$N = 1$ effective supergravities for flux compactifications

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About the speaker . . .

	Padova
Advisor:	Prof. Fabio Zwirner
Co-advisor:	Dr. Gianguido Dall'Agata

About the speaker . . .

	Padova	⟹	Torino
Advisor:	Prof. Fabio Zwirner		Prof. Pietro Fré
Co-advisor:	Dr. Gianguido Dall'Agata		. . .

Motivation

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$\longrightarrow$ (Superstring/M-theory)

Outline

1. Supersymmetry and supergravity
2. Compactification and dimensional reduction
3. Fluxes
4. Original part: type IIB supergravity

I. Supersymmetry and supergravity

Why supersymmetry?

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- Candidate for Dark Matter
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\{Q_\alpha, \overline{Q}_{\dot\beta}\} &= 2\,\sigma^\mu_{\alpha\dot\beta}\,P_\mu\,, \\
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\tag{1}
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$\Longrightarrow$ describes gravitational interactions: supergravity

The supergravity action

Complete action depends on three functions:

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- gauge kinetic function $f_{(ab)}$

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$$\mathcal{L}_{pure} = -\frac{1}{2}\, e\, R + e\, \epsilon^{\mu\nu\rho\sigma}\, \overline{\Psi}_\mu \overline{\sigma}_\nu \partial_\rho \Psi_\sigma + \dots$$

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$$V(z^i, \bar{z}^{\bar{\jmath}}) = e\, e^K \left[K^{i\bar{\jmath}} (D_i W)(D_{\bar{\jmath}} \overline{W}) - 3|W|^2 \right] + \ldots$$

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$\Downarrow$

Need for an effective theory in $d = 4$

Starting point: supergravities in $D = 10, 11$

Theory	Bosonic fields					
▸ $D = 11$	G_{MN}		$A_{(3)}$			
▸ $N = 1$	G_{MN}	Φ	$B_{(2)}$	$A_{(1)}$ gauge		
▸ IIA	G_{MN}	Φ	$B_{(2)}$	$C_{(1)} \leftrightarrow C_{(7)}$	$C_{(3)} \leftrightarrow C_{(5)}$	$C_{(9)} \leftrightarrow C_{(-1)}$
▸ IIB	G_{MN}	Φ	$B_{(2)}$	$C_{(0)} \leftrightarrow C_{(8)}$	$C_{(2)} \leftrightarrow C_{(6)}$	$C_{(4)} \leftrightarrow C_{(4)}$

II. Compactification and dimensional reduction

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- $\longrightarrow$ Result: $d = 4$ theory for a finite numer of fields

Compactification and dimensional reduction

Simplest example

Free massless complex scalar field in $D = 5$ on a circle of length L

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- Identification $y \equiv y + L$

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- Periodicity conditions

$$\varphi(x, y + L) \equiv \varphi(x, y)$$

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Free massless complex scalar field in $D = 5$ on a circle of length L

- Fourier series expansion

$$\varphi(x,y) = \frac{1}{\sqrt{L}} \sum_{n\in\mathbb{Z}} \varphi_n(x)\, e^{i\left(\frac{2\pi n}{L}\right) y}$$

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- Infinite $d = 4$ scalars

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- Kaluza-Klein tower of states

$$(m_n)^2 = \left(\frac{2\pi n}{L}\right)^2$$

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- φ_0 is massless

Next-to-simplest example

Introduction of the Scherk-Schwarz twist in the previous example

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In general

$$\varphi(y + L) \equiv T\,\varphi(y)$$

T symmetry of the action

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- General solution

$$\varphi(x, y) = e^{i\mu y} \frac{1}{\sqrt{L}} \sum_{n \in \mathbb{Z}} \varphi_n(x)\, e^{i\left(\frac{2\pi n}{L}\right) y}$$

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Observation

Dimensional reduction $\sim$ local redefinition

$$\varphi(x, y) = e^{i\mu y} \, \varphi(x)$$

Supergavity compactifications

Aim:

$N = 1,\ d = 4$
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Study:

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Study: compactifications on $\mathbb{T}^k/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, $k = 6, 7$ (in some cases with a further $\mathbb{Z}_2$ projection)

Supergravity compactifications

Six cases studied (bosonic sector)

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5. M-theory ($\rightarrow$ IIA) Go

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5. M-theory ($\longrightarrow$ IIA) Go
6. M-theory ($\longrightarrow$ heterotic) Go

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- Weyl rescaling: string frame $\longrightarrow$ Einstein frame

$$G_{\mu\nu} = \hat{s}^{-1}\,\widetilde{G}_{\mu\nu}$$

▸ Torus compactification

Orbifold compactifications

Orbifold action $\mathbb{Z}_2 \times \mathbb{Z}_2'$ on the coordinates

	x^5	x^6	x^7	x^8	x^9	x^{10}
$\mathbb{Z}_2$	$-$	$-$	$-$	$-$	$+$	$+$
$\mathbb{Z}_2'$	$+$	$+$	$-$	$-$	$-$	$-$
$\mathbb{Z}_2\mathbb{Z}_2'$	$-$	$-$	$+$	$+$	$-$	$-$

$\Longrightarrow$ three invariant two-tori $\mathbb{T}^6 = \mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$

Orbifold compactifications

Metric components:

$$G_{MN} = \begin{cases} blockdiag\,(\hat{s}^{-1}\widetilde{G}_{\mu\nu}, G_{i_1j_1}, G_{i_2j_2}, G_{i_3j_3}) & D = 10 \\ & \end{cases} \tag{3}$$

where

$$G_{i_Aj_A} = \frac{\hat{t}_A}{\hat{u}_A}\begin{pmatrix} (\hat{u}_A^2 + \hat{\nu}_A^2) & \hat{\nu}_A \\ \hat{\nu}_A & 1 \end{pmatrix} \tag{4}$$

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Orbifold compactifications

Consequence:

- $D = 10$,

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- $D = 11$,

$$\hat{v} = \left(\frac{\hat{s}}{\hat{t}_1 \hat{t}_2 \hat{t}_3} \right)^2$$

▸ Orbifold compactification

Supergavity compactifications

▸ Het ▸ IIAO6 ▸ IIBO3O7 ▸ IIBO5O9 ▸ MtoIIA ▸ MtoHet

Result:

- $N = 1$, $d = 4$ effective theories

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Result:

- $N = 1,\ d = 4$ effective theories
- 7 complex scalar fields $S,\ T_A,\ U_B\ (A, B = 1, 2, 3)$ ▸ Summary

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Result:

- $N = 1,\ d = 4$ effective theories
- 7 complex scalar fields $S,\ T_A,\ U_B\ (A, B = 1, 2, 3)$ ▸ Summary
- Kähler potential K:

$$K(S_{,A}, U_B) \;=\; -\log(S+\overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) + \\ - \sum_{B=1}^{3} \log(U_A + \overline{U}_A)$$

Supergavity compactifications

Het IIA06 IIBO3O7 IIBO5O9 MtoIIA MtoHet

Result:

- $N = 1,\ d = 4$ effective theories
- 7 complex scalar fields $S,\ T_A,\ U_B\ (A, B = 1, 2, 3)$ Summary
- Kähler potential K:

$$K(S,_A, U_B) = -\log(S + \overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) + - \sum_{B=1}^{3} \log(U_A + \overline{U}_A)$$

- Superpotential $W = 0 \iff V = 0$

Problems

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Idea

generate a scalar potential by using fluxes

$\Longrightarrow$ moduli stabilization?

