Neutrino Masses and the See-saw Mechanism

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Introduction

- Standard Model (SM): three ν_L
- Neutrinos oscillate $\Longrightarrow$

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- Neutrinos oscillate $\Longrightarrow$ are massive
- Evidence for Physics beyond the Standard Model

Outline

Neutrino Masses

- quarks and charged leptons Dirac mass
- both left- and right-handed components required
- SM: there are only left-handed neutrinos
- But ν are massive

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- both left- and right-handed components required
- SM: there are only left-handed neutrinos
- But ν are massive
- $\Longrightarrow$ two possibilites: Dirac mass or Majorana mass

Dirac Neutrino Masses

Extension of the SM mechanism:

$$L'_L \equiv \begin{pmatrix} L'_{eL} \\ L'_{\mu L} \\ L'_{\tau L} \end{pmatrix} , \quad L'_{\alpha L} \equiv \begin{pmatrix} \nu'_{\alpha L} \\ l'_{\alpha L} \end{pmatrix} , \quad \alpha = e, \mu, \tau$$

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$$l'_R = \begin{pmatrix} l'_{eR} \\ l'_{\mu R} \\ l'_{\tau R} \end{pmatrix}$$

Introduce right-handed neutrinos (restrict to 3 ν_R)

$$\nu'_R = \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

Mass term from

$$\mathcal{L}_{Yuk,lept} = -\overline{L'_L}\, Y'^{l}\, \Phi\, l'_R - \overline{L'_L}\, Y'^{\nu}\, \tilde{\Phi}\, \nu'_R + h.c.$$

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In the unitary gauge

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \quad \tilde{\Phi} = i\tau_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix}$$

Hence

$$\mathcal{L}_{Yuk,lept} = -\left(\frac{v+h}{\sqrt{2}}\right) \left[\overline{l'_L}\, Y'^{l}\, l'_R + \overline{\nu'_L}\, Y'^{\nu}\, \nu'_R\right] + h.c.$$

Matrices Y'^{l} and Y'^{ν} diagonalized by four unitary matrices:

$$l'_L = V^l_L \, l_L \,, \quad l'_R = V^l_R \, l_R \,,$$

$$\nu'_L = V^\nu_L \, n_L \,, \quad \nu'_R = V^\nu_R \, n_R$$

Eigenvalues:

$$V^{l\dagger}_L \, Y'^{l} \, V^l_R = Y^l \,, \quad Y^l_{\alpha\beta} = y^l_\alpha \, \delta_{\alpha\beta} \,, \quad (\alpha, \beta = e, \mu, \tau)$$

$$V^{\nu\dagger}_L \, Y'^{\nu} \, V^\nu_R = Y^\nu \,, \quad Y^\nu_{ij} = y^\nu_i \, \delta_{ij} \,, \quad (i, j = 1, 2, 3)$$

real and positive y^l_α and y^ν_i (▸ Mixing)

Therefore

$$\mathcal{L}_{Yuk,lept} \;=\; -\left(\frac{v+h}{\sqrt{2}}\right)\left[\overline{l_L}\,Y^l\,l_R + \overline{n_L}\,Y^\nu\,n_R\right] + h.c.$$

mass eigenstates l_L, l_R,

$$n_{L,R} = \begin{pmatrix} \nu_{1L,R} \\ \nu_{2L,R} \\ \nu_{3L,R} \end{pmatrix}$$

charged lepton and neutrino masses

$$m_\alpha = \frac{y^l_\alpha v}{\sqrt{2}}\,, \quad m_j = \frac{y^\nu_j v}{\sqrt{2}}$$

This approach is not completely satisfactory:

- Lepton masses $m_\alpha = \frac{y^l_\alpha v}{\sqrt{2}}$, $m_j = \frac{y^\nu_j v}{\sqrt{2}}$ are proportional to v
- Why $m_j \ll m_\alpha$ $(y^l_j \ll y^\nu_\alpha)$?

