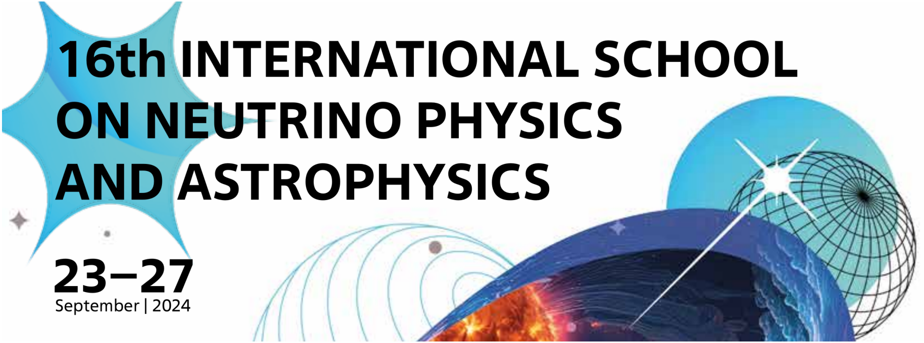
The Gallium Anomaly

Carlo Giunti

INFN, Torino, Italy



Standard Three Neutrino Mixing Paradigm

- Supported by robust, abundant, and consistent solar, atmospheric and long-baseline (accelerator and reactor) neutrino oscillation data.
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions.
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector.
- Neutrino Mixing: a Flavor Neutrino is a superposition of Massive Neutrinos:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{e2}^* & U_{e3}^* \\ U_{\mu 1}^* & U_{\mu 2}^* & U_{\mu 3}^* \\ U_{\tau 1}^* & U_{\tau 2}^* & U_{\tau 3}^* \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

- U is the 3×3 unitary Neutrino Mixing Matrix.
- $P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \qquad (\alpha, \beta = e, \mu, \tau)$
- The oscillation probabilities depend on:

U (osc. amplitude) and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ (osc. phase)

Three-Neutrino Mixing Parameters

Standard Parameterization of Mixing Matrix (as CKM)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

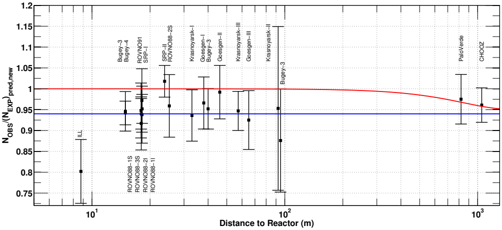
OSCILLATION PARAMETERS:
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\iff$ $|\Delta L| = 2$ processes

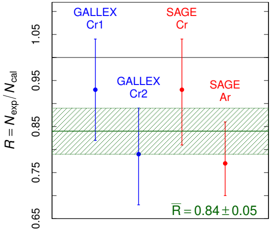
- In the standard 3ν mixing paradigm there are two independent Δm^2's:
  - $\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$ Solar Mass Splitting
  - $\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$ Atmospheric Mass Splitting
- The solar and atmospheric mass splittings generate oscillations that are detectable at the distances
  - $L^{\text{osc}}_{\text{SOL}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{SOL}}} \approx 50\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
  - $L^{\text{osc}}_{\text{ATM}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{ATM}}} \approx 1\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
- The solar and atmospheric mass splittings cannot explain neutrino oscillations at shorter distances.
- A neutrino oscillation explanation of short-baseline anomalies needs the existence of larger Δm^2's.

Historical Short-Baseline Anomalies

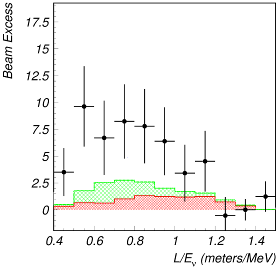
2011 Reactor Anomaly: $\bar{\nu}_e \to \bar{\nu}_x$ ($\approx 2.5\sigma$)



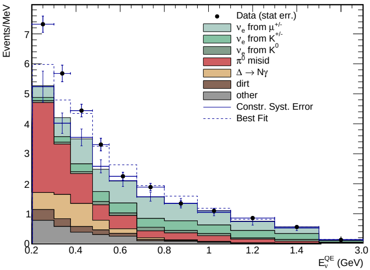
2005 Gallium Anomaly: $\nu_e \to \nu_x$ ($\approx 2.9\sigma$)



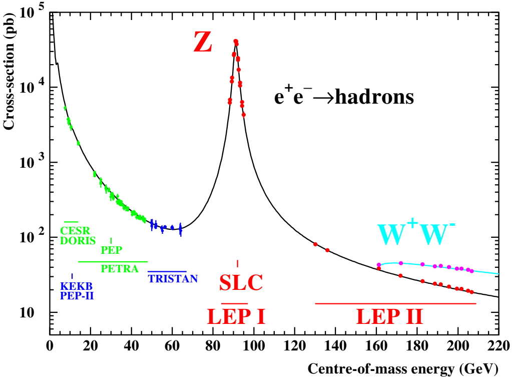
1995 LSND Anomaly: $\bar{\nu}_\mu \to \bar{\nu}_e$ ($\sim 4\sigma$)

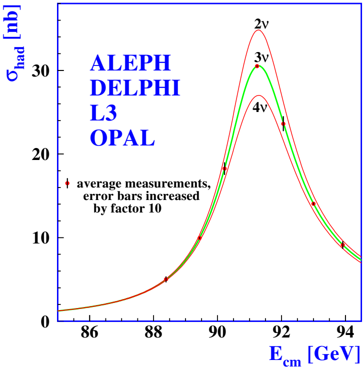


2008 MiniBooNE Anomaly: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ (4.8σ)



Number of Flavor and Massive Neutrinos?





[LEP, hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

Improved cross section: $N_\nu = 2.9975 \pm 0.0074$ [Janot, Jadach, arXiv:1912.02067]

$$e^+e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \; \nu_\mu \; \nu_\tau$$

- We know that there are 3 light active flavor neutrinos
- We do not know the number of massive neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

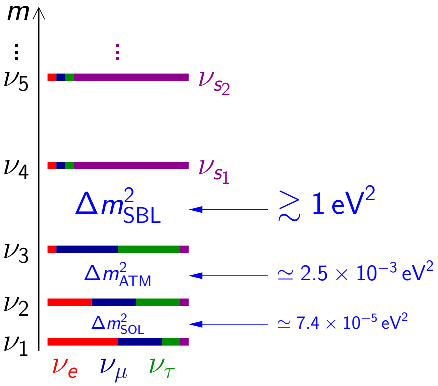
$N \geq 3$
no upper limit!

U: unitary $N \times N$ mixing matrix

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots$$

Light Sterile Neutrinos



$$L^{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2}$$

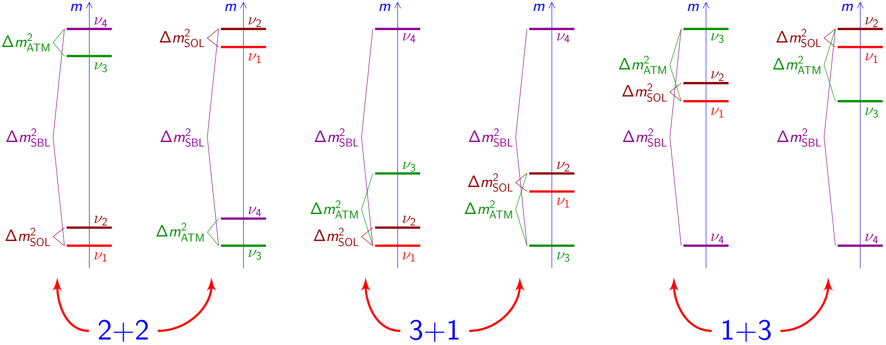
Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

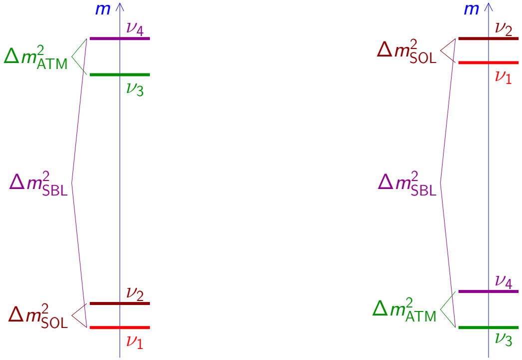
- Minimal perturbation of successfull 3ν mixing:

  effective 4ν mixing with $|U_{e4}|, |U_{\mu 4}|, |U_{\tau 4}| \ll 1$

Four-Neutrino Schemes: 2+2, 3+1 and 1+3



2+2 Four-Neutrino Schemes

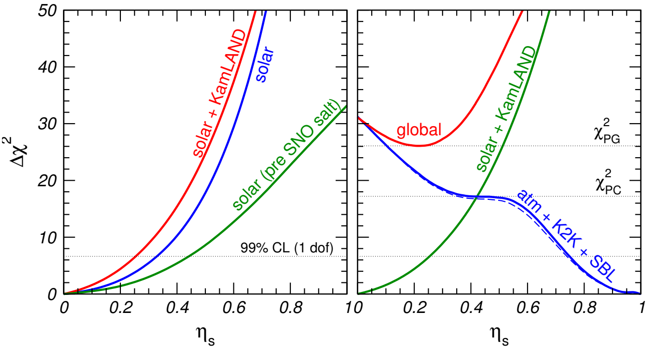


- After LSND (1995) 2+2 was preferred to 3+1, because of the 3+1 appearance-disappearance tension

[Okada, Yasuda, hep-ph/9606411; Bilenky, CG, Grimus, hep-ph/9607372]

- This is not a perturbation of 3-ν Mixing $\Longrightarrow$ Large active–sterile oscillations for solar or atmospheric neutrinos!

