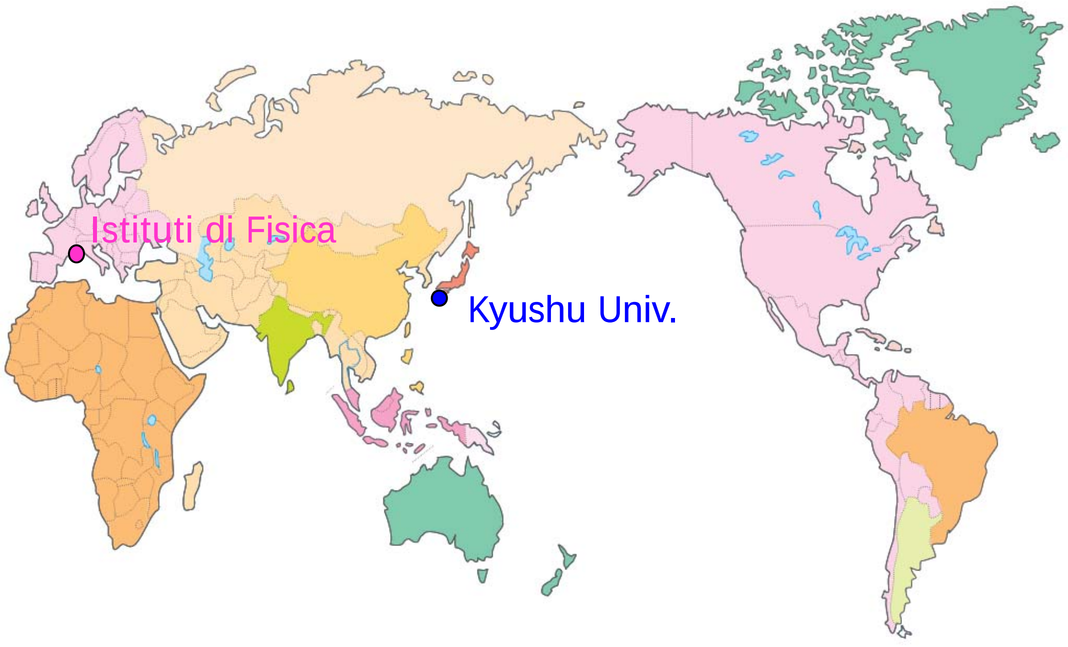
Recent development in CDCC



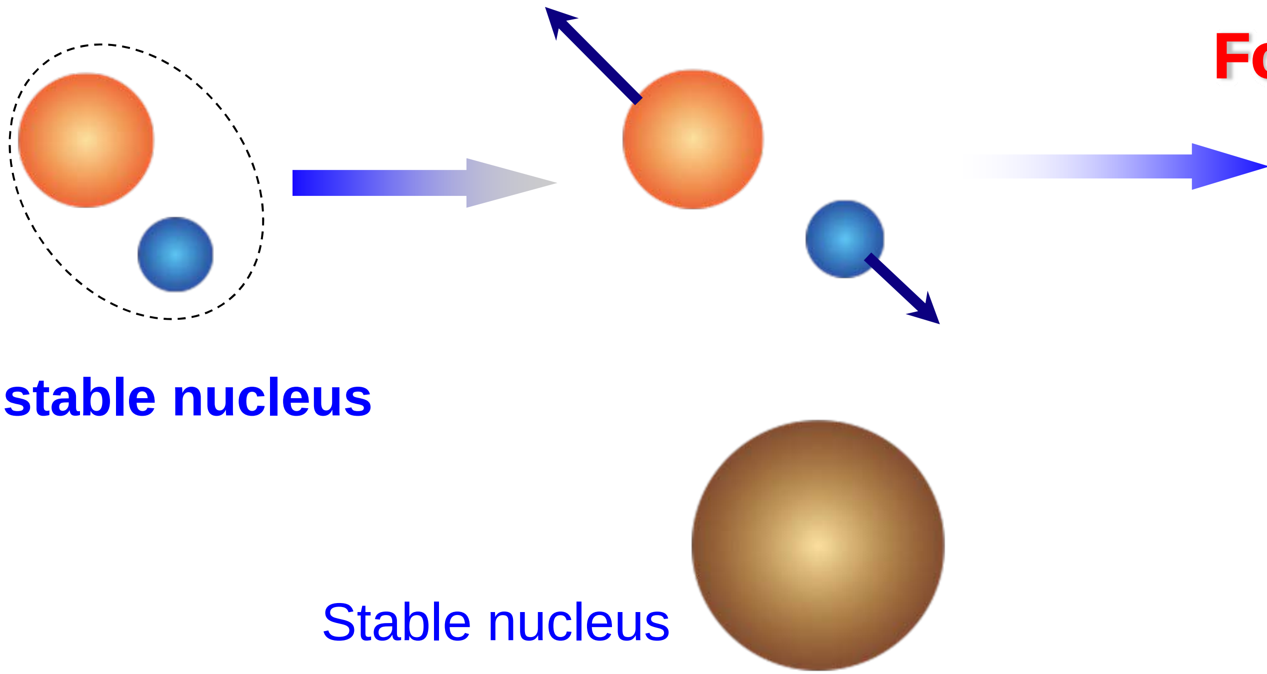
Kyushu Group

M. Yahiro

Collaborators

T. Matsumoto, T. Fukui,
K. Minomo, K. Ogata,
Y. Iseri, Y. Shimizu,
S. Hashimoto,
M. Kawai
K. Kato

Scattering of unstable nucleus

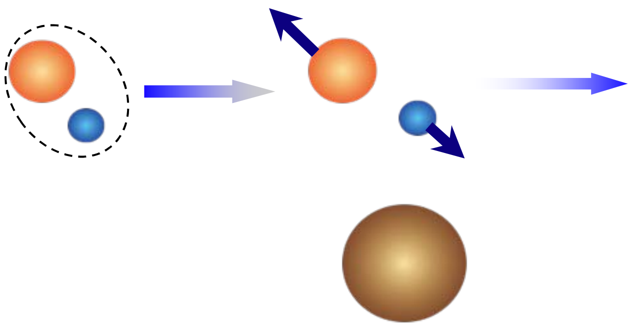


Various kinds of Reactions

- elastic scattering
- inelastic scattering
- Removal reactions
- transferred reactions
- breakup reactions

CDCC

(The method of Continuum-Discretized Coupled Channels)



Review Papers

Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.89,1(1986)

Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep. 154(1987),126.

Theoretical foundation

Austern, Yahiro and Kawai, PRL 63, 2649(1989)

Austern, Kawai and Yahiro, PRC 53, 394(1996)

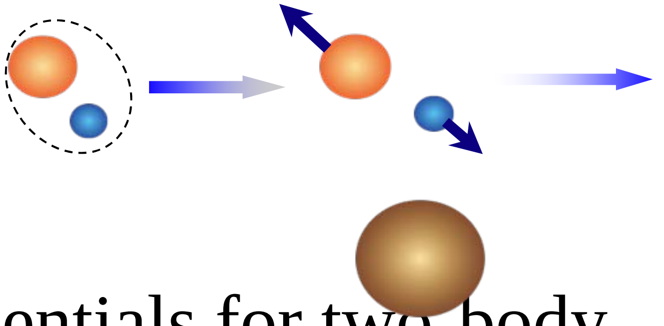
Numerical comparison between CDCC and Faddeev solutions

A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76 (2007), 064602.

Contents

1. Foundation of CDCC.
2. Latest results of CDCC;

   four-body CDCC is applied to ^{6}Li and ^{6}He scattering.



3. Input of CDCC Hamiltonian is optical potentials for two-body subsystems.

   Construction of the microscopic optical potential.

   This is discussed for proton and deuteron scattering.

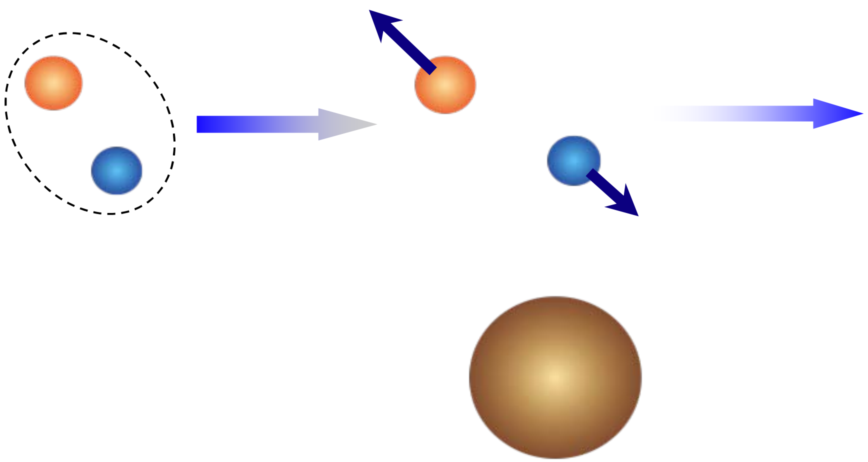
1. Foundation of CDCC

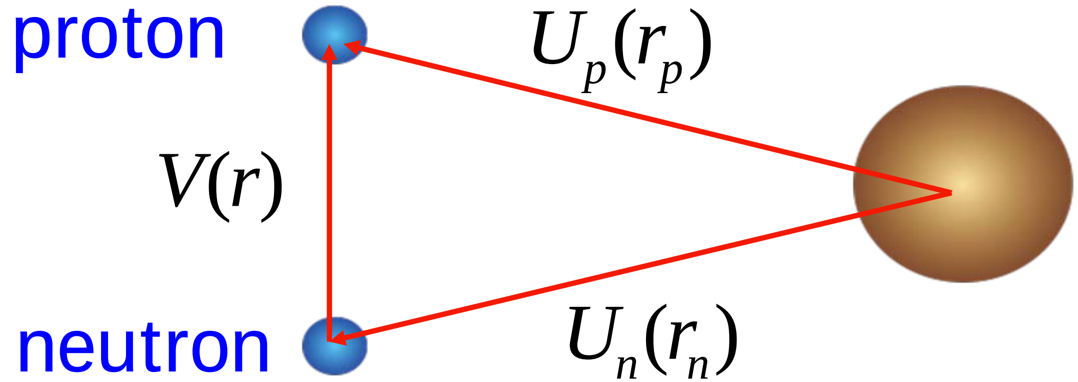
Austern,Yahiro,Kawai， PRL63， 2649（1989)

What is CDCC ?

Austern, Yahiro, Kawai， PRL63， 2649（1989)

Deuteron scattering

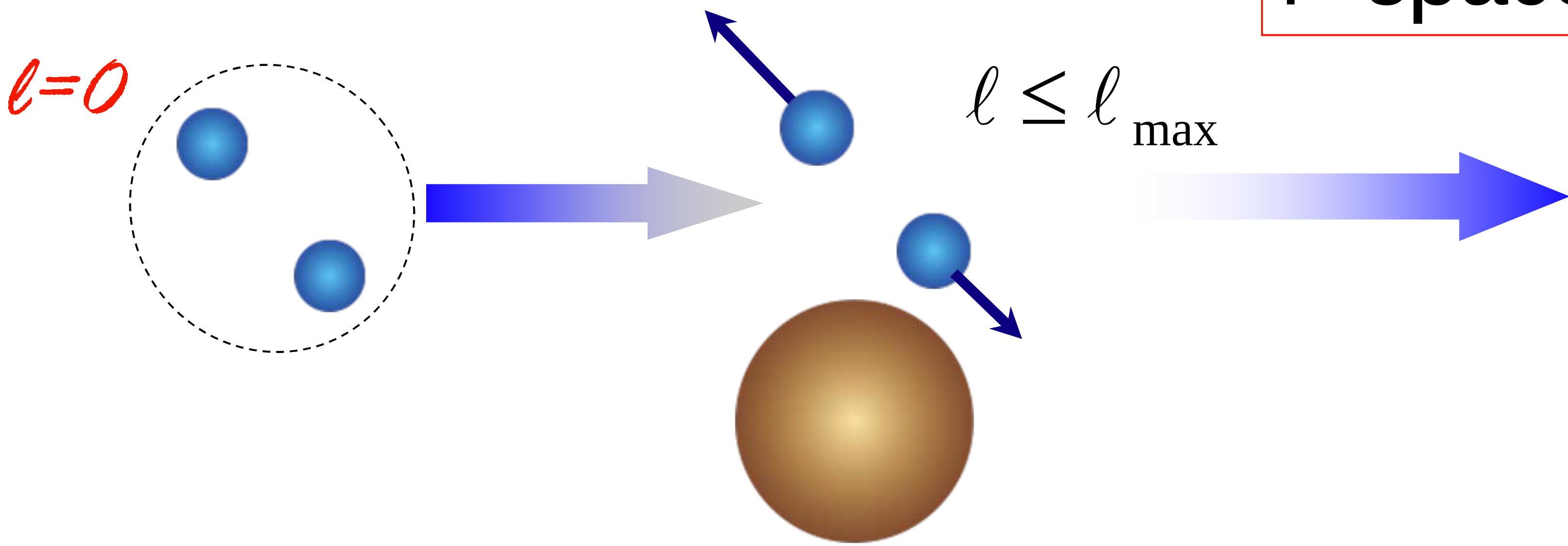




$$\left(E-K-V(r)-U_{\mathrm{p}}(r_p)-U_{\mathrm{n}}(r_n)\right)\psi=0$$

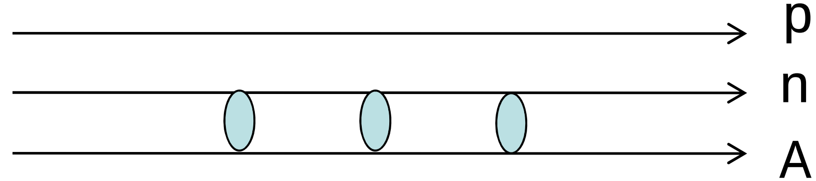
breakup process

P-space



CDCC-equation $(E - K - V - PU_{\mathrm{p}}P - PU_{\mathrm{n}}P)\psi = 0$

Disconnected diagram

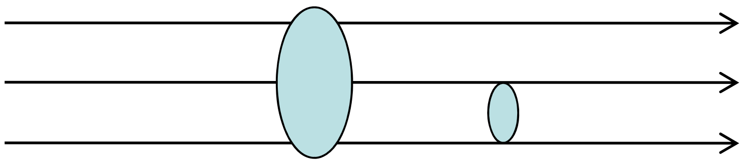


$$PU_{\mathrm{p}}P = P\ \exp[-r_{\mathrm{p}}^2]\ P = \int d\Omega_r\ \exp[-(\boldsymbol{R} - \boldsymbol{r}/2)^2] = \exp[-R^2 - r^2/4]\ j_0(Rr)$$

