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on Correlations in Reactions and Continuum
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The research of E1 mode using the Canonical-basis TDHFB

~ Pygmy resonance ? ~

Shuichiro Ebata[A,B]

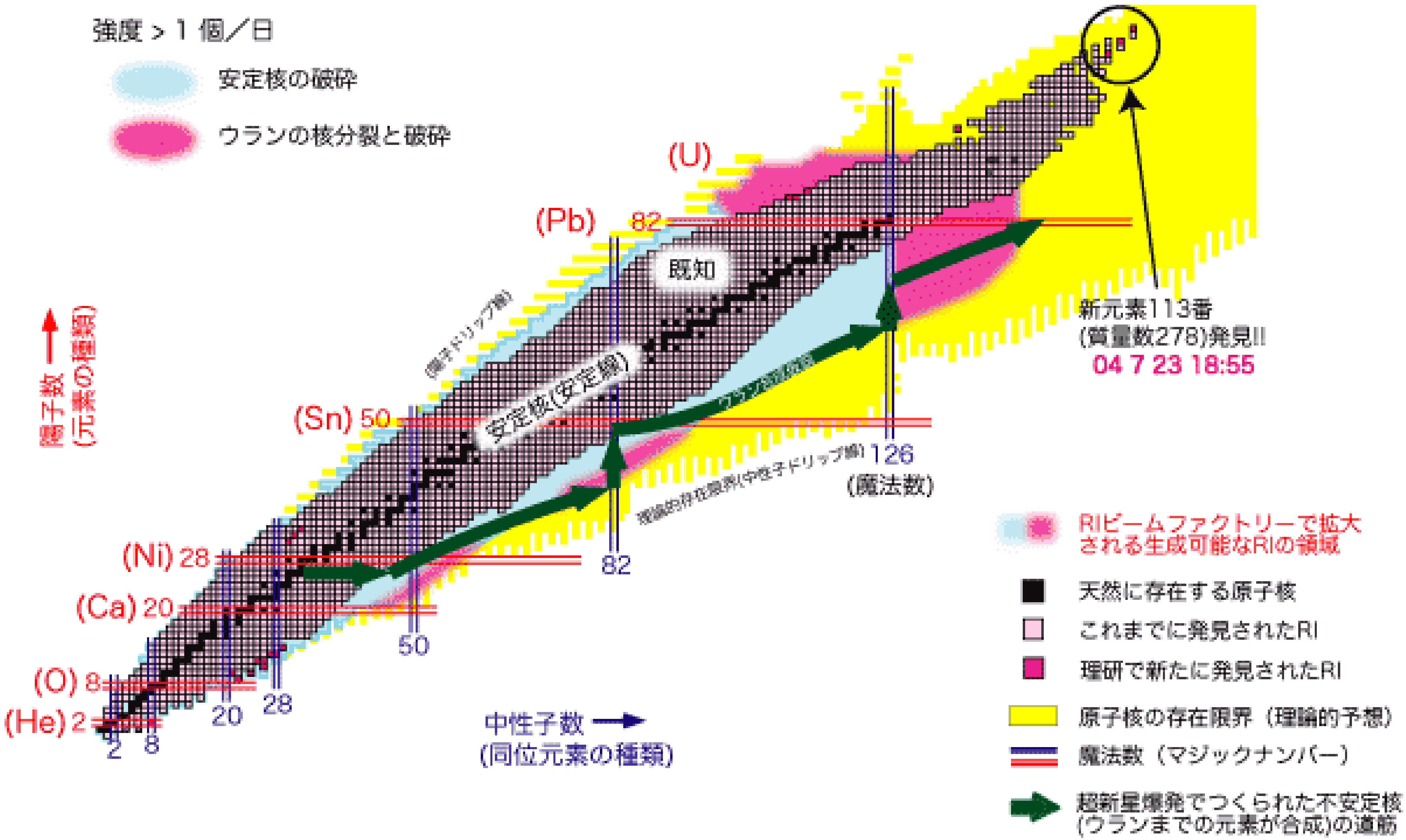
T. Nakatsukasa[B,C], T. Inakura[A,B],
K. Yoshida[B], Y. Hashimoto[A,C],
K. Yabana[A,B,C]

Main Contents

1, Derivation of Cb-TDHFB
2, Application of Cb-TDHFB for E1 mode

Graduate School of Pure and Applied Sciences, University of Tsukuba[A]
RIKEN Nishina Center Theoretical Nuclear Physics Laboratory [B]
Center for Computational Sciences, University of Tsukuba [C]

Introduction



from http://www.rarf.riken.go.jp/newcontents/contents/facility/RIBF.html

Introduction

Construction of theoretical framework
to calculate structure and response of
from light nuclei to heavy ones systematically

Requirements :

1, Applicable to **heavy** nuclei

2, **No symmetry restriction** for **any** deformed nuclei

3, Able to describe **excitations** and various **dynamics** of nuclei

4, Including effects of **Pairing Correlation**

→ **Cb-TDHFB** + **3-Dimensional coordinate-space**

Recipe for the Canonical-basis TDHFB (Cb-TDHFB)

Ebata et al, arXiv:1007.0785

1, Canonical-basis representation

$$|\Psi(t)\rangle \equiv \prod_{k>0}\Big(u_k(t) + v_k(t)\hat{c}_k^\dagger(t)\hat{c}_{\bar{k}}^\dagger(t)\Big)|0\rangle$$

$$\rho_k(t) \equiv |v_k(t)|^2$$ **: Occupation prob.**

$$\kappa_k(t) \equiv u_k(t)v_k(t)$$ **: Pair prob.**

$$\rho_{\mu\nu}(t) = \sum_{k>0}\rho_k(t)\Big(\langle\mu|\phi_k(t)\rangle\langle\phi_k(t)|\nu\rangle + \langle\mu|\phi_{\bar{k}}(t)\rangle\langle\phi_{\bar{k}}(t)|\nu\rangle\Big)$$ **: Normal density**

$$\kappa_{\mu\nu}(t) = \sum_{k>0}\kappa_k(t)\Big(\langle\mu|\phi_k(t)\rangle\langle\nu|\phi_{\bar{k}}(t)\rangle - \langle\mu|\phi_{\bar{k}}(t)\rangle\langle\nu|\phi_k(t)\rangle\Big)$$ **: Pair density**

TDHFB*

$$i\hbar\frac{\partial}{\partial t}\mathcal{R}(t) = \Big[\mathcal{H}(t),\ \mathcal{R}(t)\Big]$$

$$\mathcal{R}(t) = \begin{pmatrix} \rho(t) & \kappa(t) \\ -\kappa(t) & 1-\rho^*(t) \end{pmatrix}$$

$$\mathcal{H}(t) = \begin{pmatrix} h(t) & \Delta(t) \\ -\Delta^*(t) & -h^*(t) \end{pmatrix}$$

2, Assumption for Pairing potential

$$\Delta_{\mu\nu}(t) = -\sum_{k>0}\Delta_k(t)\Big(\langle\mu|\phi_k(t)\rangle\langle\nu|\phi_{\bar{k}}(t)\rangle - \langle\mu|\phi_{\bar{k}}(t)\rangle\langle\nu|\phi_k(t)\rangle\Big)$$

$$\Delta_k(t) = -\sum_{l>0}\kappa_l(t)\bar{V}_{k\bar{k},l\bar{l}}$$

… Pair potential is diagonal.