Majorana Neutrino Masses

- The SM conserves B, L (L_e, L_μ, L_τ)
- Neutrinos and antineutrinos are electrically neutral
- $\Longrightarrow$ can be Majorana particles $\nu = \nu^c \equiv \mathcal{C}\overline{\nu}^T$
- no need for right-handed components

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- The SM conserves B, L (L_e, L_μ, L_τ)
- Neutrinos and antineutrinos are electrically neutral
- $\Longrightarrow$ can be Majorana particles $\nu = \nu^c \equiv \mathcal{C}\overline{\nu}^T$
- no need for right-handed components
- but lepton number violation:

$$L(\nu) = +1\,, \quad L(\nu^c) = -1$$

  and we require $\nu = \nu^c$ ($\Delta L = 2$)
- source of CP violation

- A neutrino Majorana mass term

$$\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$$

  has hypercharge $Y = -2$

- Gauge invariance (and renormalizability) $\Longrightarrow$ Higgs triplet with $Y = +2$

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- SM is an effective low-energy theory
- $\Longrightarrow$ non-renormalizable terms which respect the SM gauge symmetry are allowed
- expansion in operators with dimension greater than four

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{g_5}{\mathcal{M}}\mathcal{O}_5 + \frac{g_6}{\mathcal{M}^2}\mathcal{O}_6 + \ldots$$

- The only operator with dimension 5 is a Majorana neutrino mass term

$$\begin{aligned} \mathcal{O}_5 &= \left(L_L^T \tau_2 \Phi\right) \mathcal{C}^\dagger \left(\Phi^T \tau_2 L_L\right) + h.c. = \\ &= \frac{1}{2} \left(L_L^T \mathcal{C}^\dagger \tau_2 \vec{\tau} L_L\right) \cdot \left(\Phi^T \tau_2 \vec{\tau} \Phi\right) + h.c. \end{aligned}$$

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- In the unitary gauge

$$\mathcal{L}_5 = \frac{1}{2} \frac{g_5 v^2}{\mathcal{M}} \nu_L^T \mathcal{C}^\dagger \nu_L + h.c. + \ldots$$

- Masses

$$m = \frac{g_5 v^2}{\mathcal{M}}$$

If $\mathcal{M} \sim M_{GUT} \sim 10^{15}\, GeV$, $g_5 \sim 1$, then $(v \sim 10^2\, GeV)$

$$m_{\nu_L} \sim 10^{-2}\, eV$$

For three generations of Majorana neutrinos

$$\nu'_L = \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$$

Majorana mass term

$$\mathcal{L}^{Maj}_{mass} = \frac{1}{2}\,\nu'^T_L \mathcal{C}^\dagger M^L \nu'_L + h.c.$$

where M^L is a 3×3 complex symmetric matrix in flavour space

Diagonalization with a unitary matrix V_L^ν:

$$\nu'_L = V_L^\nu \, n_L \qquad n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$(V_L^\nu)^T \; M^L \, V_L^\nu = M \,, \quad M_{jk} = m_j \, \delta_{jk} \quad (j, k = 1, 2, 3)$$

with real and positive eigenvalues m_j

$$\begin{aligned} \mathcal{L}_{mass}^{Maj} &= \frac{1}{2} \, n_L^T \, \mathcal{C}^\dagger \, M \, n_L + h.c. = \\ &= \frac{1}{2} \sum_{k=1}^{3} m_k \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + h.c. \end{aligned}$$

Dirac-Majorana Neutrino Masses for One Generation

The most general mass term for one generation is

$$\mathcal{L}_{mass}^{Dir+Maj} = \mathcal{L}_{mass}^{Dir} + \mathcal{L}_{mass}^{Maj,L} + \mathcal{L}_{mass}^{Maj,R}$$

where

$$\mathcal{L}_{mass}^{Dir} = -m_D \overline{\nu_R}\, \nu_L + h.c.$$

$$\mathcal{L}_{mass}^{Maj,L} = \frac{1}{2}\, m_L\, \nu_L^T\, \mathcal{C}^\dagger\, \nu_L + h.c.$$

$$\mathcal{L}_{mass}^{Maj,R} = \frac{1}{2}\, m_R\, \nu_R^T\, \mathcal{C}^\dagger\, \nu_R + h.c.$$

Define

$$N_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\overline{\nu_R}^T \end{pmatrix}$$

so that

$$\mathcal{L}_{mass}^{Dir+Maj} = \frac{1}{2} N_L^T \, \mathcal{C}^\dagger \, M \, N_L + h.c.$$

where

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

Diagonalization:

$$n_L = U^\dagger N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}, \quad U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

with real masses $m_{1,2} \geq 0$

$$\mathcal{L}_{mass}^{Dir+Maj} = \frac{1}{2} \sum_{k=1,2} m_k \, \nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} + h.c = -\frac{1}{2} \sum_{k=1,2} m_k \, \overline{\nu_k} \nu_k$$

$$\nu_k = \nu_{kL} + (\nu_{kL})^c \implies \nu_k = (\nu_k)^c$$

Assume that $M = M^*$ is real (CP is conserved), with positive m_R, m_D

Then M can be diagonalized by $U = O\,\rho$

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$$

where $\rho_k^2 = \pm 1$, $m_k^2 = \rho_k^2\, m_k'^2$ such that the physical masses are positive

$$O^T\,M\,O = \begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix} \qquad \tan 2\theta = \frac{2\,m_D}{m_R - m_L}$$

$$m_{2,1}' = \frac{1}{2}\left((m_L + m_R) \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right)$$

