Spreading width of doorway states and banded random matrices

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Strength function

Strength function

$$S(E) = \sum_i |\langle 0|i\rangle|^2 \delta(E - \varepsilon_i) \quad (1)$$

where
$|0>$ doorway state
ε_i and $|i>$ eigenvalues and eigenstates of the Hamiltonian H

$S(E)$ displays a broad maximum $\longrightarrow E_0$
Important quantities:
$\Gamma^{\downarrow} \longrightarrow$ spreading width ($\sim 4 \div 5$ MeV)
$d \longrightarrow$ mean spacing of nuclear levels (~ 10 eV)

Bohr-Mottelson

$$\overline{S(E)} = \frac{\Gamma^{\downarrow}/(2\pi)}{(E - E_0)^2 + (1/4)(\Gamma^{\downarrow})^2} \tag{2}$$

with

$$\Gamma^{\downarrow} = 2\pi v^2/d \tag{3}$$

Assumptions:

- constant level spacing d of the $|i>$
- coupling between background and $|0> \rightarrow V_\mu$ (Gaussian distributed with average 0 and variance v^2)

Our previous approach:

$$H = \begin{pmatrix} E_0 & V_\nu \\ V_\mu & \mathcal{H}_{\mu\nu} \end{pmatrix} \quad (4)$$

$H_{\mu\nu} \to$ GOE (real symmetric)
$|0>$ at energy E_0 coupled to N background states via V_μ ($\mu = 1..N$), real matrix elements

Results:

- spectrum of $H_{\mu\nu} \to$ semicircle of radius 2λ
- at variance with Bohr-Mottelson, spectrum **is confined**
- solution obtained via Pastur equation if the spreading width is not negligible with respect to 2λ

Then $\Gamma_{\text{eff}} > 2\pi v^2/d$, the increase being proportional to $\Gamma^{\downarrow}/\lambda$

Our new approach:

General background level distribution

$$H = \begin{pmatrix} E_0 & V_\nu \\ V_\mu & E_\mu \delta_{\mu\nu} \end{pmatrix} \quad (5)$$

V_μ Gaussian random variables with average 0 and variance v^2

No assumption on the distribution of E_μ

Some details of the calculation

$$S(E) = -\frac{1}{\pi}\,\Im\left(\langle 0|\frac{1}{E^+ - H}|0\rangle\right) \tag{6}$$

$\downarrow$

$$S(E) = -\frac{1}{\pi}\,\Im\left(\frac{1}{E^+ - E_0 - \sum_\mu V_\mu (E^+ - E_\mu)^{-1} V_\mu}\right) \tag{7}$$

$\downarrow$

$$\overline{\Sigma}^V = v^2 \sum_\mu (E^+ - E_\mu)^{-1}\,. \tag{8}$$

Finally

$$\overline{S(E)} = \frac{1}{2\pi} \frac{\Gamma^{\downarrow}}{(E - E_0 - \Delta)^2 + (1/4)(\Gamma^{\downarrow})^2} \tag{9}$$

where

$$\begin{aligned} \Gamma^{\downarrow} &= 2\pi v^2 \rho(E) \ , \\ \Delta &= v^2 \int \mathrm{d}E' \, \frac{\mathcal{P}}{E^+ - E'} \, \rho(E') \end{aligned} \tag{10}$$