2+2 Schemes are Strongly Disfavored



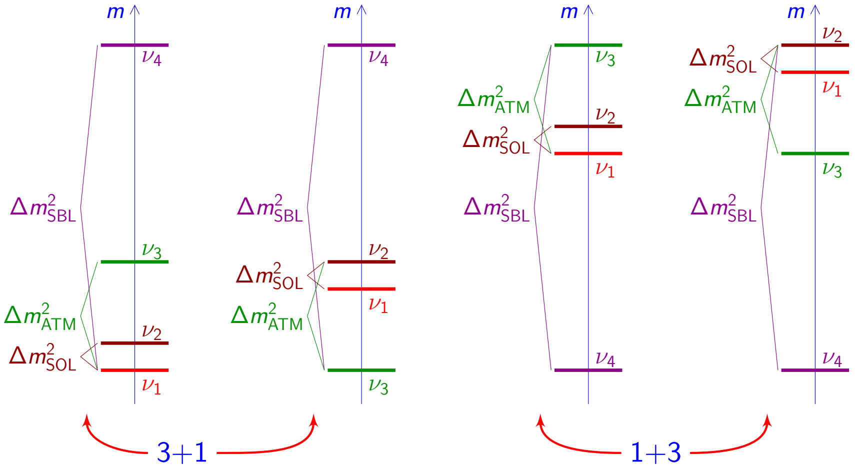
Solar: Matter Effects + SNO NC   Atmospheric: Matter Effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 = 1 - |U_{s3}|^2 + |U_{s4}|^2$$

99% CL: $\begin{cases} \eta_s < 0.25 & \text{(Solar + KamLAND)} \\ \eta_s > 0.75 & \text{(Atmospheric + K2K)} \end{cases}$

[Maltoni, Schwetz, Tortola, Valle, hep-ph/0405172]

3+1 and 1+3 Four-Neutrino Schemes



- Perturbation of 3-ν Mixing: $|U_{e4}|^2, |U_{\mu4}|^2, |U_{\tau4}|^2 \ll 1 \quad |U_{s4}|^2 \simeq 1$
- 1+3 schemes are disfavored by cosmology (ΛCDM):

  $\sum_{k=1}^{3} m_k \lesssim 0.12\,\text{eV}$ (95% CL) [Planck, arXiv:1807.06209]

Effective 3+1 SBL Oscillation Probabilities

$$|\nu_\alpha\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} |\nu_k\rangle \quad \xrightarrow{t} \quad |\nu_\alpha(t)\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} e^{-iE_k t} |\nu_k\rangle$$

$$A_{\nu_\alpha \to \nu_\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \qquad (\langle \nu_\beta | \nu_k \rangle = U_{\beta k})$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 = \left| e^{iE_1 t} \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2$$

$$= \underbrace{\left| e^{iE_1 t} \right|^2}_{1} \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2$$

$$= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2$$

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} \quad \Longrightarrow \quad E_k - E_1 \simeq \frac{\Delta m_{k1}^2}{2p}$$

$$E = p \qquad\qquad t \simeq L$$

$$P_{\nu_\alpha \to \nu_\beta} \simeq \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} \exp\left(-i \frac{\Delta m_{21}^2 L}{2E} \right) \right.$$

$$\left. + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(-i \frac{\Delta m_{41}^2 L}{2E} \right) \right|^2$$

SBL $\Longrightarrow$ $\frac{\Delta m_{21}^2 L}{2E} \ll 1 \qquad \frac{\Delta m_{31}^2 L}{2E} \ll 1$

$$P_{\nu_\alpha \to \nu_\beta}^{\text{SBL}} \simeq \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} + U_{\alpha 4}^{*} U_{\beta 4} \exp\left(-i \frac{\Delta m_{41}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^{*} U_{\beta 4}$$

$$P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq \left|\delta_{\alpha\beta} - U^*_{\alpha 4} U_{\beta 4}\left[1-\exp\left(-i\frac{\Delta m^2_{41}L}{2E}\right)\right]\right|^2$$

$$= \delta_{\alpha\beta} + |U_{\alpha 4}|^2 |U_{\beta 4}|^2 \left(2 - 2\cos\frac{\Delta m^2_{41}L}{2E}\right)$$

$$- 2\delta_{\alpha\beta}|U_{\alpha 4}|^2 \left(1 - \cos\frac{\Delta m^2_{41}L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 2|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right)\left(1 - \cos\frac{\Delta m^2_{41}L}{2E}\right)$$

$$= \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right)\sin^2\frac{\Delta m^2_{41}L}{4E}$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \sin^2\left(\frac{\Delta m^2_{41}L}{4E}\right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P^{\text{SBL}}_{\nu_\alpha\to\nu_\alpha} \simeq 1 - 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)\sin^2\left(\frac{\Delta m^2_{41}L}{4E}\right)$$

Appearance ($\alpha \neq \beta$)

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 3+3 mixing angles
- 1+2 Dirac CPV phases
- 2+1 Majorana CPV phases

- $\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$
- CP violation is not observable in SBL experiments!
- CPV is observable in LBL accelerator experiments sensitive to Δm^2_{ATM} [de Gouvea et al, arXiv:1412.1479, 1507.03986, 1605.09376; Palazzo et al, arXiv:1412.7524, 1509.03148; Kayser et al, arXiv:1508.06275, 1607.02152] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, arXiv:1304.2207]

Common Convenient Parameterization of 4ν Mixing

$$U = \left[W^{34} R^{24} W^{14} R^{23} W^{13} R^{12}\right] \mathrm{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}, e^{i\lambda_{41}}\right)$$

$$= \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ \cdots & \cdots & \cdots & c_{14}s_{24} \\ \cdots & \cdots & \cdots & c_{14}c_{24}s_{34}e^{-i\delta_{34}} \\ \cdots & \cdots & \cdots & c_{14}c_{24}c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 & 0 \\ 0 & 0 & e^{i\lambda_{31}} & 0 \\ 0 & 0 & 0 & e^{i\lambda_{41}} \end{pmatrix}$$

$$|U_{e4}|^2 = \sin^2\vartheta_{14} \quad \Rightarrow \quad \sin^2 2\vartheta_{ee} = 4|U_{e4}|^2\left(1 - |U_{e4}|^2\right) = \sin^2 2\vartheta_{14}$$

$$|U_{\mu4}|^2 = \cos^2\vartheta_{14}\sin^2\vartheta_{24} \simeq \sin^2\vartheta_{24} \Rightarrow \sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2\left(1 - |U_{\mu4}|^2\right) \simeq \sin^2 2\vartheta_{24}$$

Effective short-baseline survival probability of ν_e (Gallium) and $\bar{\nu}_e$ (reactor):

$$P_{ee}^{\text{SBL}} \simeq 1 - \sin^2 2\vartheta_{ee} \sin^2\left(\frac{\Delta m_{41}^2 L}{4E}\right)$$

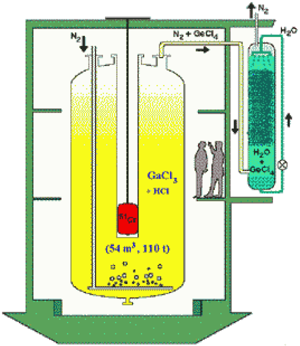
with different notations in the literature:

$$\vartheta_{ee} = \vartheta_{14} = \vartheta_{\text{new}} = \vartheta$$

and

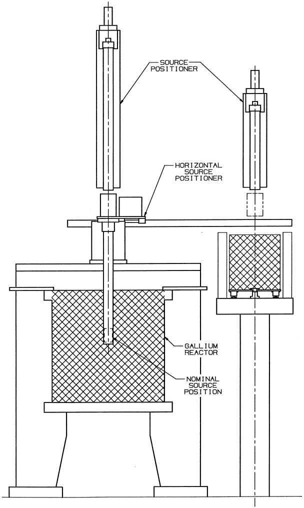
$$\Delta m_{41}^2 = \Delta m_{\text{SBL}}^2 = \Delta m_{\text{new}}^2 = \Delta m^2$$

Gallium Radioactive Source Experiments



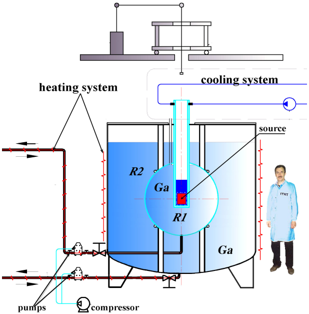
GALLEX

$\langle L \rangle_{\text{GALLEX}} \simeq 1.9\,\text{m}$



SAGE

$\langle L \rangle_{\text{SAGE}} \simeq 0.6\,\text{m}$

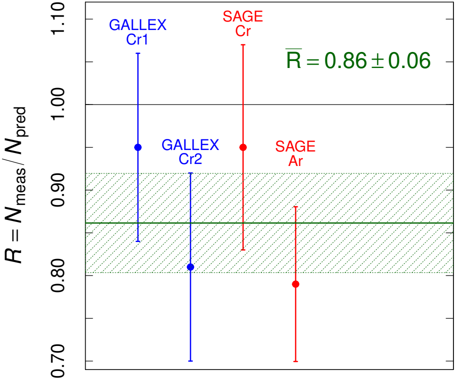


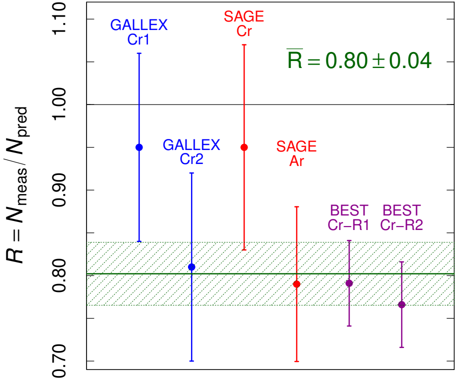
BEST

$\langle L \rangle^{\text{R1}}_{\text{BEST}} \simeq 0.7\,\text{m}$

$\langle L \rangle^{\text{R2}}_{\text{BEST}} \simeq 1.1\,\text{m}$

Gallium Anomaly



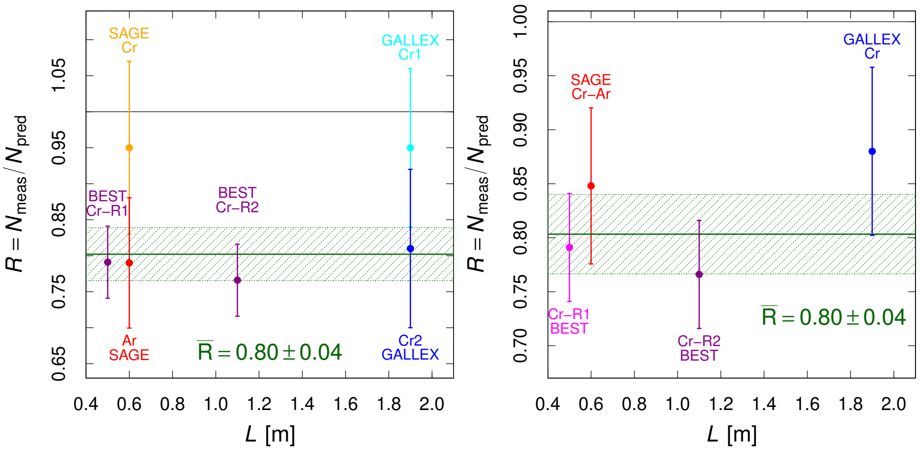


2005: $\approx 2.3\sigma$ deficit: Mild Anomaly!

2021: $\approx 5\sigma$ deficit: Strong Anomaly!

