Complex absorbing potential for the continuum in real-space calculations

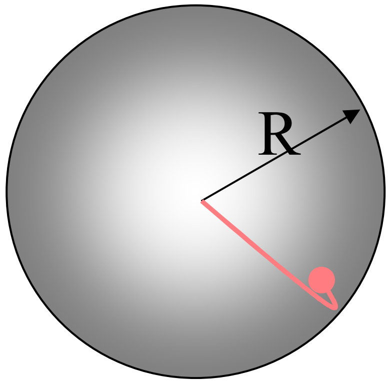
Takashi NAKATSUKASA *(RIKEN Nishina Center)*

JAPAN-ITALY EFES Workshop on Correlations in Reactions and Continuum,
Torino, Italy, 6-8, Sep. 2010

L^2 approximation of continuum wave functions

Time period of emitted particles to return

$$T = \frac{R}{v}$$

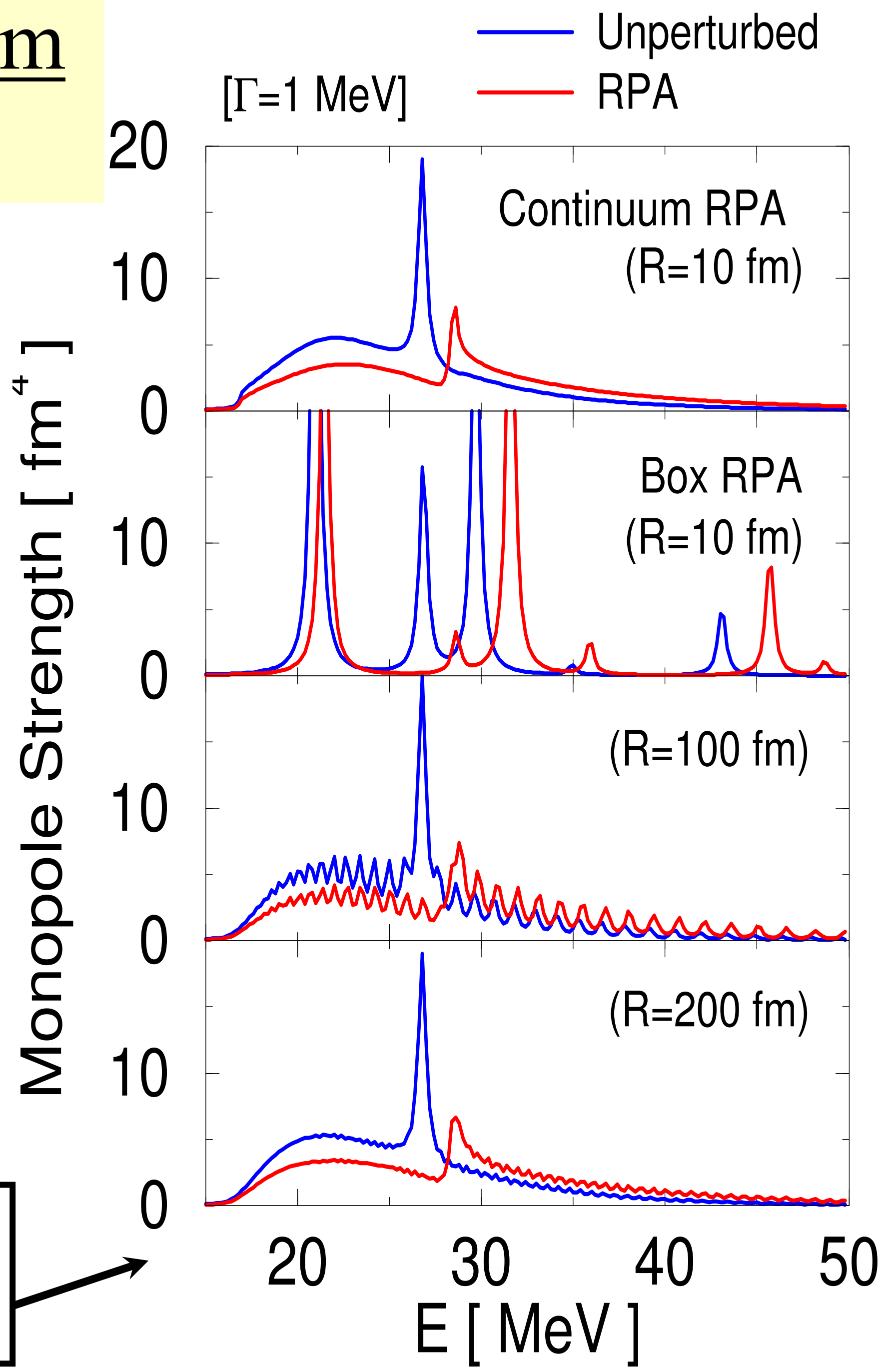


Corresponding to

$$\Delta E = \frac{2\pi}{T} \approx \frac{9}{R}\sqrt{\frac{E}{M}}$$

Therefore, in order to obtain $\Delta E < 1$ MeV, $R \geq 200$ fm is required.

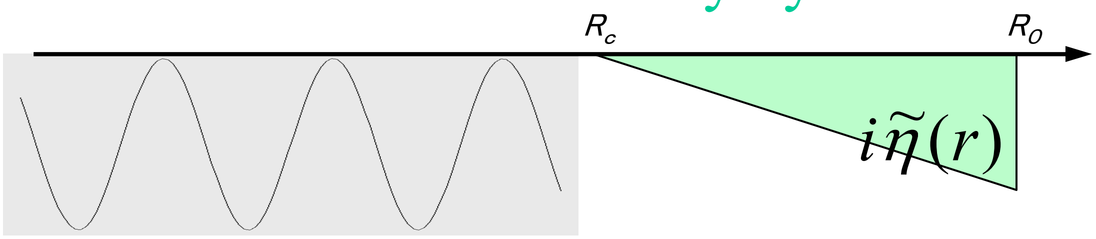
Test calculation:
GMR for ^{16}O with BKN interaction