$$(E - K - V(r) - U_p - U_n)\psi = 0$$

Faddeev decomposition

$$\psi = \psi_d + \psi_p + \psi_n$$



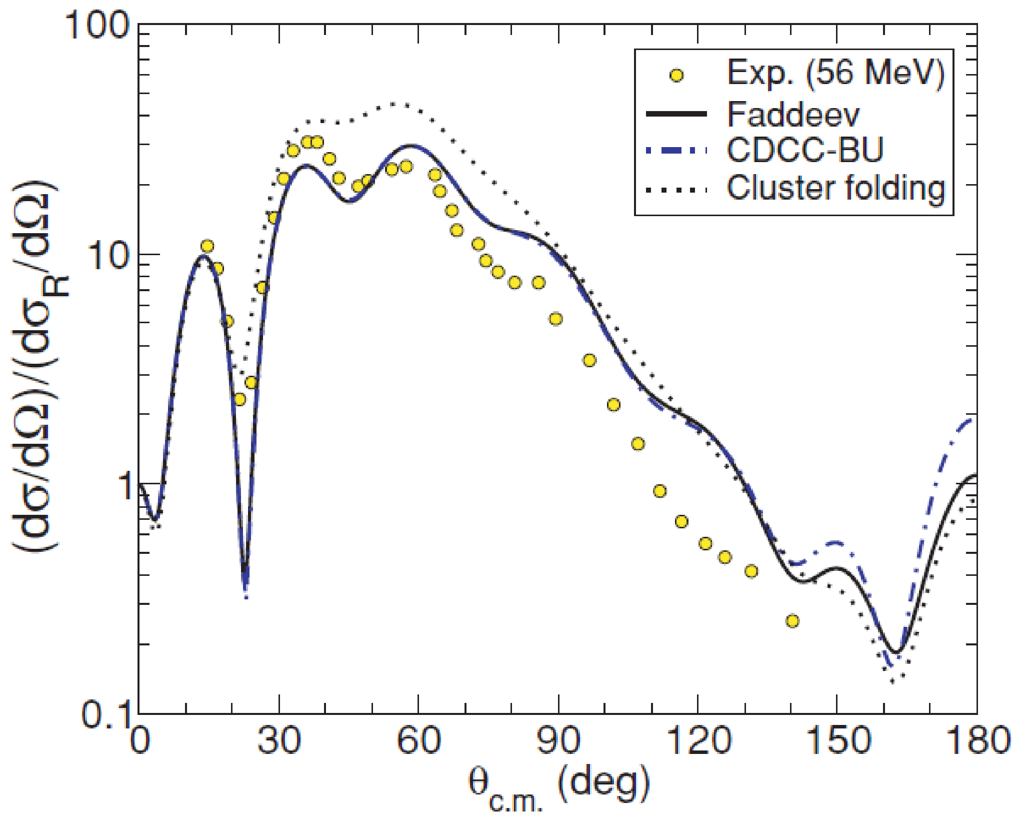
Distorted Faddeev equations

$$[E - K - V \qquad]\psi_d = 0$$

$$[E - K - U_p]\psi_p = (U_p \qquad)\psi_d + U_p \psi_n$$

$$[E - K - U_n]\psi_n = (U_n \qquad)\psi_d + U_n \psi_p$$

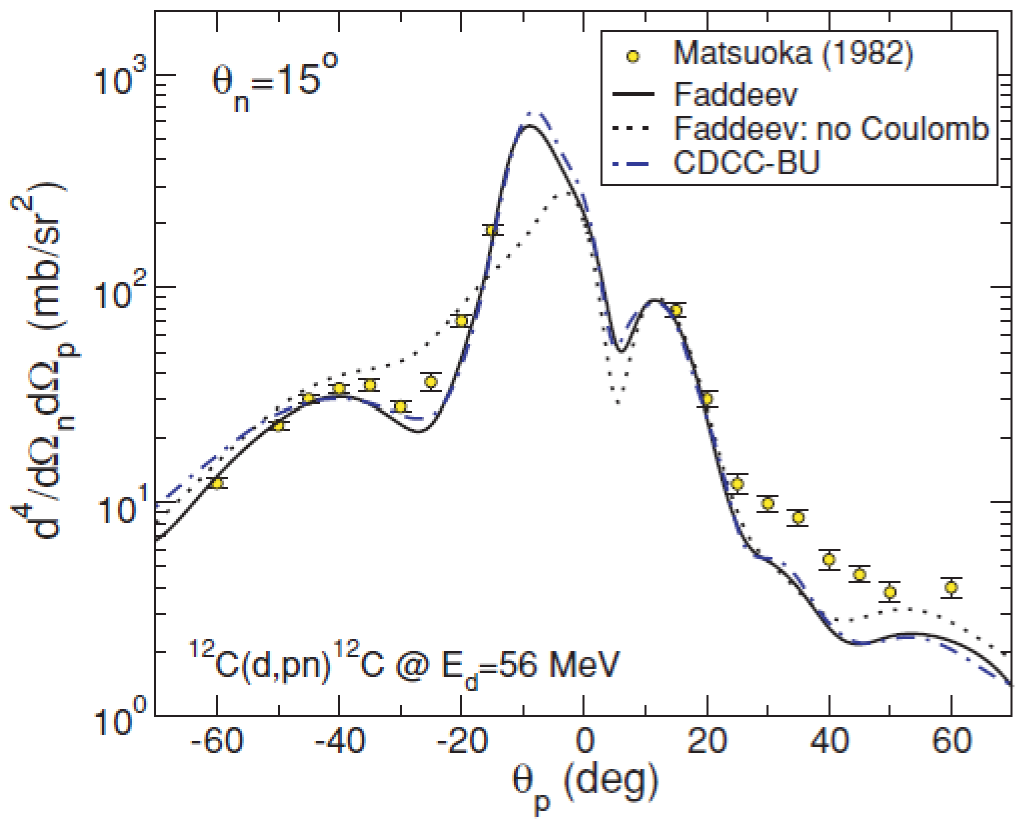
Comparison between CDCC and Faddeev solutions



d+^{12}C at 56 MeV
Elastic scattering

A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76 (2007), 064602.

$^{12}C(d,pn)$ at 56 MeV

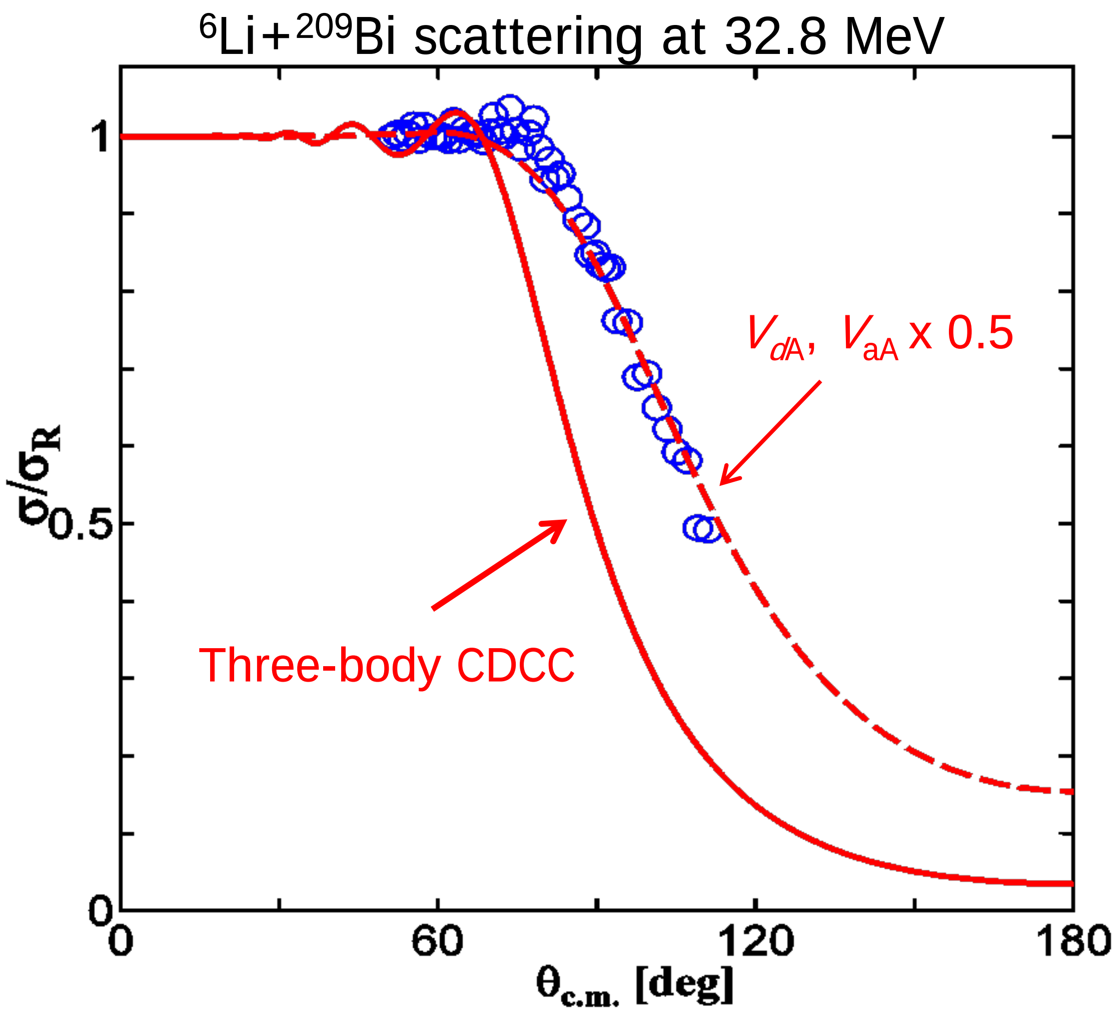


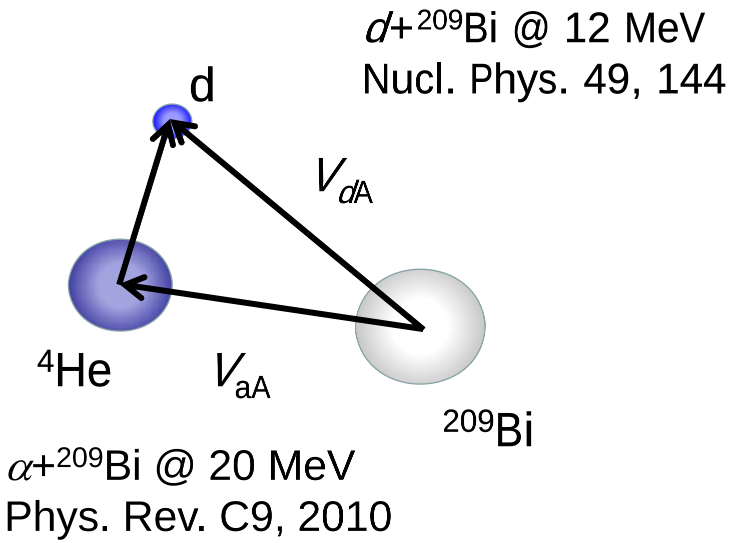
2. Four-body CDCC

T. Matsumoto

T. Matsumoto, K. Kato and M. Yahiro, arXiv:1006.0668 [nucl-th].

$^{6}Li+^{209}Bi$ scattering at 30 MeV





Three-body model does not work.