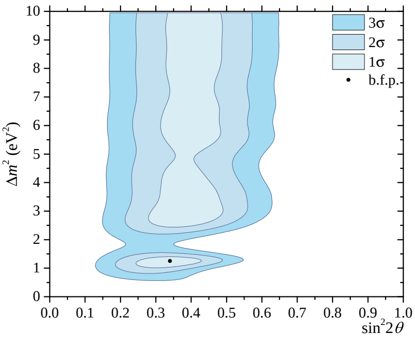
SAGE: nucl-ex/0512041, arXiv:0901.2200;
Laveder et al: NPPS 168 (2007) 344, arXiv:hep-ph/0610352, arXiv:0711.4222, 1006.3244, 1507.08204;
Kostensalo et al, arXiv:1906.10980

BEST: arXiv:2109.11482, 2109.14654, 2201.07364;
Barinov and Gorbunov: arXiv:2109.14654;
Huber et al: arXiv:2111.12530, 2209.02885;
Brdar et al: arXiv:2303.05528; Banks et al: arXiv:2311.06352;
Schwetz et al: arXiv:2303.15524, 306.09422;
Elliott et al: arXiv:2303.13623, 2306.03299;
CG et al: arXiv:2209.00916, 2212.09722, 2312.00565



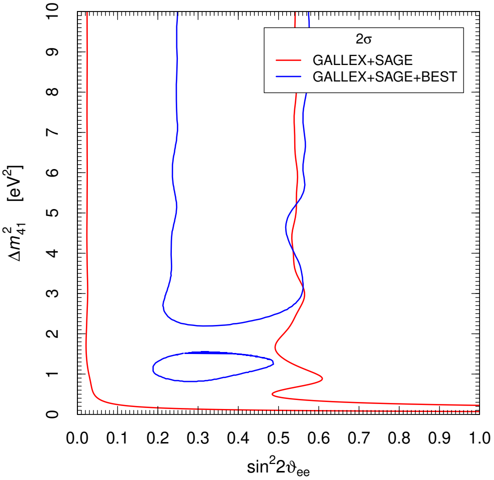
- No clear model-independent anomaly from different path lengths.
- After the BEST measurements, the Gallium Anomaly is still an anomaly based on the absolute comparison of observed and predicted rates.

3+1 Active-Sterile Neutrino Oscillations



[BEST, arXiv:2109.11482]

GALLEX+SAGE+BEST
with Bahcall cross section



Ingredients of the Gallium Anomaly

- The ν_e flux.
- The ν_e propagation probability.
- The $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ detection cross section.
- The radiochemical ${}^{71}\text{Ge}$ extraction efficency.

ν_e Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\quad$ $e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

$E \simeq 0.75\,\text{MeV}$ and $E \simeq 0.43\,\text{MeV}$ $\quad$ $E \simeq 0.81\,\text{MeV}$

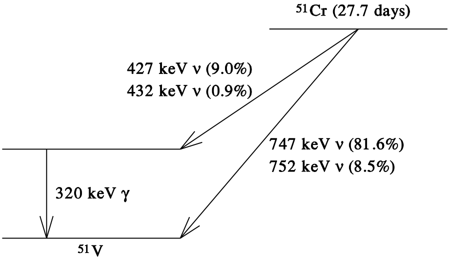
$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$

ν_e Detection

The ν_e flux

The neutrino flux is mainly evaluated by calorimetric measurements of the activity of the sources:

^{51}Cr Source [SAGE, hep-ph/9803418]



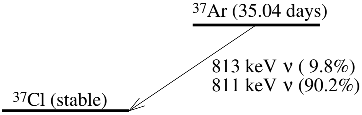
Energy release per decay:

$320\,\text{keV} \times 0.099 \simeq 31.7\,\text{keV}$

With atomic energy release:

$36.750 \pm 0.084\,\text{keV}$ [BEST, 2015]

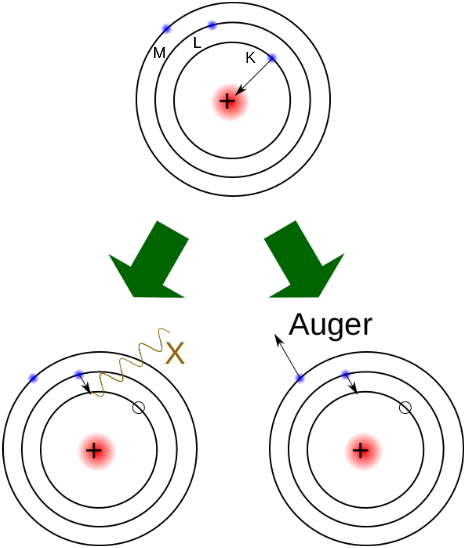
^{37}Ar Source [SAGE, nucl-ex/0512041]



Atomic energy release:

$2.751 \pm 0.021\,\text{keV}$

[SAGE, nucl-ex/0512041]


M
L
K
X
Auger

^{51}Cr Source [BEST, 2015] $T_{1/2}(^{51}\text{Cr}) = 27.704 \pm 0.003\,\text{d}$

Type of radiation	Energy, keV	Fraction of ^{51}Cr decays	Energy release per ^{51}Cr decay, keV
Gamma radiation	320.0835 (4)	0.0991 (2)	31.720 (64)
K capture	5.465	0.8919 (17)	4.874 (9)
L capture	0.628	0.0927 (14)	0.0582 (9)
M capture	0.067	0.0154	0.001
Int. bremss.	751 (max)	$3.8 \times 10^{-4} \times 0.902$ ($\pm 10\%$)	0.096 (10)
Int. bremss.	430 (max)	$1.2 \times 10^{-4} \times 0.0983$ ($\pm 10\%$)	0.001
TOTAL			**36.750 (84)**

^{37}Ar Source [SAGE, nucl-ex/0512041] $T_{1/2}(^{37}\text{Ar}) = 35.01 \pm 0.02\,\text{d}$

Decay mode	Atomic energy release (keV)	Fraction of ^{37}Ar decays	Energy per ^{37}Ar decay (keV)
K capture	2.8224	0.9017 ± 0.0024	2.5450 ± 0.0068
L capture	0.2702	0.0890 ± 0.0027	0.0240 ± 0.0007
M capture	0.0175	$0.0093^{+0.0006}_{-0.0004}$	0.0002
1*s* int. brems.	325 (average)	$\sim$0.0005	$\sim 0.16 \pm 0.02$
2*s* int. brems.	325 (average)	$\sim$0.00007	$\sim 0.021 \pm 0.002$
p int. brems.	$\sim$10 (average)	$\sim$0.00007	$\sim$ 0.0007
Total			2.751 ± 0.021

Well measured radioactivities of the sources $(1\,\text{Ci} = 3.7 \times 10^{10}\,\text{Bq})$:

- GALLEX Cr1 [PLB 1995] : $1.714^{+0.030}_{-0.043}\,\text{MCi} = 63.4^{+1.1}_{-1.6}\,\text{PBq}$ $(\approx 2.5\%\,\text{unc.})$

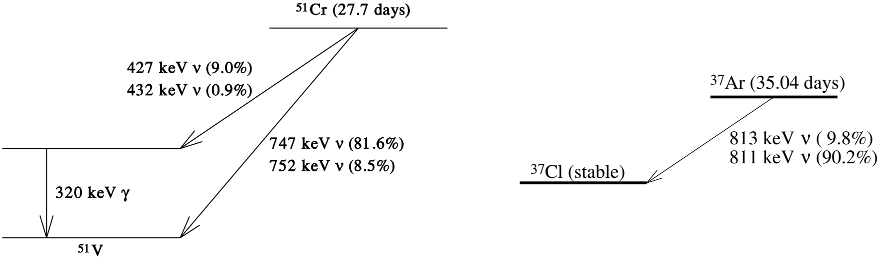
- GALLEX Cr2 [PLB 1998] : $1.868^{+0.090}_{-0.057}\,\text{MCi} = 69.1^{+3.3}_{-2.1}\,\text{PBq}$ $(\approx 3\%\,\text{unc.})$

- SAGE Cr [hep-ph/9803418] : $0.5166 \pm 0.0060\,\text{MCi} = 19.11 \pm 0.22\,\text{PBq}$ $(\approx 1.2\%\,\text{unc.})$

- SAGE Ar [nucl-ex/0512041] : $0.409 \pm 0.002\,\text{MCi} = 15.1 \pm 0.7\,\text{PBq}$ $(\approx 4.6\%\,\text{unc.})$

- BEST Cr [arXiv:2109.11482] : $3.414 \pm 0.008\,\text{MCi} = 12.632 \pm 0.030\,\text{PBq}$ $(\approx 0.3\%\,\text{unc.})$

ν_e propagation probability



	^{51}Cr				^{37}Ar	
E [keV]	747	752	427	432	811	813
B.R.	0.8163	0.0849	0.0895	0.00930	0.9020	0.098
σ [10^{-46} cm^2]	62.58	63.22	26.72	27.14	71.35	71.63

[Barinov, Cleveland, Gavrin, Gorbunov, Ibragimova, arXiv:1710.06326]

$$\langle P_{\nu_e\to\nu_e}\rangle = \frac{\int \mathrm{d}V\, L^{-2} \sum_i (\mathrm{B.R.})_i\, \sigma_i\, P_{\nu_e\to\nu_e}(L, E_i)}{\sum_i (\mathrm{B.R.})_i\, \sigma_i \int \mathrm{d}V\, L^{-2}}$$

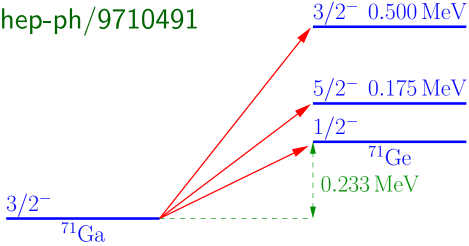
[Acero, CG, Laveder, arXiv:0711.4222]

The detection cross section

- Deficit could be due to an overestimate of

$$\sigma(\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-)$$

- First calculation: Bahcall, hep-ph/9710491



- $\sigma_{\text{G.S.}}$ from $T_{1/2}({}^{71}\text{Ge}) = 11.43 \pm 0.03\,\text{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\text{G.S.}}({}^{51}\text{Cr}) = (5.54 \pm 0.02) \times 10^{-45}\,\text{cm}^2$$

- $\sigma({}^{51}\text{Cr}) = \sigma_{\text{G.S.}}({}^{51}\text{Cr}) \left(1 + 0.669\,\frac{\text{BGT}_{175}}{\text{BGT}_{\text{G.S.}}} + 0.220\,\frac{\text{BGT}_{500}}{\text{BGT}_{\text{G.S.}}}\right)$

- The contribution of excited states is only $\sim 5\%$! [Bahcall, hep-ph/9710491]

$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ cross sections in units of $10^{-45}\,\text{cm}^2$:

