Density dependence of the nuclear symmetry energy estimated from neutron skin thickness in finite nuclei

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Phys. Rev. Lett. **102** 122502 (2009)
M. Warda, X. Viñas, X. Roca-Maza and M. Centelles,
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Phys. Rev. **C81** 054309 (2010)

Why is important the nuclear symmetry energy ?

The nuclear symmetry energy is a fundamental quantity in Nuclear Physics and Astrophysics because it governs, at the same time, important properties of very small entities like the atomic nucleus ($R \sim 10^{-15}$ m) and very large objects as neutron stars ($R \sim 10^{4}$ m)

- Nuclear Physics: Neutron skin thickness in finite nuclei, stable nuclei, Heavy-Ion collisions, Giant Resonances...
- Astrophysics: Supernova explosion, cooling of protoneutron stars, Mass-Radius relations in neutron stars, Composition of the crust of neutron stars...

Equation of State in asymmetric matter

$$e(\rho,\delta) = e(\rho,0) + c_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) \qquad \left(\delta = \frac{\rho_n - \rho_p}{\rho}\right)$$

Around the saturation density we can write

$$e(\rho,0) \simeq a_v + \frac{1}{2}K_v\epsilon^2 \quad \textit{and} \quad c_{sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2 \quad \left(\epsilon = \frac{\rho_0 - \rho}{3\rho_0}\right)$$

$$\rho_0 \approx 0.16 fm^{-3}, \quad a_v \approx -16 MeV, \quad K_v \approx 230 MeV, \quad J \approx 32 MeV$$

However, the values of

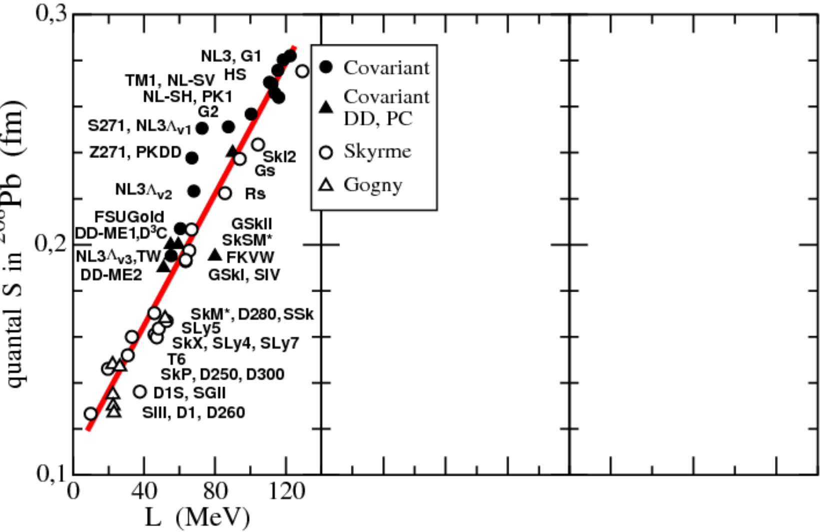
$$L = 3\rho\partial c_{sym}(\rho)/\partial\rho|_{\rho_0} \qquad \textit{and} \qquad K_{sym} = 9\rho^2\partial^2 c_{sym}(\rho)/\partial\rho^2|_{\rho_0}$$

which govern the density dependence of c_{sym} near ρ_0 are less certain and predictions vary largely among nuclear theories.

Experimental constraints

- Recent reseach in heavy-ion collisions at intermediate energy is consistent with $c_{sym}(\rho) = c_{sym}(\rho_0).(\rho/\rho_0)^\gamma$ at $\rho < \rho_0$.
- Isospin difussion $\gamma = 0.7$–1.05 ($L = 88 \pm 25$ MeV).
- Isoscaling $\gamma = 0.69$ ($L \sim 65$ MeV)
- Inferred from nucleon emision ratios $\gamma = 0.5$($L \sim 55$ MeV).
- The GDR of ^{208}Pb analyzed with Skyrme forces suggests a constraint $c_{sym}(0.1\,\mathrm{fm}^{-3})$=23.3–24.9 MeV ($\gamma \sim 0.5$–$0.65$).
- The study of the PDR in ^{68}Ni and ^{132}Sn predicts L=49-80 MeV.
- The Thomas-Fermi model of Myers and Swiatecki fitted very precisely to binding energies of 1654 nuclei predicts an EOS that yields $\gamma = 0.51$
- NEUTRON SKIN THICKNESS ?

