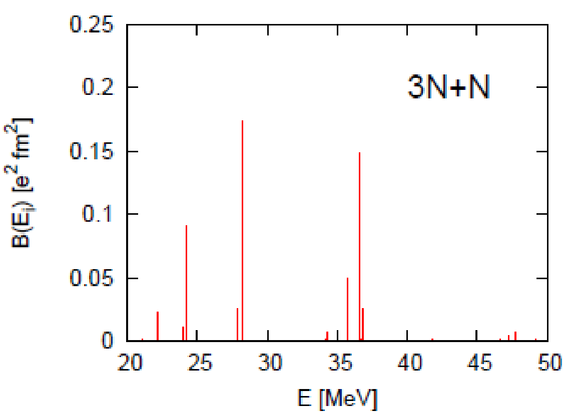
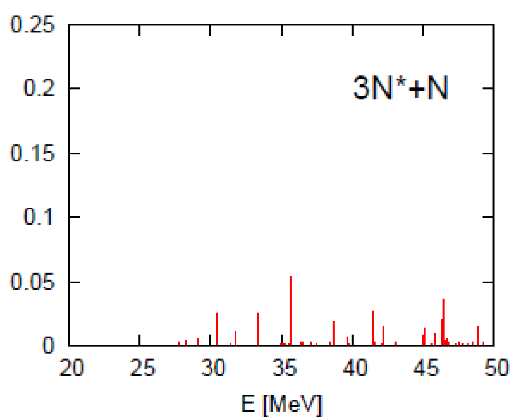
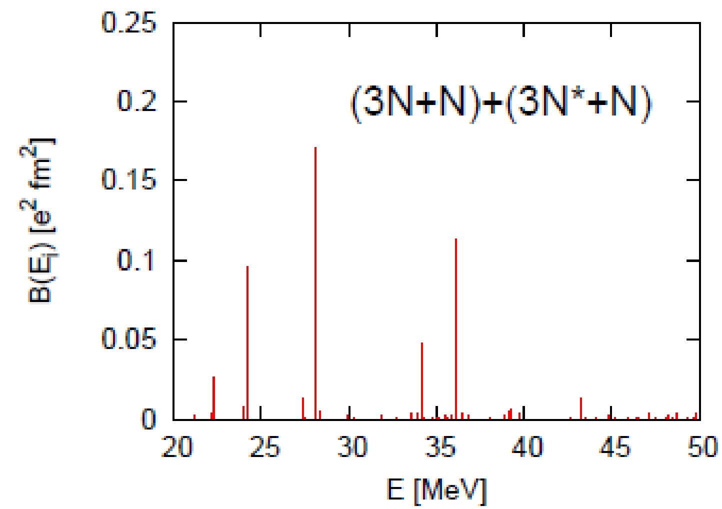
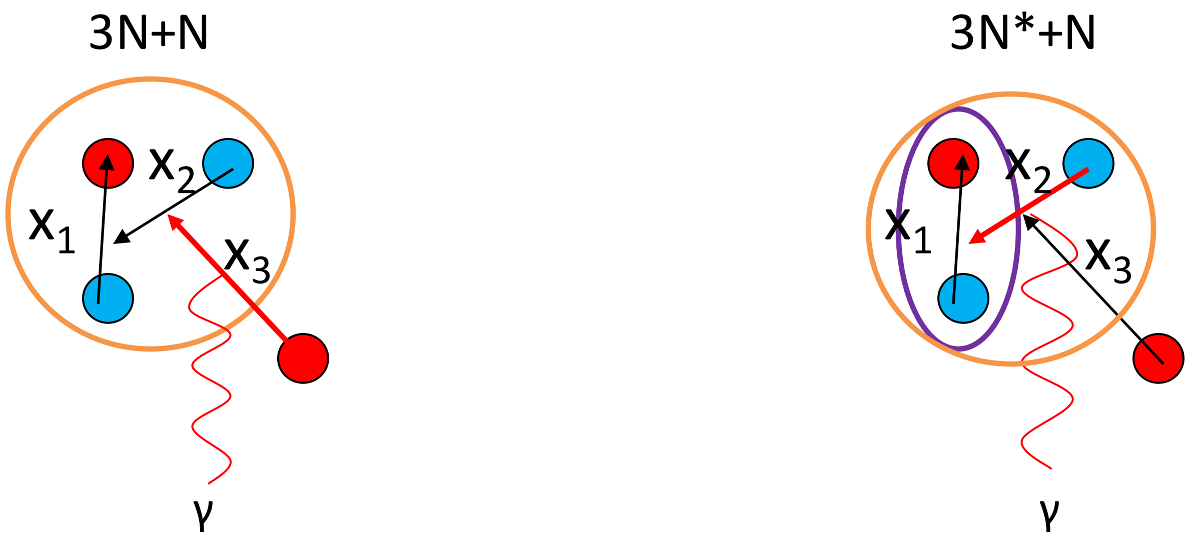
3N*-N: s-wave