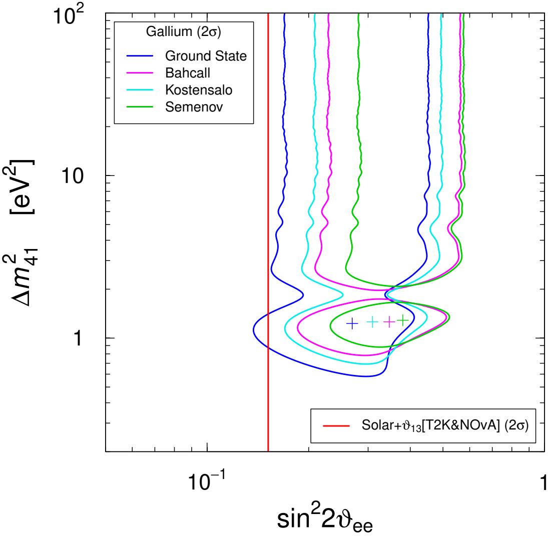
Model	Method	^{51}Cr σ_{tot}	^{51}Cr δ_{exc}	^{37}Ar σ_{tot}	^{37}Ar δ_{exc}
Ground State [Semenov, Phys.Atom.Nucl. 83 (2020) 1549]	$T_{1/2}(^{71}\text{Ge})$	5.539 ± 0.019	–	6.625 ± 0.023	–
Bahcall (1997) [hep-ph/9710491]	$^{71}\text{Ga}(p,n)^{71}\text{Ge}$	5.81 ± 0.16	4.7%	7.00 ± 0.21	5.4%
Haxton (1998) [nucl-th/9804011]	Shell Model	6.39 ± 0.65	13.3%	7.72 ± 0.81	14.2%
Frekers et al. (2015) [PRC 91 (2015) 034608]	$^{71}\text{Ga}(^3\text{He},{}^3\text{H})^{71}\text{Ge}$	5.92 ± 0.11	6.4%	7.15 ± 0.14	7.3%
Barinov et al. (2018) [arXiv:1710.06326]	$^{71}\text{Ga}(^3\text{He},{}^3\text{H})^{71}\text{Ge}$	5.910 ± 0.114	$\approx 6\%$	7.138 ± 0.146	$\approx 7\%$
Kostensalo et al. (2019) [arXiv:1906.10980]	Shell Model	5.67 ± 0.06	2.3%	6.80 ± 0.08	2.6%
Semenov (2020) [Phys.Atom.Nucl. 83 (2020) 1549]	$^{71}\text{Ga}(^3\text{He},{}^3\text{H})^{71}\text{Ge}$	5.938 ± 0.116	6.7%	7.169 ± 0.147	7.6%
Elliott et al. (2023) [arXiv:2303.13623]	$^{71}\text{Ga}(p,n)^{71}\text{Ge} + \text{SM}$	$5.69^{+0.28}_{-0.06}$	5.3%	$6.85^{+0.35}_{-0.08}$	5.8%

Size of the Gallium Anomaly

Model	Method	$\overline{R}$	GA
Ground State [Semenov, Phys.Atom.Nucl. 83 (2020) 1549]	$T_{1/2}(^{71}\mathrm{Ge})$	0.84 ± 0.03	5.0σ
Bahcall (1997) [hep-ph/9710491]	$^{71}\mathrm{Ga}(p,n)^{71}\mathrm{Ge}$	0.80 ± 0.04	5.2σ
Haxton (1998) [nucl-th/9804011]	Shell Model	0.73 ± 0.09	5.1σ
Frekers et al. (2015) [PRC 91 (2015) 034608]	$^{71}\mathrm{Ga}(^{3}\mathrm{He},{}^{3}\mathrm{H})^{71}\mathrm{Ge}$	0.79 ± 0.03	6.1σ
Barinov et al. (2018) [arXiv:1710.06326]	$^{71}\mathrm{Ga}(^{3}\mathrm{He},{}^{3}\mathrm{H})^{71}\mathrm{Ge}$	0.79 ± 0.03	6.0σ
Kostensalo et al. (2019) [arXiv:1906.10980]	Shell Model	0.82 ± 0.03	5.5σ
Semenov (2020) [Phys.Atom.Nucl. 83 (2020) 1549]	$^{71}\mathrm{Ga}(^{3}\mathrm{He},{}^{3}\mathrm{H})^{71}\mathrm{Ge}$	0.79 ± 0.03	6.1σ
Elliott et al. (2023) [arXiv:2303.13623]	$^{71}\mathrm{Ga}(p,n)^{71}\mathrm{Ge} + \mathrm{SM}$	0.82 ± 0.03	5.6σ

[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916; CG, Li, Ternes, Xin, arXiv:2212.09722]

3+1: Strong tension with the solar neutrino bound



	Solar neutrinos + ϑ_{13}[T2K&NOvA] $\Delta\chi^2_{PG}$	GoF_{PG}
Ground State	10.65	0.49%
Bahcall	14.14	0.085%
Kostensalo	12.79	0.17%
Semenov	17.24	0.018%

[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916]

- Both Gallium and solar experiments detect ν_e
- No CPT-violating solution of the tension!

[see also: Goldhagen, Maltoni, Reichard, Schwetz, arXiv:2109.14898; Berryman, Coloma, Huber, Schwetz, Zhou, arXiv:2111.12530]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

$$N_{\text{D}} = \text{Number of Data} \qquad N_{\text{P}} = \text{Number of Fitted Parameters}$$

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF}$ $\qquad \text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, dz$ $\qquad p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020 (arXiv:hep-ph/0304176)

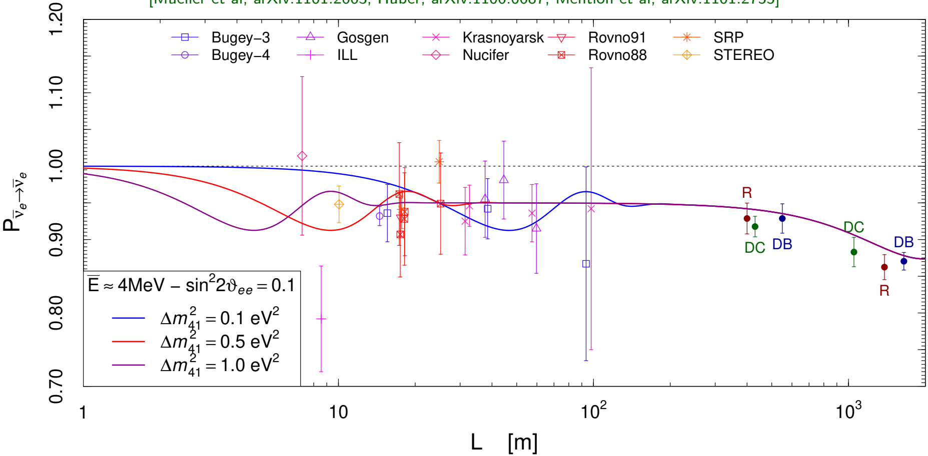
- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, dz$

Reactor Electron Antineutrino Anomaly

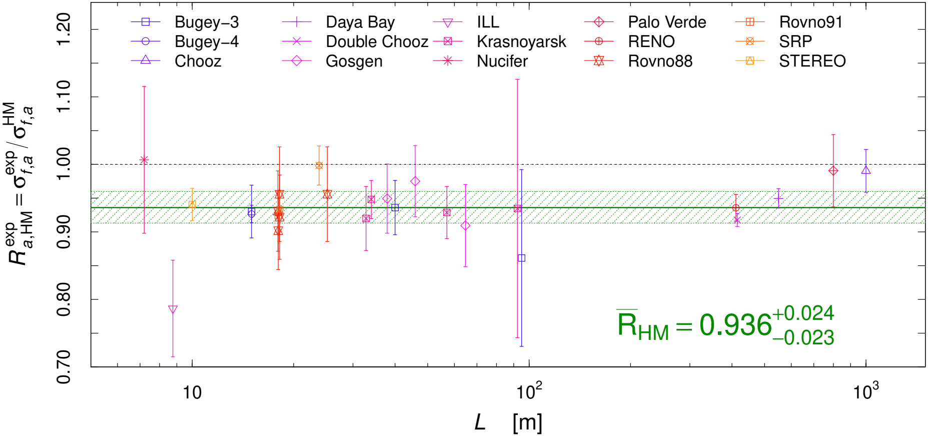
[Mueller et al, arXiv:1101.2663, Huber, arXiv:1106.0687, Mention et al, arXiv:1101.2755]



- $\Delta m^2_{\text{RAA}} = \Delta m^2_{41} \gtrsim 0.5\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$
- The Reactor Antineutrino Anomaly is model dependent: it depends on the reactor antineutrino flux calculation.

2011: HM fluxes (conversion method)

[Mueller et al, arXiv:1101.2663, Huber, arXiv:1106.0687]



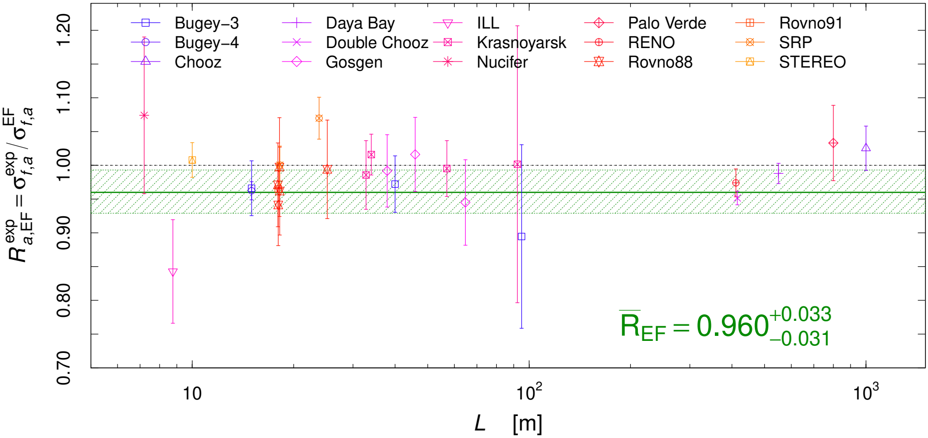
$2.5\,\sigma$ deficit $\Longrightarrow$ Anomaly! [CG, Li, Ternes, Xin, arXiv:2110.06820]

- Original 2011 Reactor Antineutrino Anomaly:

$$\overline{R}^{(2011)}_{HM} = 0.943 \pm 0.023 \Longrightarrow \approx 2.5\sigma$$ [Mention et al, arXiv:1101.2755]

2019: EF fluxes (summation method)

[Estienne, Fallot, et al, arXiv:1904.09358]



$1.2\,\sigma$ deficit $\Longrightarrow$ No Anomaly! [CG, Li, Ternes, Xin, arXiv:2110.06820]