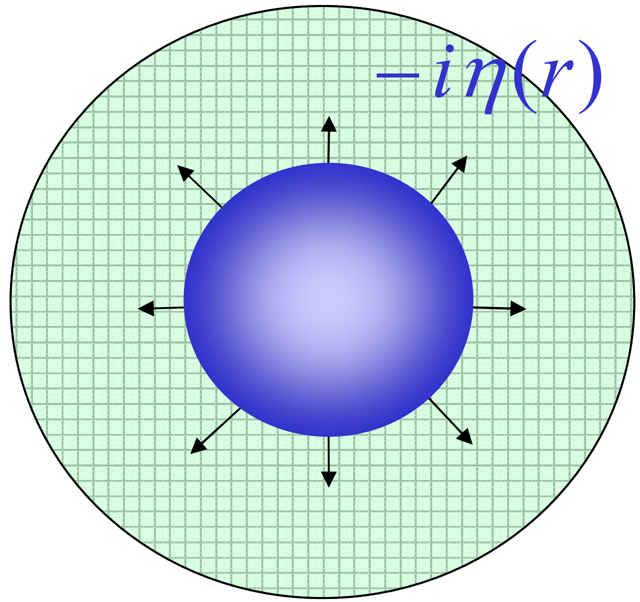


Continuum boundary simulated by ABC

- ABC (Absorbing boundary condition)

Particles are taken away by absorber





$$i\tilde{\eta}(r) = \begin{cases} 0 & \text{for } r < R_c \\ i\eta_0 (r - R_c)/(R_0 - R_c) & \text{for } r > R_c \end{cases}$$

Outgoing waves will damp as

$$e^{ikr} \rightarrow e^{ikr - \bar{k}r}$$

Criterion for a good absorber

$$7\frac{E^{1/2}}{(R_0 - R_c)(8m)^{1/2}} < \eta_0 < \frac{1}{10}(R_0 - R_c)(8m)^{1/2} E^{3/2}$$

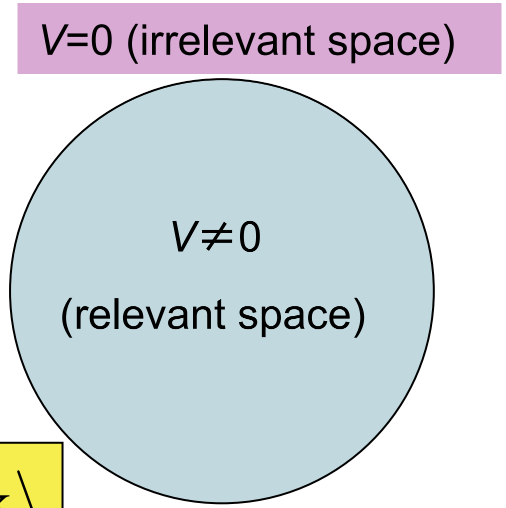
$$G^{(+)}(E) = \frac{1}{E - H + i\eta} \Rightarrow G^{(+)}(E) = \frac{1}{E - H + i\tilde{\eta}(r)}$$

Potential scattering

Wave function of an outgoing scattering state

$$\Psi^{(+)} = e^{i\mathbf{k}\cdot\mathbf{r}} + \Psi^{(+)}_{\text{scatt}} \xrightarrow{r\to\infty} e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega)\frac{e^{ikr}}{r}$$

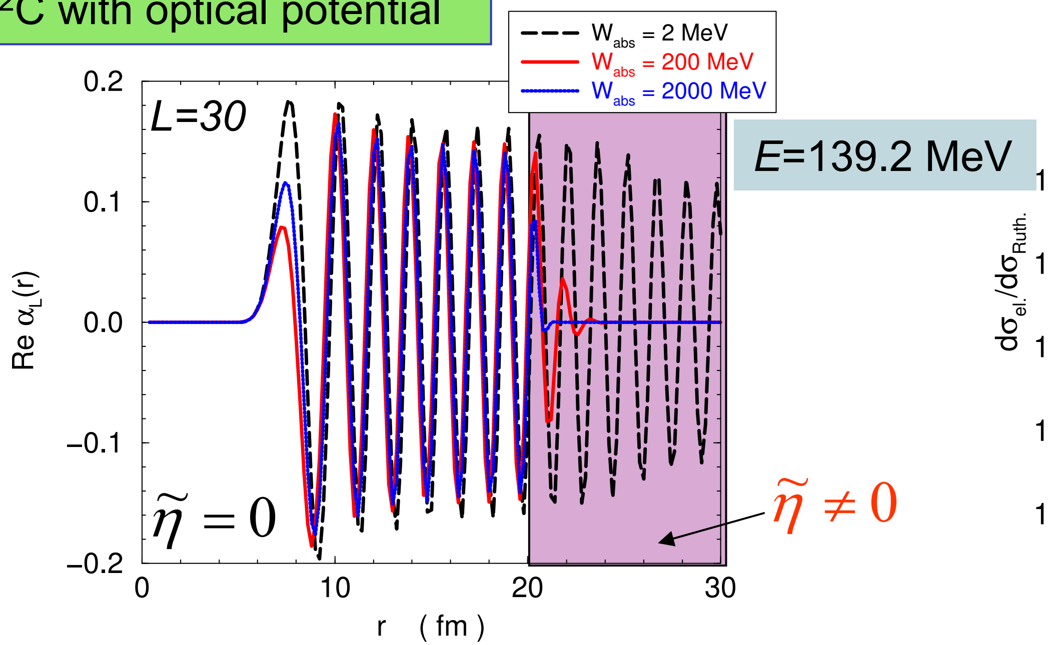
$$f(\Omega) = -\frac{m}{2\pi\hbar^2}\int d\vec{r}\, e^{-i\vec{k}_\Omega \vec{r}}\, V(r)\,\phi^{(+)}(\vec{r})$$