and

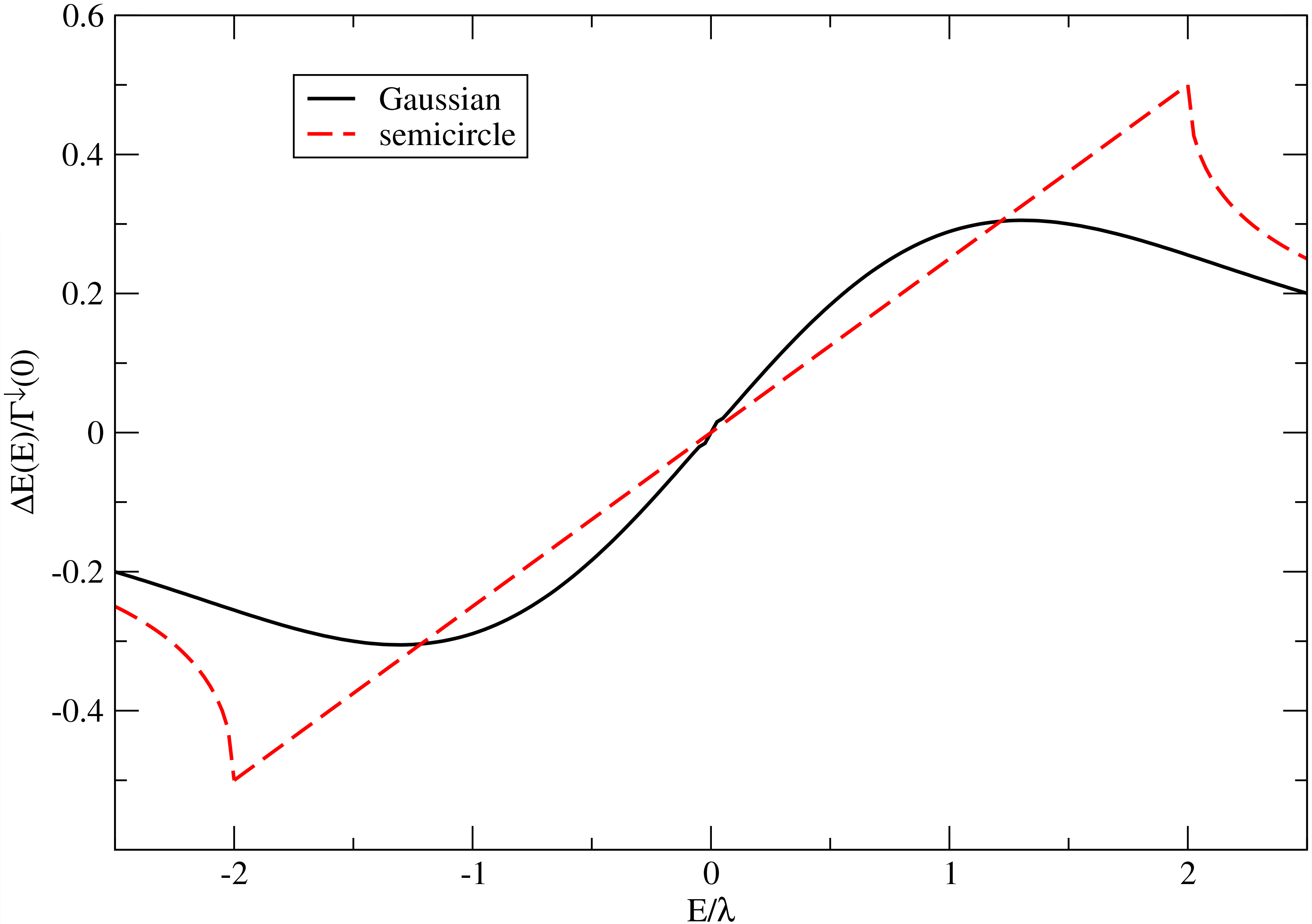
$$\rho(E') = \sum_{\mu} \delta(E' - E_{\mu}) \tag{11}$$

The shift function is energy dependent

$$\rho(E) = \frac{N}{\pi\lambda}\sqrt{1-\left(\frac{E}{2\lambda}\right)^2}\ (\text{semicircle})\ ,$$
$$\rho(E) = \frac{N}{\sqrt{2\pi}\lambda}\exp[-E^2/(2\lambda^2)]\ \ (\text{Gaussian}) \qquad (12)$$