III. Fluxes

Different kinds of fluxes

Two kinds of fluxes considered:

1. potential p-form fluxes
2. Geometric fluxes

Potential p-form fluxes

Potential p-form fluxes

- Theory with p-forms $A_{(p)}$ and field strengths $F_{(p+1)} = dA_{(p)}$

Potential *p*-form fluxes

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  - Generalization of the Zeeman effect
  - $d = 4$ Poincaré symmetry preserved

Geometric fluxes

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- Scherk-Schwarz twist with global symmetry $\longrightarrow$ mass parameter

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Geometric fluxes

- Scherk-Schwarz twist with global symmetry $\longrightarrow$ mass parameter
- effect absorbed in local redefinition
- what happens when the symmetry is local?
- Interesting case: gravity, invariant under GCT

Twisted torus

Geometric fluxes

- Generalized dimensional reduction: $\longrightarrow$ fields depend on x^i

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- Then:

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- $\Longrightarrow$ Lie Group with ${C_{ij}}^k$
- gravitational case: ${\omega_{ij}}^k \sim {C_{ij}}^k$
- $\Longrightarrow$ geometric fluxes

Scalar potential from geometric fluxes

Geometric fluxes generate a scalar potential

$$V_E = \frac{1}{8}\sqrt{-\widetilde{G}_4}\,\hat{s}^{-1}\,\left[2\,\omega_{jk}{}^{i}\,\omega_{il}{}^{j}\,G^{kl} + \omega_{jk}{}^{i}\,\omega_{mn}{}^{l}\,G_{il}\,G^{jm}\,G^{kn}\right]$$

Consistency conditions

1. No sources

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   BI: $dH \quad = 0 \qquad\qquad H = dB$

   solution:

   $H = dB \qquad +\overline{H}\,, \quad \overline{H} = const.$

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2. Jacobi identity:
   $$\omega_{i[j}{}^{k} \omega_{mn]}{}^{i} = 0$$
3. Invariance of the volume form:
   $$\omega_{ij}{}^{i} = 0$$

IV. Original part: type IIB supergravity

Original part

Study of two examples:

a) IIB with O3/O7 orientifold

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a) IIB with O5/O9 orientifold

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In each case:

- turn on some fluxes

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- computation of V and W

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In each case:

- turn on some fluxes
- computation of V and W
- study of vacua and moduli stabilization

IIB with O3/O7 orientifold

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Fluxes and superpotential

- No geometric fluxes

IIB with O3/O7 orientifold

Fluxes and superpotential

- No geometric fluxes
- Turn on: F_{579}, F_{5710}, F_{6810}, F_{6710}, H_{6710} Fluxes

IIB with O3/O7 orientifold

Fluxes and superpotential

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- Turn on: F_{579}, F_{5710}, F_{6810}, F_{6710}, H_{6710} ▸ Fluxes

Then

$$W_{O3/O7} \;=\; 2\sqrt{2}\, e^{i\alpha}\Big[i\, F_{579} - U_3 F_{5710} + U_1 U_2 U_3 F_{6810} + \\ - i\, U_1 U_3 F_{6710} + S U_1 U_3 H_{6710}\Big]$$

IIB with O3/O7 orientifold

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Independent of T_A

IIB with O3/O7 orientifold

Consequences

- Scalar potential positive semi-definite

$$V_{O3/O7} = \sqrt{-\widetilde{G}_4}\, e^K \Big[\big|(S+\overline{S})W_S - W\big|^2 + + \sum_{B=1}^{3} \big|(U_B+\overline{U}_B)W_{U_B} - W\big|^2 \Big]$$

[1] in the absence of sources

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- Vacua (susy and not) only for trivial values of the fluxes[1]

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$\Longrightarrow$ Maldacena-Nuñez no-go theorem

[1] in the absence of sources

IIB with O3/O7 orientifold

IIB with O3/O7 orientifold

Consequences

$\Longrightarrow$ Moduli not stabilized

IIB with O5/O9 orientifold

Fluxes and superpotential

- Geometric fluxes: yes

IIB with O5/O9 orientifold

Fluxes and superpotential

- Geometric fluxes: yes
- Turn on: $\omega_{57}{}^{9}$, $\omega_{59}{}^{7}$, $\omega_{610}{}^{8}$, $\omega_{68}{}^{10}$ ▸ Fluxes

IIB with O5/O9 orientifold

Fluxes and superpotential

- Geometric fluxes: yes
- Turn on: ${\omega_{57}}^{9}$, ${\omega_{59}}^{7}$, ${\omega_{610}}^{8}$, ${\omega_{68}}^{10}$ ▸ Fluxes
- Then

$$W_{O5/O9} = 2\sqrt{2}\, e^{i\alpha} \Big[{\omega_{59}}^{7}\, T_2 U_2 - i\, {\omega_{610}}^{8}\, T_2 U_1 U_3 + - {\omega_{57}}^{9}\, T_3 U_3 + i\, {\omega_{68}}^{10}\, T_3 U_1 U_2 \Big]$$

IIB with O5/O9 orientifold

Consequences

- Scalar potential accidentally positive semi-definite

$$V(z^i) = \sqrt{-\widetilde{G}_4}\, e^K \Big[\sum_{A=2}^{3} |(T_A + \overline{T}_A) W_{T_A} - W|^2 + \\ + \sum_{B=1}^{3} |(U_B + \overline{U}_B) W_{U_B} - W|^2 - |W|^2 \Big]$$

IIB with O5/O9 orientifold

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- trivial susy vacuum $\omega_{57}{}^{9} = \omega_{59}{}^{7} = \omega_{610}{}^{8} = \omega_{68}{}^{10} = 0$
- $\exists$ non susy vacua for non-trivial values of the fluxes

IIB with O5/O9 orientifold

IIB with O5/O9 orientifold

Consequences

$\Longrightarrow$ stabilization of some moduli with geometrical fluxes

Vacua in IIB with O5/O9 orientifold

Classification of non-trivial vacua:

Vacua in IIB with O5/O9 orientifold

Classification of non-trivial vacua:

i) Case ${\omega_{610}}^{8} = 0 = {\omega_{68}}^{10}$, ${\omega_{57}}^{9}/{\omega_{59}}^{7} < 0$ (non susy)

$$\begin{cases} \langle t_2 \rangle &= -\frac{{\omega_{57}}^{9}}{{\omega_{59}}^{7}} \left\langle \frac{t_3\, u_3}{u_2} \right\rangle \\ \langle \nu_2 \rangle &= \langle \nu_3 \rangle = 0 \\ \langle \tau_2 \rangle &= \langle \tau_3 \rangle = 0 \end{cases} \tag{5}$$

Vacua in IIB with O5/O9 orientifold

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Spectrum in type i) vacua

Gravitino mass

$$m_{3/2}^2 = \left\langle e^K \, |W|^2 \right\rangle = -\frac{1}{4}{\omega_{57}}^9 \, {\omega_{59}}^7 \, \langle (s t_1 u_1)^{-1} \rangle$$

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$$m_{3/2}^2 = \left\langle e^K \, |W|^2 \right\rangle = -\frac{1}{4}\omega_{57}{}^9\,\omega_{59}{}^7\,\langle (st_1 u_1)^{-1}\rangle$$