***TDHFB : Time-Dependent Hartree-Fock-Bogoliubov**

Recipe for the Cb-TDHFB

Ebata et al, arXiv:1007.0785

3, We adopt a schematic pairing functional :

$$E_{\text{pair}} \equiv -\sum_{k,l>0} G_{kl}\kappa_k^*(t)\kappa_l(t) \equiv -\sum_{k>0}\kappa_k^*\Delta_k(t) \;,\; \Delta_k(t) = \sum_{l>0} G_{kl}\kappa_l(t)$$

This pairing functional **violate gauge invariance** related to the **phase degree of freedom** of the canonical states.

4, We must choose the special gauge :

$$\eta_k(t) \Rightarrow \varepsilon_k(t) \equiv \langle\phi_k(t)|\, h(t)\, |\phi_k(t)\rangle \Leftrightarrow \left\langle \frac{\partial\phi_k(t)}{\partial t}\middle|\phi_k(t)\right\rangle = 0$$

Cb-TDHFB eqs.

$$i\hbar\frac{\partial}{\partial t}|\phi_k(t)\rangle = \big(h(t) - \varepsilon_k(t)\big)\,|\phi_k(t)\rangle$$

$$i\hbar\frac{\partial}{\partial t}\,\rho_k(t) = \kappa_k(t)\Delta_k^*(t) - \Delta_k(t)\kappa_k^*(t)$$

$$i\hbar\frac{\partial}{\partial t}\,\kappa_k(t) = \big(\varepsilon_k(t) + \varepsilon_{\bar{k}}(t)\big)\kappa_k(t) + \Delta_k(t)\big(2\rho_k(t) - 1\big)$$

Properties of Cb-TDHFB

$$d/dt\langle\hat{N}\rangle = d/dt E_{\text{Total}} = 0$$

$$d/dt\langle\phi_k(t)|\phi_{k'}(t)\rangle = 0$$

At a limit of $\Delta = 0$,

→ TDHF

At a static limit,

→ HF+BCS

Recipe for the Cb-TDHFB

Ebata et al, arXiv:1007.0785

TDHFB

1, Canonical-basis representation

2, Assumption for Pairing potential

3, We adopt a schematic pairing functional.

4, We choose the special gauge.

Cb-TDHFB

How to investigate the E1 mode ?

→ Linear response calculation of isovector dipole mode

Instantaneous external field $\hat{V}_{\rm ext}(t) \equiv -k\hat{F}\delta(t)$ → $k \ll 1$

then the equations can be automaticaly linearised with respect to V_{ext} and the density fluctuation.

Strength function $S(E;\hat{F})$

$$S(E;\hat{F}) = \sum_n \left|\langle n|\hat{F}|0\rangle\right|^2 \delta(E-E_n)$$

$$\simeq -\frac{1}{k\pi}\mathrm{Im}\int_0^{\mathrm{T}} dt\; e^{iEt/\hbar-\Gamma t}\langle\Psi(t)|\hat{F}|\Psi(t)\rangle$$

Initial state of Real-Time cal.

$$|\Psi(0_+)\rangle \equiv e^{i\hbar k\hat{F}}|\Psi(0)\rangle$$

Ground state of **HF** or **HF+BCS**

Strength function is calculated as
Fourier transformed time dependent expectation value of $\hat{F}$.

Calculation setup

Interaction : Skyrme force (SkM*)

Pairing strength : Smoothed Pairing strength $G \sim 0.6$ [MeV]

Even-even Nuclei : **$^{18\text{-}32}$Ne, $^{18\text{-}40}$Mg, $^{24\text{-}46}$Si, $^{28\text{-}50}$S, $^{32\text{-}58}$Ar, ^{144,154}Sm, ^{172}Yb** ($^{12\text{-}22}$C, $^{14\text{-}28}$O, $^{34\text{-}64}$Ca)

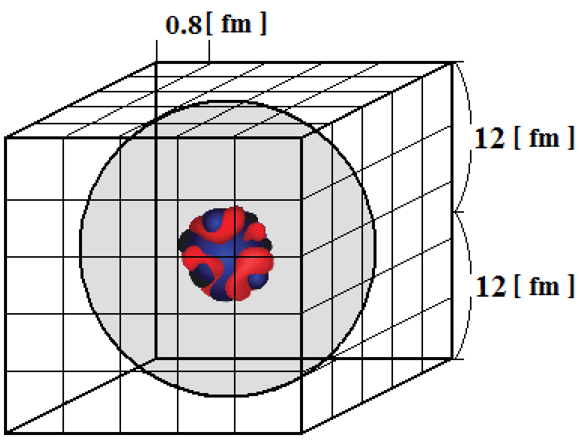
External field : Isovector Dipole

$$\hat{F}_i^{\mathrm{N}} = -(Ze/A)\hat{\boldsymbol{r}}_i \qquad \hat{F}_i^{\mathrm{P}} = (Ne/A)\hat{\boldsymbol{r}}_i$$

Cal. space (3D-Spherical box**):**

radius **12** [fm] mesh **0.8** [fm]