- if $m_L\, m_R < m_D^2$, $m_1' < 0$ and $\rho_1^2 = -1$

$$m_1 = \frac{1}{2}\left(\sqrt{(m_L - m_R)^2 + 4m_D^2} - (m_L + m_R)\right)$$

$$m_2 = \frac{1}{2}\left((m_L + m_R) + \sqrt{(m_L - m_R)^2 + 4m_D^2}\right)$$

- if $m_L\, m_R \geq m_D^2$, $m_1' \geq 0$ and $\rho_1^2 = +1$

$$m_1 = \frac{1}{2}\left((m_L + m_R) - \sqrt{(m_L - m_R)^2 + 4m_D^2}\right)$$

$$m_2 = \frac{1}{2}\left((m_L + m_R) + \sqrt{(m_L - m_R)^2 + 4m_D^2}\right)$$

In each case, oscillations between active ν_L and sterile $(\nu_R)^c$ are allowed and

$$P_{\nu_L \to (\nu_R)^c}(L, E) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

where

$$\Delta m^2 = m_2^2 - m_1^2 = (m_L + m_R)\sqrt{(m_L - m_R)^2 + 4\, m_D^2}$$

Interesting limits

- Maximal mixing $m_L = m_R$

$$m_1 = |m_L - m_D| \ , \quad m_2 = m_L + m_D$$

$$\Delta m^2 = 4 m_L m_D$$

$$\theta = \frac{\pi}{4}$$

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$$\theta = \frac{\pi}{4}$$

- Dirac limit $m_L = m_R = 0$

$$m_1 = m_2 = m_D$$

$$\Delta m^2 = 0$$

$$\theta = \frac{\pi}{4}$$

- Pseudo-Dirac limit $|m_L|, m_R \ll m_D$

$$m'_{2,1} = m_D \left[\frac{(m_L + m_R)}{2m_D} \pm \sqrt{1 + \frac{1}{4}\left(\frac{m_L - m_R}{m_D}\right)^2} \right] =$$

$$= m_D \left[\frac{(m_L + m_R)}{2m_D} \pm \left[1 + O\left(\frac{m_L - m_R}{m_D}\right)^2\right]\right]$$

$$m_1 = m_D - \frac{m_L + m_R}{2} + \ldots, \quad m_2 = m_D + \frac{m_L + m_R}{2} + \ldots$$

then

$$\Delta m^2 = 2m_D\,(m_L + m_R) + \ldots$$

$$\tan 2\theta = \frac{2\,m_D}{m_R - m_L} \gg 1 \quad \Longrightarrow \quad \theta \simeq \frac{\pi}{4}$$

Type I See-saw Mechanism

$m_L = 0,\ m_D \ll m_R$

$$
\begin{aligned}
m'_{2,1} &= \frac{m_R}{2}\left[1 \pm \sqrt{1 + 4\left(\frac{m_D}{m_R}\right)^2}\right] = \\
&= \frac{m_R}{2}\left[1 \pm \left[1 + 2\left(\frac{m_D}{m_R}\right)^2 + O\left(\left(\frac{m_D}{m_R}\right)^4\right)\right]\right]
\end{aligned}
$$

$$m_1 = \frac{m_D^2}{m_R} + \dots, \qquad m_2 = m_R + \frac{m_D^2}{m_R} + \dots$$

then

$$\Delta m^2 = m_R^2 + 2\, m_D^2 + \dots$$

$$\tan 2\theta = \frac{2\, m_D}{m_R} \ll 1 \quad \Longrightarrow \quad \theta \simeq 0$$