Field	Squared mass
$\left(\frac{t_3'-t_2'+u_3'-u_2'}{2}\right)$	$32\ m_{3/2}^2$
$\tau_2',\ \tau_3',\ \nu_2',\ \nu_3'$	$8\ m_{3/2}^2$
$\left(\frac{t_3'+t_2'+u_3'+u_2'}{2}\right)$	0
$\left(\frac{t_3'+t_2'-u_3'-u_2'}{2}\right)$	0
$\left(\frac{t_3'-t_2'-u_3'+u_2'}{2}\right)$	0
$s',\ \sigma',\ t_1',\ \tau_1',\ u_1',\ \nu_1'$	0

Summary

- Supersymmetry

Summary

- Supersymmetry $\longrightarrow$ SM extension

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- Supersymmetry $\longrightarrow$ SM extension
- Supergravity $\longrightarrow$ includes gravity
- Superstrings/M-theory $\longrightarrow$ supergravity in $D = 10$ (11)
- Compactification and dimensional reduction $\longrightarrow$ moduli
- Fluxes $\longrightarrow$ moduli stabilization
- Original part: IIB on $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

The End

$D = 11$ supergravity action

$$S_{11} = \frac{1}{2k_{11}^2} \int d^{11}x \, (-G_{11})^{1/2} \left[R_{11} - \frac{1}{2} \left| F_{(4)} \right|^2 \right] + - \frac{1}{6} \int A_{(3)} \wedge F_{(4)} \wedge F_{(4)}$$

In general, given a p-form $F_{(p)}$, we use the convention

$$|F_{(p)}|^2 = \frac{1}{p!} G^{M_1 N_1} \cdots G^{M_p N_p} F_{M_1 \cdots M_p} F_{N_1 \cdots N_p}$$

▸ Back

$N = 1$ (heterotic) supergravity action

$$S = \frac{1}{2k_{10}^2} \int d^{10}x \, (-G_{10})^{1/2} \; e^{-2\Phi} \Big[R_{10} + 4(\partial^\mu \Phi)(\partial_\mu \Phi) + - \frac{1}{2} \left| \widetilde{H}_{(3)} \right|^2 - \frac{k_{10}^2}{g_{10}^2} \mathrm{Tr}\; _V |F_{(2)}|^2 \Big]$$

where

$$\widetilde{H}_{(3)} = dB_{(2)} - \frac{k_{10}^2}{g_{10}^2} \omega_{(3)}$$

the Chern-Simons 1-form is

$$\omega_{(3)} = \mathrm{Tr}\; _V \left(A_{(1)} \wedge dA_{(1)} - \frac{2i}{3} A_{(1)} \wedge A_{(1)} \wedge A_{(1)} \right)$$

and g_{10} is a gauge coupling parameter Back

Type IIA supergravity action

$$S_{IIA} = S_{NS} + S_R + S_{CS}$$

where

$$\begin{aligned} S_{NS} &= \frac{1}{2k_{10}^2} \int d^{10}x \, (-G_{10})^{1/2} \, e^{-2\Phi} \Big[R_{10} + \\ &\quad + 4(\partial^\mu \Phi)(\partial_\mu \Phi) - \frac{1}{2} \left| H_{(3)} \right|^2 \Big] \end{aligned}$$

$$S_R = -\frac{1}{4k_{10}^2} \int d^{10}x \, (-G_{10})^{1/2} \left[M^2 + \left| F_{(2)} \right|^2 + \left| \widetilde{F}_{(4)} \right|^2 \right]$$

$$S_{CS} = -\frac{1}{4k_{10}^4} \int B_{(2)} \wedge F_{(4)} \wedge F_{(4)}$$

Type IIA supergravity action

and with

$$H_{(3)} = dB_{(2)}$$

$$F_{(2)} = dC_{(1)} + MB_{(2)}$$

$$F_{(4)} = dC_{(3)} + \frac{1}{2}MB_{(2)} \wedge B_{(2)}$$

$$\widetilde{F}_{(4)} = F_{(4)} - C_{(1)} \wedge H_{(3)}$$

▸ Back

Type IIB supergravity action

$$S_{IIB} = S_{NS} + S_R + S_{CS}$$

where

$$\begin{aligned} S_{NS} &= \frac{1}{2k_{10}^2} \int d^{10}x \, (-G_{10})^{1/2} \, e^{-2\Phi} \Big[R_{10} + \\ &\quad + 4(\partial^\mu \Phi)(\partial_\mu \Phi) - \frac{1}{2} \left| H_{(3)} \right|^2 \Big] \\ S_R &= -\frac{1}{4k_{10}^2} \int d^{10}x \, (-G_{10})^{1/2} \left[\left| F_{(1)} \right|^2 + \left| \widetilde{F}_{(3)} \right|^2 + \right. \\ &\quad \left. + \frac{1}{2} \left| \widetilde{F}_{(5)} \right|^2 \right] \end{aligned}$$

$$S_{CS} = -\frac{1}{4k_{10}^2} \int C_{(4)} \wedge H_{(3)} \wedge F_{(3)}$$

and with

$$\widetilde{F}_{(3)} = F_{(3)} - C_{(0)} \wedge H_{(3)}$$

$$\widetilde{F}_{(5)} = F_{(5)} - \frac{1}{2} C_{(2)} \wedge H_{(3)} + \frac{1}{2} B_{(2)} \wedge F_{(3)}$$

Back

Torus compactifications

$$
\begin{aligned}
S &= \frac{1}{2\kappa_D^2}\int d^D x\,\sqrt{-G_D}\,e^{-2\Phi}\left[R_D + 4(\partial^\mu\Phi)(\partial_\mu\Phi) - \frac{1}{2}e^{2\Phi}\left|F_{(p+1)}\right|^2\right] = \\
&= \frac{1}{2\kappa_4^2}\int d^4 x\,\sqrt{-\widetilde{G}_4}\,\sqrt{G_k}\,\left(e^{-2\Phi}\hat{s}^{-1}\right)\left[\widetilde{R}_4 - \frac{3}{2}\hat{s}^{-2}\,(\partial_\mu\hat{s})(\partial^\mu\hat{s}) + \right. \\
&\quad + 4\left(\partial_\mu\left(\Phi - \frac{1}{4}\log G_k\right)\right)\left(\partial^\mu\left(\Phi - \frac{1}{4}\log G_k\right)\right) + \\
&\quad - \frac{1}{4}G^{mn}G^{pq}\,\left((\partial_\mu G_{mp})(\partial^\mu G_{nq}) + (\partial_\mu B_{mp})(\partial^\mu B_{nq})\right) + \\
&\quad - \frac{1}{12}\,\hat{s}^2\,H_{\mu\nu\rho}H^{\mu\nu\rho} + \\
&\quad - \frac{1}{2(p!)}\hat{s}^2 e^{2\Phi}G^{m_4 n_4}\cdots G^{m_{p+1}n_{p+1}}\left(F_{\mu\nu\rho m_4\cdots m_{p+1}}\right)\left(F^{\mu\nu\rho}{}_{n_4\cdots n_{p+1}}\right) + \\
&\quad \left. - \frac{1}{2}e^{2\Phi}\,\frac{1}{p!}\,G^{m_2 n_2}\cdots G^{m_{p+1}n_{p+1}}\left(\partial_\mu A_{m_2\cdots m_{p+1}}\right)\left(\partial^\mu A_{n_2\cdots n_{p+1}}\right)\right]
\end{aligned}
$$