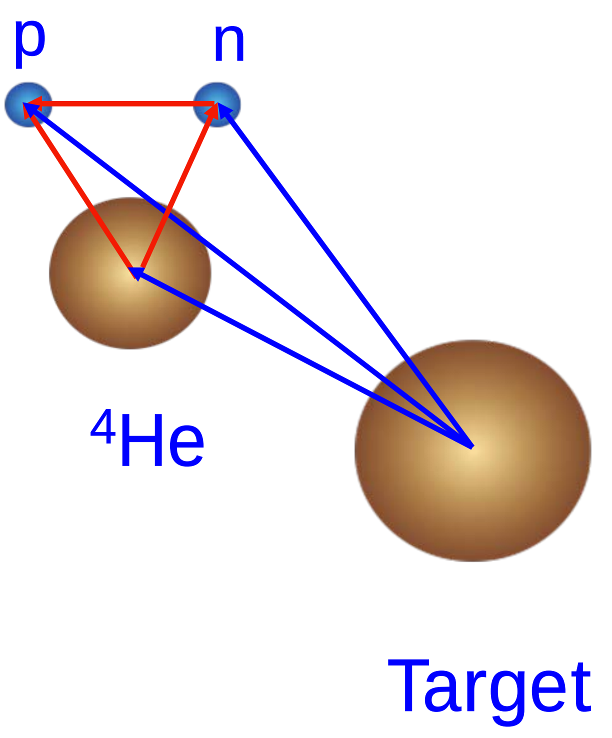
Four-body CDCC

Four-body CDCC

Four-body Schrodinger equation

$$(E - K - U - H_6) | \Psi >= 0$$

$$U = U_{pT} + U_{nT} + U_{HeT}$$



$$\langle \Phi_i | H_6 | \Phi_i \rangle = e_i \delta_{ij}$$

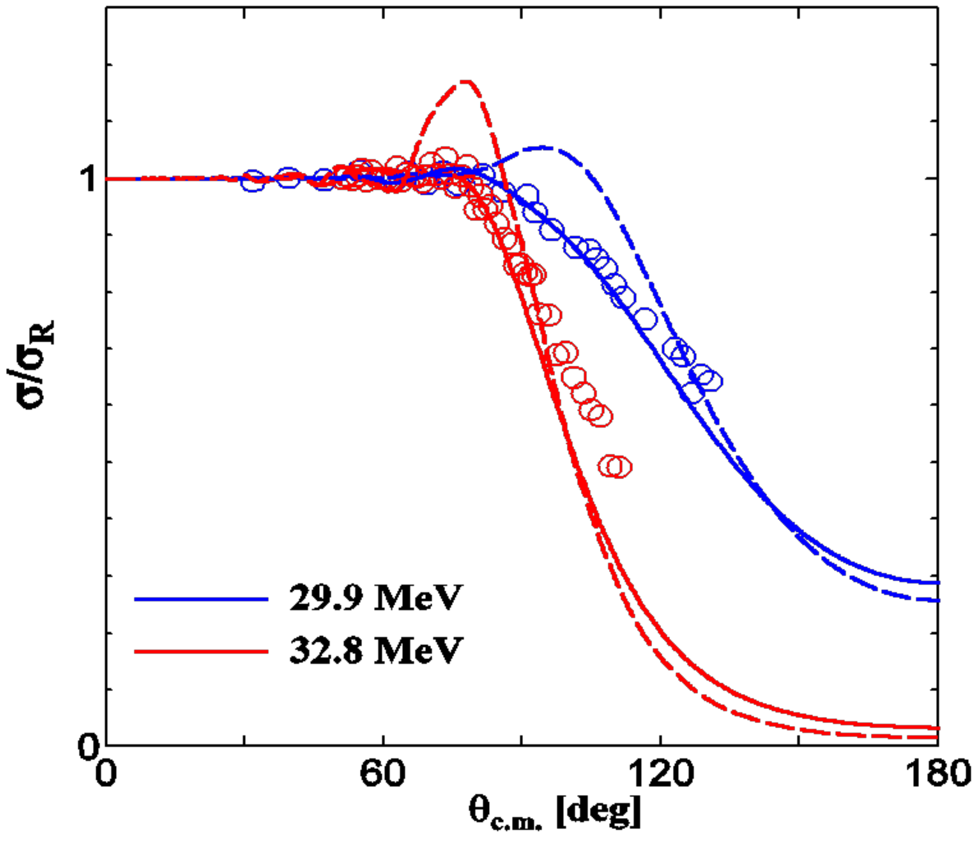
Model space $$P = \sum_i | \Phi_i >< \Phi_i |$$