Type II See-saw Mechanism

$m_L, m_D \ll m_R$

$$m'_{2,1} = \frac{m_R}{2}\left[\left(1+\frac{m_L}{m_R}\right) \pm \sqrt{\left(1-\frac{m_L}{m_R}\right)^2 + 4\left(\frac{m_D}{m_R}\right)^2}\right] =$$

$$= \frac{m_R}{2}\left[\left(1+\frac{m_L}{m_R}\right) \pm \left[\left(1-\frac{m_L}{m_R}\right) + \right.\right.$$

$$\left.\left. + \frac{1}{2}\left(\frac{m_L}{m_R}\right)^2 + 2\left(\frac{m_D}{m_R}\right)^2 + O\left(\left(\frac{m_D}{m_R}\right)^4\right)\right]\right]$$

$$m_1 = m_L - \frac{m_D^2}{m_R} + \ldots, \quad m_2 = m_R + \frac{m_D^2}{m_R} + \ldots$$

then

$$\Delta m^2 = m_R^2 + 2m_D^2 - \frac{1}{2}m_L^2 + \ldots$$

The See-Saw Mechanism

- We have seen two possible realizations
- Natural explanation of the smallness of neutrino masses
- Aesthetically appealing
- Should be included in a SM extension scheme (GUT)
- Introduce other types

Type I See-Saw

- Consider 3 generations
- Introduce 3 sterile neutrinos
- Dirac mass term
- Majorana mass term only for sterile neutrinos ($M_L = 0$)

$$\mathcal{L}_{mass}^{Dir+Maj} = -\overline{L'_L} Y'^{\nu} \tilde{\Phi} \nu'_R + \frac{1}{2} \nu'^T_R \mathcal{C}^\dagger M_R \nu'_R + h.c.$$

- Eigenvalues of M_R are not protected by the SM symmetries
- $\Longrightarrow$ order M_{GUT}
- we integrate them away

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The e.o.m. for ν'_R, in the static limit, are

$$\frac{\partial \mathcal{L}_{mass}^{Dir+Maj}}{\partial \nu'_R} = -\overline{L'_L} Y'^{\nu} \tilde{\Phi} + \nu'^{T}_R \mathcal{C}^\dagger M_R \simeq 0$$

Solving for ν'_R:

$$\nu'_R \simeq -M_R^{-1} \tilde{\Phi}^T \mathcal{C} \left(Y'^{\nu}\right)^T \left(\overline{L'_L}\right)^T$$

Substituting back $\nu'_R \simeq -M_R^{-1}\,\tilde{\Phi}^T \mathcal{C}\,(Y'^{\nu})^T \left(\overline{L'_L}\right)^T$:

$$\begin{aligned}
\mathcal{L}_{mass}^{Dir+Maj} &\simeq \frac{1}{2}\left(\overline{L'_L}\tilde{\Phi}\right)\mathcal{C}\,\left(Y'^{\nu} M_R^{-1}\,(Y'^{\nu})^T\right)\left(\tilde{\Phi}^T\left(\overline{L'_L}\right)^T\right) \\
&\quad + h.c. = \\
&= -\frac{1}{2}\Big[\left(L'^T_L \tau_2 \Phi\right)\mathcal{C}^\dagger (Y'^{\nu\dagger} M_M^{-1} Y'^{\nu *})\left(\Phi^T \tau_2 L'_L\right) \\
&\quad - \left(\overline{L'_L}\tau_2\Phi^*\right)\mathcal{C}(Y'^{\nu T} M_M^{-1} Y'^{\nu})\left(\Phi^\dagger \tau_2 (\overline{L'_L})^T\right)\Big]
\end{aligned}$$

After e.w. symmetry breaking:

$$\mathcal{L}_{mass}^{Dir+Maj} \simeq -\frac{1}{2}\nu_L'^T \mathcal{C}^\dagger (M_D^\dagger M_M^{-1} M_D^*)\nu_L' + h.c.$$

where

$$M_D \equiv \frac{Y'^\nu v}{\sqrt{2}}$$

is a Dirac mass matrix.

The effective left-handed Majorana neutrino mass matrix is

$$M_{\nu_L} = -M_D^T M_M^{-1} M_D$$

of order

$$\frac{v^2}{M_{GUT}}$$

Type II See-Saw

Add a Lorentz scalar, $SU(2)_L$ triplet Δ, with a Yukawa coupling

$$\mathcal{L}_{Yuk,\Delta} = L_L'^T \mathcal{C} Y'^{\nu} \tau_2 \Delta L_L' + h.c.$$

After symmetry breaking the triplet gets a non-vanishing v.e.v.

$$\langle \Delta \rangle \simeq \frac{M_W^2}{M_\Delta} \qquad if \;\; M_\Delta \gg M_W$$