Back

Orbifold compactifications

$$
\begin{aligned}
S \quad = \quad & \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\widetilde{G}_4}\,\sqrt{G_k}\,\left(e^{-2\Phi}\hat{s}^{-1}\right)\Big[\widetilde{R}_4 - \frac{1}{2}\hat{s}^{-2}(\partial_\mu \hat{s})(\partial^\mu \hat{s}) + \\
& - \frac{1}{4} G^{mn} G^{pq} \left((\partial_\mu G_{mp})(\partial^\mu G_{nq}) + (\partial_\mu B_{mp})(\partial^\mu B_{nq})\right) + \\
& - \frac{1}{12}\hat{s}^2 H_{\mu\nu\rho} H^{\mu\nu\rho} + \\
& - \frac{1}{2(p!)}\hat{s}^2 e^{2\Phi} G^{m_4 n_4} \cdots G^{m_{p+1} n_{p+1}} (F_{\mu\nu\rho m_4 \cdots m_{p+1}}) (F^{\mu\nu\rho}{}_{n_4 \cdots n_{p+1}}) + \\
& - \frac{1}{2} e^{2\Phi} \frac{1}{p!} G^{m_2 n_2} \cdots G^{m_{p+1} n_{p+1}} (\partial_\mu A_{m_2 \cdots m_{p+1}}) (\partial^\mu A_{n_2 \cdots n_{p+1}}) \Big]
\end{aligned}
$$

▸ Back

$N = 1$ (heterotic) supergravity (without YM)

Fields in $\mathcal{N} = 1$ supergravity		
Dilaton	:	Φ
Metric moduli	:	$\hat{t}_A$, $\hat{u}_A$, $\hat{\nu}_A$
NSNS two-form $B_{(2)}$	:	B_{56}, B_{78}, B_{910}, $B_{\mu\nu} \leftrightarrow \sigma$

$N = 1$ (heterotic) supergravity (without YM)

Define

$$\tau_1 \equiv B_{56}\,, \quad \tau_2 \equiv B_{78}\,, \quad \tau_3 \equiv B_{910}$$

Reduced action:

$$\begin{aligned} S \;=\; & \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\frac{(\partial_\mu \hat{s})(\partial^\mu \hat{s}) + (\partial_\mu \sigma)(\partial^\mu \sigma)}{\hat{s}^2} + \\ & - \frac{1}{2}\sum_{A=1}^{3}\frac{(\partial_\mu \hat{t}_A)(\partial^\mu \hat{t}_A) + (\partial_\mu \tau_A)(\partial^\mu \tau_A)}{\hat{t}_A^2} + \\ & - \frac{1}{2}\sum_{A=1}^{3}\frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{\hat{u}_A^2}\Big] \end{aligned}$$

$N=1$ (heterotic) supergravity (without YM)

Redefinitions:

$$s \equiv \hat{s}\,, \quad t_A \equiv \hat{t}_A\,, \quad u_A \equiv \hat{u}_A\,, \quad \nu_A \equiv \hat{\nu}_A\,, \quad A=1,2,3$$

Seven complex scalar fields:

$$\begin{aligned} S &= s + i\sigma \\ T_A &= t_A + i\tau_A \\ U_A &= u_A + i\nu_A \end{aligned}$$

Kähler potential

$$K = -\log(S+\overline{S}) - \sum_{A=1}^{3}\log(T_A+\overline{T}_A) - \sum_{B=1}^{3}\log(U_B+\overline{U}_B)$$

Back

IIA with O6 orientifold

$$\mathcal{R}: \quad (x^5, x^6, x^7, x^8, x^9, x^{10}) \to (-x^5, +x^6, -x^7, +x^8, -x^9, +x^{10})$$

Combined orbifold/orientifold actions							
	:	x^5	x^6	x^7	x^8	x^9	x^{10}
$\mathcal{R}$	:	$-$	$+$	$-$	$+$	$-$	$+$
$\mathbb{Z}_2\mathcal{R}$	:	$+$	$-$	$+$	$-$	$-$	$+$
$\mathbb{Z}_2'\mathcal{R}$	:	$-$	$+$	$+$	$-$	$+$	$-$
$\mathbb{Z}_2\mathbb{Z}_2'\mathcal{R}$	:	$+$	$-$	$-$	$+$	$+$	$-$

Four O6-planes:

$$(6810)\,,\ (5710)\,,\ (679)\,,\ (589)$$

IIA with O6 orientifold

Orientifold action on the fields		
Φ	$\rightarrow$	$+\Phi$
B	$\rightarrow$	$-B$
G	$\rightarrow$	$+G$
$C_{(1)}$	$\rightarrow$	$-C_{(1)}$
$C_{(3)}$	$\rightarrow$	$+C_{(3)}$

IIA with O6 orientifold

Fields in type IIA with O6	
Dilaton	Φ
Metric moduli	$\hat{t}_A,\ \hat{u}_A$
$B_{(2)}$	$B_{56},\ B_{78},\ B_{910}$
$C_{(3)}$	$C_{5710},\ C_{589},\ C_{679},\ C_{6810}$

$$G_{i_A j_A} = \begin{pmatrix} \hat{t}_A \hat{u}_A & 0 \\ 0 & \frac{\hat{t}_A}{\hat{u}_A} \end{pmatrix} \quad , \qquad A = 1, 2, 3 \tag{7}$$

IIA with O6 orientifold

$$
\begin{aligned}
S \;=\;& \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\hat{s}^{-2}\,(\partial_\mu \hat{s})(\partial^\mu \hat{s}) + \\
& -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A)}{\hat{u}_A^2} + \frac{(\partial_\mu \hat{t}_A)(\partial^\mu \hat{t}_A)}{\hat{t}_A^2}\right] + \\
& -\frac{1}{2}\Big[\frac{(\partial_\mu B_{56})(\partial^\mu B_{56})}{\hat{t}_1^2} + \frac{(\partial_\mu B_{78})(\partial^\mu B_{78})}{\hat{t}_2^2} + \frac{(\partial_\mu B_{910})(\partial^\mu B_{910})}{\hat{t}_3^2}\Big] + \\
& -\frac{1}{2}\left(\frac{e^{2\Phi}}{\hat{t}_1\hat{t}_2\hat{t}_3}\right)\Big[\frac{\hat{u}_3}{\hat{u}_1\hat{u}_2}\,(\partial_\mu C_{5710})(\partial^\mu C_{5710}) + \\
& + \frac{\hat{u}_2}{\hat{u}_1\hat{u}_3}\,(\partial_\mu C_{589})(\partial^\mu C_{589}) + \frac{\hat{u}_1}{\hat{u}_2\hat{u}_3}\,(\partial_\mu C_{679})(\partial^\mu C_{679}) + \\
& + \hat{u}_1\hat{u}_2\hat{u}_3\,(\partial_\mu C_{6810})(\partial^\mu C_{6810})\Big]
\end{aligned}
$$

IIA with O6 orientifold

$$\tau_1 \equiv B_{56}\,, \quad \tau_2 \equiv B_{78}\,, \quad \tau_3 \equiv B_{910}$$

$$\nu_1 \equiv -C_{679}\,, \quad \nu_2 \equiv -C_{589}\,, \quad \nu_3 \equiv -C_{5710}\,, \quad \sigma \equiv C_{6810}$$