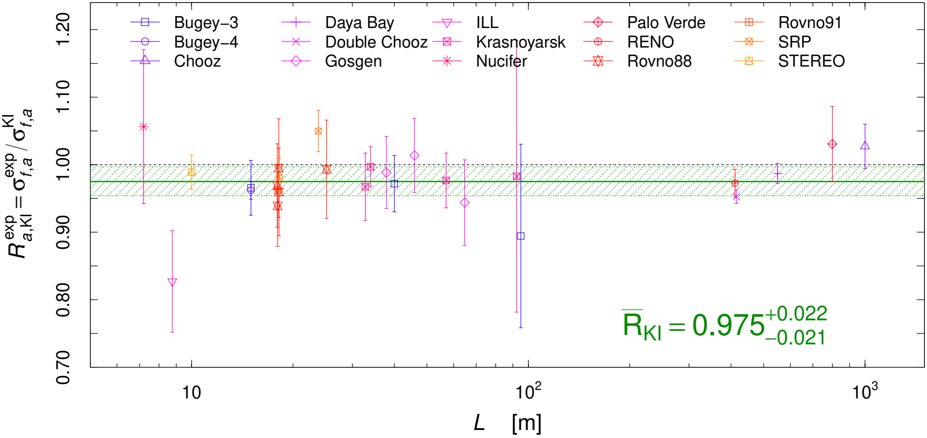
[See also: Berryman, Huber, arXiv:1909.09267, 2005.01756]

- Unknown Uncertainties! Rough estimation used in our calculations: 5% for ^{235}U, ^{239}Pu, ^{241}Pu and 10% for ^{238}U [Hayes et al., arXiv:1707.07728]

2021: KI fluxes (conversion method)

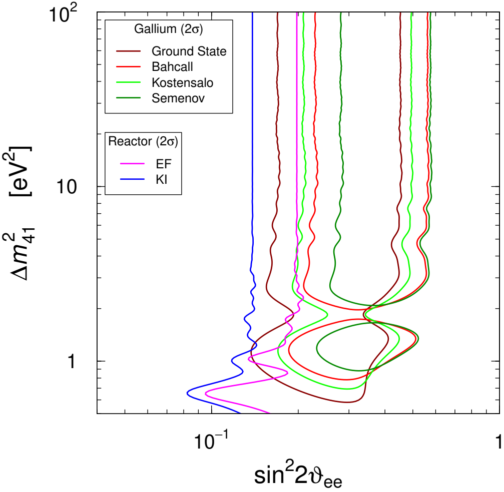
[Kurchatov Institute: Kopeikin, Skorokhvatov, Titov, arXiv:2103.01684]

$$\left(S^{(e)}_{235}/S^{(e)}_{239}\right)_{\text{ILL}} = (1.054 \pm 0.002)\left(S^{(e)}_{235}/S^{(e)}_{239}\right)_{\text{KI}}$$



$1.1\,\sigma$ deficit $\Longrightarrow$ No Anomaly! [CG, Li, Ternes, Xin, arXiv:2110.06820]

Gallium – Reactor Rates tension



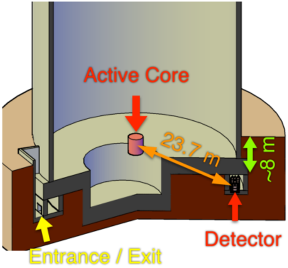
	EF		KI	
	$\Delta\chi^2_{\text{PG}}$	GoF_{PG}	$\Delta\chi^2_{\text{PG}}$	GoF_{PG}
Ground State	9.1	1.1%	11.9	0.26%
Bahcall	12.9	0.16%	16.3	0.029%
Kostensalo	11.5	0.31%	15.3	0.049%
Semenov	17.0	0.02%	22.5	0.0013%

[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916]

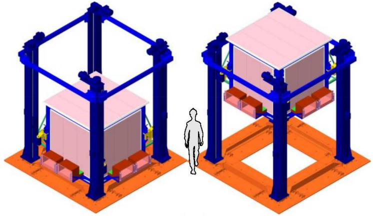
Model Indep. Measurements of Reactor ν Osc.

Ratios of spectra at different distances

NEOS

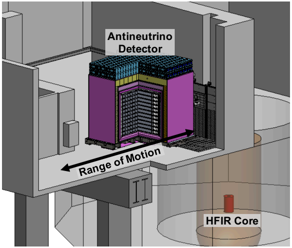


DANSS [Alekseev @ NOW 2022]

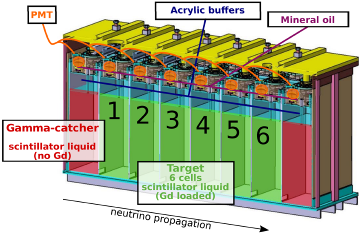


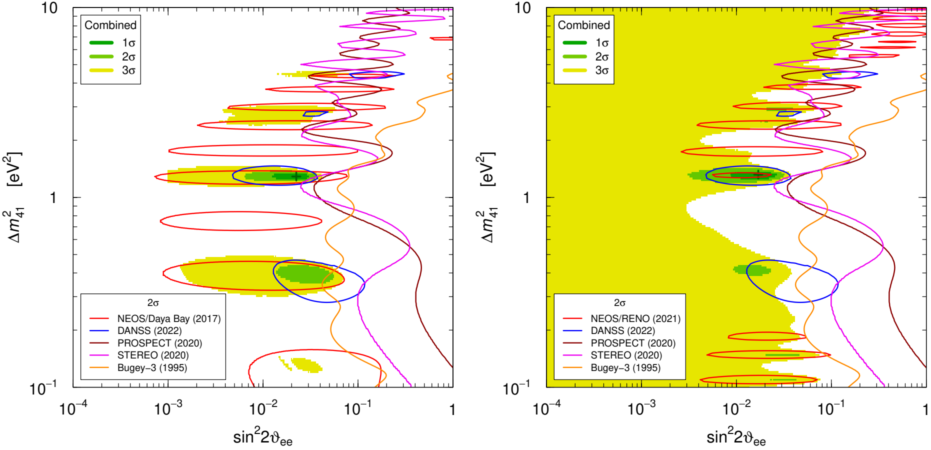
DANSS on a lifting platform

PROSPECT [Roca Catala @ NOW 2022]



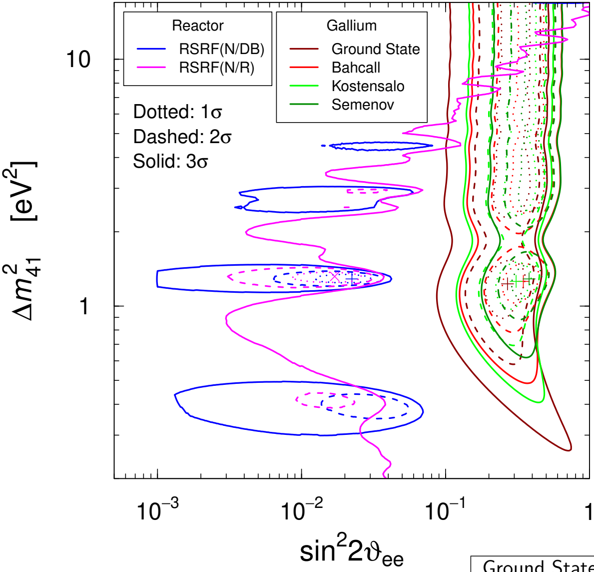
STEREO [del Amo Sanchez @ NOW 2022]





[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916]

- Fit with NEOS/Daya Bay: $\Delta\chi^2_{3\nu-4\nu} = 12.6 \Longrightarrow 3.1\ \sigma$
- Fit with NEOS/RENO: $\Delta\chi^2_{3\nu-4\nu} = 9.1 \Longrightarrow 2.6\ \sigma$



- The Reactor Spectral Ratio Fits (RSRF) prefer SBL oscillations with small mixing:

$$\sin^2 2\vartheta_{ee} \lesssim 0.1$$

for $\quad \Delta m^2_{41} \lesssim 6\,\text{eV}^2$

- Tension with the Gallium Anomaly!

[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916]

	RSRF(N/DB)		RSRF(N/R)	
	$\Delta\chi^2_{PG}$	GoF_{PG}	$\Delta\chi^2_{PG}$	GoF_{PG}
Ground State	12.95	0.15%	8.91	1.2%
Bahcall	12.86	0.16%	8.74	1.3%
Kostensalo	12.91	0.16%	8.89	1.2%
Semenov	12.88	0.16%	8.70	1.3%

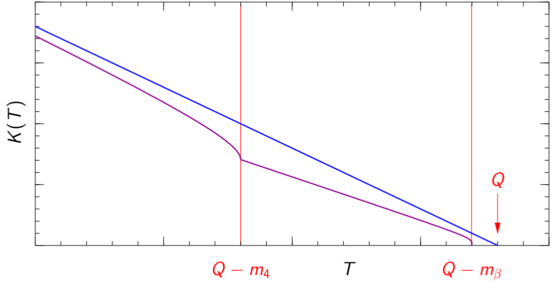
Robust kinematical probe of $\nu_e - \nu_s$ mixing

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

$$K^2(T) = (Q - T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2}\, \theta(Q - T - m_k)$$

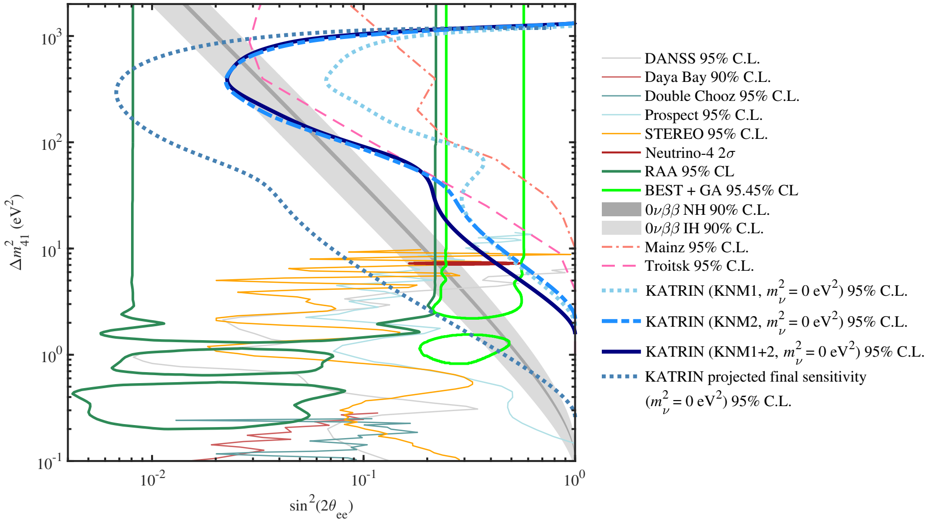
$$m_4 \gg m_{1,2,3} \Rightarrow \simeq \left(1 - |U_{e4}|^2\right) \sqrt{(Q-T)^2 - m_\beta^2}\, \theta(Q - T - m_\beta)$$

$$+ |U_{e4}|^2 \sqrt{(Q-T)^2 - m_4^2}\, \theta(Q - T - m_4)$$



$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

$$m_\beta^2 = \sum_{k=1}^{3} |U_{ek}|^2 m_k^2$$

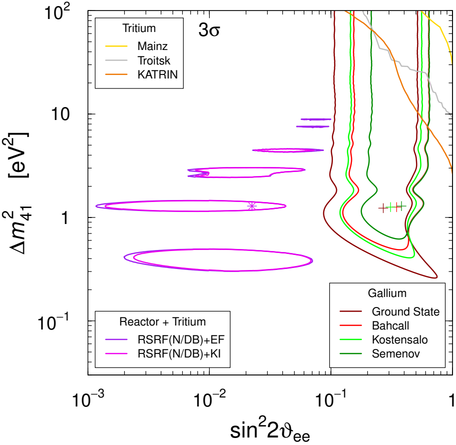


[KATRIN, arXiv:2201.11593]