$\downarrow$

$$\Delta(E)/\Gamma^{\downarrow}(0) = \frac{1}{2\pi}\int_{-2}^{+2}\mathrm{d}x'\ \frac{\mathcal{P}}{x-x'}\sqrt{1-x'^2/4}$$
$$= \frac{x}{4} - [\theta(2+x)-\theta(2-x)]\frac{1}{2}\sqrt{\frac{x^2}{4}-1},\ \ (\text{semicircle}),$$
$$\Delta(E)/\Gamma^{\downarrow}(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}x'\ \frac{\mathcal{P}}{x-x'}\exp[-x'^2/2]$$
$$= \frac{1}{2}\mathrm{e}^{-x^2/2}(-i)\mathrm{erf}(i\frac{x}{\sqrt{2}})\ \ (\text{Gaussian}) \qquad (13)$$


0.6
0.4
0.2
0
-0.2
-0.4
ΔE(E)/Γ↓(0)
Gaussian
semicircle
-2
-1
0
1
2
E/λ

Numerical simulations

(A) GOE

because of the orthogonal invariance of GOE, individual matrix elements V_μ are irrelevant, only

$$v^2 = \frac{1}{N} \sum_\mu V_\mu^2 \tag{14}$$

matters

$< H_{\mu\nu} >= 0$ and $\overline{H_{\mu\nu}^2} = (1 + \delta_{\mu\nu}) \frac{\lambda^2}{N+1} \to \frac{1}{N} \mathrm{Tr} \overline{H^2} = \lambda^2$

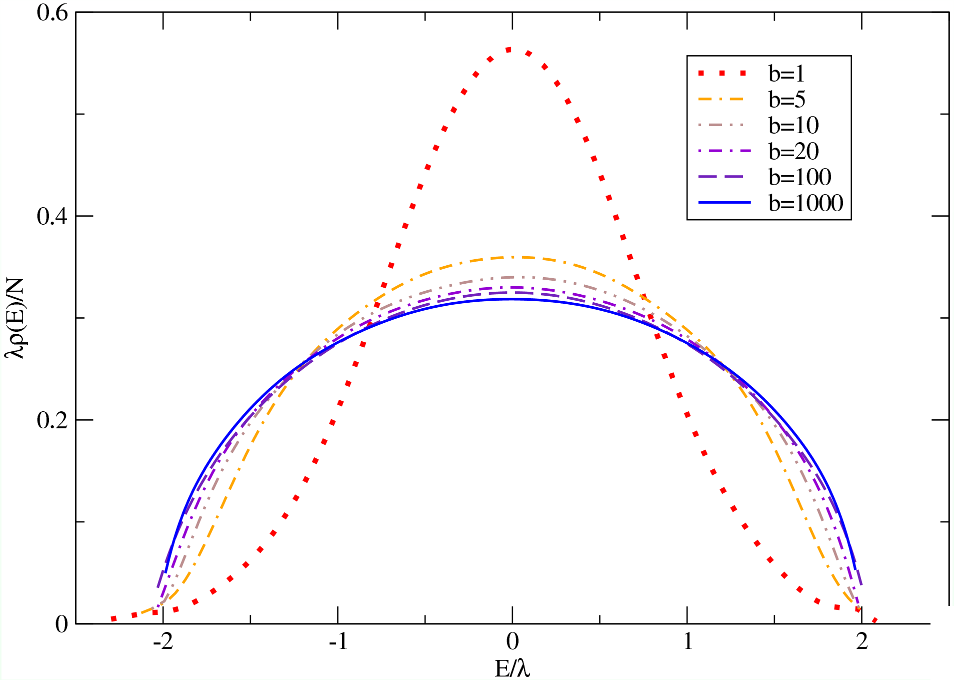
Parameters: v^2, E_0, N (=1000) (plus λ, which defines the energy scale and m (=500), the number of trials)