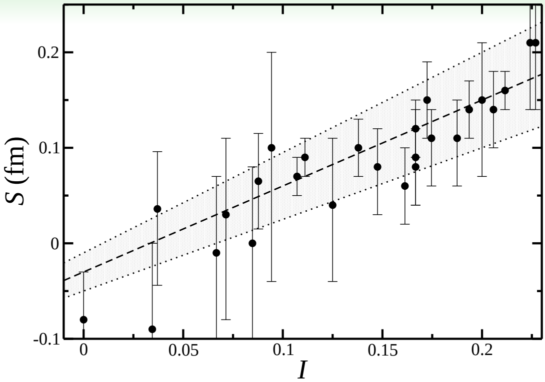
Neutron skin thickness



What is experimentally know about neutron skin thickness in nuclei ?

The neutron skin thickness is defined as $S = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$, where $\langle r_n^2 \rangle^{1/2}$ and $\langle r_p^2 \rangle^{1/2}$ are the rms radii of the neutron and proton distributions respectively.

- $\langle r_p^2 \rangle^{1/2}$ is known very accurately from elastic electron scattering measurements.
- $\langle r_n^2 \rangle^{1/2}$ has been obtained with hadronic probes such as:
  (a) Proton-nucleus elastic scattering.
  (b) Inelastic scattering excitation of the giant dipole and spin-dipole resonances.
  (c) Antiprotonic atoms: Data from antiprotonic X rays and radiochemical analysis of the yields after the antiproton annihilation.



$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02)$ fm

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

CAN S OF 26 STABLE NUCLEI, FROM ^{40}Ca TO ^{238}U, ESTIMATED USING ANTIPROTONIC ATOMS DATA BE CONSTRAINED BY THE SLOPE OF c_{sym} ?

Symmetry energy and neutron skin thickness in the Liquid Drop Model

- Symmetry Energy

$$a_{sym}(A) = \frac{J}{1 + x_A}, \quad x_A = \frac{9J}{4Q} A^{-1/3}$$

$$E_{sym}(A) = a_{sym}(A)(I + x_A I_C)^2 A$$

where

$$I = (N - Z)/A, \quad I_{\mathrm{C}} = e^2 Z/(20JR), \quad R = r_0 A^{1/3}$$

.

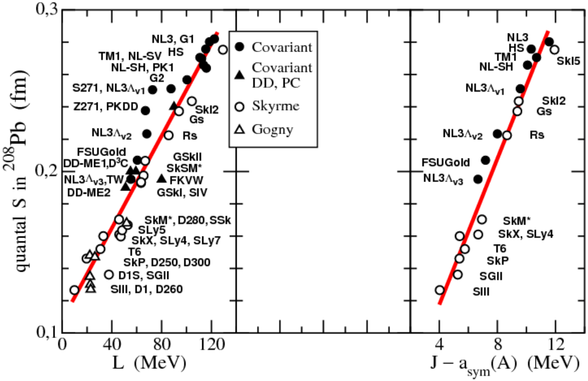
- Neutron skin thickness

$$S = \sqrt{3/5} \left[t - e^2 Z/(70J) + \frac{5}{2R}(b_n^2 - b_P^2) \right]$$

where

$$t = \frac{3r_0}{2} \frac{J/Q}{1 + x_A} (I - I_{\mathrm{C}})$$

Neutron skin thickness



$$t = \frac{2r_0}{3J}\,[J - a_{sym}(A)]\,A^{1/3}\,(I - I_{\rm C})$$

Table: Value of $a_{sym}(A)$ and density ρ that fulfils $c_{sym}(\rho) = a_{sym}(A)$ for $A = 208, 116$ and 40 in MF models. J and a_{sym} are in MeV and ρ is in fm^{-3}.

Model	J	$A = 208$		$A = 116$		$A = 40$	
		a_{sym}	ρ	a_{sym}	ρ	a_{sym}	ρ
NL3	37.4	25.8	0.103	24.2	0.096	21.1	0.083
NL-SH	36.1	25.8	0.105	24.6	0.099	21.3	0.086
FSUGold	32.6	25.4	0.098	24.2	0.090	21.9	0.075
TF-MS	32.6	24.2	0.093	22.9	0.085	20.3	0.068
SLy4	32.0	25.3	0.100	24.2	0.091	22.0	0.075
SkX	31.1	25.7	0.102	24.8	0.096	22.8	0.082
SkM*	30.0	23.2	0.101	22.0	0.093	19.9	0.078
SIII	28.2	24.1	0.093	23.4	0.088	21.8	0.077
SGII	26.8	21.6	0.104	20.7	0.096	18.9	0.082