Gallium – Reactors+Tritium tension

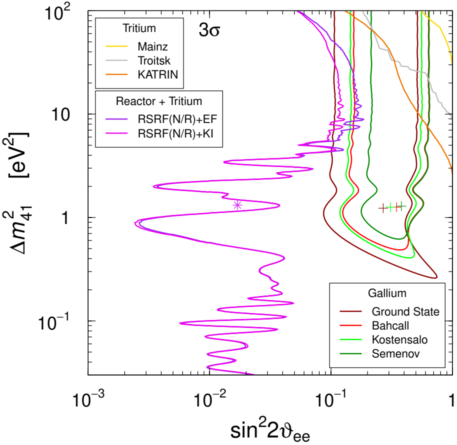
[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916]

RSRF(NEOS/Daya Bay)



	EF		KI	
	$\Delta\chi^2_{PG}$	GoF_{PG}	$\Delta\chi^2_{PG}$	GoF_{PG}
Ground State	20.82	0.003%	21.82	0.0018%
Bahcall	25.06	0.00036%	26.03	0.00022%
Kostensalo	23.83	0.00067%	27.52	0.00011%
Semenov	30.42	0.000025%	37.42	0.00000075%

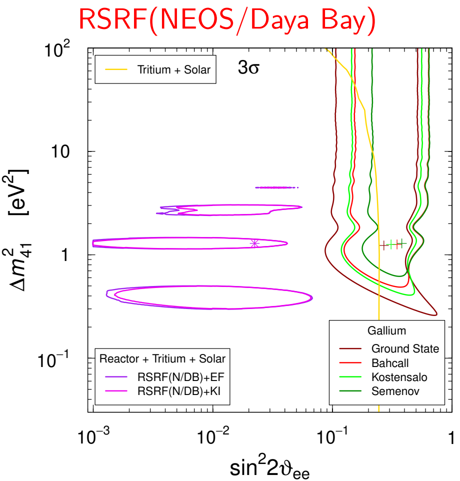
RSRF(NEOS/RENO)



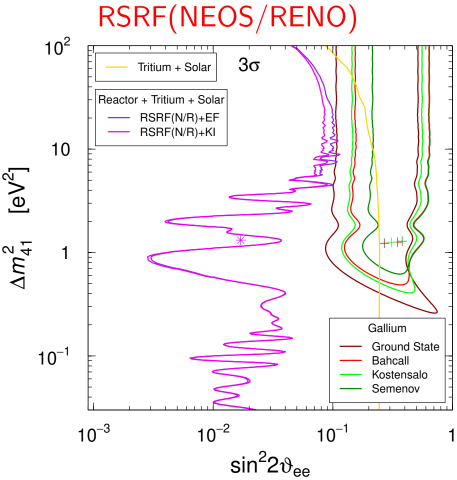
	EF		KI	
	$\Delta\chi^2_{PG}$	GoF_{PG}	$\Delta\chi^2_{PG}$	GoF_{PG}
Ground State	16.96	0.021%	21.49	0.0022%
Bahcall	21.19	0.0025%	26.08	0.00022%
Kostensalo	19.97	0.0046%	25.37	0.00031%
Semenov	26.45	0.00018%	33.56	0.0000052%

Gallium − Reactors+Tritium+Solar tension

[CG, Li ,Ternes, Tyagi, Xin, arXiv:2209.00916]



	EF		KI	
	$\Delta\chi^2_{PG}$	GoF_{PG}	$\Delta\chi^2_{PG}$	GoF_{PG}
Ground State	21.92	0.0017%	21.90	0.0018%
Bahcall	26.13	0.00021%	26.11	0.00021%
Kostensalo	27.62	0.0001%	27.60	0.0001%
Semenov	37.69	0.00000065%	38.81	0.00000037%



	EF		KI	
	$\Delta\chi^2_{PG}$	GoF_{PG}	$\Delta\chi^2_{PG}$	GoF_{PG}
Ground State	22.56	0.0013%	22.66	0.0012%
Bahcall	26.82	0.00015%	26.80	0.00015%
Kostensalo	26.27	0.0002%	28.45	0.000066%
Semenov	34.00	0.0000041%	38.24	0.0000005%

Tentative Alternative Explanations of the Gallium Anomaly

- Four-Neutrino Mixing (Active-Sterile Neutrino Oscillations) is a general possibility which can be tested in many experiments and was not invented for the Gallium Anomaly.

- Typical Alternative Explanations of the Gallium Anomaly are invented ad hoc for the Gallium Anomaly.

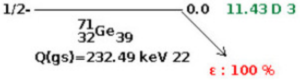
Revision of the Ground State Cross Section

[CG, Li, Ternes, Xin, arXiv:2212.09722]

- In all the cross section models the total cross section of $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ is calculated adding the relative excited states contribution to the ground state cross section:

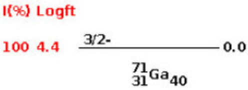
$$\sigma_{\text{tot}} = \sigma_{\text{gs}} \left(1 + \xi_{5/2^-} \frac{\text{BGT}_{5/2^-}}{\text{BGT}_{\text{gs}}} + \xi_{3/2^-} \frac{\text{BGT}_{3/2^-}}{\text{BGT}_{\text{gs}}} + \xi_{5/2^+} \frac{\text{BGT}_{5/2^+}}{\text{BGT}_{\text{gs}}} \right)$$

- The ground state cross section σ_{gs} is obtained from the measured half life $T_{1/2}({}^{71}\text{Ge})$ of ${}^{71}\text{Ge}$:



$$\sigma_{\text{gs}} = \frac{G_F^2 \cos^2 \vartheta_C}{\pi} \, g_A^2 \, \text{BGT}_{\text{gs}} \, \langle p_e E_e F(Z_{\text{Ge}}, E_e) \rangle$$

$$= \frac{\pi^2 \ln 2}{m_e^5 \, ft_{1/2}({}^{71}\text{Ge})} \, \langle p_e E_e F(Z_{\text{Ge}}, E_e) \rangle$$



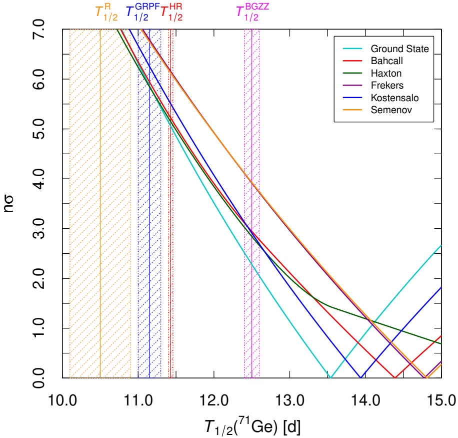
Measurements of the ^{71}Ge half life before 2023

$$T_{1/2}^{\text{BGZZ}}(^{71}\text{Ge}) = 12.5 \pm 0.1\,\text{d}$$ [Bisi, Germagnoli, Zappa, Zimmer, 1955]

$$T_{1/2}^{\text{R}}(^{71}\text{Ge}) = 10.5 \pm 0.4\,\text{d}$$ [Rudstam, 1956]

$$T_{1/2}^{\text{GRPF}}(^{71}\text{Ge}) = 11.15 \pm 0.15\,\text{d}$$ [Genz, Renier, Pengra, Fink, 1971]

$$T_{1/2}^{\text{HR}}(^{71}\text{Ge}) = 11.43 \pm 0.03\,\text{d}$$ [Hampel and Remsberg, 1985]



[CG, Li, Ternes, Xin, arXiv:2212.09722]

Recent measurements of the ^{71}Ge half life

$$T_{1/2}^{\mathrm{CY}}(^{71}\mathrm{Ge}) = 11.46 \pm 0.04\,\mathrm{d}$$ [Collar and Yoon, 2023]

$$T_{1/2}^{\mathrm{LLNL}}(^{71}\mathrm{Ge}) = 11.468 \pm 0.008\,\mathrm{d}$$ [Norman et al. (LLNL), 2024]

- Confirm the 1985 Hampel and Remsberg measurement

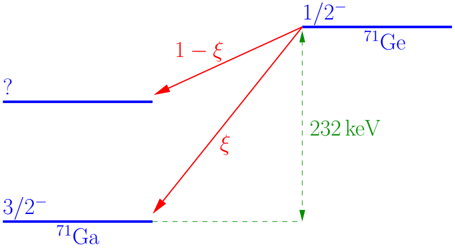
$$T_{1/2}^{\mathrm{HR}}(^{71}\mathrm{Ge}) = 11.43 \pm 0.03\,\mathrm{d}$$

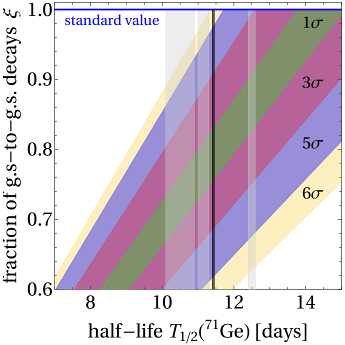
- Exclude the explanation of the Gallium Anomaly with a decrease of the Ground State Cross Section through an increase of the ^{71}Ge half life.

^{71}Ge Decay to New Excited States of ^{71}Ga?