Hence left-handed neutrinos have a Majorana mass matrix

$$M_{\nu_L} \simeq Y'^{\nu} \frac{M_W^2}{M_\Delta}$$

Type III See-Saw

Add a fermionic $SU(2)_L$ triplet ρ

$$\mathcal{L}_{Yuk,\rho} = L_L'^T \mathcal{C} Y'^{\nu} \tau_2 \rho \Phi + M_\rho \mathrm{Tr}_{\,SU(2)_L} \left(\rho^T \mathcal{C} \rho \right) + h.c.$$

symmetry breaking $\rightarrow$ Dirac mass for ρ

$$M_{D_\rho} \simeq \frac{Y'^{\nu} V}{2\sqrt{2}}$$

If $M_{D_\rho} \gg M_\rho \gg M_{D_{LR}}$, in the basis (ν_L, ν_R, ρ) the mass matrix is:

$$\begin{pmatrix} 0 & {m_{D_{LR}}}^T & 0 \\ m_{D_{LR}} & 0 & M_{D_\rho} \\ 0 & M_{D_\rho}^T & M_\rho \end{pmatrix}$$

and gives the Majorana mass matrix for ν_L

$$m_{\nu_L} = m_{D_{LR}} M_{D_\rho}^{-1} M_\rho M_{D_\rho}^{-T} m_{D_{LR}}^T$$

Double See-Saw

- Add 3 ν_R
- Add 3 neutrino singlets S which do not couple to ν_L, but only to ν_R
- Majorana mass for the singlets
- Dirac mass for neutrinos
- In the basis (ν_L, ν_R, S) the mass matrix is:

$$\begin{pmatrix} 0 & {m_{D_{LR}}}^T & 0 \\ m_{D_{LR}} & 0 & M_{D_{RS}} \\ 0 & M_{D_{RS}}^T & M_{M_{SS}} \end{pmatrix}$$

Assuming $M_{M_{SS}} \ll M_{D_{RS}}$ gives the left-handed Majorana mass matrix

$$m_{\nu_L} = m_{D_{LR}} M_{D_{RS}}^{-1} M_{M_{SS}} M_{D_{RS}}^{-T} m_{D_{LR}}^{T}$$

$\Longrightarrow$ Type-III see-saw is a special case of Double see-saw

Summary

- Neutrinos are massive, hence the SM must be extended
- There can be both a Dirac and a Majorana mass term
- Many phenomenological models, expecially based on the see-saw mechanism

"It should be clear from the above considerations that a theory of neutrino masses does no exist today. We are still lacking a unifying principle that organizes the flavour sector of particle physics. [. . .] We are of course aware that, especially in the neutrino field and in the recent past, many ideas and prejudices turned out to be wrong". (Feruglio *et al.* arXiv:0808.0812 [hep-ph])

The End

Neutrino Mixing

- Diagonalization of the neutrino mass matrix
- $\Longrightarrow$ non-diagonal CC weak interaction

$$\mathcal{L}^{CC} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + h.c.$$

with $j_W^\rho = j_{W,lept}^\rho + j_{W,quark}^\rho$

$$\begin{aligned} j_{W,lept}^\rho &= 2\overline{\nu_L'}\gamma^\rho l_L' = 2\overline{n_L}\, V_L^{\nu\dagger}\gamma^\rho V_L^l\, l_L = \\ &= 2\overline{n_L}\, U_{PMNS}^\dagger \gamma^\rho l_L \end{aligned}$$

where (diagonalization)

$$U_{PMNS} \equiv V_L^{l\dagger}\, V_L^\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Left-handed flavour neutrino fields

$$\nu_L \equiv U_{PMNS}\, n_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

Sterile neutrinos

are the right-handed neutrinos

Parametrization of the Neutrino Mixing Matrix

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$$U_{PMNS} = U^D \, D^M$$

$$U^D = R_{23} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}\, e^{i\delta_{13}} \\ 0 & 1 & 0 \\ -\sin\theta_{13}\, e^{-i\delta_{13}} & 0 & \cos\theta_{13} \end{pmatrix} R_{12}$$

$$D^M = \begin{pmatrix} e^{i\lambda_1} & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix} \qquad \lambda_1 = 0$$

Approximations for the Neutrino Mixing Matrix

- Trimaximal mixing ($\theta_{12} = \theta_{23} = \pi/4$, $\sin\theta_{13} = 1/\sqrt{3}$, $\sin\delta_{13} = \pm 1$)

$$U^D = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\frac{\pi}{6}} & e^{\mp i\frac{\pi}{6}} & 1 \\ e^{\mp i\frac{\pi}{6}} & -e^{\pm i\frac{\pi}{6}} & 1 \end{pmatrix}$$

- Trilarge mixing $(\theta_{13} = 0)$

$$U^D = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}$$

- Tri-bimaximal mixing $(\sin^2\theta_{13} = 0,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{12} = 1/3)$

$$U^D_{tb} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$