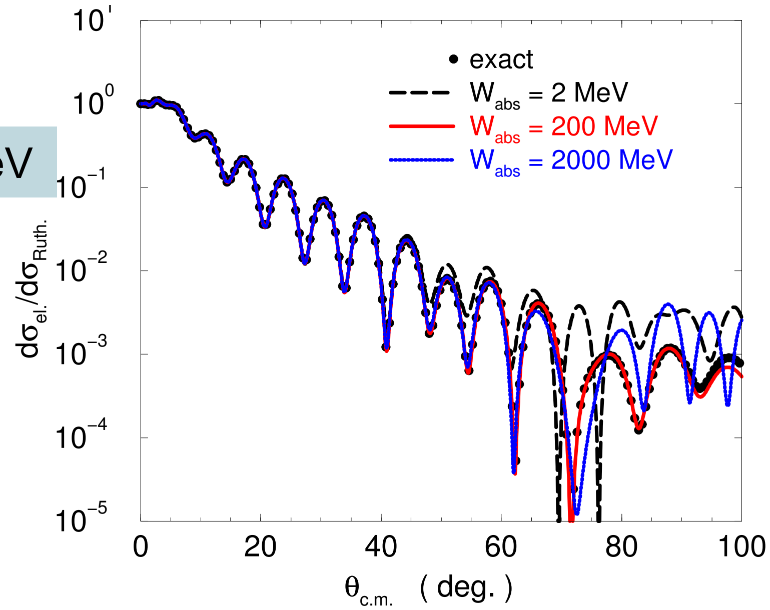


All we need is the scattering wave function in the interacting region of $V\neq 0$

$$\left|\Psi^{(+)}_{\text{scatt}}\right\rangle = \frac{1}{E-H+i\eta}V\left|\mathbf{k}\right\rangle \Rightarrow \boxed{\left(E-H+i\tilde{\eta}(\mathbf{r})\right)\left|\Psi^{(+)}_{\text{scatt}}\right\rangle = V\left|\mathbf{k}\right\rangle}$$

$^{16}O+^{12}C$ with optical potential

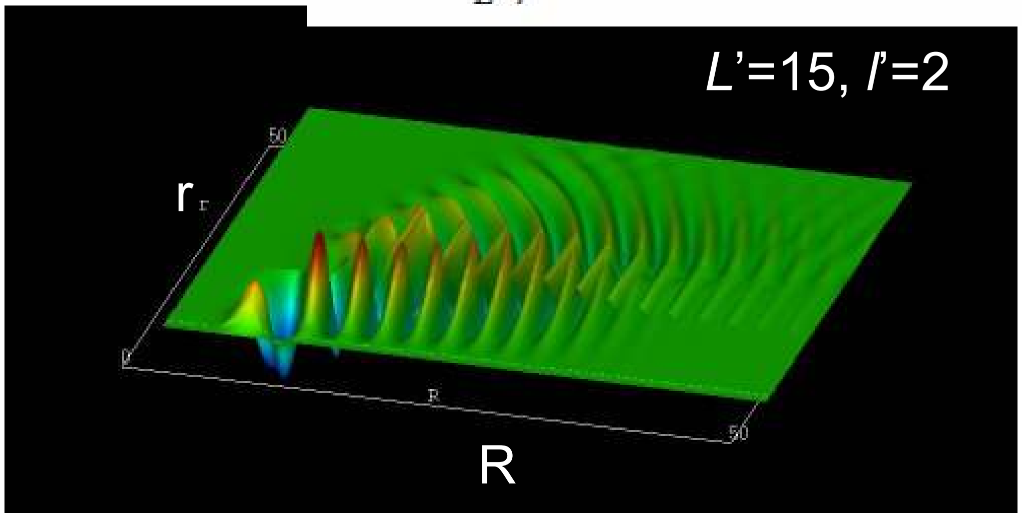


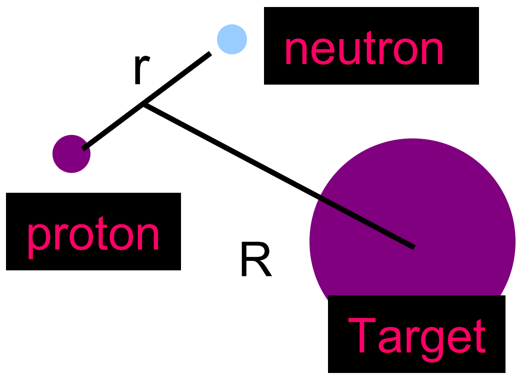


Three-body model
(Deuteron breakup reaction)

Ueda, Yabana, TN, PRC 67, 014606 (2003)