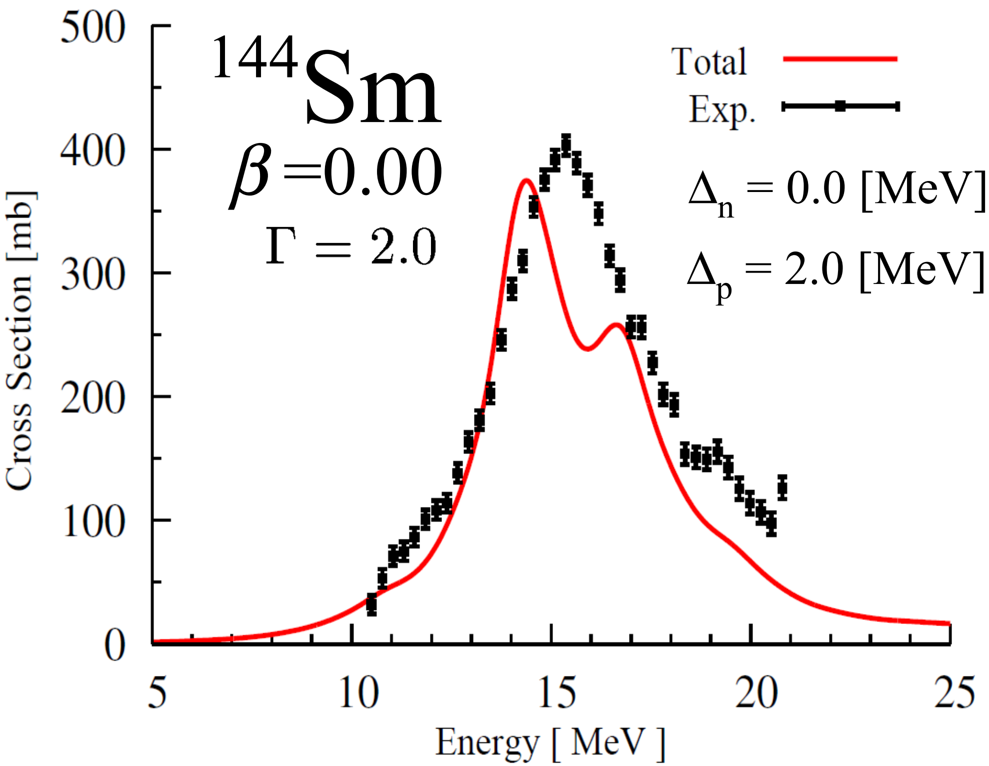
Pairing model space ⟺ **Energy cutoff**



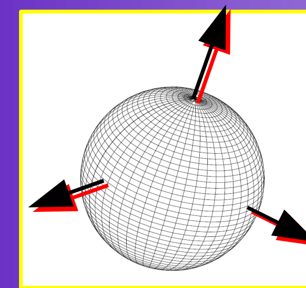
Comparison with HF+RPA calculation

How dose the **pairing correlation** work for the appearance of the E1 strength in **Low-energy** region ?

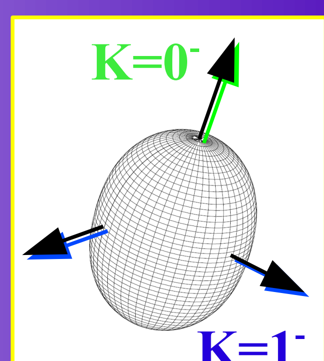
Example of E1 mode for deformed nuclei (Cross section)

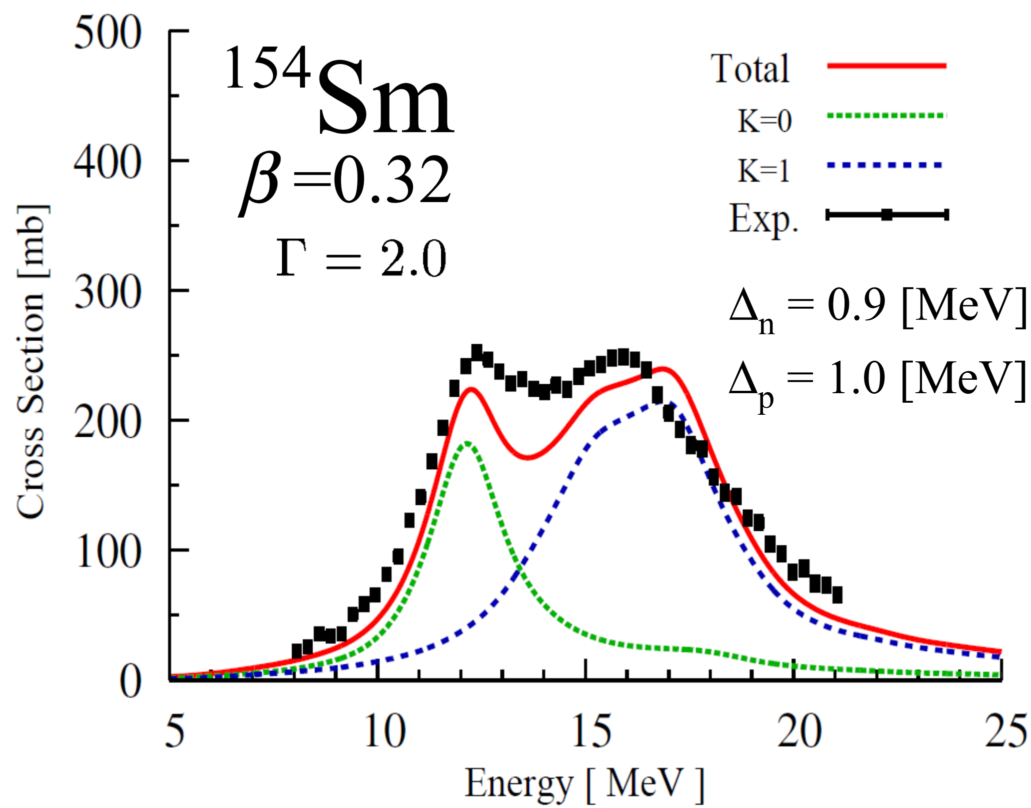


In Spherical case,
the Giant Dipole Resonance(GDR) will be a concentrated peak.