The $c_{sym}(\rho)$-$a_{sym}(A)$ correlation

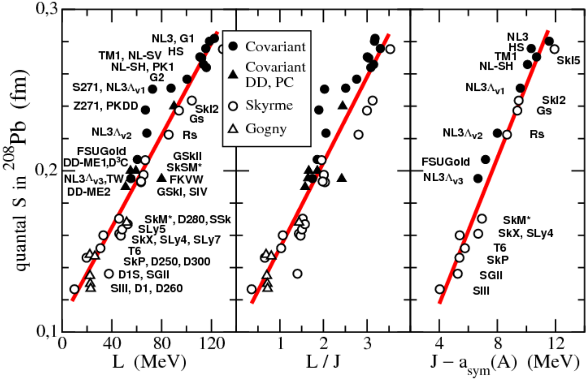
- There is a genuine relation between the symmetry energy coefficients of the EOS and of nuclei: $c_{sym}(\rho)$ equals $a_{sym}(A)$ of heavy nuclei like ^{208}Pb at a density $\rho = 0.1 \pm 0.01\,\mathrm{fm}^{-3}$.
- A similar situation occurs down to medium mass numbers, at lower densities.
- We find that this density can be very well simulated by

$$\rho \approx \rho_A = \rho_0 - \rho_0/(1 + cA^{1/3}) \,,$$

  where c is fixed by the condition $\rho_{208} = 0.1\,\mathrm{fm}^{-3}$.
- Using the equality $c_{sym}(\rho) = a_{sym}(A)$ and the LDM , the neutron skin thickness can be finally written as:

$$t = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1 - \epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I - I_{\mathrm{C}})$$

Neutron skin thickness



$$t = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1 - \epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I - I_{\mathrm{C}})$$

Fitting procedure and results

- We optimize

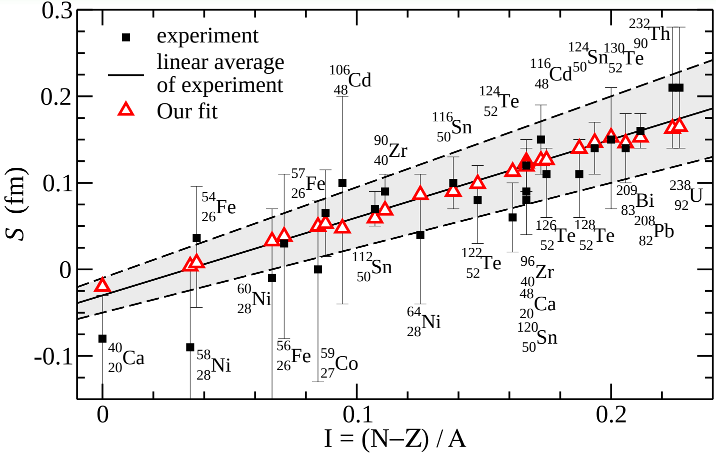
$$t = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1 - \epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I - I_{\mathrm{C}})$$

  using

$$c_{sym} = 31.6(\frac{\rho}{\rho_0})^{\gamma} MeV, \quad \epsilon = \frac{1}{3(1 + cA^{1/3})}, \quad \rho_0 = 0.16 fm^{-3}$$

  and taking as experimental baseline the neutron skins measured in 26 antiprotonic atoms.

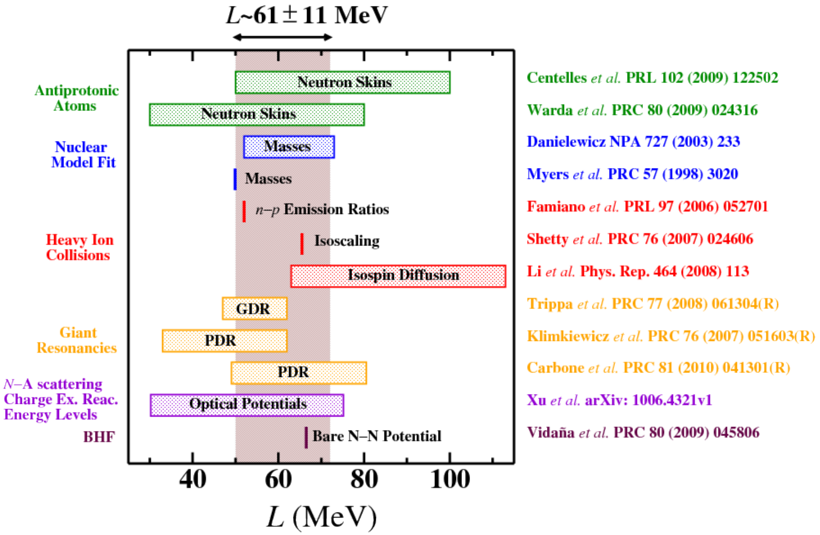
- We predict $(b_n \approx b_p)$: $L = 75 \pm 25$ MeV