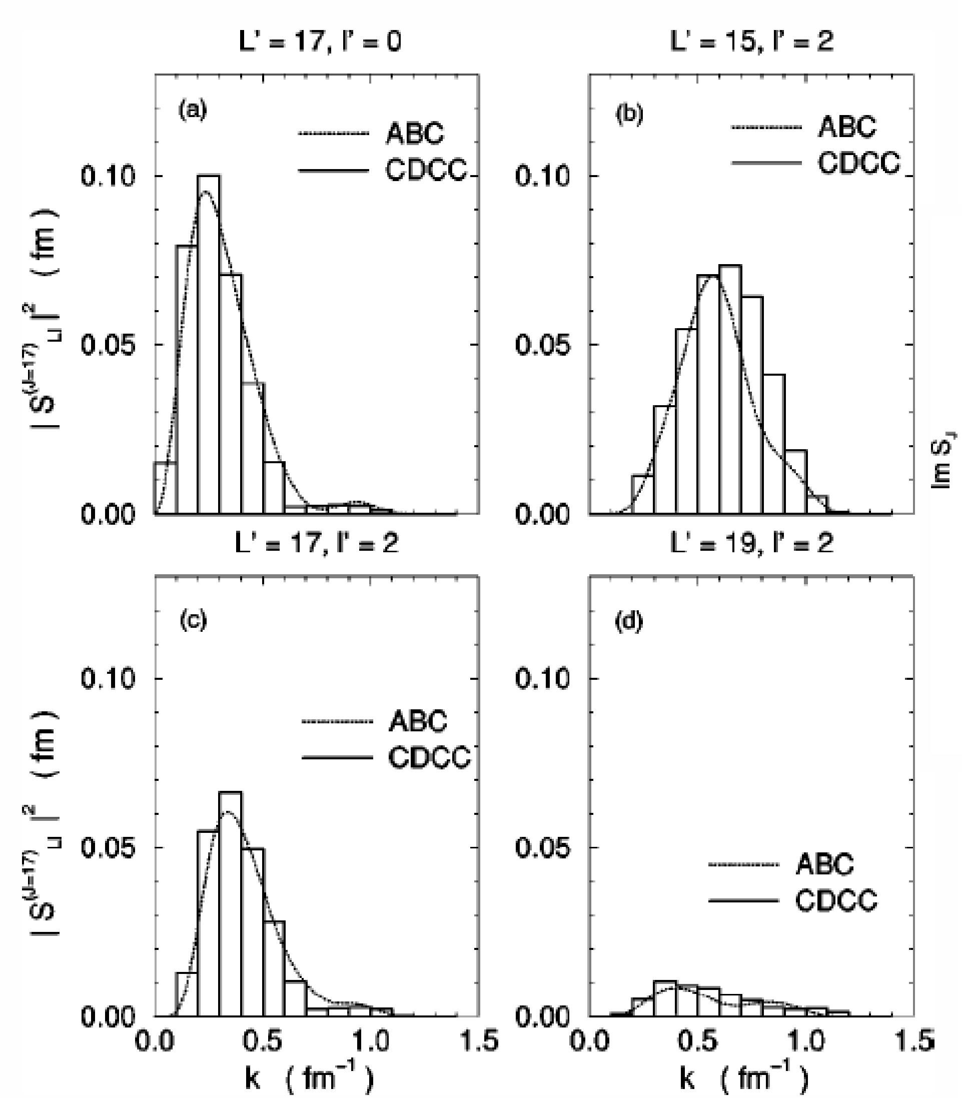
$$\left\{E + i\epsilon_{nC}(r) + i\epsilon_{pT}(R) - \left[-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + \frac{\hbar^2 L'(L'+1)}{2\mu R^2} - \frac{\hbar^2}{2m}\frac{\partial}{\partial r^2} + \frac{\hbar^2 l'(l'+1)}{2mr^2} + V_{nC}(r)\right]\right\}\alpha^J_{Ll_\alpha,L'l'}(R,r) - \sum_{L''l''} V^J_{L'l',L''l''}(R,r)\,\alpha^J_{Ll_\alpha,L''l''}(R,r)$$





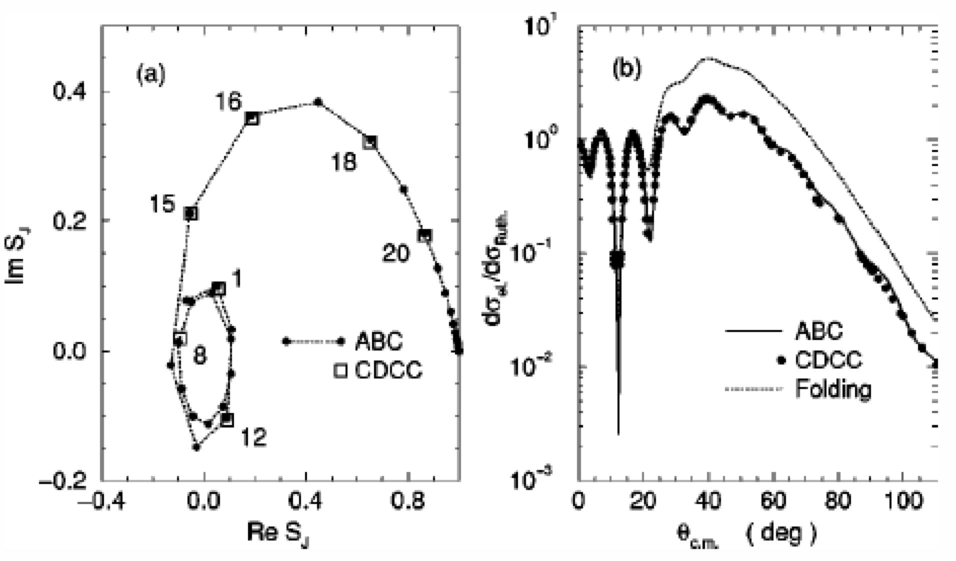
Comparison with CDCC

Deuteron breakup S-matrix



d+^{58}Ni at E_d=80 MeV

Elastic S-matrix & cross section



CDCC calculation

Yahiro et al, PTPS 89, 32 (1986)

Time-dependent picture

$$\phi^{(+)}(\vec{r}) = e^{ikz} + \frac{1}{E+i\varepsilon-T}V(r)\phi^{(+)}(\vec{r})$$
$$= e^{ikz} + \frac{1}{E+i\varepsilon-H}V(r)e^{ikz}$$

Scattering wave

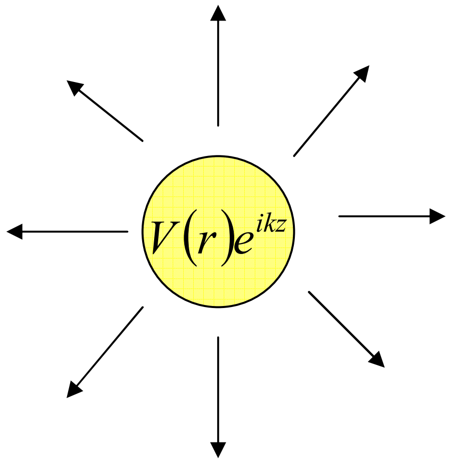
$$\chi(\vec{r}) = \frac{1}{E+i\varepsilon-H}V(r)e^{ikz} \quad \underset{r\to\infty}{\to} f(\Omega)\frac{e^{ikr}}{r}$$

$$\chi(\vec{r}) = \frac{1}{E+i\varepsilon-H}V(r)e^{ikz} = \frac{1}{i\hbar}\int_0^\infty dt\, e^{i(E+i\varepsilon)t/\hbar}e^{-iHt/\hbar}V(r)e^{ikz}$$

Time-dependent scattering wave

$$\psi(\vec{r},t=0) = V(r)e^{ikz}$$ (Initial wave packet)