CDCC equation

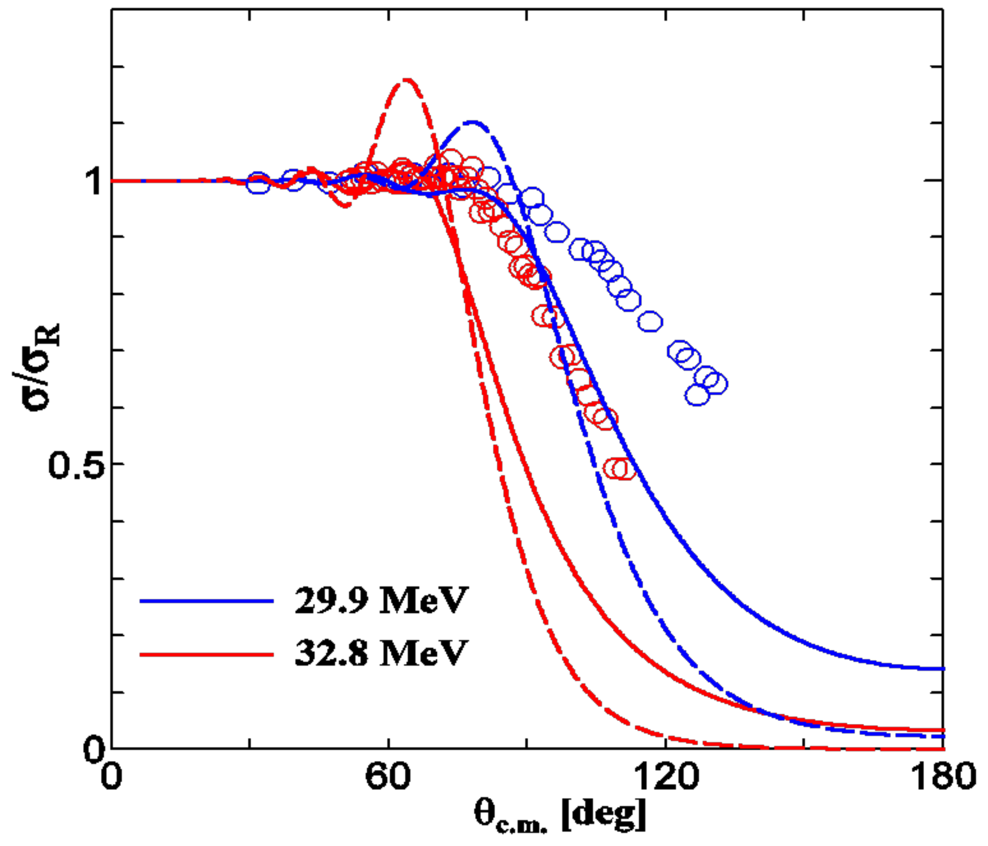
$$P(E - K - U - H_6) P | \Psi >= 0$$

$^{6}Li+^{209}Bi$ scattering at 30 MeV

Four-body ($n+p+^{4}He+A$) CDCC



Three-body ($d+^{4}He+A$) CDCC



^{6}He elastic scattering

Four-body Schrodinger equation

$$(E - K - U - H_6)\,|\,\Psi >$$

$$U = U_{nT} + U_{nT} + U_{HeT}$$

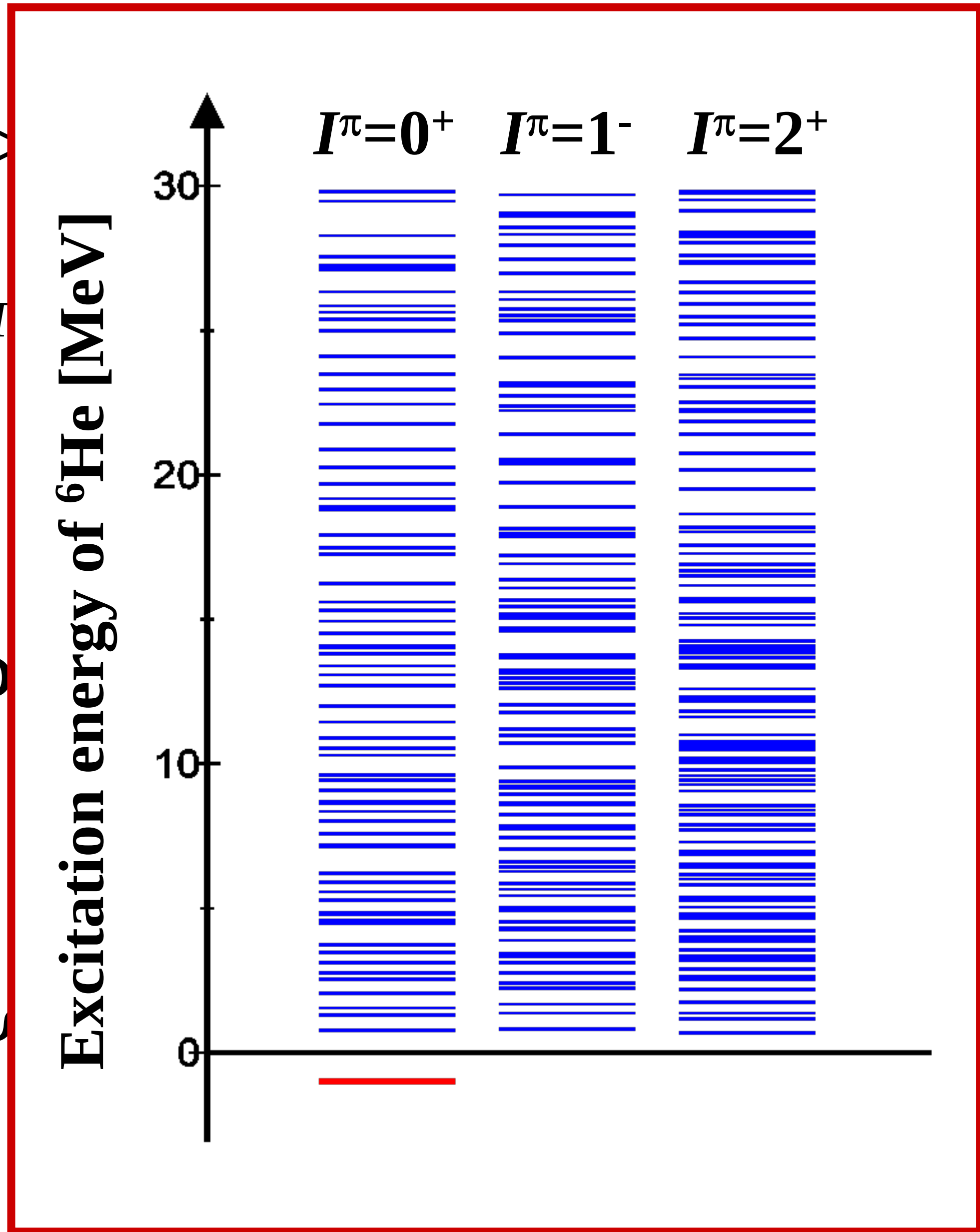
Gaussian basis functions

$$\left\langle \Phi_i \,|\, H_6 \,|\, \Phi_j \right\rangle = e_i \delta_{ij}$$

Model space $\quad P = \sum_i |\,\Phi$

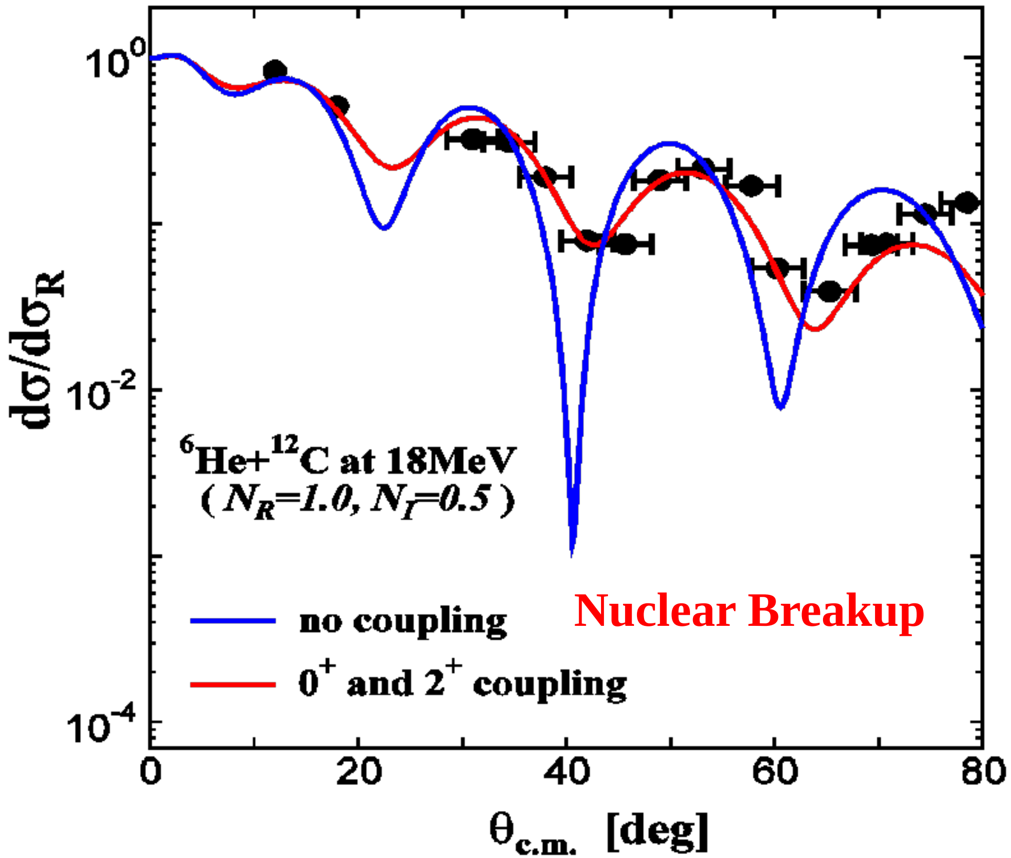
CDCC equation

$$P(E - K - U - H_6)P\,|\,\Psi$$

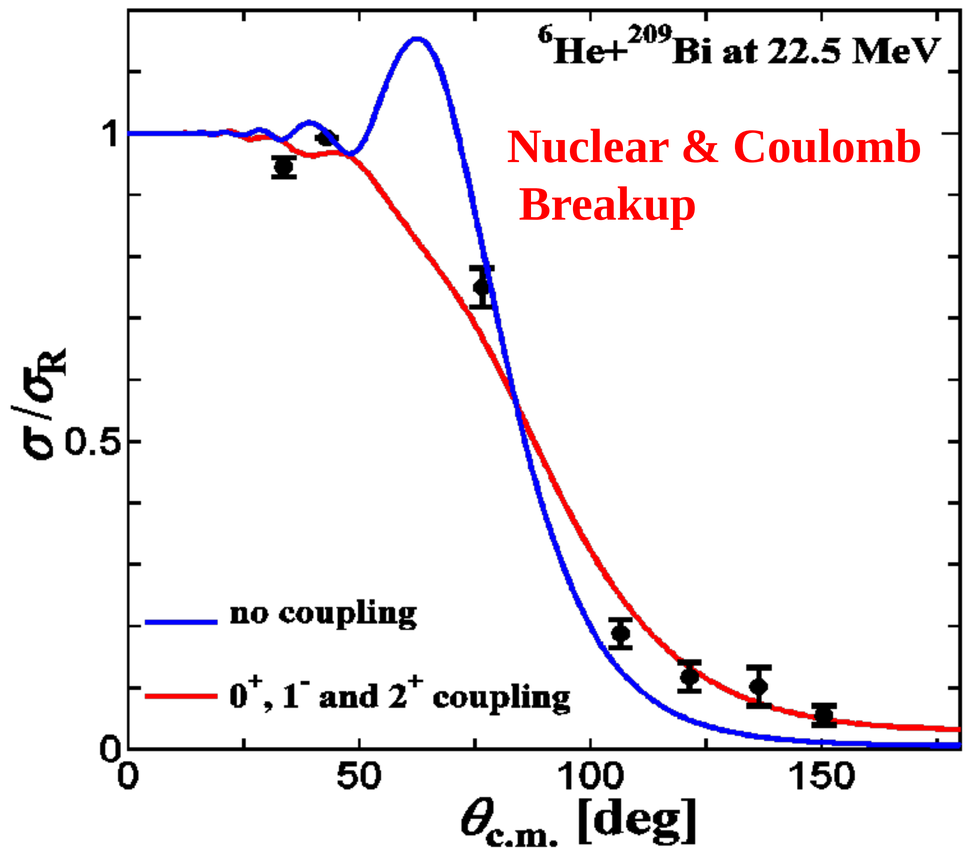


Elastic Cross Section

$^6He+^{12}C$ scattering at 18 MeV



$^6He+^{209}Bi$ scattering at 22.5 MeV

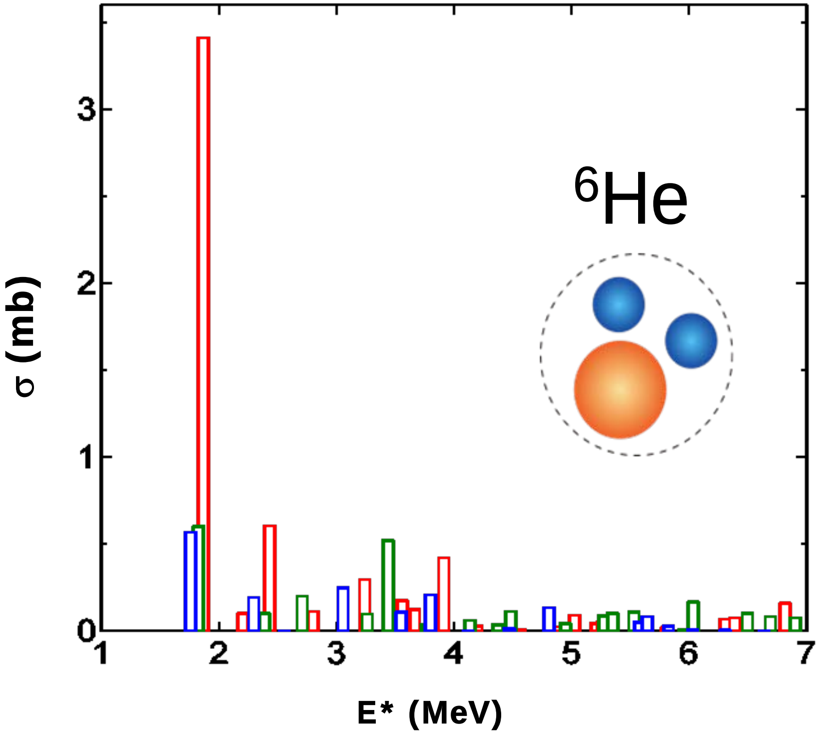


For *elastic scattering*, CDCC well reproduces the experimental data.

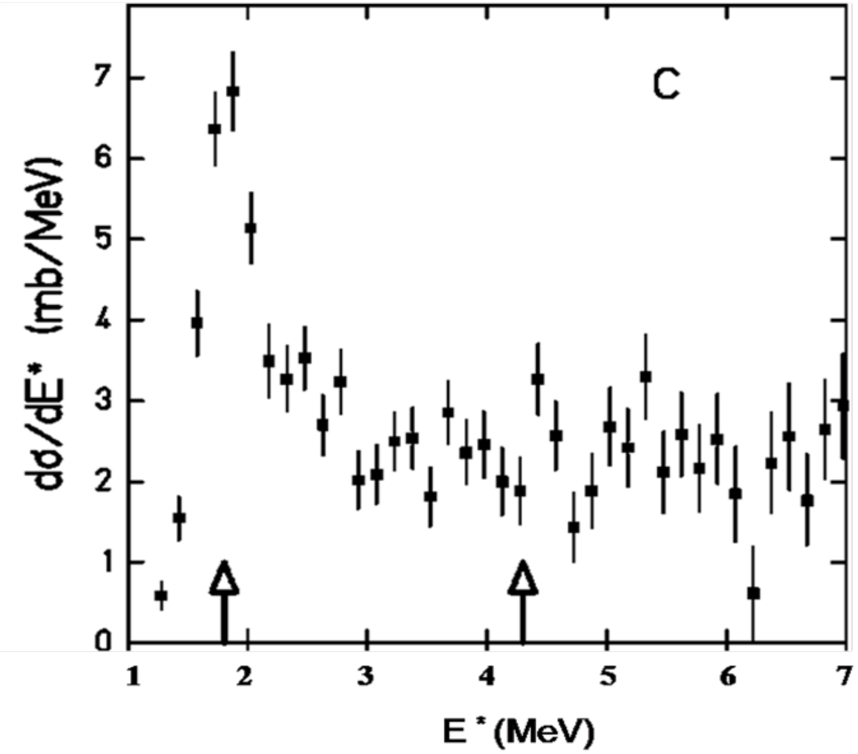
Breakup Cross Section of ^{6}He scattering

^{6}He+^{12}C scattering at 240 MeV/nucl.

4-body CDCC calc.



PRC59, 1252(1999), T. Aumann *et al*.



How to smooth the discrete spectrum

New Smoothing Procedure with *Complex Scaling Method*

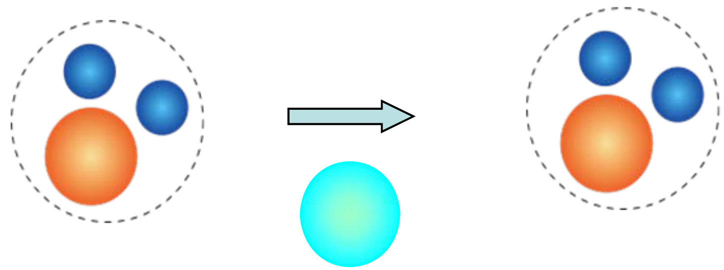
T. Matsumoto, K. Kato and M. Yahiro, arXiv:1006.0668 [nucl-th].

$$\frac{d\sigma}{dE} = \int d\vec{k}' d\vec{p}' d\vec{P}' \delta(E - E') \left| T(E) \right|^2 = \frac{1}{\pi} \mathrm{Im}\, R(E)$$

$$T(E) \quad = \quad \langle \psi^{(-)}(E, \xi) \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle$$

Continuum state of ^{6}He with positive E

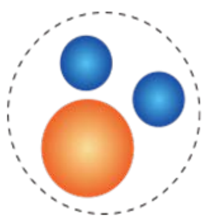
$E = E_0$



$E > 0$

Response function

$$\mathcal{R}(E) \quad = \quad \int d\xi d\xi' \langle \Psi^{(+)}(\xi, \mathbf{R}) | V^* | \chi_C^{(-)}(\mathbf{R}) \rangle_{\mathbf{R}} \mathcal{G}^{(-)}(E, \xi, \xi') \langle \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle_{\mathbf{R}}$$



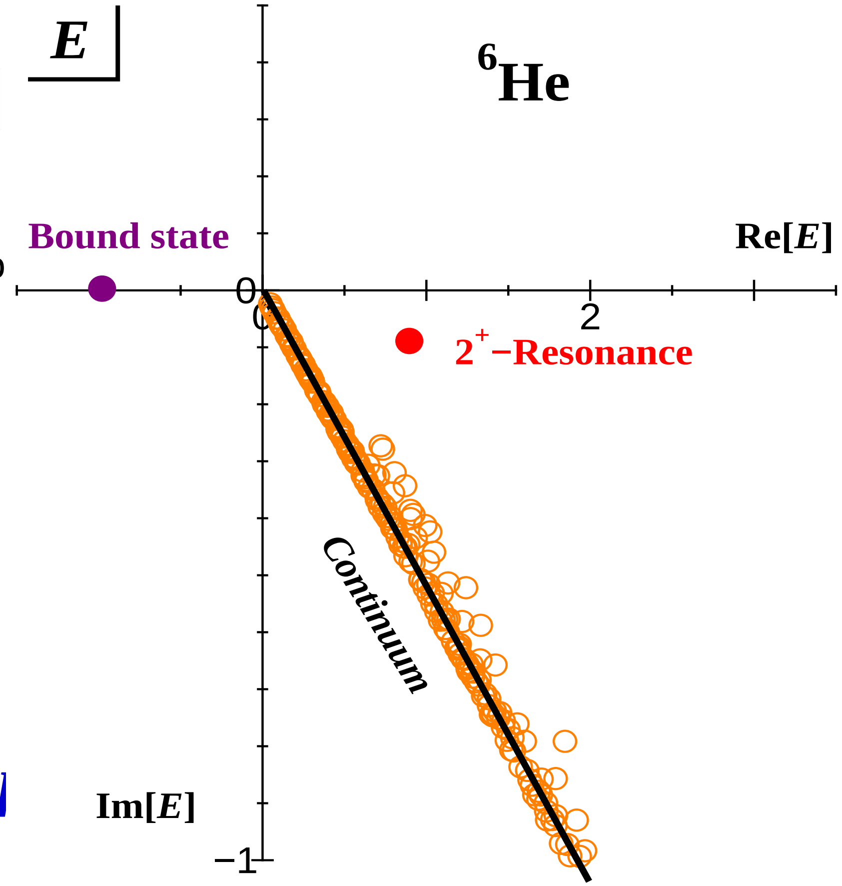
$E > 0$

$$G^{(-)}(E) = \frac{1}{E - H_6 - i\varepsilon}$$

Complex scaling

T. Matsumoto, K. Kato and M. Yahiro, arXiv:1006.0668 [nucl-th].

T. Matsumoto, T. Egami, K. Ogata and M. Yahiro, Kikuchi, Myo
Prog. Theor. Phys. 121(2009), 885-894.



Scaling operator $U(\theta)$

$$\langle \vec{r} \,|U(\theta)|\, f \rangle = e^{i3\theta/2} f\left(\vec{r} e^{i\theta}\right)$$