[Brdar, Gehrlein, Kopp, arXiv:2303.05528]

- "we speculate on the possibility that there is an additional, yet undiscovered, low-lying excited state of ^{71}Ga"
- "If a $\sim 20\%$ fraction of ^{71}Ge went into this state, the nuclear matrix element for ground state-to-ground state transitions (which enters the calculation of $\sigma(\nu_e + {}^{71}\text{Ga})$) would have been overestimated by the same amount"
- "Of course, it is unclear how the existence of such an excited state could have been missed"

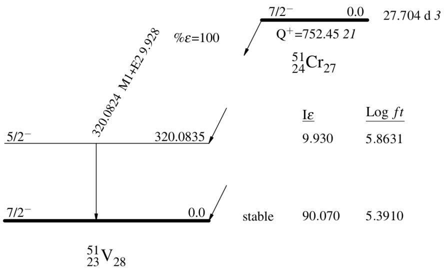




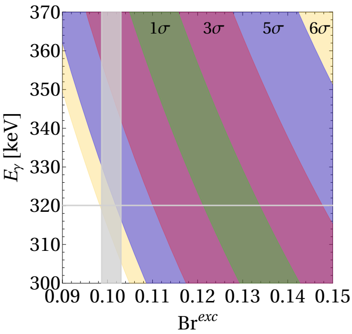
Source: ^{51}Cr Branching Ratios 1

[Brdar, Gehrlein, Kopp, arXiv:2303.05528]

- "the measurement of the source intensity is based on only $\sim$ 10% of all decays"
- "if the true branching ratio for decays to the excited state BR^{exc} was larger by only $\sim$ 2%, the source intensity would have been overestimated by $\sim$ 20%, enough to explain the gallium anomaly"
- But, "different measurements of BR^{exc} differ by 0.5%, but are consistent with each other at the 3σ level"



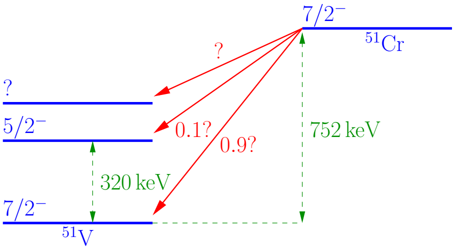
[Nuclear Data Sheets for A=51]

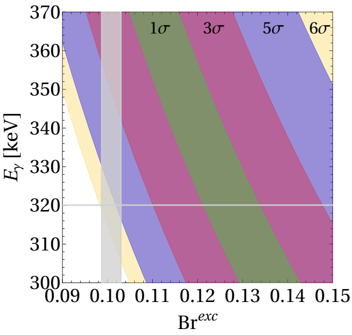


Source: ^{51}Cr Branching Ratios 2

[Brdar, Gehrlein, Kopp, arXiv:2303.05528]

- "Another avenue for attaining larger energy per decay is increasing E_γ"
- "But since measurements of nuclear excitation energies are rather robust, the only reasonable way to achieve such a large shift would be to postulate the existence of another, yet undiscovered, excited state"
- "there is weak evidence for a state at 470 keV ... these observations from the 1960s and 70s have not found further support in more recent measurements ... the relative intensity of the 470 keV line is $< 10^{-5}$"

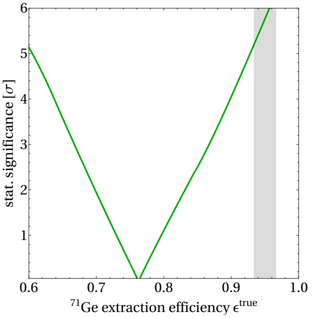




Radiochemical ^{71}Ge Extraction Efficiency

- BEST outer volume:
  10 extractions of $\sim 100\ {}^{71}\mathrm{Ge}$ atoms in $\sim 40\,\mathrm{t} \sim 3 \times 10^{29}$ atoms
- For each experiment, the ^{71}Ge extraction efficiency ϵ^{cal} was obtained with special calibration measurements.
- An overestimate of the extraction efficiency can obviously explain the Gallium Anomaly:

$$\epsilon^{\mathrm{true}} < \epsilon^{\mathrm{cal}} \quad \Longrightarrow \quad R^{\mathrm{true}} = \frac{N_{\mathrm{meas}}}{N_{\mathrm{pred}}} \frac{\epsilon_{\mathrm{cal}}}{\epsilon_{\mathrm{true}}} > \frac{N_{\mathrm{meas}}}{N_{\mathrm{pred}}} = R$$



"resolving the anomaly would require the calibration to be off by around 20%"

[Brdar, Gehrlein, Kopp, arXiv:2303.05528]

Checks of the ^{71}Ge Extraction Efficiency

- GALLEX performed a hot-atom chemistry* test introducing ^{71}As and counting the ^{71}Ge atoms produced in $e^- + {}^{71}\text{As} \to {}^{71}\text{Ge} + \nu_e$:

  $100 \pm 1\%$ recovery [GALLEX, PLB 436 (1998) 158]

- SAGE introduced about $700\,\mu g \sim 6 \times 10^{18}$ atoms of stable natural Ge carrier at the beginning of each of the 8 exposures:

  $95 \pm 3\%$ recovery [SAGE, arXiv:0901.2200]

- BEST introduced about $175\,\mu g \sim 1.5 \times 10^{18}$ atoms of stable natural Ge carrier at the beginning of each of the 10+10 exposures:

  $95 \pm 1.6\%$ recovery [BEST,arXiv:2109.11482]

- More: see the experimental papers and the review

  Elliott, Gavrin, Haxton, arxiv:2306.03299

- Conclusion: a 20% calibration overestimate seems very unlikely

---

* The ^{71}Ge recoil energy is close to that in $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ for a ^{51}Cr source and comparable to the $\sim 3 - 4\,\text{eV}$ chemical binding energy of the extracted $GeCl_4$.

scenario	comments	our rating
Explanations within the Standard Model		
increased ^{71}Ge half-life (section 2.1 and ref. [39])	would lead to smaller matrix element for $\nu + {}^{71}$Ga; but the ^{71}Ge half-life has been measured many times with different methods in [38], all of which yield consistent results. So it is hard to imagine a bias in these measurements.	★★☆☆☆
new ^{71}Ga excited state (section 2.2)	would imply a bias in the extraction of the $\nu + {}^{71}$Ga matrix element from the measured ^{71}Ge half-life. Some very old experiments claim the existence of such a state, but this has not been confirmed in more recent observations.	★★☆☆☆
increased BR(^{51}Cr $\rightarrow$ ^{51}V*) (section 3)	would cause a bias in translating the heat output of the source to a neutrino production rate. Measurements of BR(^{51}Cr $\rightarrow$ ^{51}V*) show some tension, but it is far less than the shift required to explain the gallium anomaly.	★★★☆☆
^{71}Ge extraction efficiency (section 4)	one of SAGE's calibration runs has revealed a large bias. Could a small, unnoticed, bias have been present in all gallium experiments?	★★★★☆

[Brdar, Gehrlein, Kopp, arXiv:2303.05528]

Explanations beyond the Standard Model

ν_s coupled to ultralight DM (MSW resonance, section 5.1.1)	several exotic ingredients; somewhat tuned MSW resonance; new ν_4 decay channel required for cosmology.	★★★★☆
ν_s coupled to dark energy (MSW resonance, section 5.1.2)	several exotic ingredients; somewhat tuned MSW resonance; cosmology similar to the previous scenario.	★★★☆☆
ν_s coupled to ultralight DM (param. resonance, section 5.1.3)	several exotic ingredients; somewhat tuned parametric resonance; cosmology requires post-BBN DM production via misalignment.	★★★★☆
decaying ν_s (section 5.2)	difficult to reconcile with reactor and solar data; regeneration of active neutrinos in ν_s decays alleviates tension, but does not resolve it.	★★☆☆☆
vanilla eV-scale ν_s (refs. [17, 18])	preferred parameter space is strongly disfavored by solar and reactor data.	★☆☆☆☆
ν_s with CPT violation (ref. [130])	avoids constraints from reactor experiments, but those from solar neutrinos cannot be alleviated.	
extra dimensions (refs. [131–133])	neutrinos oscillate into sterile Kaluza-Klein modes that propagate in extra dimensions; in tension with reactor data.	
stochastic neutrino mixing (ref. [134])	based on a difference between sterile neutrino mixing angles at production and detection (see also [135, 136]); fit worse than for vanilla ν_s.	
decoherence (refs. [137, 138])	non-standard source of decoherence needed; known experimental energy resolutions constrain wave packet length, making an explanation by wave packet separation alone challenging.	
ν_s coupled to ultralight scalar (ref. [139])	ultralight scalar coupling to ν_s and to ordinary matter affects sterile neutrino parameters; can not avoid reactor constraints	

Decoherence Explanation of the Gallium Anomaly

[Farzan, Schwetz, arXiv:2306.09422]

- "Our proposal does not require the presence of sterile neutrinos but implies a modification of the standard quantum mechanical evolution equations for active neutrinos"
- However, "the decoherence that we postulate here requires exotic new physics"
- "conventional decoherence based on particle localisation leads only to tiny effects which are negligible for all oscillation experiments considered here"
- $P_{ee} = \sum_{i=1}^{3} |U_{ei}|^4 + \sum_{i \neq j} |U_{ei}|^2 |U_{ej}|^2 e^{-i\phi_{ij}} e^{-\gamma_{ij} L}$ with $\phi_{ij} = \frac{\Delta m^2_{ji} L}{2E_\nu}$ and

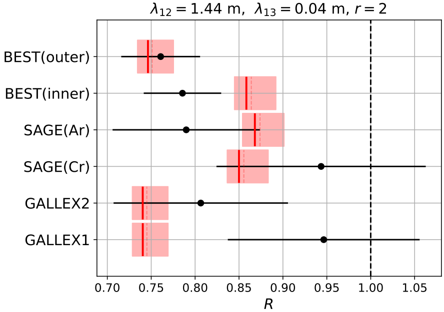
$$\gamma_{ij} = \frac{1}{\lambda_{ij}} \left(\frac{0.75\,\text{MeV}}{E_\nu} \right)^r \quad \text{with} \quad \gamma_{23} = \gamma_{12} + \gamma_{13} \pm 2\sqrt{\gamma_{12}\gamma_{13}}$$

- In Gallium experiments $\phi_{ij} \ll 1 \Rightarrow e^{-i\phi_{ij}} \approx 1$. Best fit: $\lambda_{13} \to 0 \Rightarrow \gamma_{23} = \gamma_{12}$