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = H\psi(\vec{r},t)$$ (Propagation)



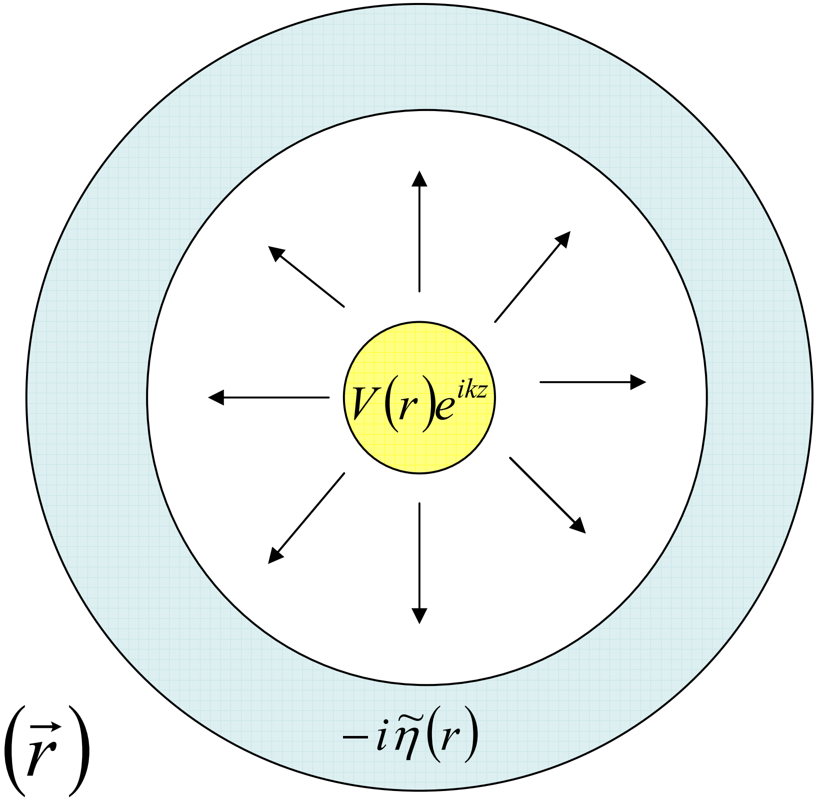
Projection on *E* : $$\chi(\vec{r}) = \frac{1}{i\hbar}\int_0^\infty dt\, e^{iEt/\hbar}\psi(\vec{r},t)$$

Time-dependent approaches

Absorb all outgoing waves outside the interacting region

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)=\left(H-i\tilde{\eta}(r)\right)\psi(\vec{r},t)$$

$$f(\Omega)=-\frac{m}{2\pi\hbar^2}\int d\vec{r}\,e^{-i\vec{k}_\Omega\vec{r}}V(r)\phi^{(+)}(\vec{r})$$



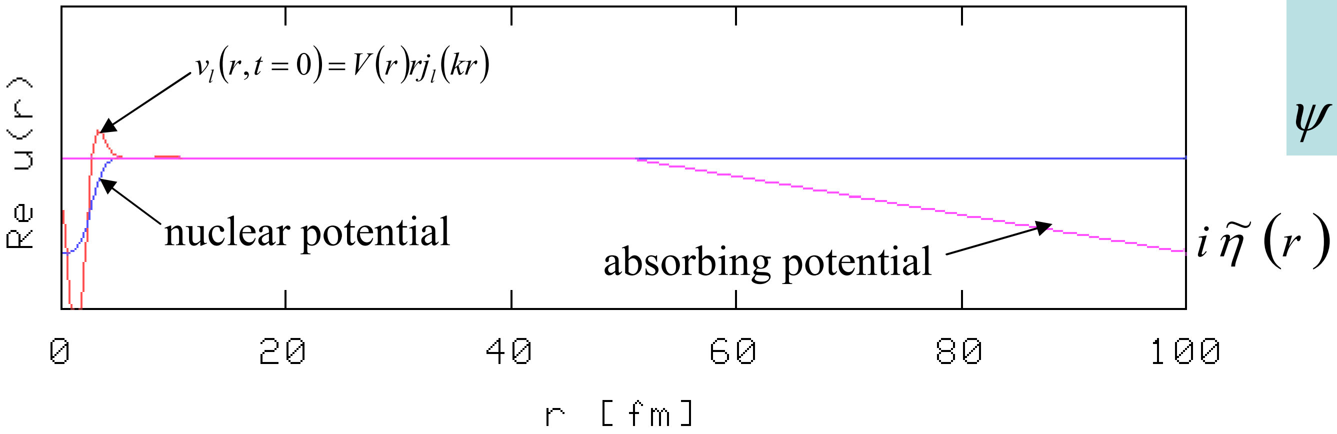
Finite time period up to *T*

Time evolution can stop when all the outgoing waves are absorbed.