Green's function with Complex-Scaling Met

$$G^{(-)}(E) = \frac{1}{E - H_6 - i\varepsilon} = U^{-\theta} \frac{1}{E - H^\theta - i\epsilon} U^\theta \approx \sum_\nu U^{-\theta} \frac{|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|}{E - E_\nu^\theta} U^\theta$$

$$1 = U^{-1}(\theta) U(\theta)$$

$$H^\theta = U^{-1} H_6 U$$

$$1 = \sum_i \left|\Phi_i\right\rangle\left\langle\Phi_i\right|$$

$$\left\langle \tilde{\Phi}_\mu^\theta \middle| H^\theta \middle| \Phi_\nu^\theta \right\rangle = E_\nu^\theta \delta_{\mu\nu} \qquad \left\langle \Phi_i \middle| H_6 \middle| \Phi_j \right\rangle = E_i \delta_{ij}$$

New Smoothing Procedure with Complex Scaling Method

Green's function with Complex-Scaling Method

$$\mathcal{G}^{(-)}(E,\xi,\xi') \approx \sum_{\nu}\sum_{i,j} |\Phi_i\rangle \frac{\langle\Phi_i|U^{-\theta}|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|U^\theta|\Phi_j\rangle}{E-E_\nu^\theta}\langle\Phi_j|$$

Response function

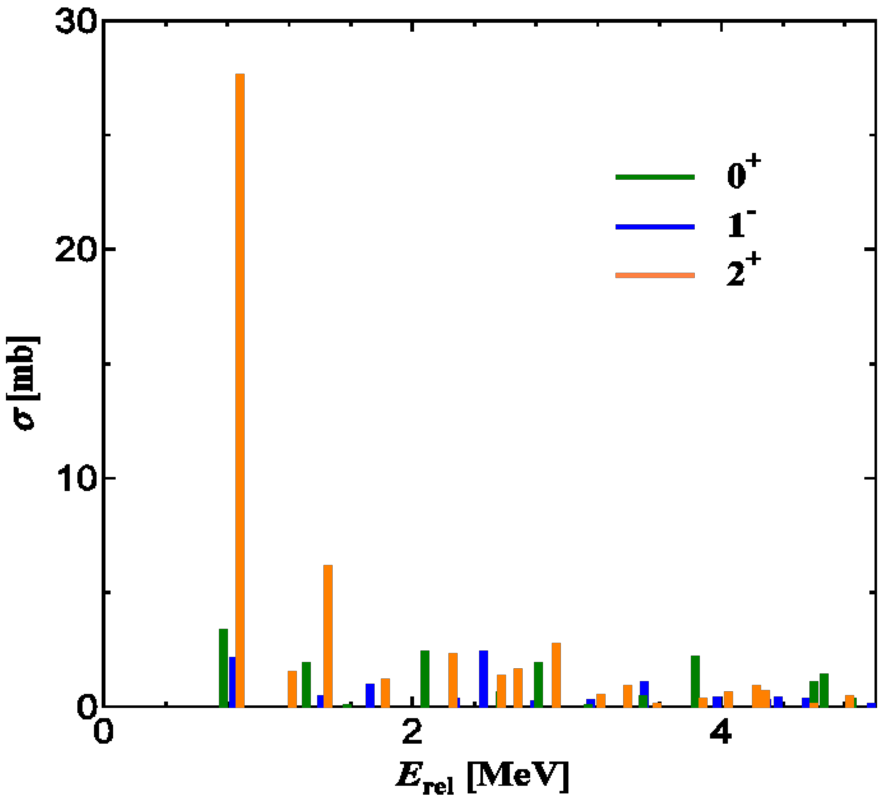
$$\mathcal{R}(E) = \int d\xi d\xi' \langle\Psi^{(+)}(\xi,\mathbf{R})|V^*|\chi_C^{(-)}(\mathbf{R})\rangle_{\mathbf{R}}\mathcal{G}^{(-)}(E,\xi,\xi')\langle\chi_C^{(-)}(\mathbf{R})|V|\Psi^{(+)}(\xi,\mathbf{R})\rangle_{\mathbf{R}}$$

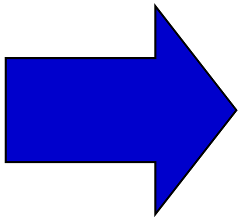
$$\mathcal{R}(E) = \sum_{\nu}\sum_{i,j} \boxed{\langle\Psi^{(+)}|V^*|\chi_C^{(-)}\Phi_i\rangle} \frac{\langle\Phi_i|U^{-\theta}|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|U^\theta|\Phi_j\rangle}{E-E_\nu^\theta} \boxed{\langle\Phi_j\chi_C^{(-)}|V|\Psi^{(+)}\rangle}$$

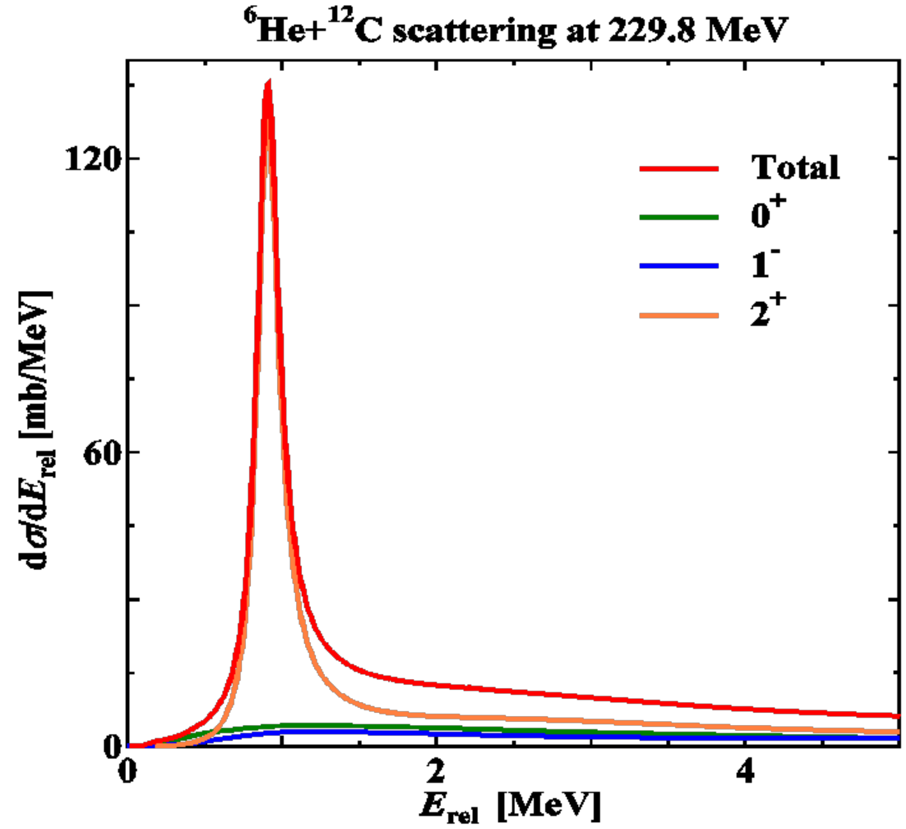
T-matrix calculated by CDCC

Breakup Spectrum

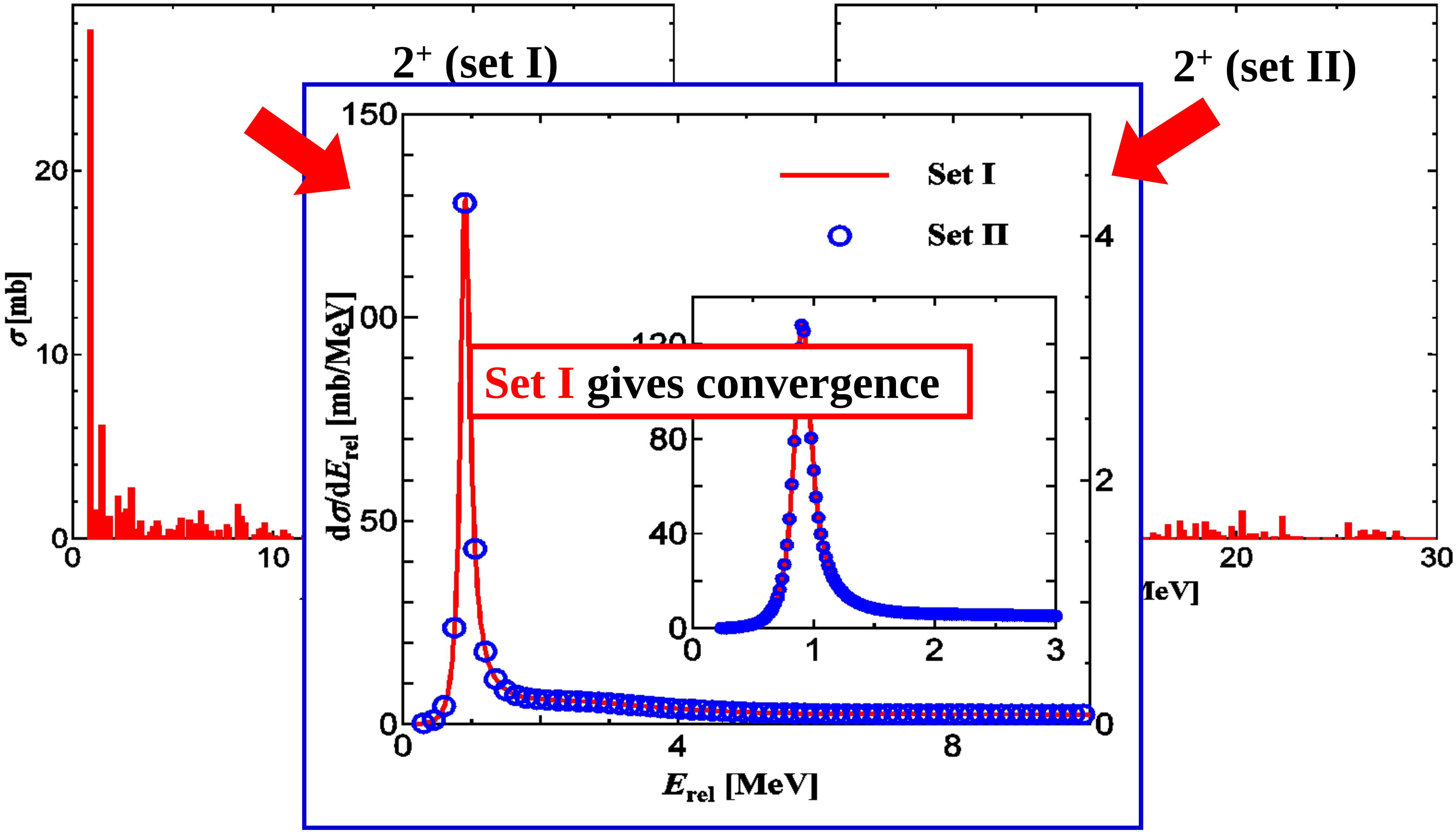
$$\frac{d\sigma}{dE} = \frac{1}{\pi}\mathrm{Im}\sum_{\nu}\sum_{i,j} T_i^{\mathrm{CDCC}\dagger}\frac{\langle\Phi_i|U^{-\theta}|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|U^\theta|\Phi_j\rangle}{E - E_\nu^\theta}T_j^{\mathrm{CDCC}}$$