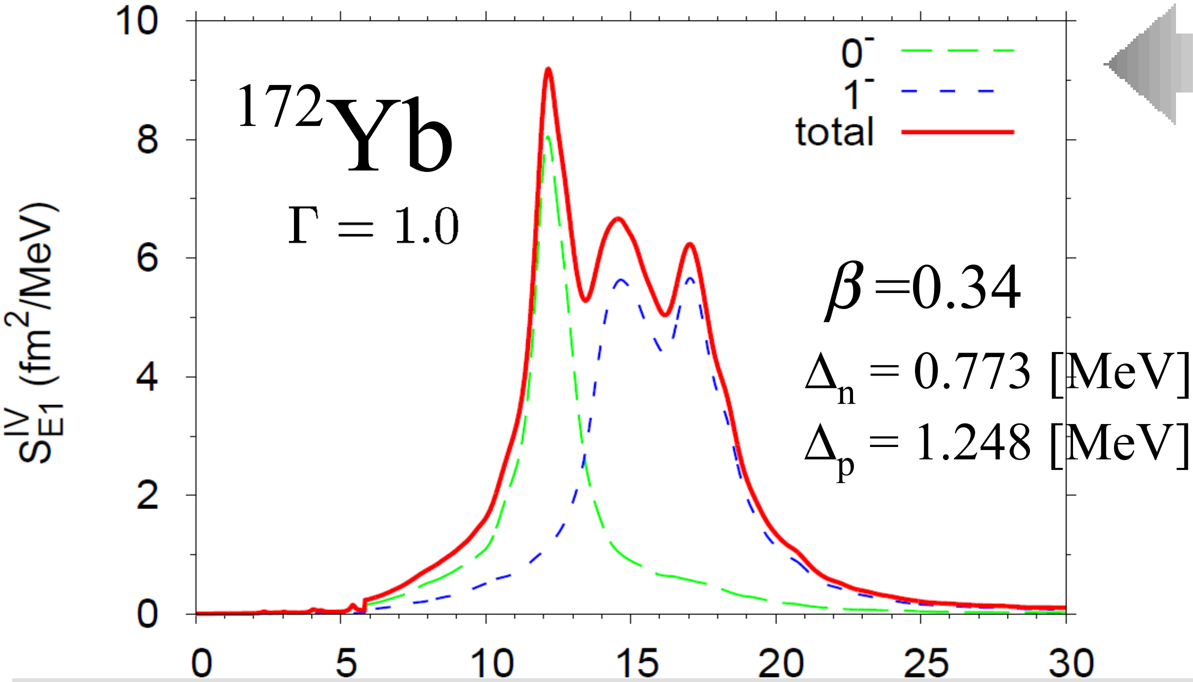
In Quadrupole deformed case,
the GDR have two components, K=0$^-$, 1$^-$





Experimental data from Nucl.Phys. A225, 171 (1974)

E1 strength functions ^{172}Yb (for computational cost)

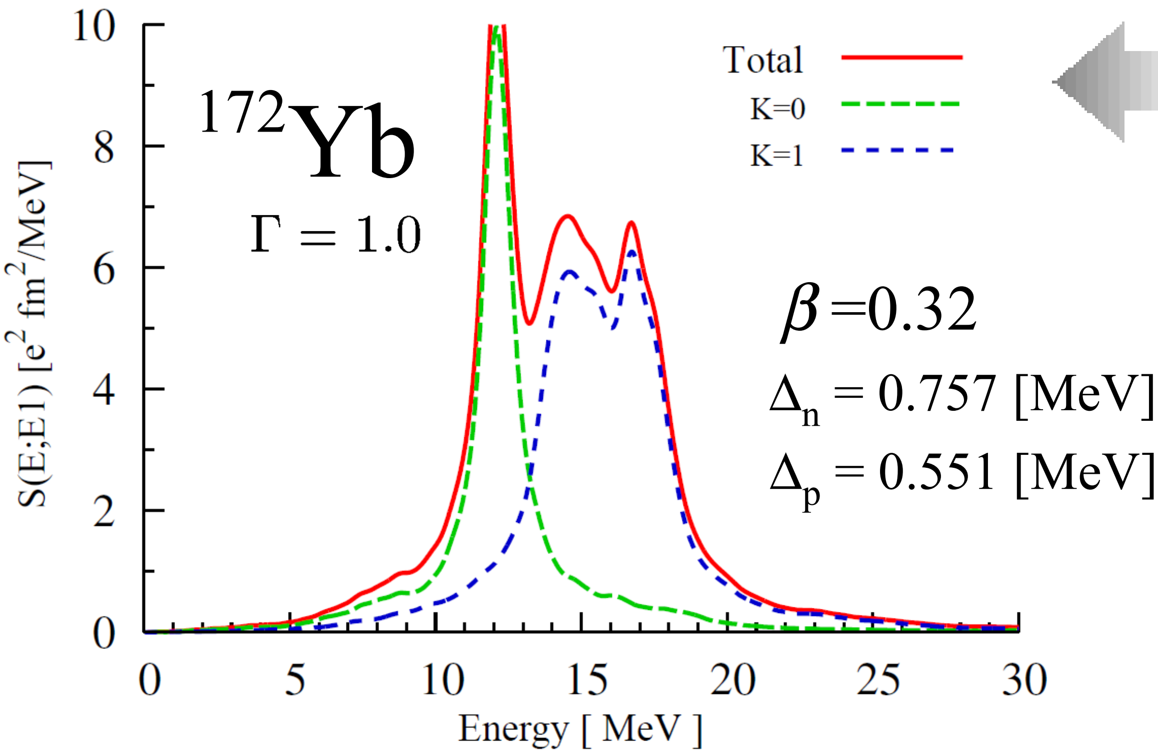


J. Terasaki and J. Engel
arXiv:1006.0010

Box Size : $\rho = z_{\pm}$=20[fm], b-spline (Cylindrical)

Single-quasiparticle space (g.s. HFB) :
5348 states for neutron,
4648 states for proton

Total time : **136,000 CPU hours**
(with **Kraken**; Super computer of ORNL)