Non-linear redefinitions:

$$u_A = \sqrt{\frac{\hat{s}\,\hat{u}_1\,\hat{u}_2\,\hat{u}_3}{\hat{u}_A^2}}$$

$$s = \sqrt{\frac{\hat{s}}{\hat{u}_1\,\hat{u}_2\,\hat{u}_3}}$$

IIA with O6 orientifold

Reduced action:

$$
\begin{aligned}
S \quad = \quad & \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Bigg[\widetilde{R}_4 - \frac{1}{2}\,\frac{(\partial_\mu s)(\partial^\mu s)+(\partial_\mu \sigma)(\partial^\mu \sigma)}{s^2} + \\
& -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu u_A)(\partial^\mu u_A)+(\partial_\mu \nu_A)(\partial^\mu \nu_A)}{u_A^2}\right] + \\
& -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu \hat{t}_A)(\partial^\mu \hat{t}_A)+(\partial_\mu \tau_A)(\partial^\mu \tau_A)}{\hat{t}_A^2}\right]\Bigg]
\end{aligned}
$$

IIA with O6 orientifold

Redefinitions:

$$t_A \equiv \hat{t}_A \,, \qquad A = 1, 2, 3$$

The reduced action describes, besides the supergravity multiplet, seven complex scalar fields:

$$S = s + i\sigma \,, \quad T_A = t_A + i\tau_A \,, \quad U_A = u_A + i\nu_A$$

with Kähler potential

$$K = -\log(S + \overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) - \sum_{B=1}^{3} \log(U_B + \overline{U}_B)$$

▸ Back

IIB with O3/O7 orientifold

$$\mathcal{R}: \quad (x^5, x^6, x^7, x^8, x^9, x^{10}) \rightarrow (-x^5, -x^6, -x^7, -x^8, -x^9, -x^{10})$$

Combined orbifold/orientifold actions							
	:	x^5	x^6	x^7	x^8	x^9	x^{10}
$\mathcal{R}$	:	$-$	$-$	$-$	$-$	$-$	$-$
$\mathbb{Z}_2\mathcal{R}$	:	$+$	$+$	$+$	$+$	$-$	$-$
$\mathbb{Z}_2'\mathcal{R}$	:	$-$	$-$	$+$	$+$	$+$	$+$
$\mathbb{Z}_2\mathbb{Z}_2'\mathcal{R}$	:	$+$	$+$	$-$	$-$	$+$	$+$

1 invariant O3-plane and 3 invariant O7-planes:

$$(5678)\,,\ (78910)\,,\ (56910)$$

IIB with O3/O7 orientifold

Orientifold action on the fields		
Φ	$\rightarrow$	$+\Phi$
B	$\rightarrow$	$-B$
G	$\rightarrow$	$+G$
$C_{(0)}$	$\rightarrow$	$+C_{(0)}$
$C_{(2)}$	$\rightarrow$	$-C_{(2)}$
$C_{(4)}$	$\rightarrow$	$+C_{(4)}$

Fields in type IIB with O3/O7	
Dilaton	Φ
Metric moduli	$\hat{t}_A$, $\hat{u}_A$, $\hat{\nu}_A$
RR 0-form	$C_{(0)}$
$C_{(4)}$	C_{5678}, C_{56910}, C_{78910}

IIB with O3/O7 orientifold

$$
\begin{aligned}
S \quad = \quad & \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\hat{s}^{-2}\,(\partial_\mu \hat{s})(\partial^\mu \hat{s}) + \\
& - \frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu \hat{t}_A)(\partial^\mu \hat{t}_A)}{\hat{t}_A^2} + \frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{\hat{u}_A^2}\right] + \\
& - \frac{1}{2}\left(\frac{e^{2\Phi}}{\hat{t}_1\hat{t}_2\hat{t}_3}\right)\Big[\frac{\hat{t}_3}{\hat{t}_1\hat{t}_2}\,(\partial_\mu C_{5678})(\partial^\mu C_{5678}) + \\
& + \frac{\hat{t}_2}{\hat{t}_1\hat{t}_3}\,(\partial_\mu C_{56910})(\partial^\mu C_{56910}) + \frac{\hat{t}_1}{\hat{t}_2\hat{t}_3}\,(\partial_\mu C_{78910})(\partial^\mu C_{78910}) + \\
& + \hat{t}_1\hat{t}_2\hat{t}_3\,(\partial_\mu C_0)(\partial^\mu C_0)\Big]\Big]
\end{aligned}
$$

IIB with O3/O7 orientifold

Axions:

$$\tau_1 \equiv C_{78910}\,, \;\; \tau_2 \equiv C_{56910}\,, \;\; \tau_3 \equiv C_{5678}$$

$$\nu_A \equiv \hat{\nu}_A\,, \quad \sigma \equiv -C_{(0)}\,, \quad A = 1, 2, 3$$

Define

$$t_A = \sqrt{\frac{\hat{s}\hat{t}_1\hat{t}_2\hat{t}_3}{\hat{t}_A^2}}$$

$$s = \sqrt{\frac{\hat{s}}{\hat{t}_1\hat{t}_2\hat{t}_3}}$$

IIB with O3/O7 orientifold

$$
\begin{aligned}
S &= \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\,\frac{(\partial_\mu \hat{s})(\partial^\mu \hat{s}) + (\partial_\mu \sigma)(\partial^\mu \sigma)}{s^2} + \\
&\quad - \frac{1}{2}\sum_{A=1}^{3}\Big[\frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{\hat{u}_A^2} + \\
&\quad + \frac{(\partial_\mu t_A)(\partial^\mu t_A) + (\partial_\mu \tau_A)(\partial^\mu \tau_A)}{t_A^2}\Big]\Big]
\end{aligned}
$$

IIB with O3/O7 orientifold

Finally

$$u_A \equiv \hat{u}_A\,, \qquad s \equiv \hat{s}$$

$$S = s + i\sigma\,, \qquad T_A = t_A + i\tau_A\,, \qquad U_A = u_A + i\nu_A$$

Kähler potential

$$K = -\log(S + \overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) - \sum_{B=1}^{3} \log(U_B + \overline{U}_B)$$

Back

IIB with O5/O9 orientifold

$$\mathcal{R}: \quad (x^5, x^6, x^7, x^8, x^9, x^{10}) \rightarrow (+x^5, +x^6, +x^7, +x^8, +x^9, +x^{10})$$

Combined orbifold/orientifold actions							
	:	x^5	x^6	x^7	x^8	x^9	x^{10}
$\mathcal{R}$	:	$+$	$+$	$+$	$+$	$+$	$+$
$\mathbb{Z}_2\mathcal{R}$	:	$-$	$-$	$-$	$-$	$+$	$+$
$\mathbb{Z}_2'\mathcal{R}$	:	$+$	$+$	$-$	$-$	$-$	$-$
$\mathbb{Z}_2\mathbb{Z}_2'\mathcal{R}$	:	$-$	$-$	$+$	$+$	$-$	$-$

One invariant O9-plane and three invariant O5-planes:

$$(5678910)\,,\ (910)\,,\ (56)\,,\ (78)$$

IIB with O5/O9 orientifold

Orientifold action on the fields		
Φ	$\rightarrow$	$+\Phi$
B	$\rightarrow$	$-B$
G	$\rightarrow$	$+G$
$C_{(0)}$	$\rightarrow$	$-C_{(0)}$
$C_{(2)}$	$\rightarrow$	$+C_{(2)}$
$C_{(4)}$	$\rightarrow$	$-C_{(4)}$