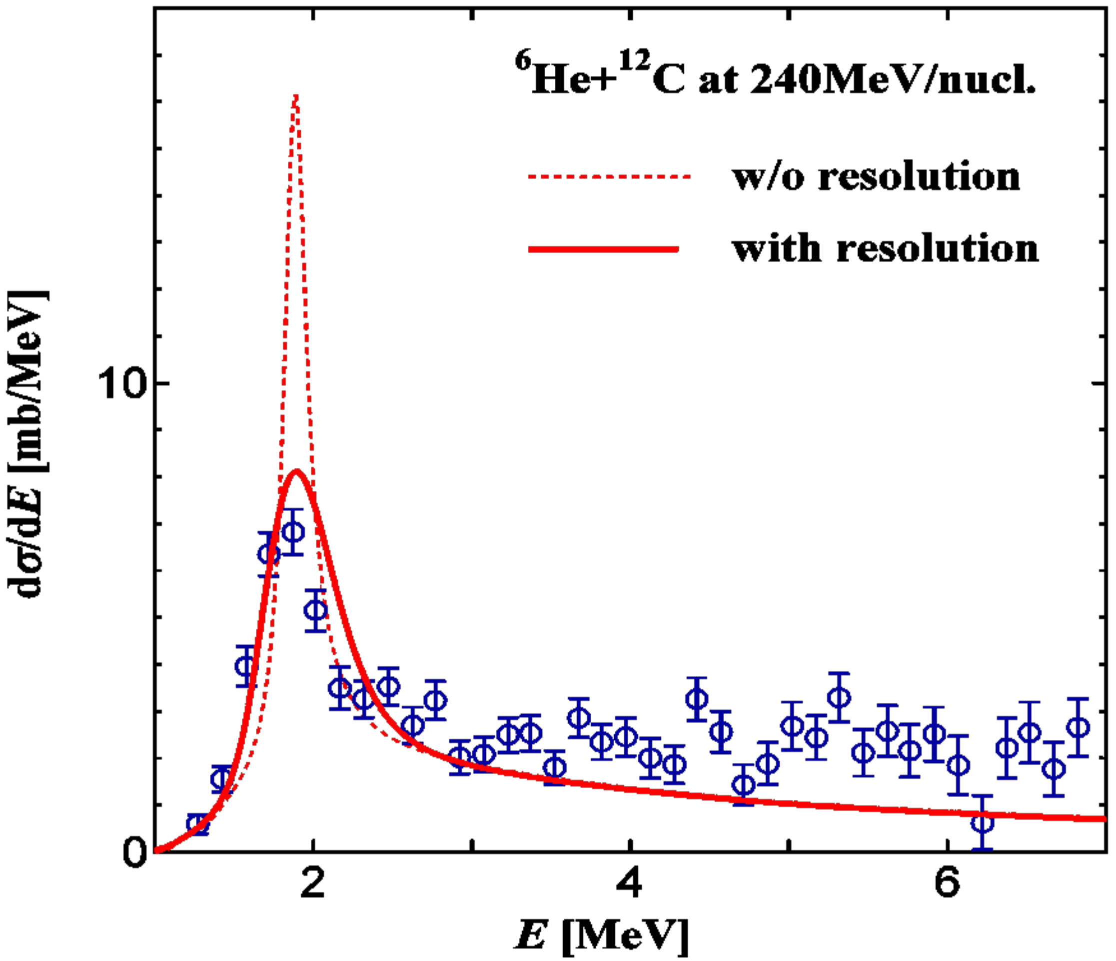




Convergence of Breakup-spectrum (2^+)



^{6}He+^{12}C scattering @ 240 MeV/nucl.



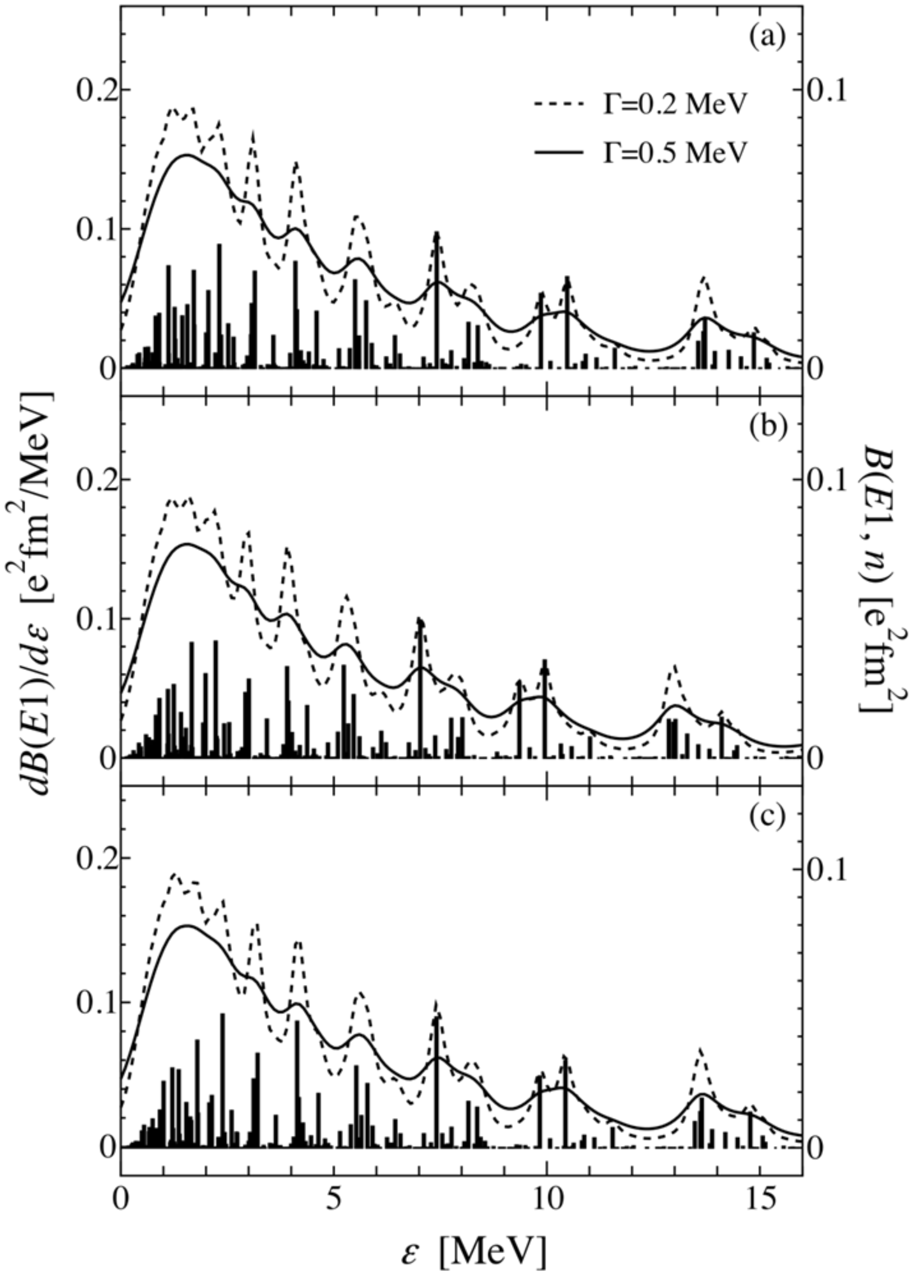
Exp. data from PRC59, 1252 (1999), T. Aumann *et al.*

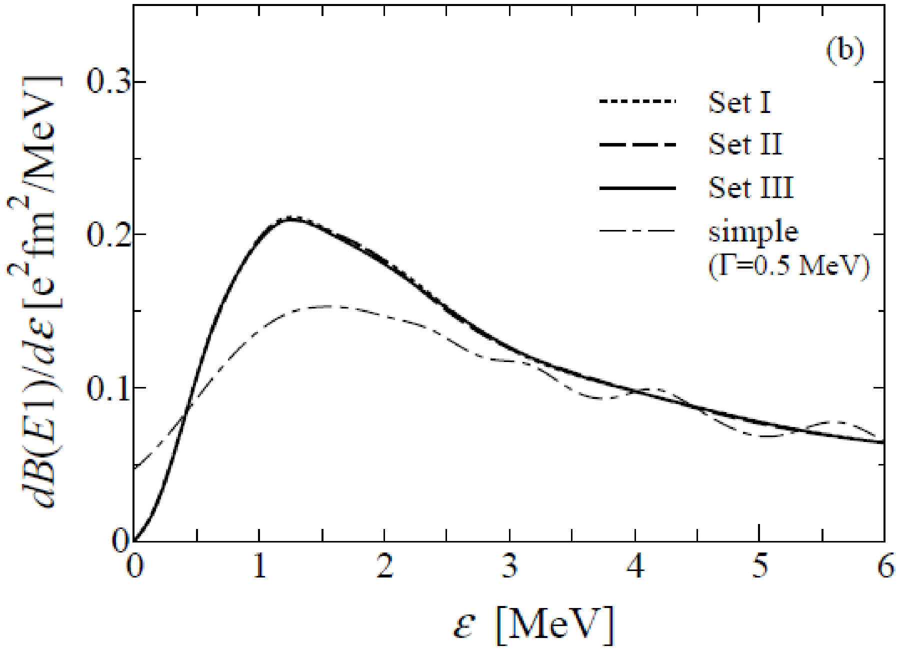
Coupling potential:

- **N-^{12}C potential folded with ^{6}He transition densities**
- **Without Coulomb breakup**

Calculation:

- **non-relativistic**
- **Coupled-channel calculation**





Simple smoothing method
→ Lorentzian distribution

$$\frac{dB(E1)}{d\varepsilon} = \sum_n \frac{\Gamma}{\pi} \frac{1}{(\varepsilon - \varepsilon_n)^2 + \Gamma^2} B(E1;n)$$

$$B(E1;n) = \sum_{\mu,m} |\langle \hat{\Phi}_{n1m} | \mathcal{O}(E1) | \Phi_0 \rangle|^2$$

T. Egami, T. Matsumoto, K. Ogata and M. Yahiro, Prog. Theor. Phys. 121(2009), 789-807.

Papers on 4-body CDCC

1) Kyushu-group

T. Matsumoto, E. Hiyama, K. Ogata, Y. Iseri, M. Kamimura, S. Chiba and M. Yahiro, Phys. Rev. C70(2004), 061601.

T. Egami, T. Matsumoto, K. Ogata and M. Yahiro, Prog. Theor. Phys. 121(2009), 789-807.

T. Matsumoto, T. Egami, K. Ogata and M. Yahiro, Prog. Theor. Phys. 121(2009), 885-894.

2) Another group

M. Rodriguez-Gallardo, J. M. Arias, J. Gomez-Camacho, R. C. Johnson, A. M. Moro, I. J. Thompson, and J. A. Tostevin, Phys. Rev. C 77, 064609 (2008).

M. Rodriguez-Gallardo, J. M. Arias, J. Gomez-Camacho, A. M. Moro, I. J. Thompson, and J. A. Tostevin, Phys. Rev. C 80, 051601(R) (2009).

3. Microscopic CDCC

K. Minomo

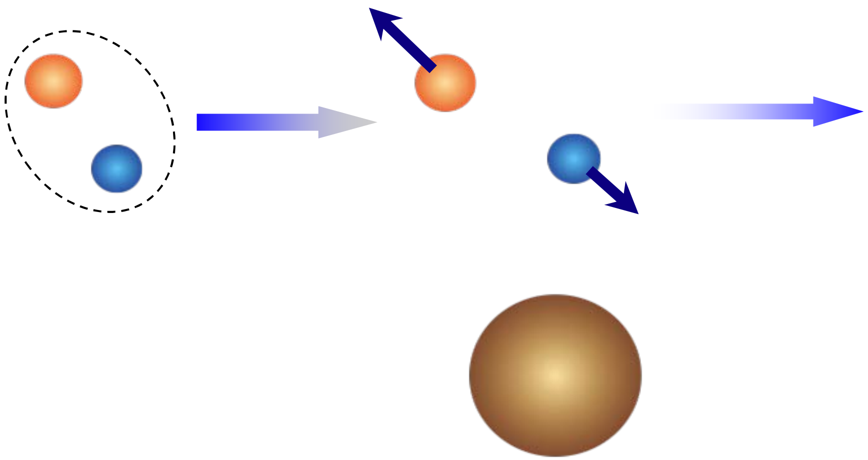
K. Minomo, K. Ogata, M. Kohno, Y. R. Shimizu, and M. Yahiro, J. Phys. G (arXiv:nucl-th0911.1184)

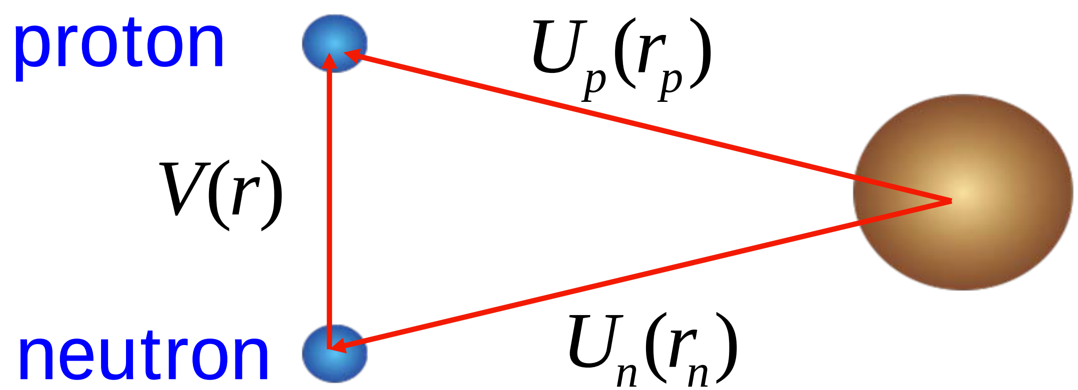
M. Yahiro, K. Minomo, K. Ogata, M. Kawai, Prog. Theor. Phys., 120(2008), 767

Deuteron scattering

Deuteron scattering

$$H = K + V(r) + U_{\mathrm{p}}(r_p) + U_{\mathrm{n}}(r_n)$$





Microscopic optical potential for Nucleon-nucleus scattering

Nucleon-nucleus folding potential

$$U_{opt} = < \Phi_A \,|\, \sum_{j \in A} g_{0j} \,|\, \Phi_A >$$

G-matrix: K. Amos et al.,
(Melbourne group)
Adv. Nucl. Phys. Vol.25 (2000) 275

⇧

Bonn-B NN interaction
+ phenomenological imaginary potential.