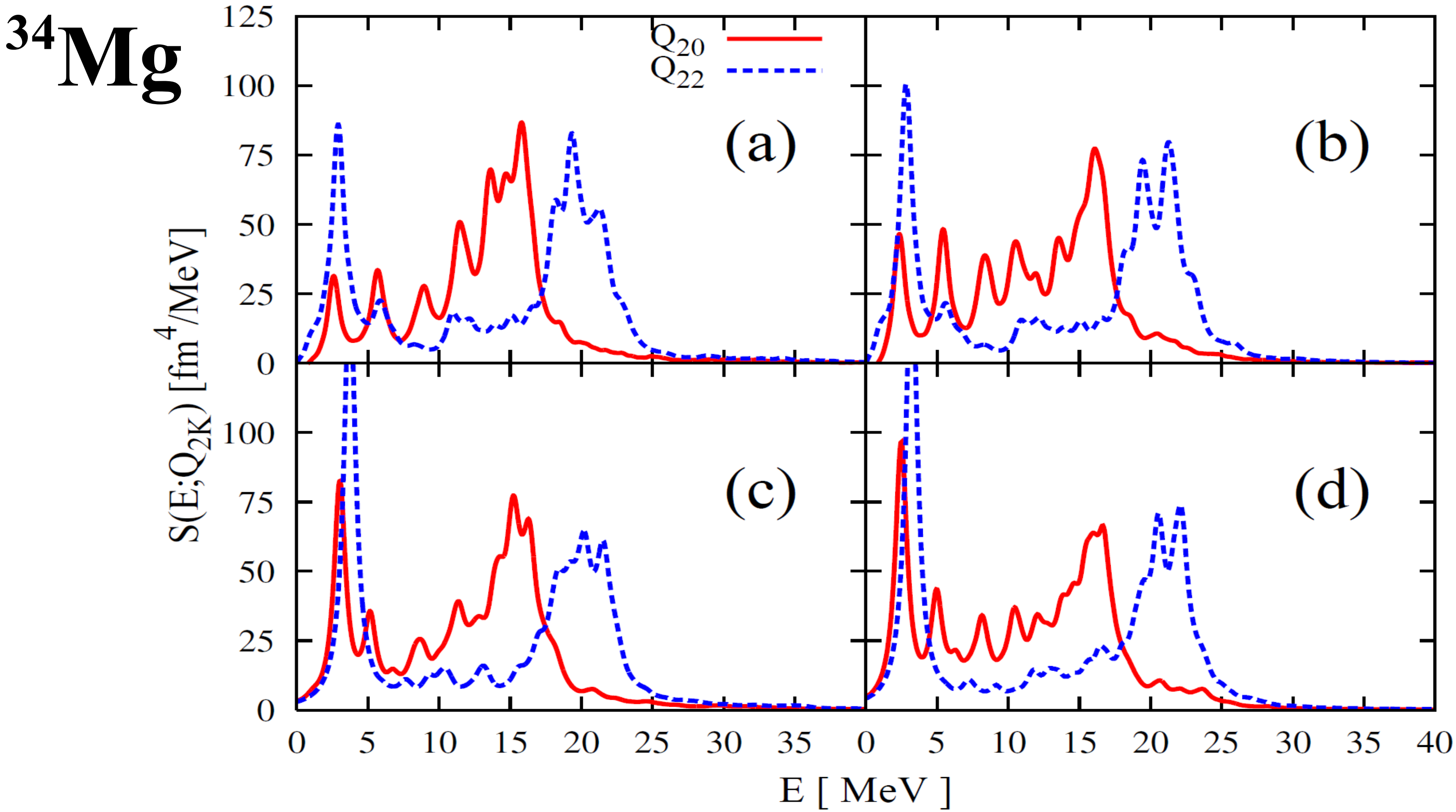
S. Ebata using Cb-TDHFB
(based on arXiv:1007.0785)

Box size : R=15[fm], mesh=1[fm] (3D-Spherical)

Canonical basis space (g.s. HF+BCS) :
146 states for neutron,
98 states for proton

Total time : **300 CPU hours**
(with **ONE CPU**; Intel Core i7 3.0 GHz)

Comparison with QRPA (IS Quadrupole) for ^{34}Mg



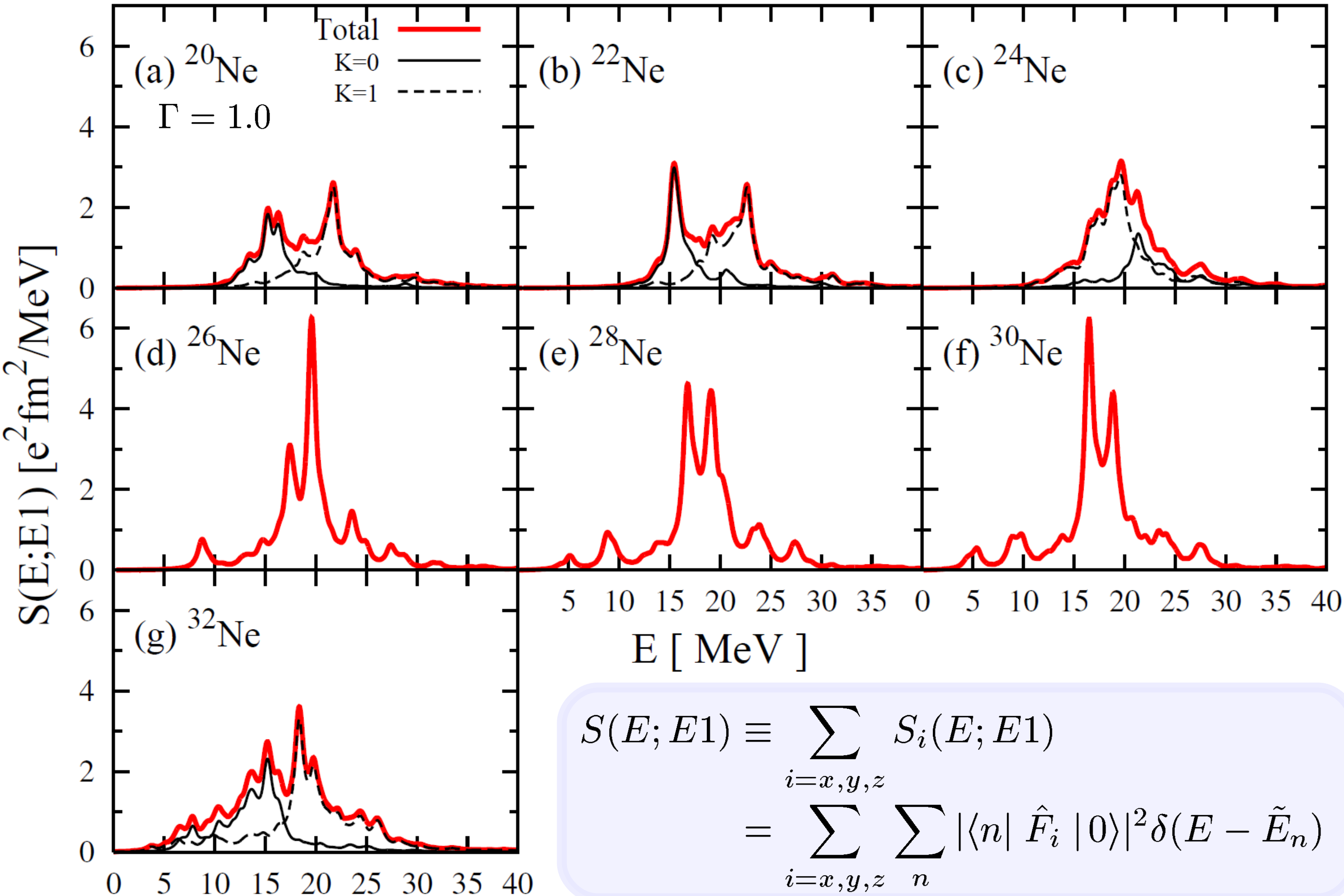
(a) Cb-TDHFB with fixed LS & Coulomb potentials