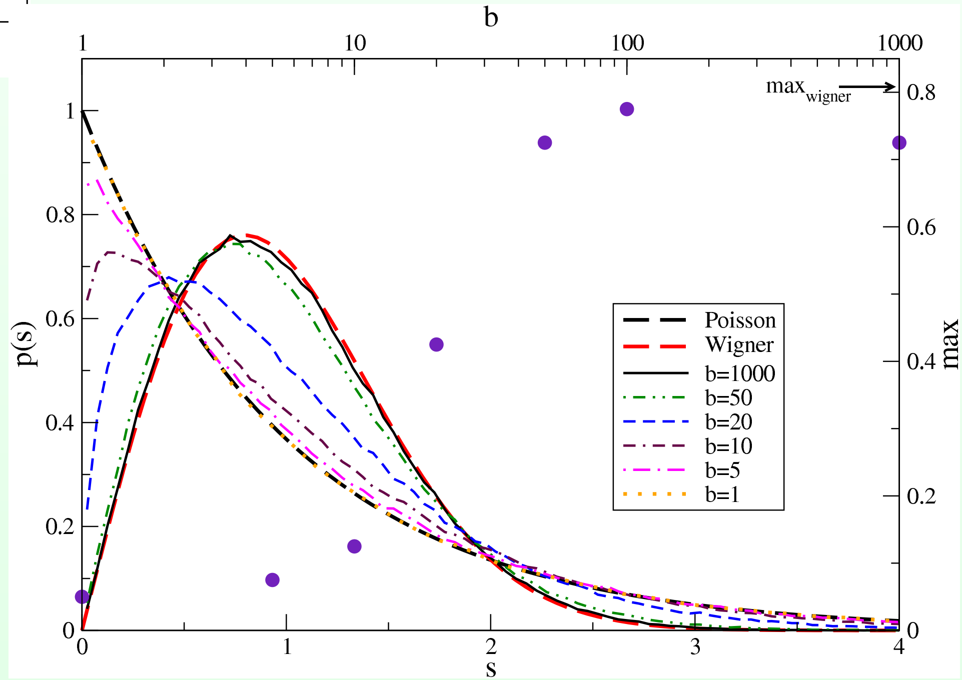
Numerical simulations

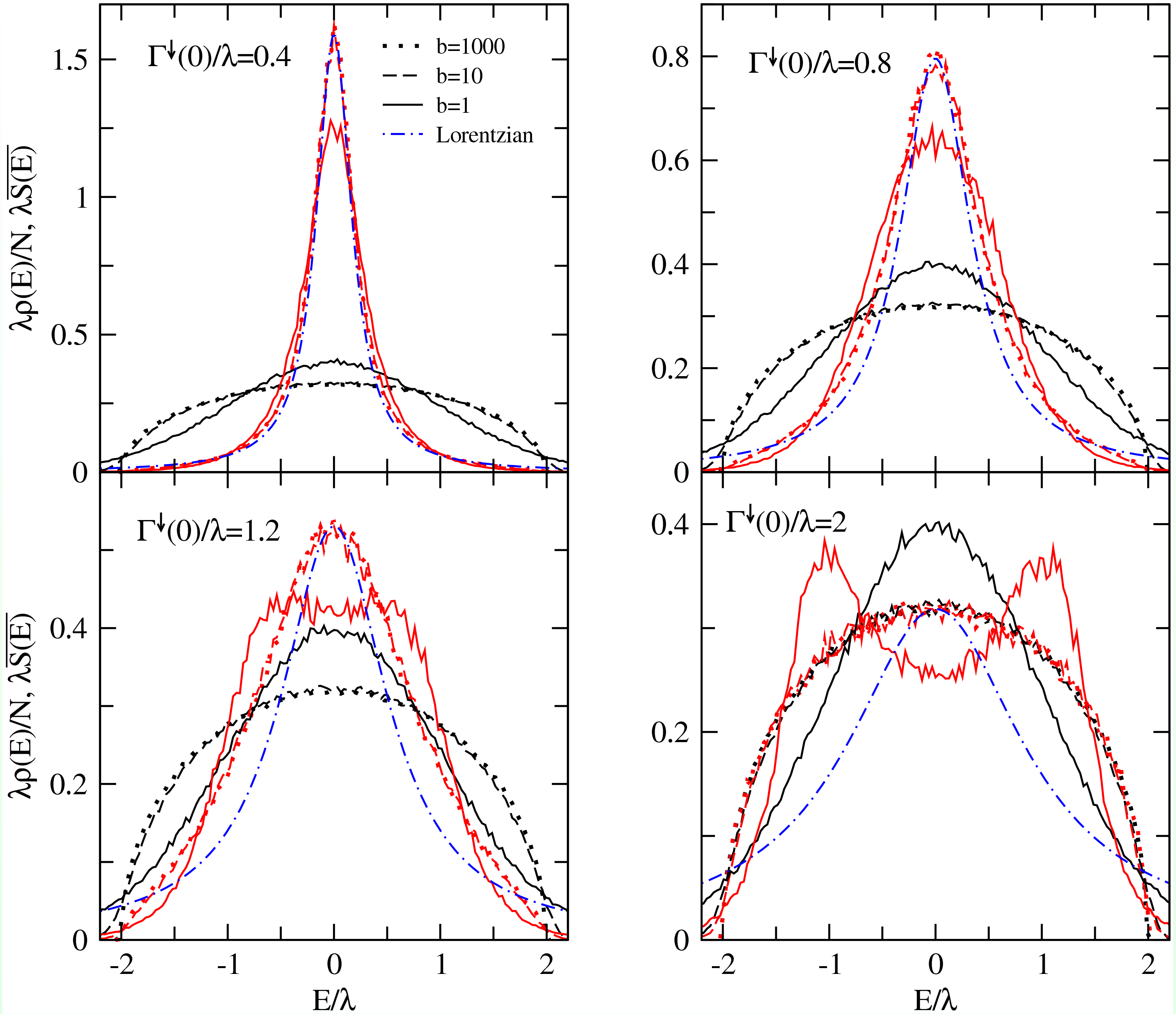