Hatree-Fock cal.
with Gogny-force (D1S)

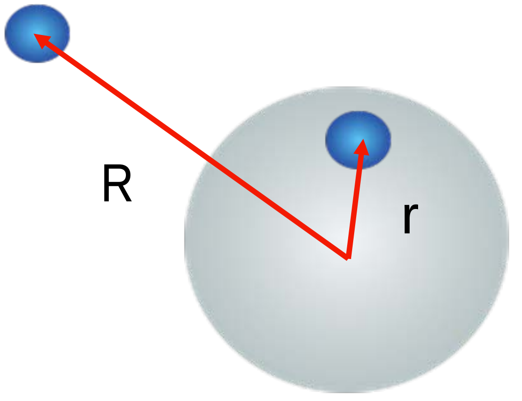
⇩

The structure of stable nuclei.

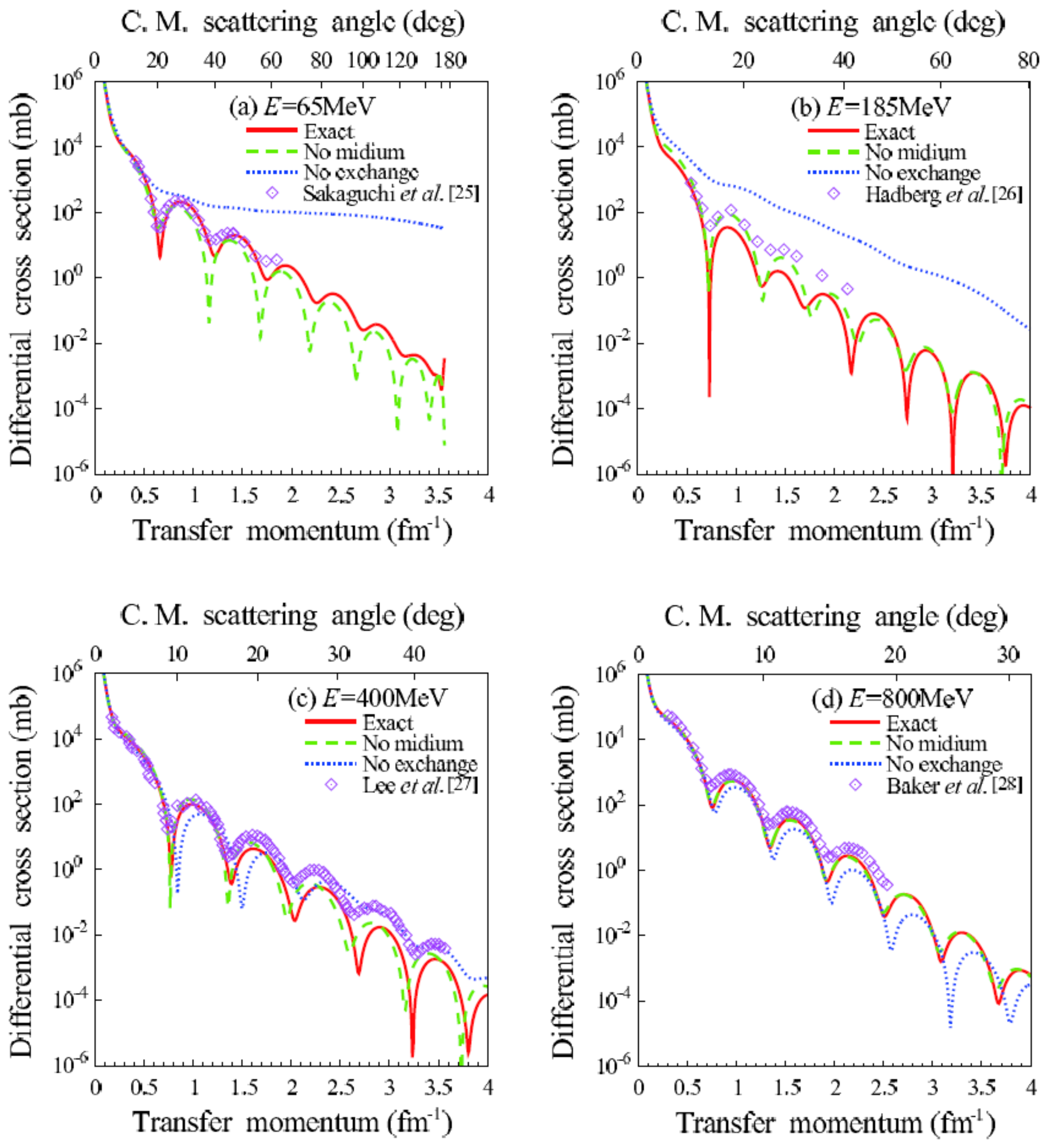
$$g_{0j} = g(r_{0j})(1 + P_{EX})$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla_R^2 + U^{\mathrm{DR}}(\mathbf{R}) + V_{\mathrm{c}}(R)\,\delta_{-1/2}^{\nu_1} - E\right]\chi_{\mathbf{K},\nu_1}(\mathbf{R}) = \int U^{\mathrm{EX}}(\mathbf{R},\mathbf{r})\chi_{\mathbf{K},\nu_1}(\mathbf{r})\,d\mathbf{r}$$

Schroedinger equation for proton scattering



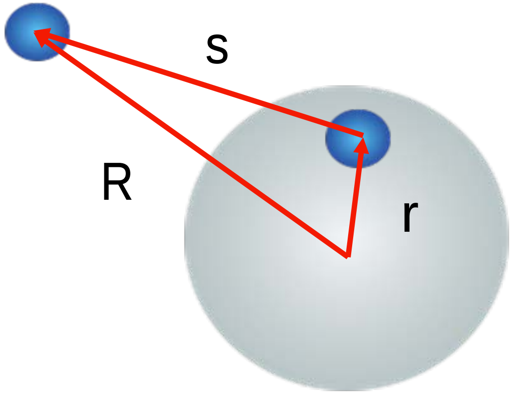
Proton scattering from ^{90}Zr



The Brieva-Rook localization

Nucl. Phys. A291,317

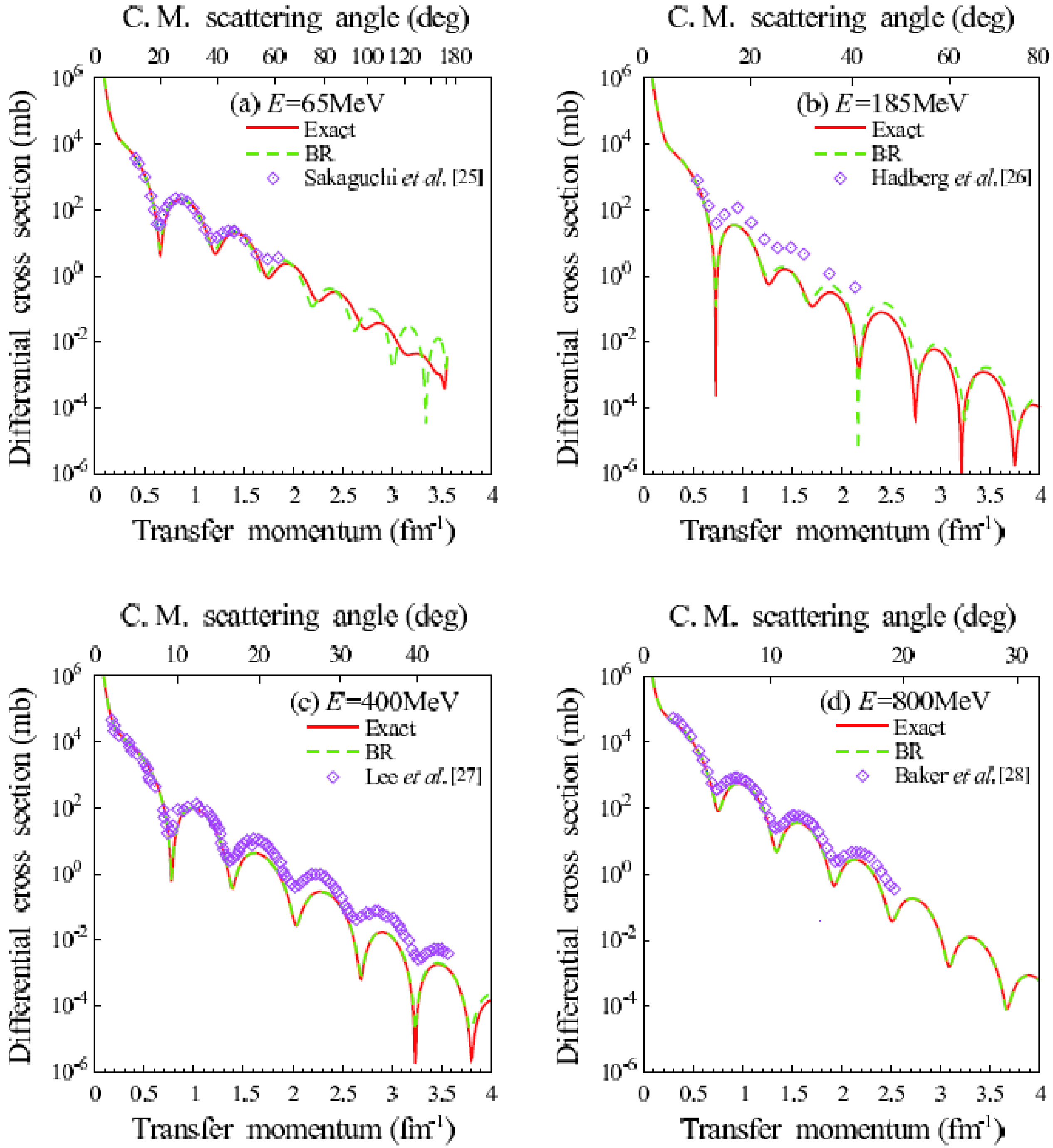
$$U_{\rm BR}^{\rm EX}(R) = \sum_{\nu_2,T_z} \int \rho_{\nu_2}^{\rm LFG}(\mathbf{R},\mathbf{r}) g_{T_z}^{\rm EX}(s;\rho_{\nu_2}(r_g)) j_0(K(R)s) d\mathbf{s}.$$



The mixed density

$$\rho_{\nu_2}(\mathbf{R},\mathbf{r}) = \sum_{nljj_z} \int \varphi^*_{\nu_2;nljj_z}(\mathbf{r},\xi)\, \varphi_{\nu_2;nljj_z}(\mathbf{R},\xi)\, d\xi,$$

K. Minomo, K. Ogata, M. Kohno, Y.R. Shimizu, M. Yahiro, arXiv:0911.1184 [nucl-th]



C. M. scattering angle (deg)
(a) E=65MeV
Exact
BR
Sakaguchi et al. [25]
(b) E=185MeV
Exact
BR
Hadberg et al. [26]
(c) E=400MeV
Exact
BR
Lee et al. [27]
(d) E=800MeV
Exact
BR
Baker et al. [28]
Differential cross section (mb)
Transfer momentum (fm-1)

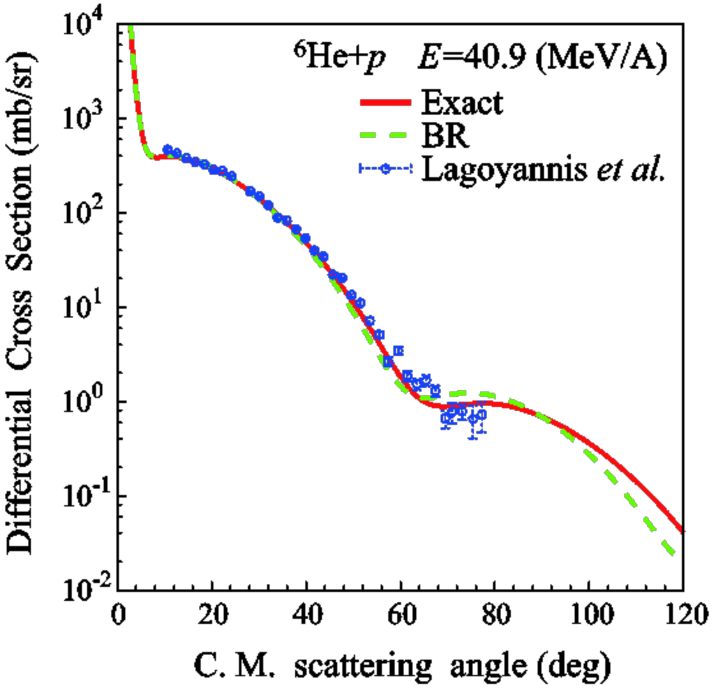
$^{6,8}He$ scattering from proton target