(b) Full Cb-TDHFB

(c) QRPA **without** residual LS & Coulomb interaction (delta-pairing)

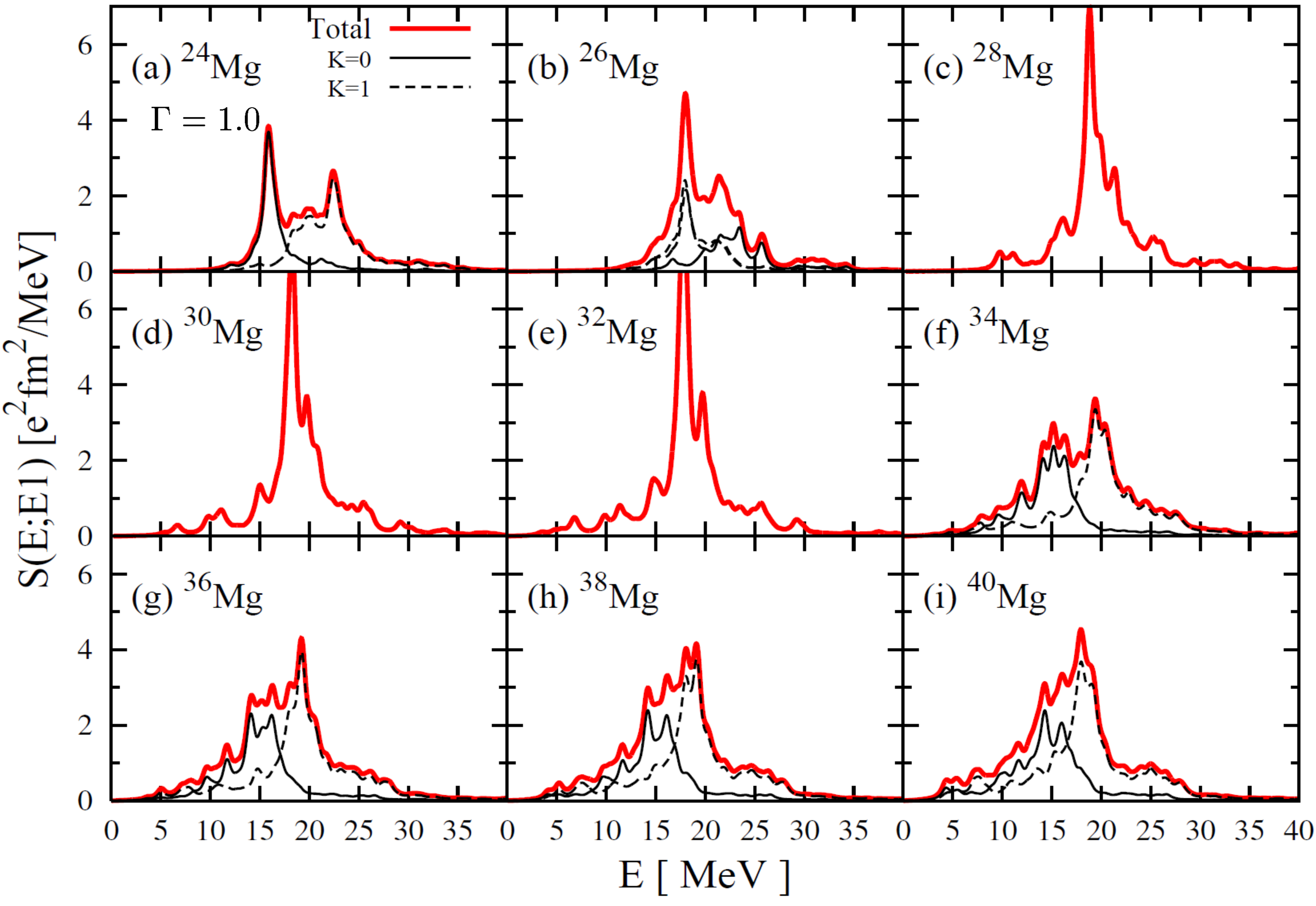
(d) QRPA (delta-pairing)
C. Losa *et al.*PRC81, 064307 (2010)

E1 strength functions for Ne isotopes



$$S(E;E1) \equiv \sum_{i=x,y,z} S_i(E;E1)$$
$$= \sum_{i=x,y,z} \sum_n |\langle n| \hat{F}_i |0\rangle|^2 \delta(E - \tilde{E}_n)$$