Fields in type IIB with O5/O9	
Dilaton	Φ
Metric moduli	$\hat{t}_A,\ \hat{u}_A,\ \hat{\nu}_A$
$C_{(2)}$	$C_{56},\ C_{78},\ C_{910},\ C_{\mu\nu} \leftrightarrow \sigma$

IIB with O5/O9 orientifold

$$
\begin{aligned}
S \quad = \quad & \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\widetilde{G}_4} \left[\widetilde{R}_4 - \frac{1}{2} \frac{(\partial_\mu \hat{s})(\partial^\mu \hat{s})}{\hat{s}^2} - \frac{1}{2} e^{-2\Phi} \frac{(\partial_\mu \sigma)(\partial^\mu \sigma)}{\hat{s}^2} + \right. \\
& - \frac{1}{2} \sum_{A=1}^{3} \left[\frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{\hat{u}_A^2} \right] + \\
& - \frac{1}{2} \sum_{A=1}^{3} \frac{(\partial_\mu \hat{t}_A)(\partial^\mu \hat{t}_A)}{\hat{t}_A^2} + \\
& - \frac{1}{2} e^{2\Phi} \left[\frac{(\partial_\mu C_{56})(\partial^\mu C_{56})}{\hat{t}_1^2} + \frac{(\partial_\mu C_{78})(\partial^\mu C_{78})}{\hat{t}_2^2} + \right. \\
& \left. \left. + \frac{(\partial_\mu C_{910})(\partial^\mu C_{910})}{\hat{t}_3^2} \right] \right]
\end{aligned}
$$

IIB with O5/O9 orientifold

Axions:

$$\tau_1 \equiv C_{56}\,, \quad \tau_2 \equiv C_{78}\,, \quad \tau_3 \equiv C_{910}\,, \quad \nu_A \equiv \hat{\nu}_A\,, \quad \sigma$$

Definitions:

$$t_A = \sqrt{\frac{\hat{s}\hat{t}_A^2}{\hat{t}_1\hat{t}_2\hat{t}_3}}$$

$$s = \sqrt{\hat{s}\hat{t}_1\hat{t}_2\hat{t}_3}$$

IIB with O5/O9 orientifold

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-\widetilde{G}_4}\Big[\widetilde{R}_4 - \frac{1}{2}\frac{(\partial_\mu \hat{s})(\partial^\mu \hat{s}) + (\partial_\mu \sigma)(\partial^\mu \sigma)}{s^2} + \\ - \frac{1}{2}\sum_{A=1}^{3}\Big[\frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \nu_A)(\partial^\mu \nu_A)}{\hat{u}_A^2} + \\ + \frac{(\partial_\mu t_A)(\partial^\mu t_A) + (\partial_\mu \tau_A)(\partial^\mu \tau_A)}{t_A^2}\Big]\Big]$$

IIB with O5/O9 orientifold

Finally

$$u_A \equiv \hat{u}_A\,, \qquad s \equiv \hat{s}$$

Seven complex scalar fields

$$S = s + i\sigma\,, \qquad T_A = t_A + i\tau_A\,, \qquad U_A = u_A + i\nu_A$$

Kähler potential

$$K = -\log(S + \overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) - \sum_{B=1}^{3} \log(U_B + \overline{U}_B)$$

Back

M-theory ($\to$ IIA)

$$\mathcal{R}: \quad (x^5, x^6, x^7, x^8, x^9, x^{10}, x^{11}) \to (-x^5, +x^6, -x^7, +x^8, -x^9, +x^{10}, -x^{11})$$

Combined orbifold actions								
	:	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}
$\mathcal{R}$	:	$-$	$+$	$-$	$+$	$-$	$+$	$-$
$\mathbb{Z}_2\mathcal{R}$	:	$+$	$-$	$+$	$-$	$-$	$+$	$-$
$\mathbb{Z}_2'\mathcal{R}$	:	$-$	$+$	$+$	$-$	$+$	$-$	$-$
$\mathbb{Z}_2\mathbb{Z}_2'\mathcal{R}$	:	$+$	$-$	$-$	$+$	$+$	$-$	$-$

M-theory ($\to$ IIA)

$\mathcal{R}$ action on the fields		
G	$\to$	$+G$
$A_{(3)}$	$\to$	$+A_{(3)}$

Fields in M-theory ($\to$ IIA)	
Metric moduli	$\hat{t}_A$, $\hat{u}_A$
$A_{(3)}$	A_{5611}, A_{5710}, A_{589}, A_{679}, A_{6810}, A_{7811}, A_{91011}

M-theory ($\longrightarrow$ IIA)

$$
\begin{aligned}
S &= \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\hat{s}^{-2}\,(\partial_\mu\hat{s})(\partial^\mu\hat{s}) + \\
&\quad - \frac{1}{4}G^{mn}G^{pq}\,(\partial_\mu G_{mp})(\partial^\mu G_{nq}) - \frac{1}{12}G^{mn}G^{pq}G^{rs}\,(\partial_\mu A_{mpr})(\partial^\mu A_{nqs})\Big]
\end{aligned}
$$

Axions:

$$
\tau_1 \equiv A_{5611}\,, \quad \tau_2 \equiv A_{7811}\,, \quad \tau_3 \equiv A_{91011}
$$

$$
\nu_1 \equiv -A_{5710}\,, \quad \nu_2 \equiv -A_{589}\,, \quad \nu_3 \equiv -A_{679}\,, \quad \sigma \equiv A_{6810}
$$

M-theory ($\longrightarrow$ IIA)

$$
\begin{aligned}
S \;=\; & \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Bigg[\widetilde{R}_4 - \frac{1}{2}\hat{s}^{-2}\,(\partial_\mu\hat{s})(\partial^\mu\hat{s}) - \frac{1}{4}\,\frac{(\partial_\mu\hat{v})(\partial^\mu\hat{v})}{\hat{v}^2} + \\
& -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu\hat{t}_A)(\partial^\mu\hat{t}_A)}{\hat{t}_A^2} + \frac{(\partial_\mu\hat{u}_A)(\partial^\mu\hat{u}_A)}{\hat{u}_A^2}\right] + \\
& -\frac{1}{2}\sum_{A=1}^{3}\frac{1}{\hat{t}_A^2\,\hat{v}}\,(\partial_\mu\tau_A)(\partial^\mu\tau_A) + \\
& -\frac{1}{2}\,\frac{1}{\hat{t}_1\hat{t}_2\hat{t}_3}\Bigg[\frac{\hat{u}_3}{\hat{u}_1\hat{u}_2}\,(\partial_\mu\nu_3)(\partial^\mu\nu_3) + \frac{\hat{u}_2}{\hat{u}_1\hat{u}_3}\,(\partial_\mu\nu_2)(\partial^\mu\nu_2) + \\
& + \frac{\hat{u}_1}{\hat{u}_2\hat{u}_3}\,(\partial_\mu\nu_1)(\partial^\mu\nu_1) + \hat{u}_1\hat{u}_2\hat{u}_3\,(\partial_\mu\sigma)(\partial^\mu\sigma)\Bigg]\Bigg]
\end{aligned}
$$

M-theory ($\to$ IIA)

Redefinitions: introduce r such that

$$r^{-1} \equiv \sqrt{\hat{v}}\hat{s}^{-1} \Longleftrightarrow \hat{s} = \sqrt{\hat{v}}r$$

and t_A such that

$$t_A^2 \equiv \hat{t}_A^2 \hat{v} \Longleftrightarrow t_A = \sqrt{\hat{v}}\hat{t}_A$$