(B) banded matrices

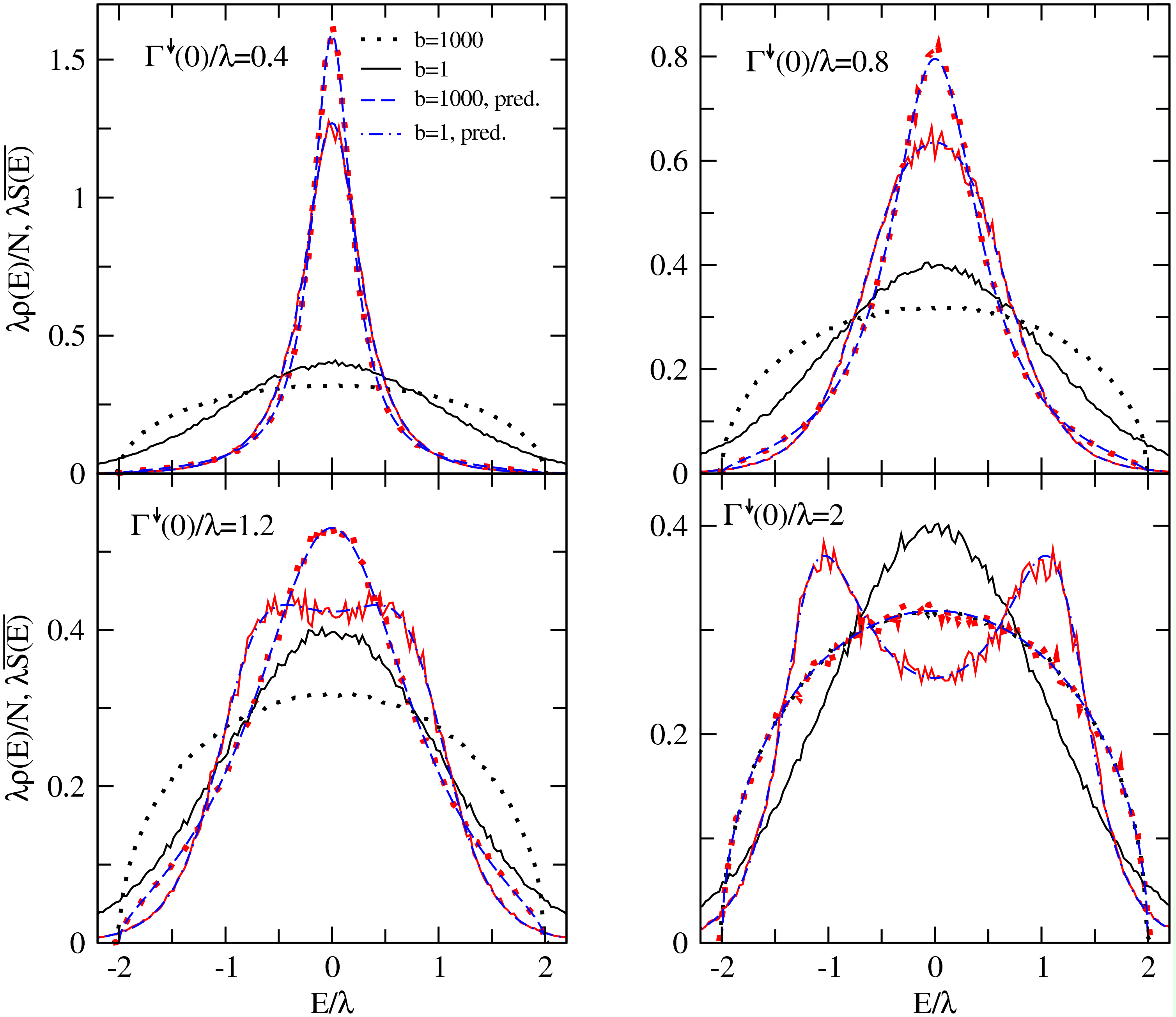
all matrix elements $|\mu - \nu| \geq b$ vanish

$$\overline{H_{\mu\nu}^2} = (1 + \delta_{\mu\nu})\beta^2 \leftarrow \frac{1}{N}\mathrm{Tr}\overline{H^2} = \lambda^2$$