E1 strength functions for Mg isotopes

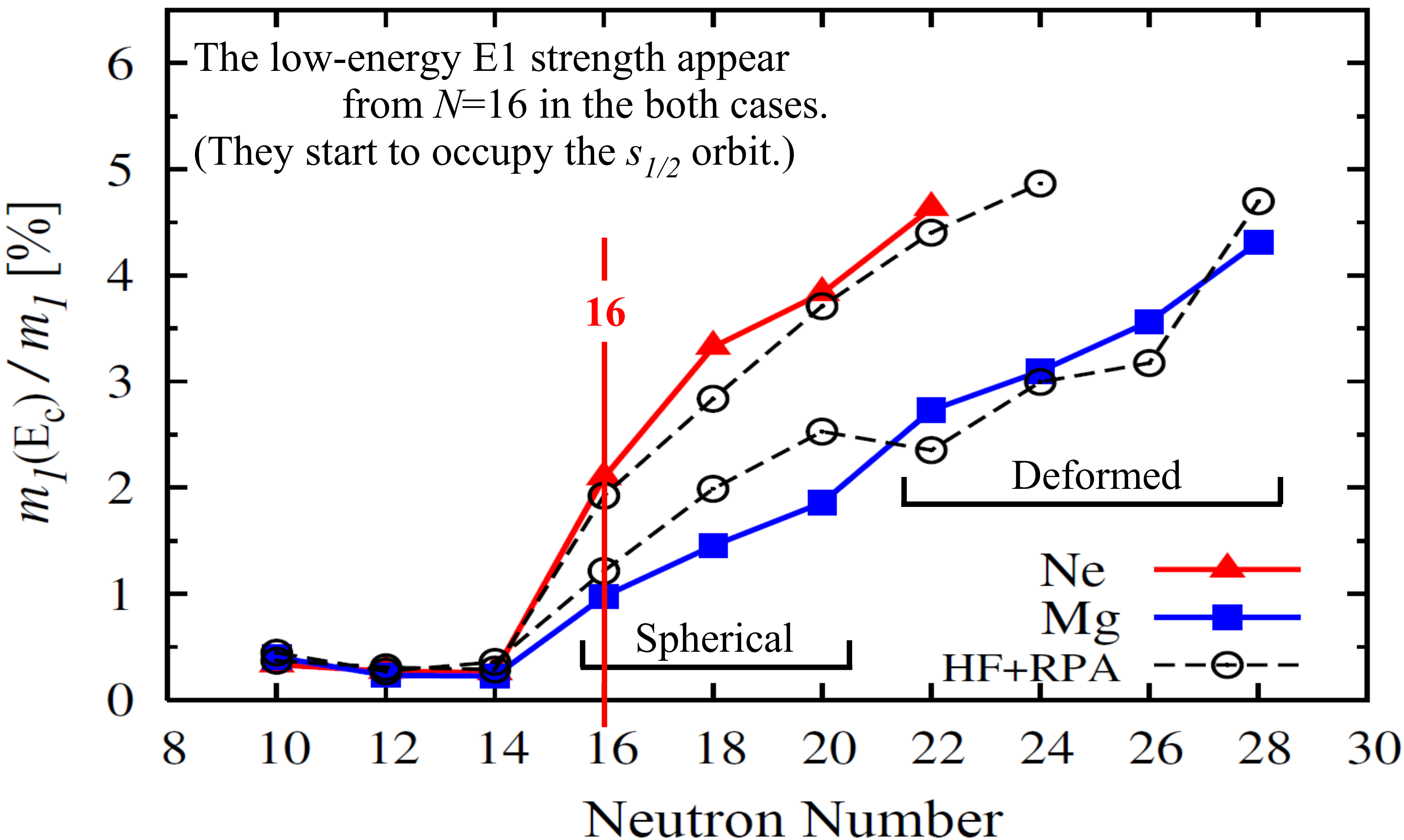


The ratio of the E1 strength in Low-energy region

$$\frac{m_1(E_c=10)}{m_1} \equiv \frac{\int_0^{10[\mathrm{MeV}]} E\ S(E;E1)dE}{\int E\ S(E;E1)dE}$$

Low-energy region < 10 [MeV]

For Ne, Mg isotopes



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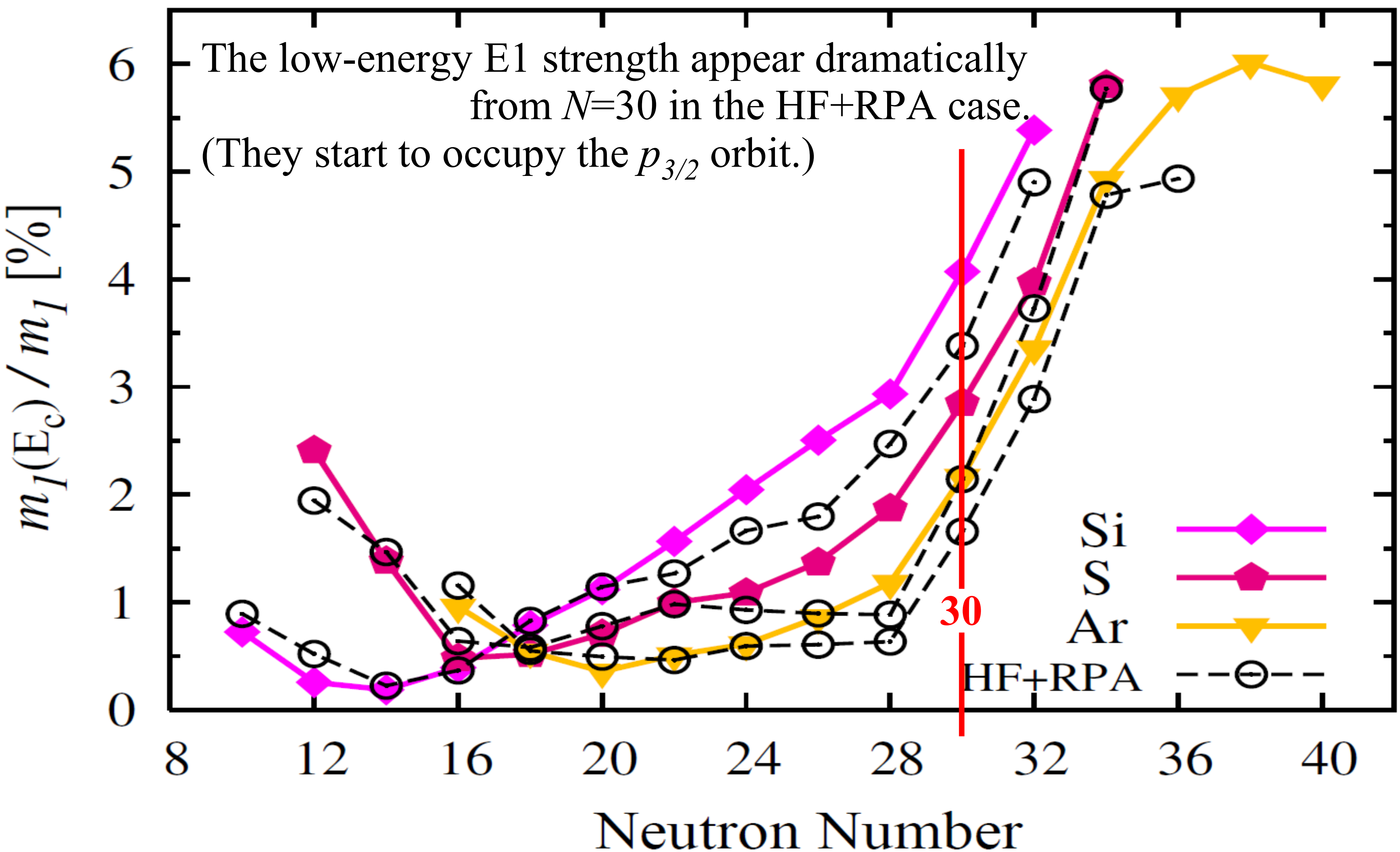
Low-energy region < 10 [MeV]

For Si, S, Ar isotopes



Summary

Derivation of **Cb-TDHFB** eqs. from TDHFB

Linear Response calculations (for C,O,Ne,Mg,Si,S,Ar,Ca,Sm,Yb) of Isovector Dipole mode **systematically** using **Cb-TDHFB**.

Comparison with **HF+RPA** results for **Ne, Mg, Si, S, Ar** isotopes.