$$\chi(\vec{r})=\frac{1}{i\hbar}\int_0^T e^{iEt/\hbar}\psi(\vec{r},t)$$

s-wave

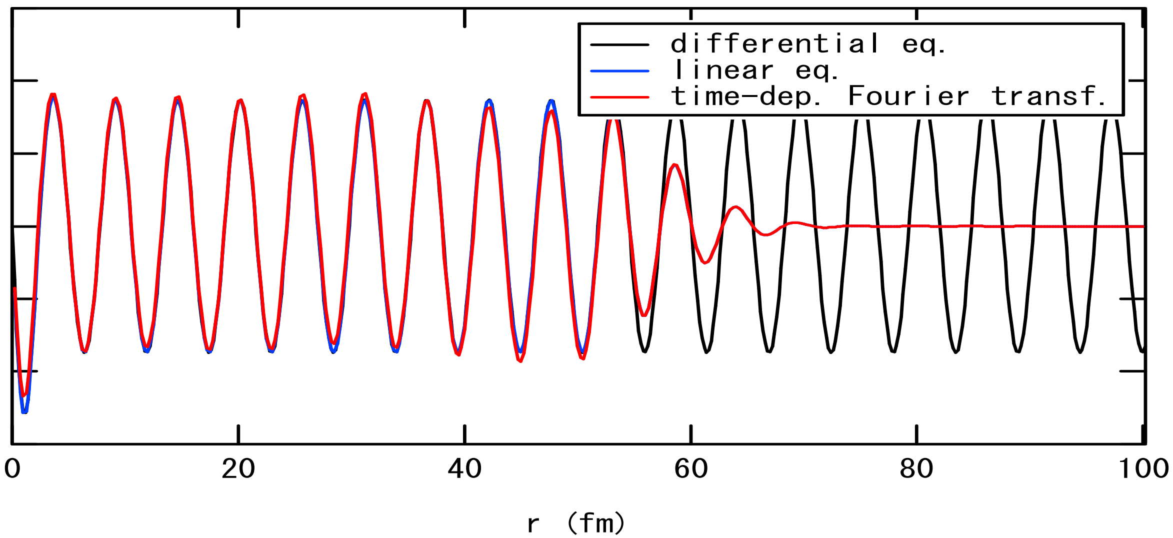
$$\mathrm{Re}\left[\psi\left(\vec{r},t\right)_{\ell=0}\right]$$



$$i\hbar\frac{\partial}{\partial t}\psi\left(\vec{r},t\right)=H\psi\left(\vec{r},t\right)$$

$$\psi\left(\vec{r},t=0\right)=V\left(r\right)e^{ikz}$$

$$\mathrm{Re}\left[\chi\left(\vec{r}\right)_{\ell=0}\right]$$



$$\chi\left(\vec{r}\right)=\frac{1}{i\hbar}\int_0^\infty dt\,e^{iEt/\hbar}\psi\left(\vec{r},t\right)$$

Strength function in the continuum

$$\frac{dB(F,E)}{dE} = \sum_{lm'} \int dE' \delta(E-E') \left| \langle \phi_{E',lm'} | F | \phi_0 \rangle \right|^2$$

$$= \langle \phi_0 | F^+ \delta(E-H) F | \phi_0 \rangle$$

$$= -\frac{1}{\pi} \mathrm{Im} \langle \phi_0 | F^+ \frac{1}{E+i\varepsilon-H} F | \phi_0 \rangle$$

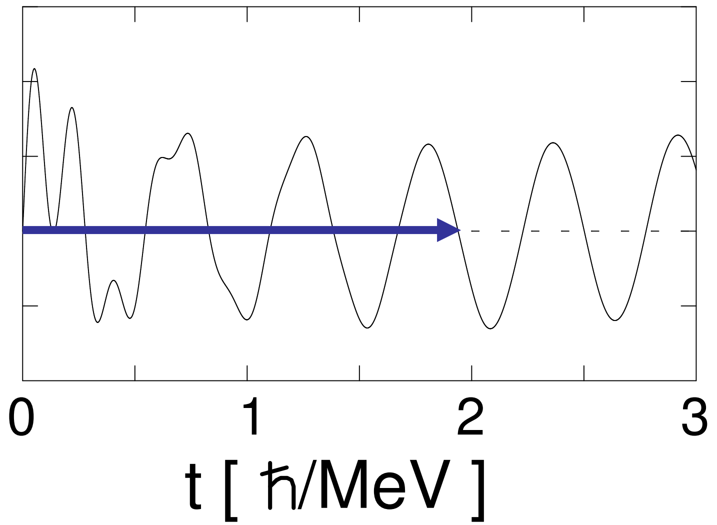
$$\frac{dB(F,E)}{dE} = -\frac{1}{\pi} \mathrm{Im} \sum_m \frac{1}{i\hbar} \int_0^\infty dt e^{iEt/\hbar} \langle \phi_0 | F^+ e^{-iHt/\hbar} F | \phi_0 \rangle$$

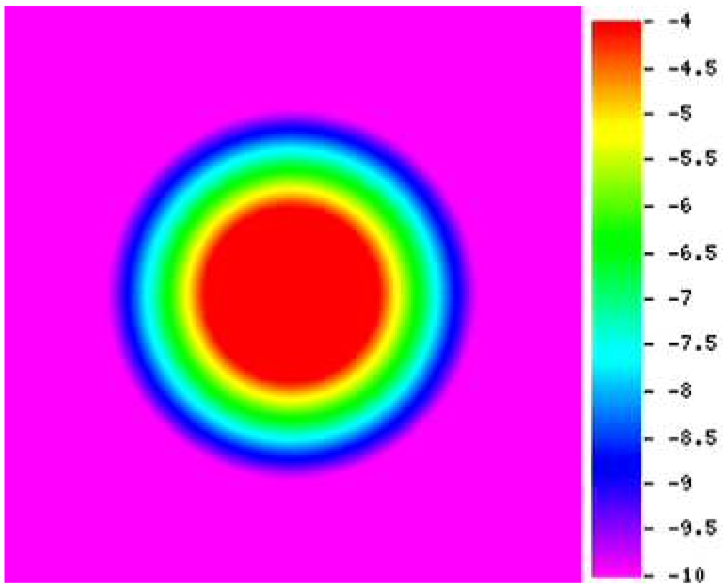
$$= \frac{1}{\pi\hbar} \mathrm{Re} \int_0^\infty dt e^{iEt/\hbar} \sum_m \psi_F^*(\vec{r},0) \psi_F(\vec{r},t)$$

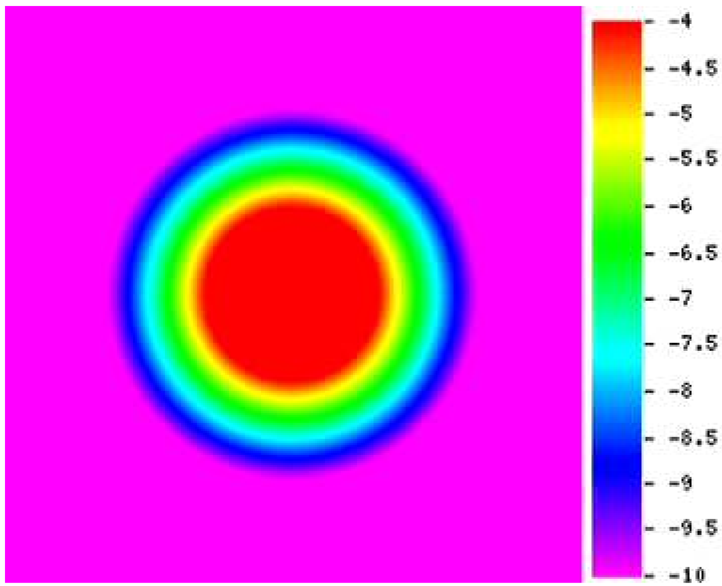
Solved for three-body systems in the time-dependent manner.