Then

$$\hat{v} = r^{-\frac{2}{3}} \left(t_1 t_2 t_3\right)^{\frac{2}{3}}$$

$$\hat{s} = r^{\frac{2}{3}} \left(t_1 t_2 t_3\right)^{\frac{1}{3}}$$

M-theory ($\longrightarrow$ IIA)

$$
\begin{aligned}
S \quad = \quad & \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\widetilde{G}_4} \left[\widetilde{R}_4 - \frac{1}{2} r^{-2} (\partial_\mu r)(\partial^\mu r) + \right. \\
& - \frac{1}{2} \sum_{A=1}^{3} \left[\frac{(\partial_\mu t_A)(\partial^\mu t_A)}{t_A^2} + \frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A)}{\hat{u}_A^2} \right] + \\
& - \frac{1}{2} \sum_{A=1}^{3} \frac{1}{\hat{t}_A^2 \hat{v}} (\partial_\mu \tau_A)(\partial^\mu \tau_A) + \\
& - \frac{1}{2} \frac{1}{\hat{t}_1 \hat{t}_2 \hat{t}_3} \left[\frac{\hat{u}_3}{\hat{u}_1 \hat{u}_2} (\partial_\mu \nu_3)(\partial^\mu \nu_3) + \frac{\hat{u}_2}{\hat{u}_1 \hat{u}_3} (\partial_\mu \nu_2)(\partial^\mu \nu_2) + \right. \\
& \left. \left. + \frac{\hat{u}_1}{\hat{u}_2 \hat{u}_3} (\partial_\mu \nu_1)(\partial^\mu \nu_1) + \hat{u}_1 \hat{u}_2 \hat{u}_3 (\partial_\mu \sigma)(\partial^\mu \sigma) \right] \right]
\end{aligned}
$$

M-theory ($\to$ IIA)

Redefinitions:

$$\hat{u}_A = \sqrt{\frac{u_1 u_2 u_3}{s u_A^2}}, \quad r = \sqrt{s u_1 u_2 u_3}$$

Then

$$\begin{aligned} S &= \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\widetilde{G}_4} \Big[\widetilde{R}_4 - \frac{1}{2} \frac{(\partial_\mu s)(\partial^\mu s) + (\partial_\mu \sigma)(\partial^\mu \sigma)}{s^2} + \\ & - \frac{1}{2} \sum_{A=1}^{3} \left[\frac{(\partial_\mu u_A)(\partial^\mu u_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{u_A^2} \right] + \\ & - \frac{1}{2} \frac{(\partial_\mu t_A)(\partial^\mu t_A) + (\partial_\mu \tau_A)(\partial^\mu \tau_A)}{t_A^2} \Big] \end{aligned}$$

M-theory ($\rightarrow$ IIA)

$$\nu_A \equiv \hat{\nu}_A\,, \quad A = 1,2,3$$

Seven complex scalar fields

$$S = s + i\sigma\,, \quad T_A = t_A + i\tau_A\,, \quad U_A = u_A + i\nu_A$$

and Kähler potential

$$K = -\log(S+\overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) - \sum_{B=1}^{3} \log(U_A + \overline{U}_A)$$

Back

M-theory ($\to$ heterotic)

$$\mathcal{R}: \quad (x^5, x^6, x^7, x^8, x^9, x^{10}, x^{11}) \to (+x^5, +x^6, +x^7, +x^8, +x^9, +x^{10}, -x^{11})$$

Action on the coordinates								
	:	x^5	x^6	x^7	x^8	x^9	x^{10}	x^{11}
$\mathcal{R}$	:	$+$	$+$	$+$	$+$	$+$	$+$	$-$
$\mathbb{Z}_2\mathcal{R}$	:	$-$	$-$	$-$	$-$	$+$	$+$	$-$
$\mathbb{Z}_2'\mathcal{R}$	:	$+$	$+$	$-$	$-$	$-$	$-$	$-$
$\mathbb{Z}_2\mathbb{Z}_2'\mathcal{R}$	:	$-$	$-$	$+$	$+$	$-$	$-$	$-$

M-theory ($\to$ heterotic)

Invariance of the Chern-Simons term under the $\mathcal{R}$ projection calls for a non-trivial action on the fields:

Action on the fields		
G	$\to$	$+G$
$A_{(3)}$	$\to$	$-A_{(3)}$

Fields in M-theory ($\to$ heterotic)	
Metric moduli	$\hat{t}_A$, $\hat{u}_A$, $\hat{\nu}_A$
$A_{(3)}$	A_{5611}, A_{7811}, A_{91011}, $A_{\mu\nu 11} \leftrightarrow \sigma$

M-theory ($\rightarrow$ heterotic)

Axions:

$$\tau_1 \equiv A_{5611}\,, \quad \tau_2 \equiv A_{7811}\,, \quad \tau_3 \equiv A_{91011}$$

$$\begin{aligned} S \;=\; & \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\hat{s}^{-2}\,(\partial_\mu \hat{s})(\partial^\mu \hat{s}) - \frac{1}{4}\,\frac{(\partial_\mu \hat{v})(\partial^\mu \hat{v})}{\hat{v}^2} + \\ & -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu \hat{t}_A)(\partial^\mu \hat{t}_A)}{\hat{t}_A^2} + \frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{\hat{u}_A^2}\right] + \\ & -\frac{1}{2}\sum_{A=1}^{3}\frac{1}{\hat{t}_A^2\,\hat{v}}\,(\partial_\mu \tau_A)(\partial^\mu \tau_A) - \frac{1}{2}s^{-2}\,(\partial_\mu \sigma)(\partial^\mu \sigma)\Big] \end{aligned}$$

M-theory ($\rightarrow$ heterotic)

Introduce a field r such that

$$r^{-1} \equiv \sqrt{\hat{v}}\hat{s}^{-1} \Longleftrightarrow \hat{s} = \sqrt{\hat{v}}r$$

and t_A such that

$$t_A^2 \equiv \hat{t}_A^2\hat{v} \Longleftrightarrow t_A = \sqrt{\hat{v}}\hat{t}_A$$

Then

$$\hat{v} = r^{-\frac{2}{3}} \left(t_1 t_2 t_3\right)^{\frac{2}{3}}$$

$$\hat{s} = r^{\frac{2}{3}} \left(t_1 t_2 t_3\right)^{\frac{1}{3}}$$

M-theory ($\rightarrow$ heterotic)

$$
\begin{aligned}
S \quad = \quad & \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-\widetilde{G}_4}\,\Big[\widetilde{R}_4 - \frac{1}{2}\frac{(\partial_\mu r)(\partial^\mu r) + (\partial_\mu \sigma)(\partial^\mu \sigma)}{r^2} + \\
& - \frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu \hat{u}_A)(\partial^\mu \hat{u}_A) + (\partial_\mu \hat{\nu}_A)(\partial^\mu \hat{\nu}_A)}{\hat{u}_A^2}\right] + \\
& - \frac{1}{2}\frac{(\partial_\mu t_A)(\partial^\mu t_A) + (\partial_\mu \tau_A)(\partial^\mu \tau_A)}{t_A^2}\Big]
\end{aligned}
$$

M-theory ($\longrightarrow$ heterotic)