- The low-energy E1 strength is sensitive to **low-momentum orbits** and **nuclear deformation**. The **Pairing** correlation should be include for this energy region.
- The appearance of the low-energy E1 strength of nearly magic number is smoothed by the **continuous occupation** of orbitals caused **Pairing** correlation.

Comparison with **deformed HFB+QRPA** results for **^{172}Yb, ^{34}Mg**.

- **Cb-TDHFB significantly** reduces the computational cost.
  For the E1 strength function of ^{172}**Yb**: **136,000 → 300** CPU hours
- The results of **Cb-TDHFB** well agree with other deformed QRPA cals. in isoscalar quadrupole modes, except for height of the lowest peak.
  (caused by using a schematic pairing functional ?)

Summary

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Linear Response calculations (for C,O,Ne,Mg,Si,S,Ar,Ca,Sm,Yb) of Isovector Dipole mode **systematically** using **Cb-TDHFB**.

Comparison with **HF+RPA** results for **Ne, Mg, Si, S, Ar** isotopes.

Comparison with **deformed HFB+QRPA** results for **^{172}Yb, ^{34}Mg**.

Future work

Application of **Cb-TDHFB** to systematic calculation with other modes(ISQ, ISO, IVM, etc.)

Application of **Cb-TDHFB** to **heavy-ion collision**

Thank you !!

Energy cutoff function $f(\varepsilon)$

$$\Delta_k(t) = \sum_{l>0} G_{kl}\kappa_l(t) \quad \Rightarrow \quad \Delta(t) = \bar{G}\sum_{l>0} f(\varepsilon_l)\kappa_l(t)$$

$$f(\varepsilon) = \left(1 + \exp\left[\frac{\varepsilon - \varepsilon_\tau^{\mathrm{c}}}{0.5}\right]\right)^{-1/2} \theta(e_\tau^{\mathrm{c}} - \varepsilon)$$

$$\varepsilon_\tau^{\mathrm{c}} = \lambda_{\mathrm{HF}} + 5.0\ [\mathrm{MeV}]$$

$$e_\tau^{\mathrm{c}} = \varepsilon_\tau^{\mathrm{c}} + 2.3\ [\mathrm{MeV}]$$

[N.Tajima *et al*. NPA**603**(1996)23**]**

Gap equation

$$\Delta = \frac{G}{2}\sum_{l>0} \frac{f^2(\bar{\varepsilon}_l)\Delta}{\sqrt{(\varepsilon_l - \lambda)^2 + f^2(\bar{\varepsilon}_l)\Delta^2}}$$

Particle number equation

$$N = \sum_{l>0}\left(1 - \frac{\varepsilon_l - \lambda}{\sqrt{(\varepsilon_l - \lambda)^2 + f^2(\bar{\varepsilon}_l)\Delta^2}}\right)$$

Smoothed Pairing $\bar{G}_\tau$

$$\bar{\Delta} = \frac{\bar{G}_\tau}{2}\bar{\Delta}\int_{-\infty}^{\infty} d\varepsilon \frac{f_\tau^2(\varepsilon)\bar{D}_\tau(\varepsilon)}{\sqrt{(\varepsilon - \bar{\lambda}_\tau)^2 + f_\tau^2(\varepsilon)\bar{\Delta}^2}} \qquad \bar{N}_\tau = \int_{-\infty}^{\infty} d\varepsilon \frac{(\varepsilon - \bar{\lambda}_\tau)^2\bar{D}_\tau(\varepsilon)}{\sqrt{(\varepsilon - \bar{\lambda}_\tau)^2 + f_\tau^2(\varepsilon)\bar{\Delta}^2}}$$

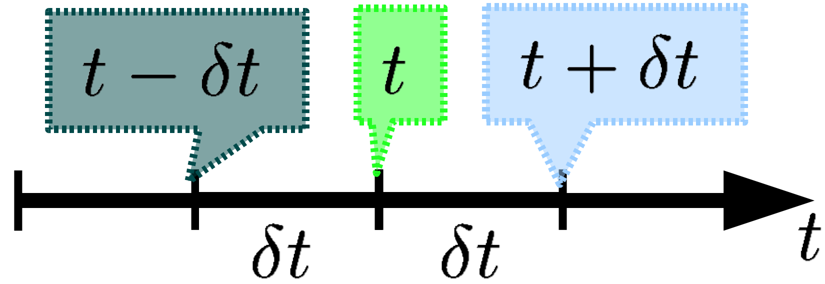
$$\bar{\Delta} = 12A^{-1/2} \qquad \bar{D}_\tau(\varepsilon) = \frac{1}{2\pi^2}\int dr \left(\frac{2m_\tau^*(r)}{\hbar^2}\right)^{3/2} (\varepsilon - V_\tau(r))^{1/2}\Theta(\varepsilon - V_\tau)$$

How to calculate time development ?

$$i\hbar\dot{\phi}_l(t) = \Big(\hat{h}(t) - \varepsilon_l(t)\Big)\phi_l(t)$$

$$i\hbar\dot{\rho}_l(t) = \kappa_l(t)\Delta^*(t) - \Delta(t)\kappa_l^*(t)$$

$$i\hbar\dot{\kappa}_l(t) = (\varepsilon_l(t) + \varepsilon_{\bar{l}}(t))\kappa_l(t) + \Delta(t)(2\rho_l(t) - 1)$$



δt ⇒

$$\phi_l(t+\delta t) = \phi_l(t-\delta t) - \frac{i}{\hbar}2\delta t\Big(\hat{h}(t) - \varepsilon_l(t)\Big)\phi_l(t)$$

$$\rho_l(t+\delta t) = \rho_l(t-\delta t) - \frac{i}{\hbar}2\delta t\Big(\kappa_l(t)\Delta^*(t) - \Delta(t)\kappa_l^*(t)\Big)$$

$$\kappa_l(t+\delta t) = \kappa_l(t-\delta t) - \frac{i}{\hbar}2\delta t\Big\{(\varepsilon_l(t) + \varepsilon_{\bar{l}}(t))\kappa_l(t) + \Delta(t)(2\rho_l(t) - 1)\Big\}$$

$$\rho_l, \kappa_l \;\Rightarrow v_l$$

$$\kappa_l = u_l v_l = v_l\sqrt{1 - |v_l|^2} = |v_l|\, e^{i\zeta}\sqrt{1 - |v_l|^2}$$

ζ **: phase of** v_l

⇒ $$v_l = |v_l|\frac{\kappa_l}{|\kappa_l|}$$

$$|v_l| \Leftarrow \rho_l$$