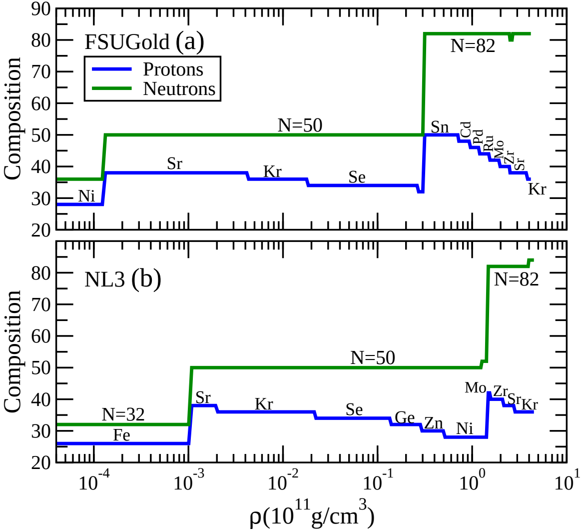
$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02)$ fm

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

Constraints on the slope of the symmetry energy



Structure and composition of a neutron star crust



X. Roca-Maza and J. Piekarewicz Phys. Rev. C **78** 025807 (2008)

The larger the slope of the symmetry energy, the larger the neutron skin of ^{208}Pb, the more exotic the composition of the outer crust

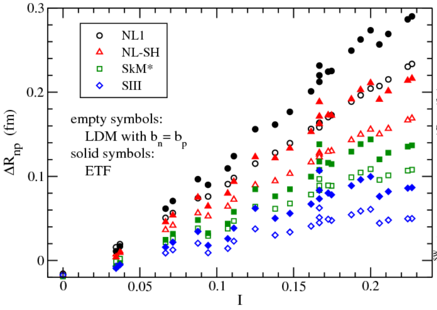
Summary and Conclusions

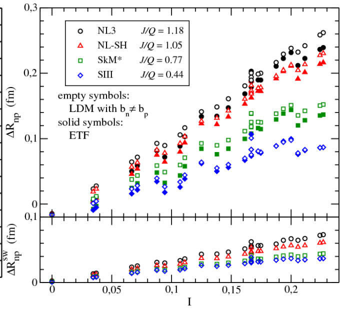
- We have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation.
- We take advantage of this relation to explore constraints on $c_{sym}(\rho)$ from neutron skins measured in antiprotonic atoms. These constraints points towards a soft symmetry energy.
- We discuss the L values constrained by neutron skins in comparison with most recent observations from reactions and giant resonances.
- We learn that in spite of present error bars in the data of antiprotonic atoms, the size of the final uncertainties in L is comparable to the other analyses.

Thank you for your attention

Extra material

Influence of the surface width ($b_n \neq b_p$)