TN, Yabana, Ito, Eur. Phys. J. Special Topics 156, 249 (2008)

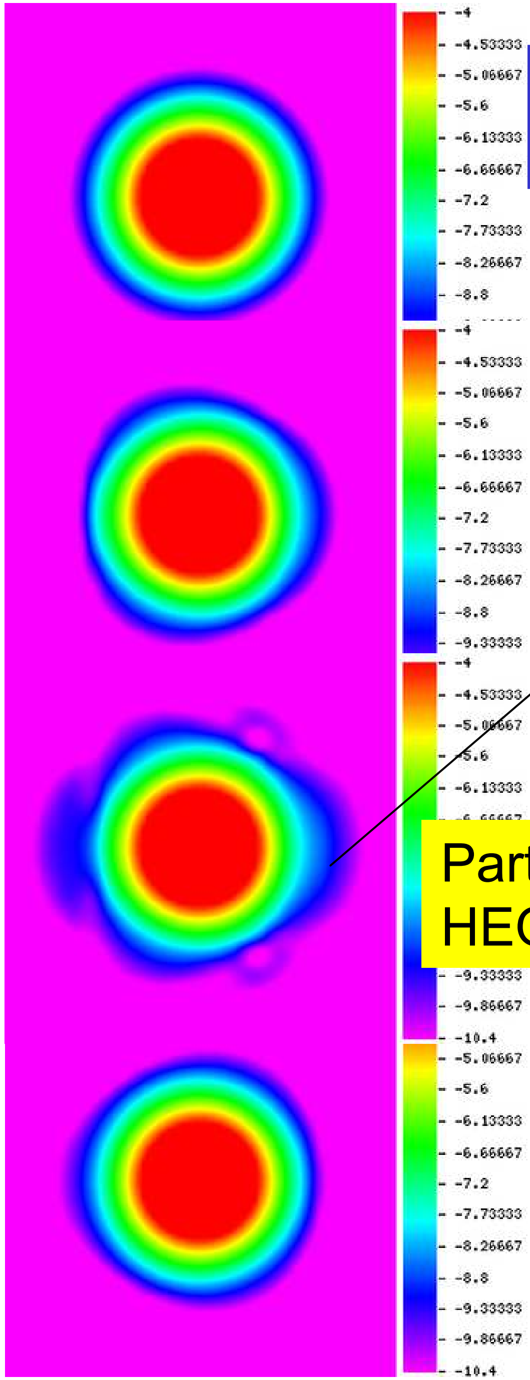
TDHF (TDDFT) with an absorbing potential



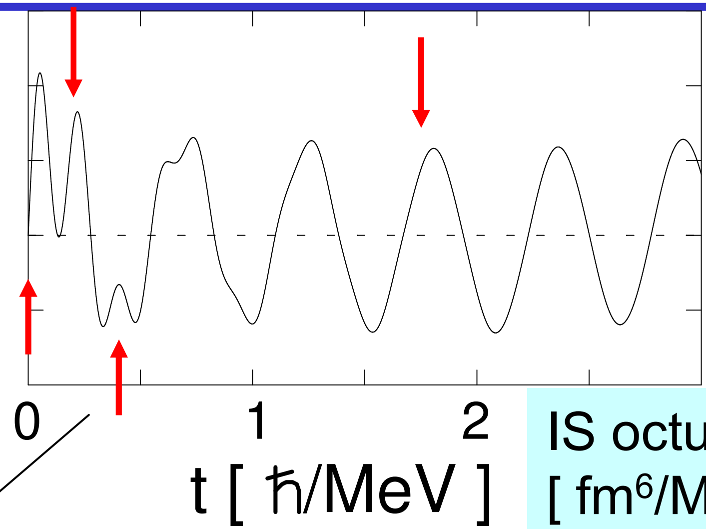




IS octupole resonances in ^{16}O

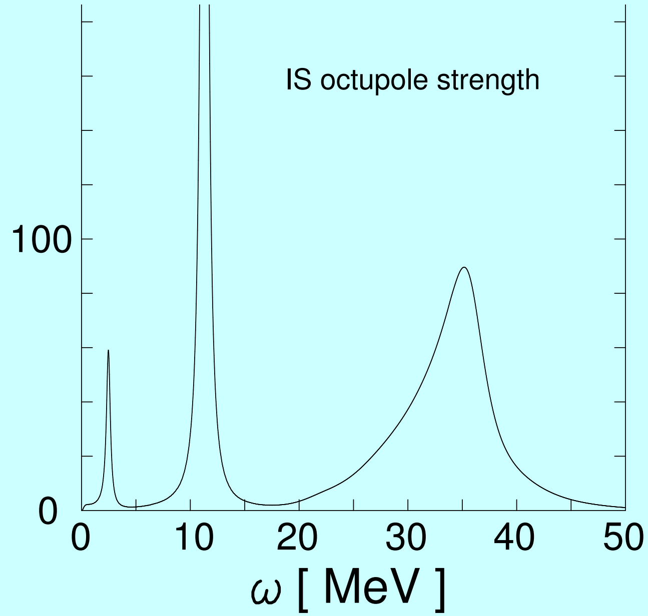


Particle decay of HEOR



Time evolution of IS octupole moment

IS octupole strength function [fm^6/MeV]