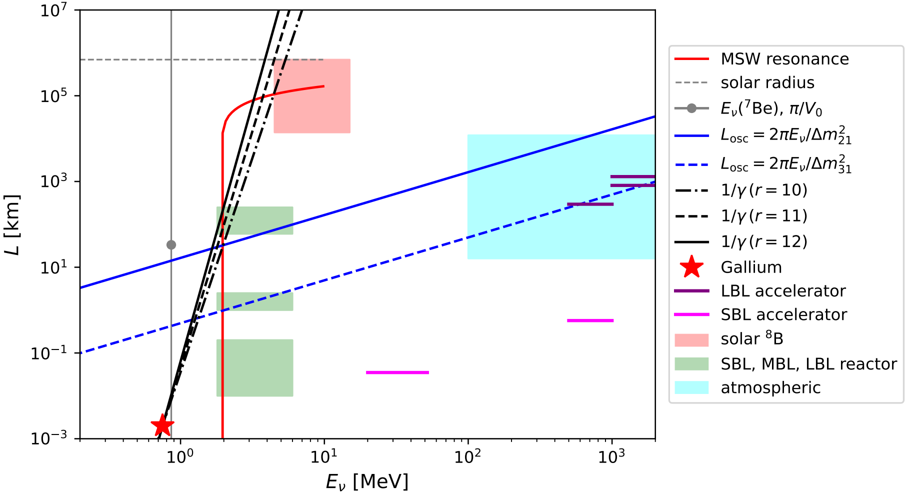
$$P_{ee}^{\text{gal}} \approx 1 - \frac{1}{2}\sin^2 2\theta_{13} - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_{12}\left(1 - e^{-\gamma_{12} L}\right)$$

- "Numerically we have $0.5\cos^4\theta_{13}\sin^2 2\theta_{12} \approx 0.404$. Hence, we need partial decoherence in the 12 sector to obtain $P_{ee}^{\text{gal}} \simeq 0.8$"

	$r=2$					$r=12$				
	χ^2_{min}/dof	p-val.	$\Delta\chi^2$	$\#\sigma$	λ_{12} [m]	χ^2_{min}/dof	p-val.	$\Delta\chi^2$	$\#\sigma$	λ_{12} [m]
CS1, BEST	2.0/1	0.16	30.1	5.1	1.44	1.7/1	0.19	30.4	5.2	1.44
CS1, all	7.7/5	0.17	28.6	5.0	1.74	8.3/5	0.14	28.0	4.9	2.10
CS2, BEST	2.6/1	0.11	32.1	5.3	1.19	2.2/1	0.14	32.5	5.4	1.44
CS2, all	8.4/5	0.14	30.0	5.1	1.44	9.2/5	0.10	29.2	5.0	1.74

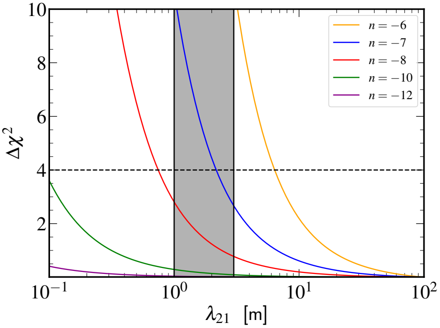


- “While this is a huge improvement compared to the p-values of the null hypothesis the fit is not perfect”
- “This is related to the partial decoherence in the 12 sector”
- “It leads to a distance dependence on the scale of gallium experiments which in particular predicts different event ratios in the inner and outer detector volumes of the BEST experiment”



- Solar and KamLAND oscillations $\Longrightarrow$ $r \gtrsim 10 - 12$ Extreme!
- "A testable prediction of our scenario is a distance dependent deficit at the radioactive source experiments"

Constraints on the decoherence length λ_{21} from reactor rate data using the KI flux model:



- The shaded region is the approximate preferred region for the explanation of the Gallium Anomaly.
- At 2σ this explanation of the Gallium Anomaly is not in conflict with reactor rate data for

$$r = -n \gtrsim 7$$

[CG, Ternes, arXiv:2312.00565]

Other Proposed Solutions of the Gallium Anomaly

- Active-sterile mixing with wavepacket-induced decoherence

  [Arguelles, Bertolez-Martinez, Salvado, arXiv:2201.05108, Hardin et al, arXiv:2211.02610]

  Extremely small wave packet size $\sigma \approx 6.7 \times 10^{-5}\,\text{nm}$ in tension with:

  - a theoretical estimation: $\approx (2 - 14) \times 10^{2}\,\text{nm}$ [Akhmedov, Smirnov, arXiv:2208.03736]
  - phenomenological bounds: $\sigma > 2.1 \times 10^{-4}\,\text{nm}$ at 90% C.L.

    [de Gouvea, De Romeri, Ternes, arXiv:2005.03022, 2104.05806]

  In tension with the reactor rates [CG, Ternes, arXiv:2312.00565]

- ν_4 decay [Hardin et al, arXiv:2211.02610; Brdar, Gehrlein, Kopp, arXiv:2303.05528]

  In tension with the reactor rates [CG, Ternes, arXiv:2312.00565]

- Broad ν_4 mass distribution [Banks, Kelly, McCullough, Zhou, arXiv:2311.06352]

  In tension with the reactor rates [CG, Ternes, arXiv:2312.00565]

Compatibility with Solar Neutrinos

- The Gallium experiments SAGE and GALLEX/GNO measured the solar neutrino flux with about 5% uncertainty $(66.1 \pm 3.1\,\mathrm{SNU})$

- Taking into account the results of other solar neutrino experiments and neutrino oscillations [SAGE, arXiv:0901.2200]

  $$\Phi_{pp}^{\mathrm{Gallium}} = (6.0 \pm 0.8) \times 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$: about 13% uncertainty

- In agreement with the Borexino measurement [Borexino, Nature 562 (2018) 505]

  $$\Phi_{pp}^{\mathrm{Borexino}} = (6.1 \pm 0.5^{+0.3}_{-0.5}) \times 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$: about 12% uncertainty

  and with the Standard Solar Model (SSM) predictions

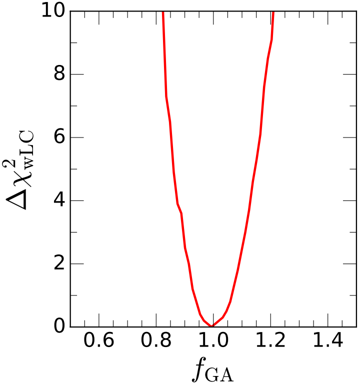
  $$\Phi_{pp}^{\mathrm{HZ}} = 5.98(1 \pm 0.006) \times 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$
  $$\Phi_{pp}^{\mathrm{LZ}} = 6.03(1 \pm 0.005) \times 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$$

  [Vinyoles et al, arXiv:1611.09867]

- The $\sim 20\%$ reduction of the detection efficiency of the Gallium experiments indicated by the Gallium Anomaly, which implies a $\sim 20\%$ increase of $\Phi_{pp}^{\mathrm{Gallium}}$, would be in tension with Borexino and SSM

Global Fit of Solar Neutrino Data

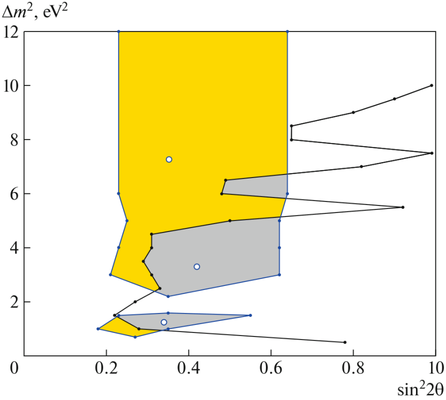
[Gonzalez-Garcia, Maltoni, Pinheiro, Serenelli,arXiv:2311.16226]



- $f_{\rm GA}$ multiplies the predicted event rates of all solar fluxes in the Gallium experiments
- "the global analysis of the solar experiments do not support a modification of the neutrino capture cross section in Gallium (or any other effect inducing an energy-independent reduction of the detection efficiency in the Gallium experiments)"
- "This is so because the global fit implies a rate of *pp* and ^{7}Be neutrinos in the Gallium experiment which is in good agreement with the luminosity constraint as well as with the rates observed in Borexino"

Future Proposals

- Perform another high-intensity experiment with ^{51}Cr or another source
- BEST-2: 50 tons of ^{71}Ga divided into 3 zones, irradiated with a ^{65}Zn EC source [Gorbachev, Gavrin, Ibragimova, Phys.Atom.Nucl. 86 (2023) 1385]



Sensitivity for a 0.5 Mci ^{65}Zn source

- Longer half life: 244.01 ± 0.09 d
  Allows more extractions
- Higher ν_e energy: 1350 keV
  Probes larger values of Δm^2_{41}
- Larger detection cross section
  Bigger event rate
- Problem: larger contribution of model-dependent transitions to higher energy excited states of ^{71}Ge

- Real-time detection of ν_e from a ^{51}Cr source with a Cerium-doped Gadolinium Aluminum Gallium Garnet (Ce:GAGG) crystal scintillator detector: [Huber, arXiv:2209.02885]
  - $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ and $\nu_e + e^- \to \nu_e + e^-$
  - "With 1.5 tons of scintillator and 10 source runs of 3.4 MCi, each, we obtain about 1700 gallium capture events with a purity of 90% and 680,000 neutrino electron scattering events"
  - "this configuration would allow us to test the gallium anomaly at more than 5σ in an independent way"

- A strong ν_e or $\bar{\nu}_e$ source inside or near a liquid scintillator detector as in the cancelled projects SOX with the Borexino detector [SOX, arXiv:1304.7721] and CeLAND with the KamLAND detector [arXiv:1312.0896] :
  - JUNO: [arXiv:1507.05613]
    - A monocromatic ^{51}Cr or ^{37}Ar source of ν_e detected with $\nu_e + e^- \to \nu_e + e^-$
    - A ^{144}Ce-^{144}Pr source of $\bar{\nu}_e$ with a continuous β spectrum detected with the Inverse Beta Decay (IBD) reaction $\bar{\nu}_e + p \to n + e^+$
    - A cyclotron-produced ^{8}Li source (IsoDAR) of $\bar{\nu}_e$ with a continuous β spectrum detected with the IBD reaction

Conclusions

- Light Sterile Neutrinos can be powerful messengers of BSM New Physics.
- Historically, the existence of light sterile neutrinos is motivated by the LSND, Gallium, and Reactor Short-Baseline Anomalies.
- The Reactor Antineutrino Anomaly, discovered in 2011, is fading away.
- The Gallium Neutrino Anomaly, discovered in 2007, has been revived by the BEST results. It is now the most significant anomaly in neutrino pshysics.
- As 13 years ago, before the discovery of the Reactor Antineutrino Anomaly, there is a Reactor Antineutrino–Gallium Neutrino tension.
- CPT violation explanation of this tension? [CG, Laveder, arXiv:1008.4750]
  - Theoretically challenging.
  - Cannot resolve the tension between the the Gallium Neutrino Anomaly and the solar neutrino bound.
- No convincing SM explanation of the Gallium Anomaly has been found.
- Difficulty: known BSM explanations of the Gallium Anomaly affect also solar and reactor neutrinos if not fine-tuned with ad-hoc assumptions.