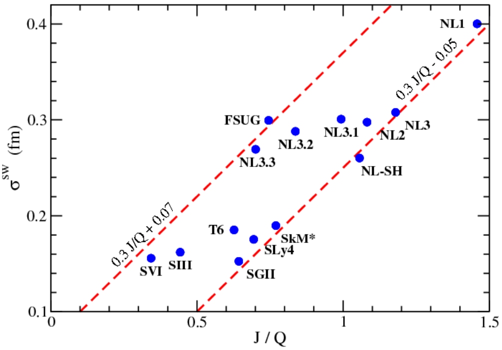




$$S = \sqrt{3/5}\left[t - e^2 Z/(70J) + \frac{5}{2R}(b_n^2 - b_P^2)\right]$$

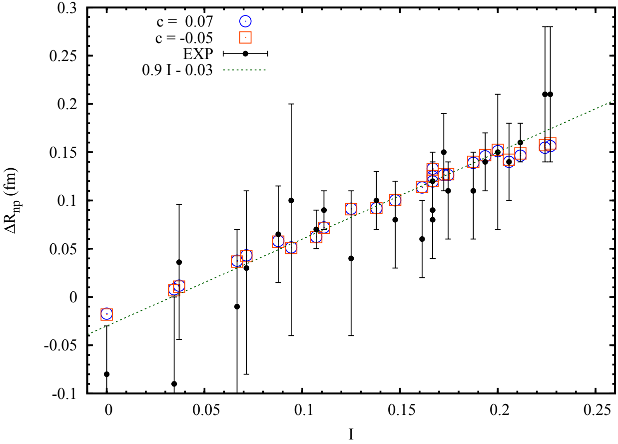
b_n and b_p are obtained semiclassically at ETF level

Surface contribution to the neutron skin thickness



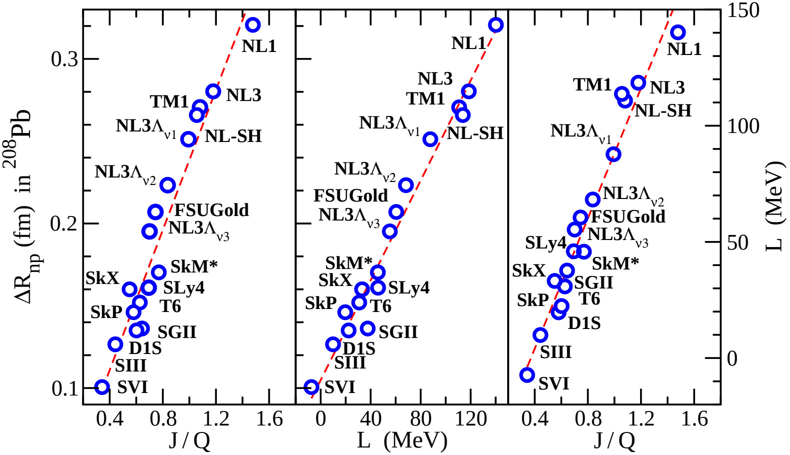
$$\sqrt{\frac{3}{5}}\frac{5}{2R}(b_n^2 - b_p^2) = \sigma^{sw} I = (0.3\frac{J}{Q} + c) I$$

Fit and results



$$\frac{J}{Q} = 0.6 - 0.9$$

Neutron skin thickness



$L = 30 - 80$ MeV

Some technical details

- The surface stiffness coeficient Q and the surface widths b_n and b_p are obtained from self-consistent calculations of the neutron and proton density profiles in asymmetric semi-infinite nuclear matter.
- To this end one has to minimize the total energy per unit area with the constraint of conservation of the number of protons and neutrons with respect to arbitrary variations of the densities.

$$\frac{E_{\rm const}}{S} = \int_{-\infty}^{\infty} \left[\varepsilon(z) - \mu_n \rho_n(z) - \mu_p \rho_p(z)\right] dz,$$

  where $\varepsilon(z)$ is the nuclear energy density functional.
- In the non-relativistic framework the densities ρ_n and ρ_p obey the coupled local Euler-Lagrange equations:

$$\frac{\delta\varepsilon(z)}{\delta\rho_n} - \mu_n = 0, \qquad \frac{\delta\varepsilon(z)}{\delta\rho_p} - \mu_p = 0.$$

  The relative neutron excess $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is a function of the z-coordinate. When $z \to -\infty$, the densities ρ_n and ρ_p approach the values of asymmetric uniform nuclear matter in equilibrium with a bulk neutron excess δ_0.

- From the calculated density profiles one computes:

$$z_{oq} = \frac{\int_{-\infty}^{\infty} z\rho_q'(z)dz}{\int_{-\infty}^{\infty} \rho_q'(z)dz},$$

$$b_q^2 = \frac{\int_{-\infty}^{\infty}(z - z_{0q})^2\rho_q'(z)dz}{\int_{-\infty}^{\infty} \rho_q'(z)dz}.$$

- From the relation

$$t = z_{0n} - z_{0p} = \frac{3r_0}{2}\frac{J}{Q}\delta_0,$$

one can evaluate Q from the slope of t at $\delta_0 = 0$.

- The distance t and the surface widths b_n and b_p in finite nuclei with neutron excess $I = (N - Z)/A$ are obtained using δ_0 given by:

$$\delta_0 = \frac{I + \frac{3}{8}\frac{c_1}{Q}\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}}.$$