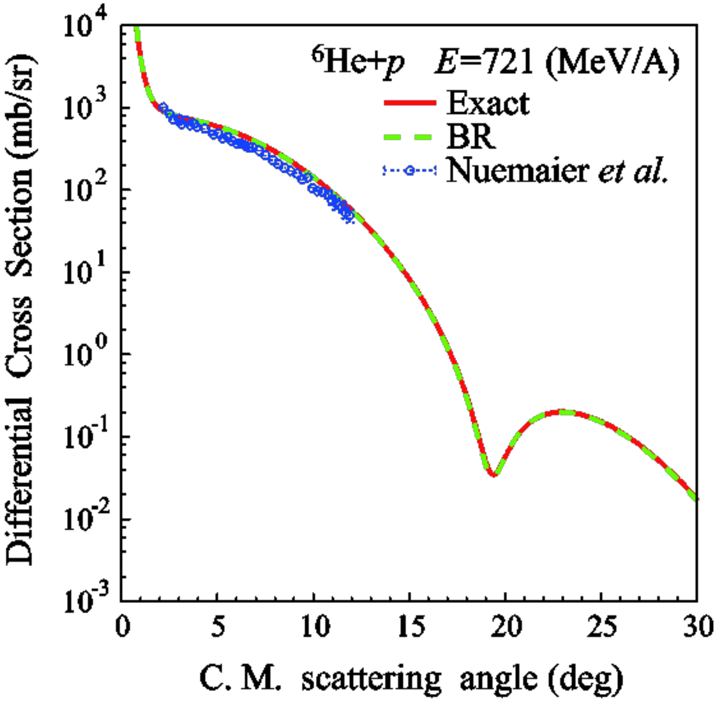
Table of binding energies

	^{6}He	^{8}He
exp	29.268	31.408
HF	29.466	31.905

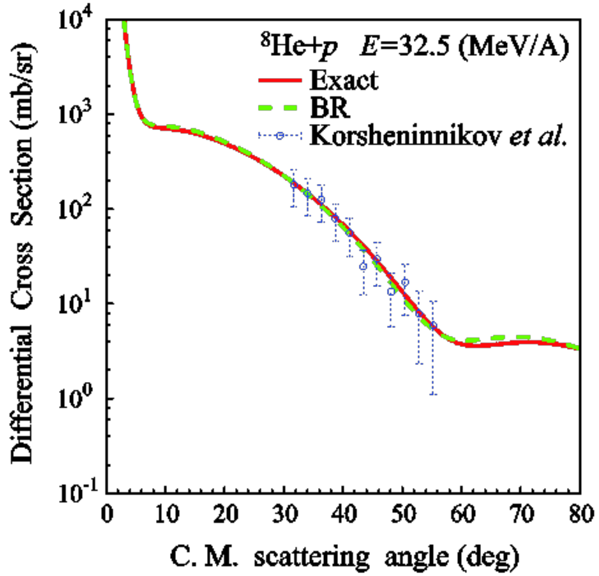
Hatree-Fock cal. with Gogny-force (D1S)

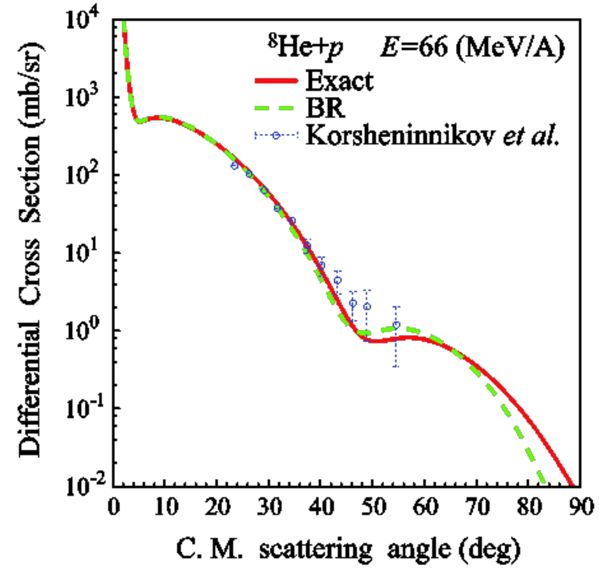
^{6}He+p scattering

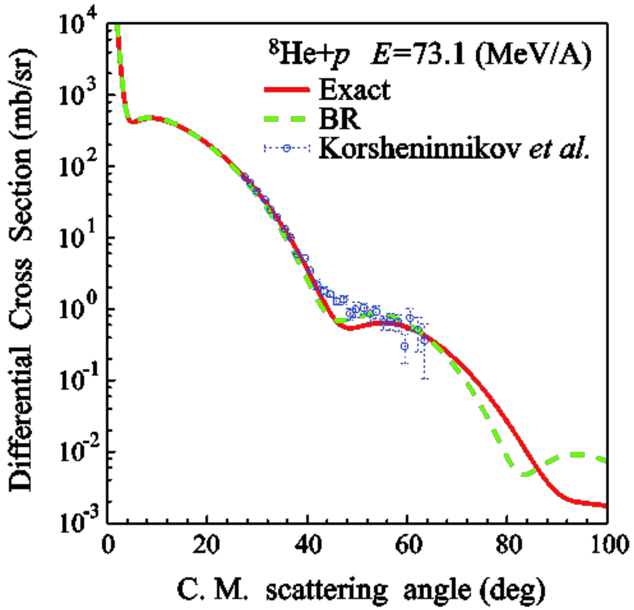


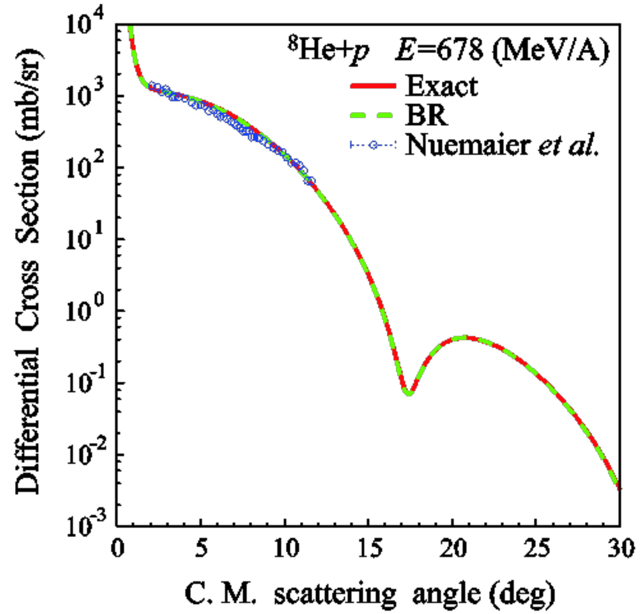


^8He+p scattering







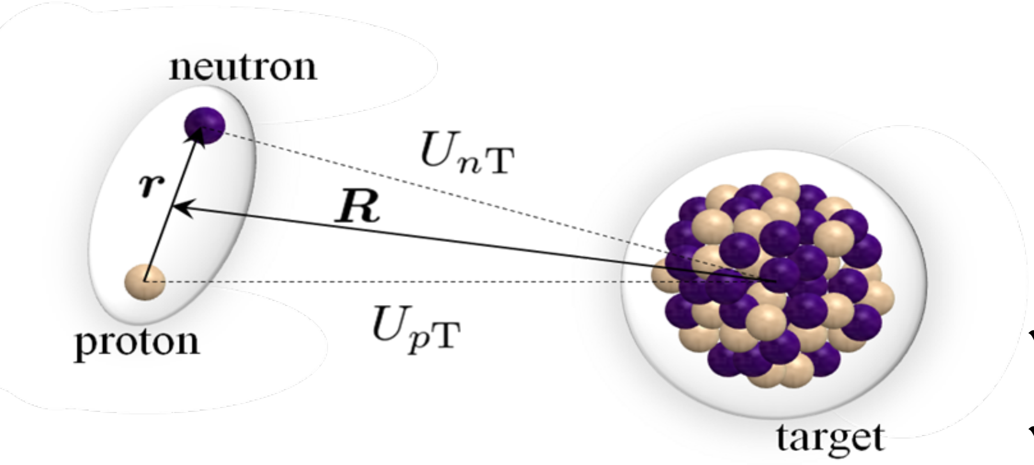


Application

- For deuteron induced reaction

$$\left[K + h_{pn} + h_{\mathrm{T}} + \sum_{j\in \mathrm{T}} \left(\tau_{pj} + \tau_{nj}\right) - E\right]\Psi = 0$$

<u>three-body model</u>



Optical potentials as an input

$$U_{p\mathrm{T}} = \left\langle \varphi_{\mathrm{T}} \middle| \sum_{j\in \mathrm{T}} \tau_{pj} \middle| \varphi_{\mathrm{T}} \right\rangle$$

$$U_{n\mathrm{T}} = \left\langle \varphi_{\mathrm{T}} \middle| \sum_{j\in \mathrm{T}} \tau_{nj} \middle| \varphi_{\mathrm{T}} \right\rangle$$

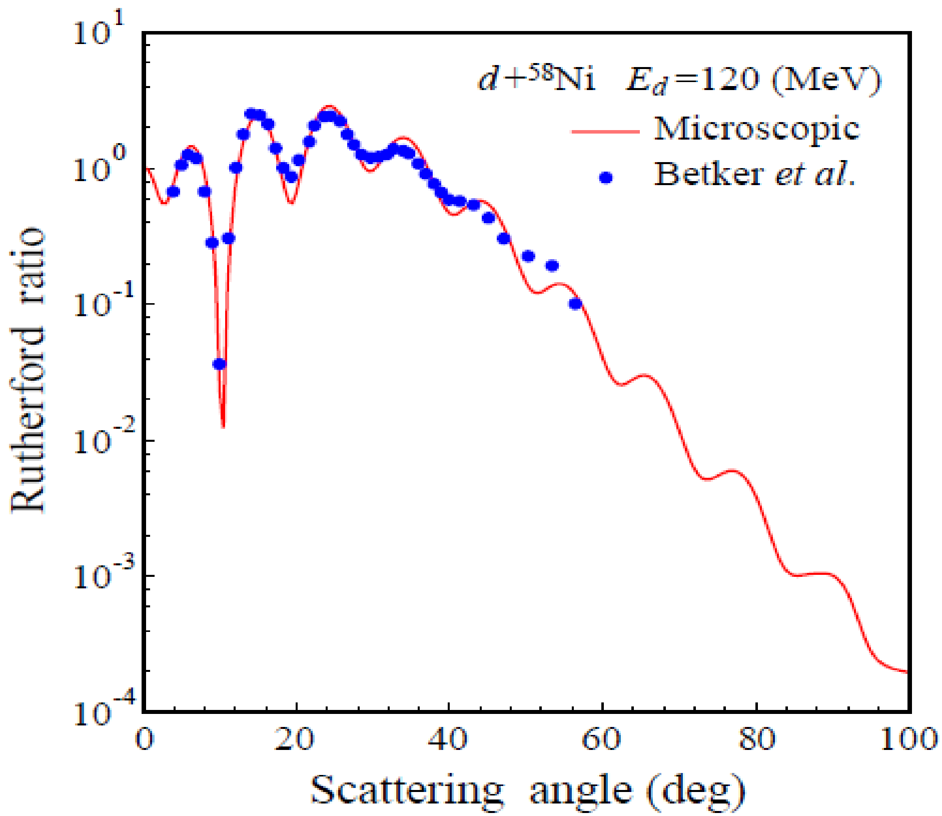
✓ ~~non-local potential~~ complicated

✓ localized potential（BR method） useful

✓ **Continuum-Discretized Coupled-Channels method (CDCC)**

It is a standard direct reaction theory to describe real and virtual breakup.

$d+^{58}$Ni elastic scattering



Our microscopic calculation reproduces the data.

A success of
Microscopic CDCC

in the case of the scattering of unstable nuclei

- ✓ ~~phenomenological potential~~ unavailable
- ✓ ~~non-local potential~~ complicated
- ✓ localized potential（BR method） useful

Summary

1. CDCC is an accurate method for treating the projectile breakup process. The Austern-Yahiro-Kawai theory gives a theoretical foundation of CDCC.
2. Four-body CDCC is feasible also for four-body systems. In principle, this formulation is applicable for N-body system.
3. Microscopic non-local nucleon-nucleus optical potential can be localized with good accuracy by the Brieva-Rook method. CDCC with the local microscopic optical potential well describes the deuteron scattering.