Finally:

$$u_A \equiv \hat{u}_A\,, \quad \nu_2 \equiv \hat{\nu}_A\,, \quad s \equiv r\,, \quad A = 1,2,3$$

$$S = s + i\sigma\,, \quad T_A = t_A + i\tau_A\,, \quad U_A = \hat{u}_A + i\hat{\nu}_A$$

Kähler potential

$$K = -\log(S + \overline{S}) - \sum_{A=1}^{3} \log(T_A + \overline{T}_A) - \sum_{B=1}^{3} (U_B + \overline{U}_B)$$

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Supergravity compactifications

N = 1 (heterotic) supergravity (without YM) ▸ Go

$\tau_1 \equiv B_{56}$
$\tau_2 \equiv B_{78}$
$\tau_3 \equiv B_{910}$
$s \equiv \hat{s}$
$t_A \equiv \hat{t}_A$
$u_A \equiv \hat{u}_A$
$\nu_A \equiv \hat{\nu}_A$

IIA with O6 orientifold ▸ Go

$\tau_1 \equiv B_{56}$
$\tau_2 \equiv B_{78}$
$\tau_3 \equiv B_{910}$
$\nu_1 \equiv -C_{679}$
$\nu_2 \equiv -C_{589}$
$\nu_3 \equiv -C_{5710}$
$\sigma \equiv C_{6810}$
$u_A = \sqrt{\frac{\hat{s}\hat{u}_1\hat{u}_2\hat{u}_3}{\hat{u}_A^2}}$
$s = \sqrt{\frac{\hat{s}}{\hat{u}_1\hat{u}_2\hat{u}_3}}$
$t_A \equiv \hat{t}_A$

IIB with O3/O7 orientifold ▸ Go

$\tau_1 \equiv C_{78910}$
$\tau_2 \equiv C_{56910}$
$\tau_3 \equiv C_{5678}$
$\nu_A \equiv \hat{\nu}_A$
$\sigma \equiv -C_{(0)}$
$t_A = \sqrt{\frac{\hat{s}\hat{t}_1\hat{t}_2\hat{t}_3}{\hat{t}_A^2}}$
$s = \sqrt{\frac{\hat{s}}{\hat{t}_1\hat{t}_2\hat{t}_3}}$
$u_A \equiv \hat{u}_A$
$s \equiv \hat{s}$

IIB with O5/O9 orientifold ▸ Go

$\tau_1 \equiv C_{56}$
$\tau_2 \equiv C_{78}$
$\tau_3 \equiv C_{910}$
$\nu_A \equiv \hat{\nu}_A$
σ
$t_A = \sqrt{\frac{\hat{s}\hat{t}_A^2}{\hat{t}_1\hat{t}_2\hat{t}_3}}$
$s = \sqrt{\hat{s}\hat{t}_1\hat{t}_2\hat{t}_3}$
$u_A \equiv \hat{u}_A$
$s \equiv \hat{s}$

M-theory (→ IIA) ▸ Go

$\tau_1 \equiv A_{5611}$
$\tau_2 \equiv A_{7811}$
$\tau_3 \equiv A_{91011}$
$\nu_1 \equiv -A_{5710}$
$\nu_2 \equiv -A_{589}$
$\nu_3 \equiv -A_{679}$
$\sigma \equiv A_{6810}$
$r^{-1} \equiv \sqrt{\hat{\nu}}\hat{s}^{-1}$
$t_A^2 \equiv \hat{t}_A^2\hat{\nu}$
$\hat{\nu} = r^{-\frac{2}{3}}\ (t_1t_2t_3)^{\frac{2}{3}}$
$\hat{s} = r^{\frac{2}{3}}\ (t_1t_2t_3)^{\frac{1}{3}}$
$\hat{u}_A = \sqrt{\frac{u_1u_2u_3}{su_A^2}}$
$r = \sqrt{su_1u_2u_3}$
$\nu_A \equiv \hat{\nu}_A$

M-theory (→ heterotic) ▸ Go

$\tau_1 \equiv A_{5611}$
$\tau_2 \equiv A_{7811}$
$\tau_3 \equiv A_{91011}$
$r^{-1} \equiv \sqrt{\hat{\nu}}\hat{s}^{-1}$
$t_A^2 \equiv \hat{t}_A^2\hat{\nu}$
$\hat{\nu} = r^{-\frac{2}{3}}\ (t_1t_2t_3)^{\frac{2}{3}}$
$\hat{s} = r^{\frac{2}{3}}\ (t_1t_2t_3)^{\frac{1}{3}}$
$u_A \equiv \hat{u}_A$
$\nu_2 \equiv \hat{\nu}_A$
$s \equiv r$

▸ Back

Orbifold compactifications

After some non-linear re-definitions

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-\widetilde{G}_4}\Big[\widetilde{R}_4 - \frac{1}{2}\frac{(\partial_\mu s)(\partial^\mu s)+(\partial_\mu\sigma)(\partial^\mu\sigma)}{s^2} + \\ -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu u_A)(\partial^\mu u_A)+(\partial_\mu\nu_A)(\partial^\mu\nu_A)}{u_A^2}\right] + \\ -\frac{1}{2}\sum_{A=1}^{3}\left[\frac{(\partial_\mu\hat{t}_A)(\partial^\mu\hat{t}_A)+(\partial_\mu\tau_A)(\partial^\mu\tau_A)}{\hat{t}_A^2}\right]\Big]$$

Orbifold compactifications

The reduced action describes, besides the supergravity multiplet, seven complex scalar fields:

- $S = s + i\sigma$
- $T_A = t_A + i\tau_A$
- $U_B = u_B + i\nu_B$

$(A, B = 1, 2, 3)$

IIB with O3/O7 orientifold

- No geometric fluxes
- 8 3-form fluxes from the NSNS sector,

$$H_{mnr} : H_{579},\ H_{5710},\ H_{589},\ H_{5810},\ H_{679},\ H_{6710},\ H_{689},\ H_{6810}$$

- 8 3-form fluxes from the NSNS sector

$$F_{(3)} : F_{579},\ F_{5710},\ F_{589},\ F_{5810},\ F_{679},\ F_{6710},\ F_{689},\ F_{6810}$$

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24 geometric fluxes

$$\omega_{nr}{}^{m}:\quad \begin{matrix} \omega_{79}{}^{5} & \omega_{95}{}^{7} & \omega_{57}{}^{9} \\ \omega_{710}{}^{5} & \omega_{105}{}^{7} & \omega_{57}{}^{10} \\ \omega_{89}{}^{5} & \omega_{95}{}^{8} & \omega_{58}{}^{9} \\ \omega_{810}{}^{5} & \omega_{105}{}^{8} & \omega_{58}{}^{10} \\ \omega_{79}{}^{6} & \omega_{96}{}^{7} & \omega_{67}{}^{9} \\ \omega_{710}{}^{6} & \omega_{106}{}^{7} & \omega_{67}{}^{10} \\ \omega_{89}{}^{6} & \omega_{96}{}^{8} & \omega_{68}{}^{9} \\ \omega_{810}{}^{6} & \omega_{106}{}^{8} & \omega_{68}{}^{10} \end{matrix} \tag{8}$$

8 3-form fluxes from the RR sector:

$$F_{(3)}:\ F_{579},\ F_{5710},\ F_{589},\ F_{5810},\ F_{679},\ F_{6710},\ F_{689},\ F_{6810}$$

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