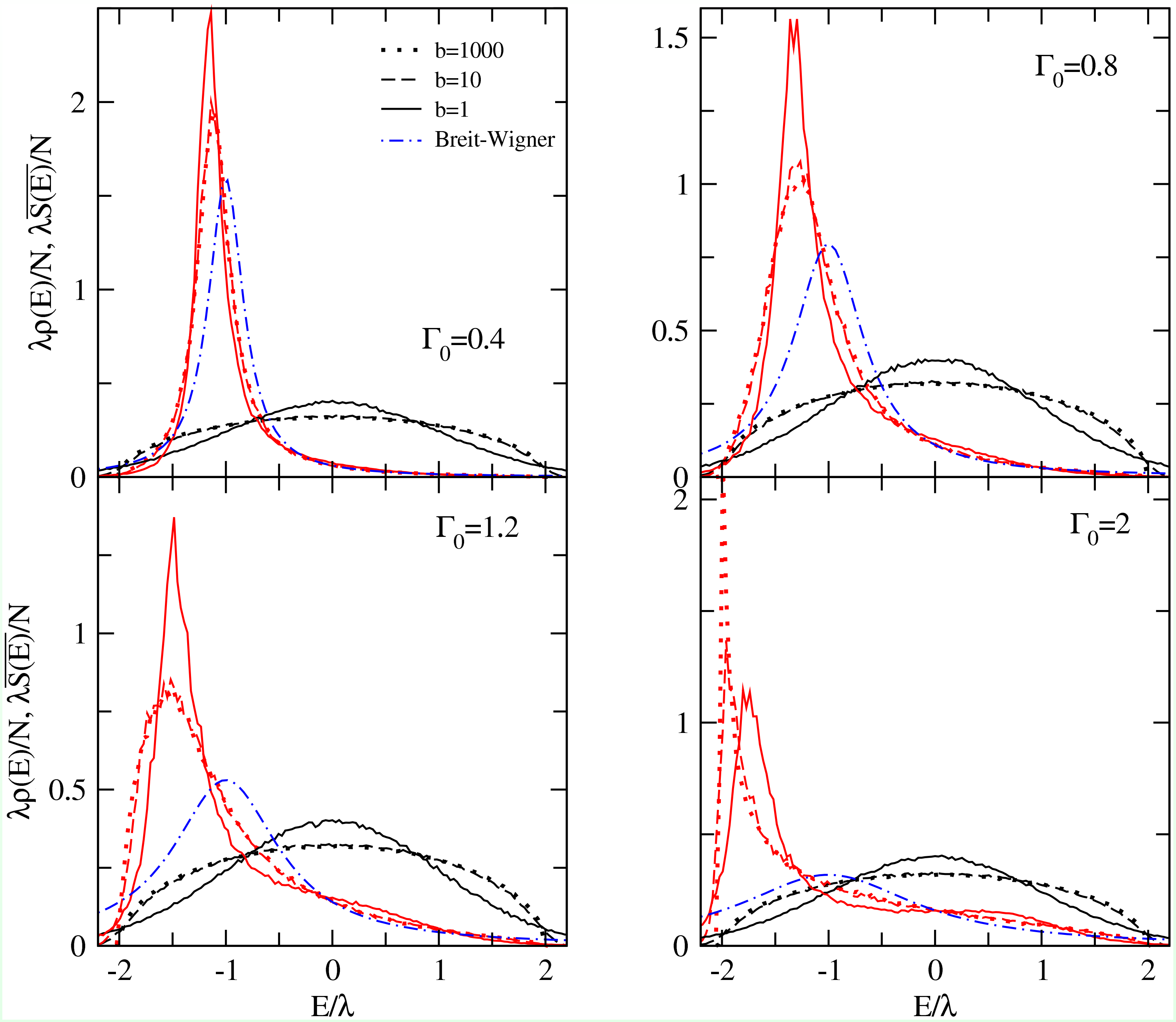

0.6
0.4
0.2
0
λρ(E)/N
b=1
b=5
b=10
b=20
b=100
b=1000
-2
-1
0
1
2
E/λ

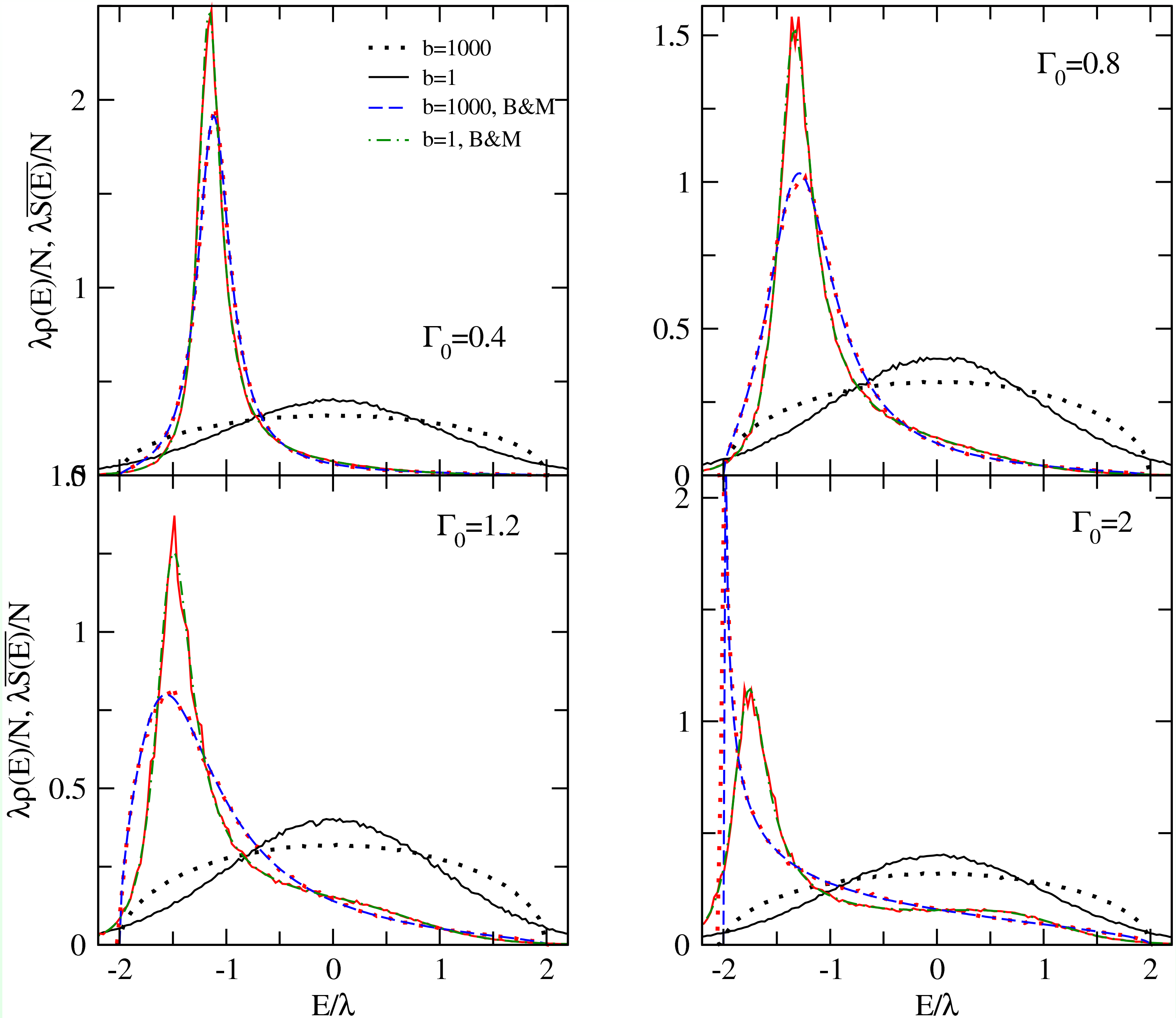

b
1
10
100
1000
max$_{wigner}$
p(s)
max
Poisson
Wigner
b=1000
b=50
b=20
b=10
b=5
b=1
s


Γ↓(0)/λ=0.4
b=1000
b=10
b=1
Lorentzian
Γ↓(0)/λ=0.8
Γ↓(0)/λ=1.2
Γ↓(0)/λ=2
λρ(E)/N, λ$\overline{S(E)}$
E/λ
1.5
1
0.5
0
0.8
0.6
0.4
0.2
-2
-1
0
1
2



Γ↓(0)/λ=0.4
b=1000
b=1
b=1000, pred.
b=1, pred.
Γ↓(0)/λ=0.8
Γ↓(0)/λ=1.2
Γ↓(0)/λ=2
λρ(E)/N, λS(E)
E/λ


b=1000
b=10
b=1
Breit-Wigner
Γ_0=0.4
Γ_0=0.8
Γ_0=1.2
Γ_0=2
$\lambda\rho(E)/N, \lambda\overline{S(E)}/N$
$\lambda\rho(E)/N, \lambda\overline{S(E)}/N$
E/λ
E/λ



b=1000
b=1
b=1000, B&M
b=1, B&M
Γ_0=0.4
Γ_0=0.8
Γ_0=1.2
Γ_0=2
$\lambda\rho(E)/N, \lambda\overline{S(E)}/N$
$\lambda\rho(E)/N, \lambda\overline{S(E)}/N$
E/λ
E/λ

Localization

Inverse participation ratio

$$P = \sum_i \overline{|\langle 0|i\rangle|^4} \tag{15}$$