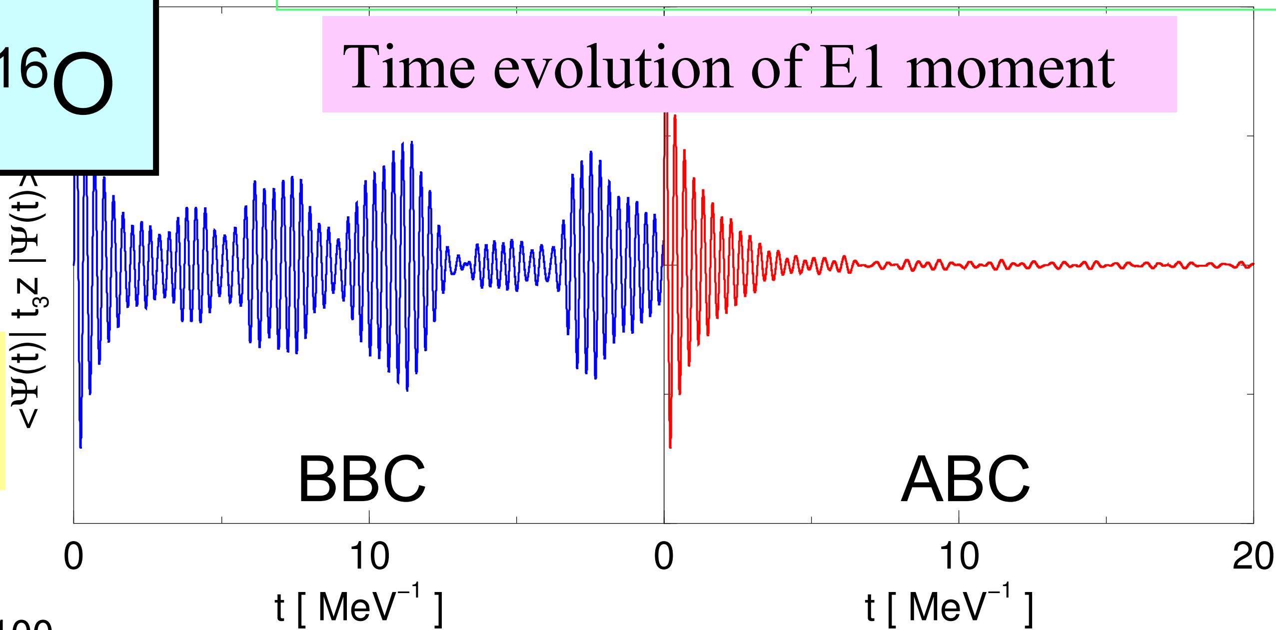
Simple BKN interaction is used.

TN and Yabana, J. Chem. Phys. **114**, 2550 (2001); Phys. Rev. C **71**, 024301 (2005)

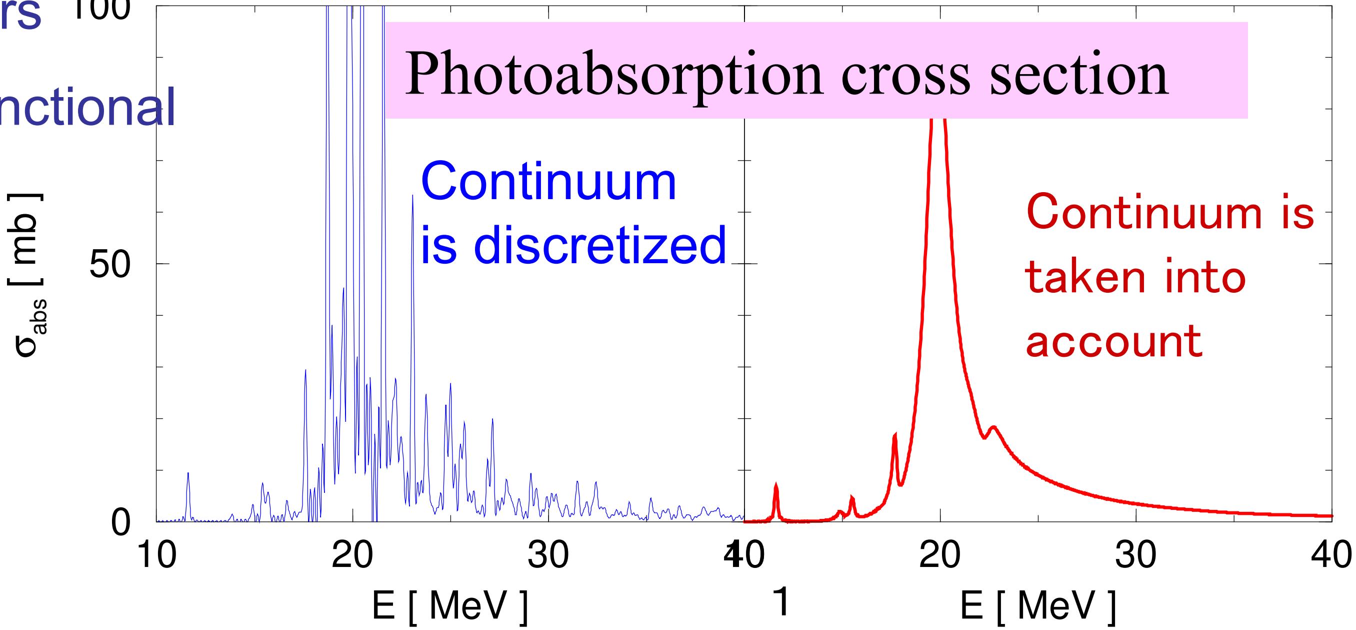
TDHF for ^{16}O

$$V_{ext}(t) = kV_{ext}\,\delta(t)$$

$$V_{ext} = \frac{Z}{A}\hat{z}_n - \frac{N}{A}\hat{z}_p$$



- •SGII parameters
- •Full Skyrme functional
- ・Γ =0.1 MeV
- ・T=30 ħ/MeV



1

Summary

- The complex absorbing potential is a simple alternative for calculation of the scattering properties.
- The real-space representation is intuitive and conceptually simple, but requires considerable computational cost.
- Combination with time-dependent approaches provides us with a powerful tool.
- No continuum discretization is involved.