$$\Psi_{1M}(3N+N) = \mathcal{A}\left[[\Psi_{L_{123}}(3N) f_1(x_3)]_L \left[\chi_{S_{12},S_{123}}(123)\chi_{1/2}(4)\right]_S\right]_{1M} ,$$

$$\begin{aligned}\Psi_{1M}(3N^*+N) &= \mathcal{A}\left[[\Psi_L(3N^*) g_0(x_3)]_L \left[\chi_{1,S_{123}}(123)\chi_{1/2}(4)\right]_S\right]_{1M} , \\ &= \mathcal{A}\left[[[\Psi_{L_{12}}(2N) f_1(x_2)]_L\, g_0(x_3)]_L \left[\left[\chi_1(12)\chi_{1/2}(3)\right]_{S_{123}} \chi_{1/2}(4)\right]_S\right]_{1M}\end{aligned}$$

Four-nucleon system

$$\Phi_{(LS)JM_JTM_T}$$
$$= \mathcal{A}\{\mathrm{e}^{-\frac{1}{2}\tilde{x}Ax}[[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_L\chi_S]_{JM_J}\eta_{TM_T}\},$$

J^π	(LS)
0^+	(00), (22) ; (11)
1^+	(01), (21), (22) ; (10), (11), (12), (32)
0^-	(11) ; (22)
1^-	(10), (11), (12), (32) ; (21), (22)
2^-	(11), (12), (31), (32) ; (20), (21), (22), (42).

$$\mathrm{e}^{-\frac{1}{2}\tilde{x}Ax} = \exp\left[-\frac{1}{2}\sum_{i<j}\left(\frac{\boldsymbol{r}_i - \boldsymbol{r}_j}{b_{ij}}\right)^2\right] \qquad A \rightleftarrows (b_{12},\, b_{13},\, \ldots,\, b_{34})$$

Correlated Gaussian with global vectors (GVR: Global vector representation)

- L, parity $(-1)^L$

$$\exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)\mathcal{Y}_{LM}(\widetilde{u_1}\boldsymbol{x})$$

$$\tilde{\boldsymbol{x}}A\boldsymbol{x} = \sum_{i,j=1}^{N-1} A_{ij}\boldsymbol{x}_i \cdot \boldsymbol{x}_j \qquad A_{ij} \neq 0$$

$$\exp(A_{ij}\boldsymbol{x}_i \cdot \boldsymbol{x}_j) \rightarrow \sum_n (\boldsymbol{x}_i \cdot \boldsymbol{x}_j)^n \sim \sum_{\ell=n,n-2,\ldots} [y_\ell(\boldsymbol{x}_i) y_\ell(\boldsymbol{x}_j)]_{00}$$

$$\widetilde{u_1}\boldsymbol{x} = \sum_{i=1}^{N-1} u_{1_i}\boldsymbol{x}_i$$ Global vector

$$\mathcal{Y}_{LM_L}(u_1 x_1 + u_2 x_2) = \sum_{\ell=0}^{L} \sqrt{\frac{4\pi(2L+1)!}{(2\ell+1)!(2L-2\ell+1)!}}\, u_1^\ell u_2^{L-\ell} [\mathcal{Y}_\ell(\boldsymbol{x}_1)\mathcal{Y}_{L-\ell}(\boldsymbol{x}_2)]_{LM_L}$$

- L, parity $(-1)^{L+1}$

$$\exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_L(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_1(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Algorithm of the SVM

Possibility of the stochastic optimization
1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

1. **Generate** $(A_k^1, A_k^2, \cdots\cdots, A_k^m)$ **randomly**
2. **Get the eigenvalues** $(E_k^1, E_k^2, \cdots\cdots, E_k^m)$
3. **Select** A_k^n corresponding to the lowest E_k^n and **Include** it in a basis set
4. **k → k+1**

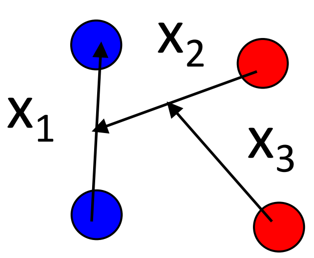
Y. Suzuki and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).
K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

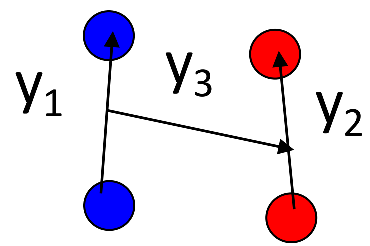
Advantages of GVR

Variational parameters A, u

→ Stochastically selected

- No need to specify intermediate angular momenta.
  - Just specify total angular momentum L
- Easy to include various rearrangement channels.





$$y = Tx \implies \tilde{y}By = \tilde{x}\tilde{T}BTx \qquad \tilde{v}y = \widetilde{\tilde{T}v}x$$